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The distribution of the geostrophic flow in a stratified surface layer

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ABSTRACT

We consider the drainage of a low-density surface layer from a semi-enclosed deep basin. The flow is geostrophically controlled and occurs in a frontal region where the isopycnals bend upwards to meet the surface. A useful fact for modellers is that the total volume flow in the frontal region is completely determined from the stratification inside the region covered by the surface layer. In this paper, we will discuss how the geostrophic flow is divided between different parts of the surface layer. Such information is needed for the calculation of the flow of heat, salt and chemical constituents that are not well mixed within the surface layer. For this purpose, the structure of the front has to be considered. A general formula for the distribution of the flow with respect to density is derived, and some useful qualitative properties are demonstrated which may be helpful for modelling purposes. The distribution of the flow depends first on the density stratification of the undisturbed surface layer inside the basin. Secondly, it depends on how the separation of the isopycnals vary through the front. If, e.g., the isopycnals converge as they reach the surface, the flow of the uppermost part of the surface layer is favoured. The distribution of the flow, however, does not depend on the width of the front. Nor does it depend on the shape of the isopycnals as long as the vertical distance between the isopycnals does not vary through the front. Assuming a standard, or close to standard, behaviour of the front, we derive certain simplified formulas for the calculation of the distribution of the flow with density.

1. Introduction

Oceanographic modelling often involves calculations of both the water flow between different basins and the flow of various constituents such as heat, salt, nutrients, etc., which are transported by the water. In many situations to be modelled or described, the flow of water can be considered, as a good approximation, to be geostrophically balanced. To mention one recent investigation of the applicability of geostrophy, Johns et al. (1989) found a quite good agreement between measured and calculated, geostrophically balanced, transport of the Gulf Stream.

The use of the geostrophic approximation in circulation models is very widespread. We do not intend to give any comprehensive review of its use here, but only to refer to some arbitrarily chosen examples. Stigebrandt (1981) applied the geostrophic balance to the upper layer of the Arctic Ocean to calculate the outflow of water and salt. In this model, the salinity of the upper layer was assumed to be constant. In a later work, Stigebrandt (1987) used a similar approach to compute the flow of dense water into the Baltic Sea from hydrographical observations in the Arkona Basin. Björk (1989) calculated the outflow of polar water and salt from the upper Arctic Ocean in a one-dimensional model by applying the geostrophic balance to a continuous vertical stratification, i.e., to the same situation as we are discussing in this paper. The procedure used by Björk is however not unique but implies a specific choice of the distribution of the flow with respect to salinity.

A slightly different approach was used by Schlitzer (1988) in a model of nutrient and carbon cycles of the North Atlantic. Here the geostrophic approximation was used to calculate the velocity of the water, which together with the observed spatial distribution of different constituents was used to calculate the required flows.

Because of the frequent use and applicability of the geostrophic approximation, it would be of interest to investigate the behaviour of a geostrophically balanced flow in more detail. In doing so, the discussion will be focused on the special but important dynamic situation where the outflow from a region is confined to a stratified surface layer in geostrophic balance on top of a deep, stagnant and homogeneous bottom water.

Before going any further, let us first draw attention to the fact that in many situations, the water flow as well as the concentration of constituents may vary within the water body. In order to calculate the transport of heat, salt or nutrients, etc., the distribution of the constituent in question, as well as the distribution of the water flow, has to be known, i.e., we have to express both the variation of the water flow and the distribution of the constituent as functions of the same independent variables. If the distribution of the constituent is correlated with the distribution of some other character of the water, such as the density, the temperature or the salinity, this correlation may be employed to reduce the number of independent variables. Since it is not uncommon that such a correlation exists with density, we have here chosen the density, ϱ, as the independent variable. The concentration of an arbitrary constituent, γ, will thus be represented by $C\gamma(\varrho)$.

The next step is to express the water flow as a function of the same variable. For this purpose we define:

$M(\varrho)$ the transport of water with a density greater than ϱ

$m(\varrho)$ $= -\partial M(\varrho)/\partial \varrho$; the transport of water per unit density interval

$M\gamma(\varrho)$ the transport of γ in water with density greater than ϱ.

This procedure leads to the result that the total transport of the property γ can be calculated from

$$M\gamma_0 = \int_0^\infty m(\varrho)\, C\gamma(\varrho)\, \mathrm{d}\varrho. \tag{1}$$

The same approach was used by Walin (1977) in a theoretical framework for calculations of the general circulation in estuaries. Here we will concentrate on some properties of the distribution functions $M(\varrho)$ and $m(\varrho)$. In Section 2, some general properties of the distribution of a geostrophically balanced flow is discussed. In Section 3, the formal aspects of the calculation of $M(\varrho)$ and $m(\varrho)$ are considered. In Section 4, the foregoing discussion is illustrated by calculations of $m(\varrho)$ for different cases of density distribution.

2. Geostrophic flow in a stratified surface layer

The Gulf Stream, the Greenland Current, the outflow of surface water from the Skagerrak are all examples of the type of flow we will discuss here, namely a stratified surface layer in approximate geostrophic balance on top of a homogeneous and stagnant bottom layer. The bottom water reaches to the surface at some distance away from the region where the light surface water is found, as illustrated in Fig. 1.

Let us consider a hydrographical section which we assume to be bounded by two known, vertical density profiles. The first profile, in the following referred to as station 1, is located in the area

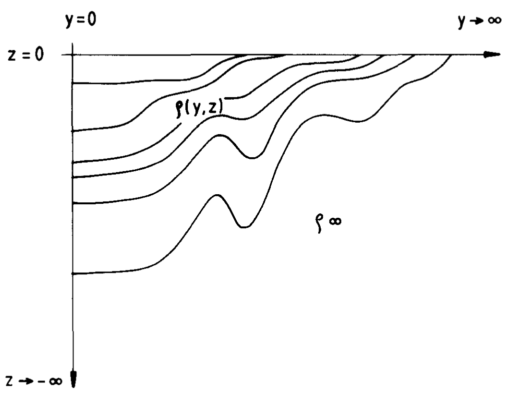

Fig. 1. Isopycnals of a stratified surface layer on top of homogeneous bottom water. The constant density in the bottom water is denoted by ϱ_∞. All the isopycnals reach the surface as $y \to \infty$. It is assumed that the vertical density stratification is known at $y = 0$ and at $y \to \infty$. These locations are referred to as stations 1 and 2, respectively.

where the light surface water is found (at $y = 0$ in Fig. 1). The other profile, referred to as station 2, is located at a position outside the light surface water area, where the bottom water reaches all the way to the surface. Formally, we may consider the ocean to be infinitely deep and station 2 to be located infinitely far away. Our section, in the following referred to as A_0, is thus defined by (cf. Fig. 1)

$$x = 0, \quad 0 < y < \infty, \quad -\infty < z < 0.$$

If the surface layer is in geostrophic balance (i.e., the horizontal pressure gradient in the surface water is balanced by the Coriolis force) and the boundary condition, $u = 0$ when $z \to -\infty$ is fulfilled, the velocity can be calculated according to the well-known equation

$$u(y, z) = g/f\varrho_0 \int_{-\infty}^{z} \partial\varrho(y, z')/\partial y \, dz', \qquad (2)$$

where

$u(y, z)$ is the x-component of the velocity, i.e., the velocity perpendicular to the section

$\varrho(y, z)$ is the density of the section

ϱ_0 is some reference density (the Boussinesq approximation is applied).

The total volume flow, M_0, i.e., $M(\varrho = 0)$, through the section is given by the integral

$$M_0 = \int_{-\infty}^{0} \int_{0}^{\infty} u(y, z) \, dy \, dz. \qquad (3)$$

From (2) and (3), changing the order of integration and performing the integration with respect to y, we obtain

$$M_0 = g/f\varrho_0 \int_{-\infty}^{0} \int_{-\infty}^{z} [\varrho_\infty - \varrho(0, z')] \, dz' \, dz \qquad (4)$$

where we recall that $\varrho(y \to \infty) = \varrho(z \to -\infty) = \varrho_\infty$.

Eq. (4) demonstrates the well-known fact that the total flow depends only on the density stratification at $y = 0$, i.e., the total flow can be determined from one density profile. The problem is, however, that we still do not know how the flow is distributed within the surface layer. While the total flow, M_0, is completely determined from $\varrho(0, z)$, we do not know the shape of the distribution functions $M(\varrho)$ and $m(\varrho)$. This lack of knowledge is of great concern, since the flow of, e.g., buoyancy (heat or fresh water), may vary greatly even though $\varrho(0, z)$, and thus M_0, remain the same.

This indeterminacy can be illustrated by a simplified case, where the surface layer consists of only two different water masses, as illustrated in Fig. 2. Let us as earlier assume that $u = 0$ when $z \to -\infty$. The geostrophic volume flow will thus occur in the two surface layers and its total magnitude will be fully determined by the vertical density stratification at $z = 0$. The distribution of the flow between the two layers is, however, not determined unless we know the density field between the two stations. Just by changing the shape of the isopycnals, the distribution of the flow can be changed in an arbitrary way, subject only to the constraint that the total volume flow remains constant. In the cases illustrated here, the net volume flow is restricted to one of the two surface layers. In Fig. 2a, all the transport takes place in the uppermost layer (assuming that the lower isopycnal is horizontal to the left of the point where the isopycnals meet). In Fig. 2b, the net flow is confined to the lower layer, if the downslope of the lower interface is adjusted to achieve compensation in the top layer, i.e., to cancel the pressure gradient in the top layer.

After this brief discussion on how the distribution of the volume flow might differ even if the total transport does not, the following section will be devoted to the question of how to calculate $M(\varrho)$.

3. Theory of the geostrophic transport as a function of density

In Section 1, the function $M(\varrho)$ was defined as the transport of water having a density greater than ϱ. Let ϱ_* denote a constant but arbitrary value of ϱ, which lies in the interval $[\varrho(0,0); \varrho_\infty]$. The function $M(\varrho_*)$ can thus be expressed as

$$M(\varrho_*) = \iint_{A_*} u(y, z) \, dy \, dz, \qquad (5)$$

where the integration is performed over the area A_*, inside A_0, in which $\varrho > \varrho_*$, A_0 being the region $0 < y < \infty$, $-\infty < z < 0$ as before.

Inserting the expression for the geostrophic velocity, u, from (2) we have:

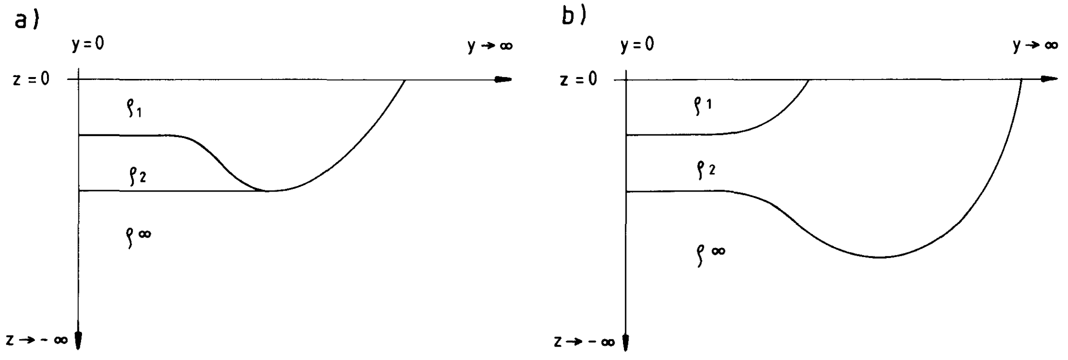

Fig. 2. A three-layer case where the bottom layer is assumed to be stagnant. The net volume flow is exactly the same in the two cases (a) and (b). (a) This density structure leads to the result that the net geostrophic flow occurs entirely in the upper surface layer. (b) If the density structure is changed to the case illustrated here, keeping everything else constant, the net geostrophic flow occurs entirely in the lower surface layer.

$$M(\varrho_*) = g/f\varrho_0 \int\int_{A_*}\int_{-\infty}^{z} [\partial\varrho(y,z')/\partial y]\,\mathrm{d}z'\,\mathrm{d}y\,\mathrm{d}z. \tag{6}$$

Since the limits of integration depend on the structure of the density field, it can easily be seen that the calculation of (6) is quite laborious. One way to make the calculation more manageable is to look for a function $Q(\varrho_*, z)$, through which $M(\varrho_*)$ can be expressed as

$$M(\varrho_*) = \int_{-\infty}^{0} Q(\varrho_*, z)\,\mathrm{d}z. \tag{7}$$

$Q(\varrho_*, z)$ thus represents that part of the flow per unit depth where $\varrho(y,z) > \varrho_*$. The function $Q(\varrho_*, z)$ may be expressed as

$$Q(\varrho_*, z) = \int_{0}^{\infty} u(y,z)\,H(\varrho - \varrho_*)\,\mathrm{d}y, \tag{8}$$

where u is the geostrophic velocity and $H(\varrho - \varrho_*)$ is the Heaviside function.

With the intention of finding an expression for $Q(\varrho_*, z)$, let us define

$$B(y,z) \equiv g/f\varrho_0 \int_{y}^{\infty}\int_{-\infty}^{z} [\partial\varrho(y',z')/\partial y']\,\mathrm{d}z'\,\mathrm{d}y', \tag{9}$$

which can be rewritten as

$$B(y,z) = g/f\varrho_0 \int_{-\infty}^{z} [\varrho_\infty - \varrho(y,z')]\,\mathrm{d}z'. \tag{10}$$

In the case of a geostrophically balanced flow function, $B(y,z)$ represents the flow per unit depth offshore the point (y,z). Eq. (9) can accordingly be written

$$B(y,z) = \int_{y}^{\infty} u(y',z)\,\mathrm{d}y'. \tag{11}$$

For the flow per unit depth between two points (y_1, z) and (y_2, z), we thus have:

$$B_1 - B_2 = \int_{y_1}^{y_2} u(y,z)\,\mathrm{d}y, \tag{12}$$

where $B_1 = B(y_1, z)$ and $B_2 = B(y_2, z)$.

We can now make use of the function $B(y,z)$ to calculate $Q(\varrho_*, z)$. In Fig. 3, we try to visualize this calculation graphically. It should be noted that the value of the y_n's in $B_n = B(y_n, z)$ are the successive crossings between the curve $\varrho_* = \varrho(y,z)$ and the horizontal line z. The sign preceding the B_n's in eq. (13) can be determined by the sign of $\partial\varrho/\partial y$ at the crossing. If $\partial\varrho/\partial y < 0$ at the point of intersection, B_n is preceded by a minus sign and vice versa. Furthermore, if $\varrho(y=0,z) < \varrho_*$ the calculation of $Q(\varrho_*, z)$ will start with $B_1 = B(y_1, z)$, instead of $B_0 = B(0,z)$ as it otherwise does.

From the properties of $B(y,z)$ as expressed by eqs. (11) and (12), we can now write

$$Q(\varrho_*, z) = \begin{bmatrix} B_0 - B_1 + B_2 - \cdots & \text{if } \varrho(0,z) > \varrho_* \\ B_1 - B_2 + B_3 - \cdots & \text{if } \varrho(0,z) < \varrho_*, \end{bmatrix} \tag{13}$$

where $B_0 = B(0,z)$, $B_n = B(y_n, z)$, y_n is determined from the equation $\varrho_* = \varrho(y_n, z)$.

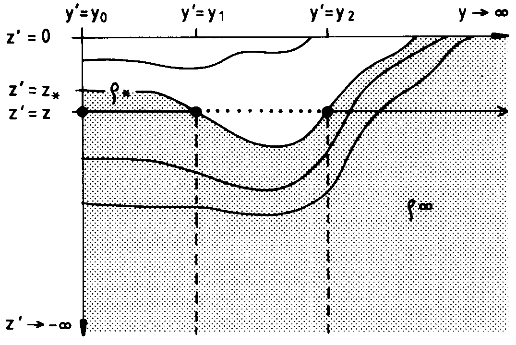

Fig. 3. The geostrophic flow per unit depth, with $\varrho > \varrho_*$, is here given by $Q(\varrho_*, z) = B(y_0, z) - B(y_1, z) + B(y_2, z)$. The function $B(y_n, z)$ is calculated by vertically integrating the density difference between (y_n, z') and (y_∞, z') from $z' \to -\infty$ to $z' = z$. y_0 indicates $y = 0$, while the other y_n's are given by the equation $\varrho_* = \varrho(y_n, z)$, i.e., y_n is the y-coordinate of the crossing between the curve $\varrho = \varrho_*$ and the line $z' = z$. The shaded area represents A_*.

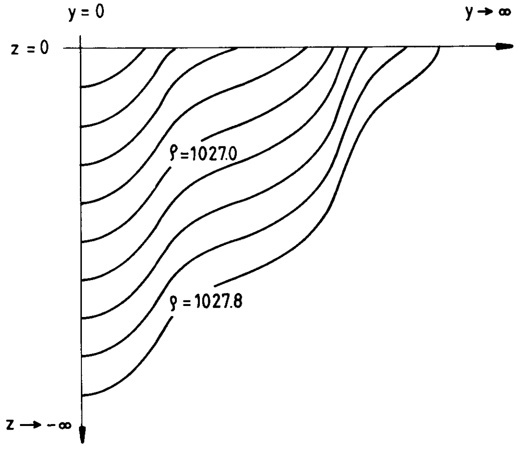

Fig. 4. The isopycnals between stations 1 and 2 in a standard case. The vertical distance between any two isopycnals is constant as they reach the surface. The density increases linearly with depth in the surface layer.

Eqs. (7), (13) and the definition of $B(y, z)$ given by (9) form the basis for the calculation of the function $M(\varrho)$.

3.1. *Some properties of $M(\varrho)$ and normalization of $m(\varrho)$*

The function $M(\varrho)$ has at least two properties which in some cases can be used to simplify the calculations. The first, (a), is that $M(\varrho)$ remains invariant when the density field is stretched in the y-direction. The second property (b) is that if the vertical distance between any two isolines is constant, as the isolines reach the surface, $M(\varrho)$ depends only on the stratification at $y = 0$.

(a) The stretching of a density field ϱ_1 can be expressed as

$$\varrho_2(y, z) = \varrho_1(y + \delta(y), z), \tag{14}$$

where $\delta(y)$ is an arbitrary function of y under the constraint that $(y + \delta(y))$ is monotonic with y and $\delta(0) = 0$. The invariance of $M(\varrho)$ under the operation $y \to y + \delta(y)$ follows from the fact that B_0 and B_n in (13) remain unchanged.

(b) When the vertical distance between any two isolines is independent of y, the density field will be of the form

$$\varrho = g[z + \delta(y)], \tag{15}$$

where $\delta(y) = 0$ when $y = 0$.

In this case, the density stratification below any point on the same isopycnal is the same, as shown in Fig. 4. This implies that $B(y, z)$ remains constant when we move along an isopycnal. Thus, all but the first $B(y_n, z)$-term (B_0 or B_1) in eq. (13) will cancel. Furthermore, since $B(y, z)$ is constant along the isopycnals, we have

$$B(y_1, z) = B(0, z_*), \tag{16}$$

where z_* is given by $\varrho(0, z_*) = \varrho_*$.

Inserting eq. (16) in the expression for $Q(\varrho_*, z)$, we obtain

$$Q(\varrho_*, z) = \begin{bmatrix} B(0, z) & \text{if} \quad \varrho(0, z) > \varrho_*, \\ B(0, z_*) & \text{if} \quad \varrho(0, z) < \varrho_*, \end{bmatrix} \tag{17}$$

which leads to the following expression for $M(\varrho_*)$:

$$M(\varrho_*) = \int_{-\infty}^{z_*} B(0, z)\,\mathrm{d}z + (-z_*)\,B(0, z_*), \tag{18a}$$

where z_* is the depth to the isoline $\varrho = \varrho_*$ at $y = 0$.

Performing a partial integration, we obtain the alternative form of (18a)

$$M(\varrho_*) = -g/f\varrho_0 \int_{-\infty}^{z_*} z[\varrho_\infty - \varrho(0, z)]\,\mathrm{d}z. \tag{18b}$$

These expressions, which are applicable when the vertical distance between any two isolines is constant, depend only on the density stratification at $y = 0$.

Since our main interest is to investigate how the structure of the density field within the front influences the distribution of the flow, we wish to eliminate the influence of the density stratification of the undisturbed surface water in the basin behind the front, i.e., the density stratification at $y = 0$. Thinking of practical applicability, the ideal would be if the influence of the density structure within the front could be totally separated from the influence of the depth of each density interval at $y = 0$. This would make it possible to employ a standard procedure together with a clever guess of the structure of the density field in the front, in situations where only the density distribution at $y = 0$ is known. For this purpose, we define the function $v(\varrho)$ as

$$v(\varrho) \equiv (\partial \varrho(0, z)/\partial z)^{-1}, \tag{19}$$

implying that $v(\varrho)\Delta\varrho$ is proportional to the amount of water in the density interval $\varrho, \varrho + \Delta\varrho$ at $y = 0$. We may then expect that the function

$$\mu(\varrho) = m(\varrho)/v(\varrho) \tag{20}$$

is less sensitive to variations of the structure of $\varrho(0, z)$. The ideal for practical use would be if a standard function $\mu_S(\varrho)$ could be found, such that the flow $m(\varrho)$ could be approximated as

$$m(\varrho) \approx \mu_S(\varrho)\, v(\varrho)\, N. \tag{21}$$

Note that following such a procedure, we will generally have an unknown constant N, which has to be found from the requirement

$$N \int_0^\infty \mu_S(\varrho)\, v(\varrho)\, \mathrm{d}\varrho = M_0. \tag{22}$$

4. Illustration of the effect on $M(\varrho)$ and $m(\varrho)$ of changes in the density field

The geostrophic calculations discussed here aim at determining (a) the total volume flow, i.e., $M(\varrho)$ for $\varrho = 0$; (b) the distribution of the flow with respect to the density, i.e., the function $M(\varrho)$ or $m(\varrho)$ for all values of ϱ which are present in the flow.

We may consider separately the influence of (a) the vertical density distribution at station 1, i.e., $\varrho(0, z)$; (b) the density field between stations 1 and 2, i.e., the manner in which the isopycnals reach the surface from their depth at station 1.

This division is motivated by the fact that we often know only the density profile $\varrho(0, z)$. We thus need to make the best possible use of this information as we have to rely on guesses regarding the density distribution between station 1 and 2. Our strategy will be: (a) select a standard case, i.e., a certain density profile $\varrho(0, z)$ combined with a certain way in which the isopycnals reach the surface; (b) consider deviations from this case caused by changes in $\varrho(0, z)$; (c) consider deviations caused by changes in the rest of the density field, keeping $\varrho(0, z)$ constant.

4.1. Standard case

As a standard case, we choose $\varrho(0, z)$ to be a linear function of z down to the bottom of the surface layer. The isopycnals are considered to proceed to the surface with a constant vertical separation. This gives rise to independence of $M(\varrho)$ from the density field, as discussed in Subsection 3.1. Furthermore, $v(\varrho)$ will be constant, which leads to $\mu(\varrho)$ being proportional to $m(\varrho)$. The structure of the density field between stations 1 and 2 is illustrated in Fig. 4. Fig. 5 shows the resulting, non-normalized distribution function $m(\varrho)$.

The two functions $m(\varrho)$ and $\mu(\varrho)$ turn out to have a parabolic shape, with their maximum in the middle of the density interval. The small transports in the deepest layers (i.e., largest values of ϱ) can be seen as being due to the small horizontal density differences there. When moving upwards (decreasing ϱ), the horizontal density differences increase as the area between the two successive isolines decreases, which gives the result that the maximum transport occurs in the middle of the density interval.

4.2. Variations of $\varrho(0, z)$

To consider deviations from the standard case caused by changes in $\varrho(0, z)$, $m(\varrho)$ and $\mu(\varrho)$ are calculated for three different cases. The chosen density profiles at station 1 are shown in Fig. 6. First we let the changes in $\varrho(0, z)$ be small compared to the profile of the standard case. This profile is indicated by 1 in Fig. 6. The small deviations appear just as kinks on the vertical density profile. Secondly, we let $\varrho(0, z)$ deviate

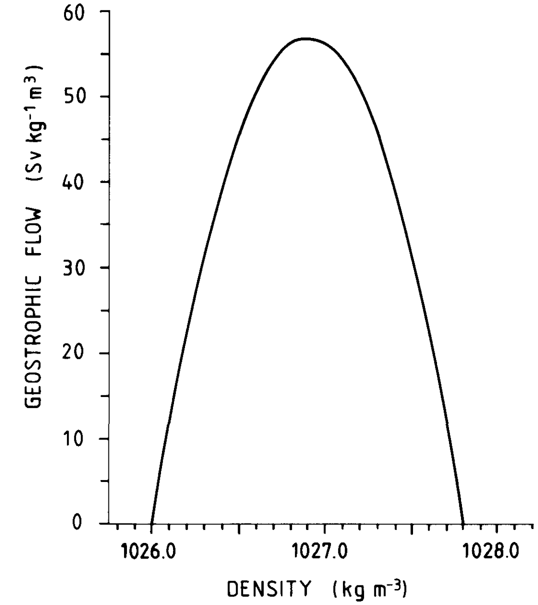

Fig. 5. The geostrophic flow per unit density interval in the surface layer of the standard case. The distribution function $m(\varrho)$ for this linear stratification turns out to have a parabolic shape.

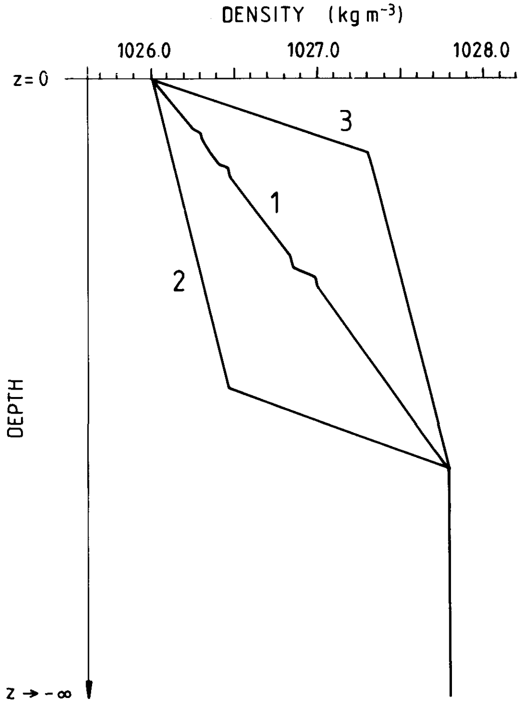

Fig. 6. The three different density profiles at station 1, chosen to investigate the influence on $m(\varrho)$ of deviations from a linear density profile (i.e., from our standard case profile). In profile 1, the changes in $\varrho(0, z)$ are small (the deviations appear as kinks on the profile). Profiles 2 and 3 deviate considerably from the standard case. The isopycnals are considered to reach the surface with constant vertical distance between any two isopycnals (i.e., as illustrated in Fig. 4).

considerably from the standard case (profiles 2 and 3 in Fig. 6). In all cases, the isopycnals reach the surface as in the standard case, i.e., with constant vertical distance (cf. Fig. 4). $\varrho(0, 0)$ and ϱ_∞ have the same value as in the standard case in all three cases. The resulting distribution functions $\mu(\varrho)$ and $m(\varrho)$ are shown in Figs. 7 and 8 respectively. The numbers in the figures refer to the respective density profiles in Fig. 6.

It is obvious that we can expect the absolute values of $m(\varrho)$ to be different in the different cases. As illustrated by Figs. 7 and 8, the shape of the normalized function $\mu(\varrho)$ is less sensitive to changes in $\varrho(0, z)$ than $m(\varrho)$ is, though the effects are not fully compensated. The results indicate that the flow is partly transferred to the density interval that occupies the largest depth interval. In case 2, this implies that the maximum transport is moved upwards in physical space, i.e., to smaller values of the density. In case 3, the flow is moved downwards in physical space, which here corresponds to larger ϱ.

Furthermore, it turns out that the respective $m(\varrho)$'s are quite well reproduced by the procedure proposed in Subsection 3.1, i.e., by a standard function $\mu_S(\varrho)$ and the respective normalization function $\nu(\varrho)$. As the standard function, $\mu_S(\varrho)$, we employ the function $\mu(\varrho)$ obtained from our so-called standard case (cf. Fig. 4). The reason to do this is that the $\mu(\varrho)$ of the standard case is computed for equidistant isopycnals in the front, which is also the assumption underlaying the calculation of the $m(\varrho)$'s shown in Fig. 8. The result of this calculation, is shown in Fig. 9. The respective functions have been multiplied with a constant, chosen so that $M(\varrho = 0)$ is the same as in the first calculation of each case.

4.3. *Variation of the density structure at $y > 0$*

We now keep $\varrho(0, z)$ as given in our standard

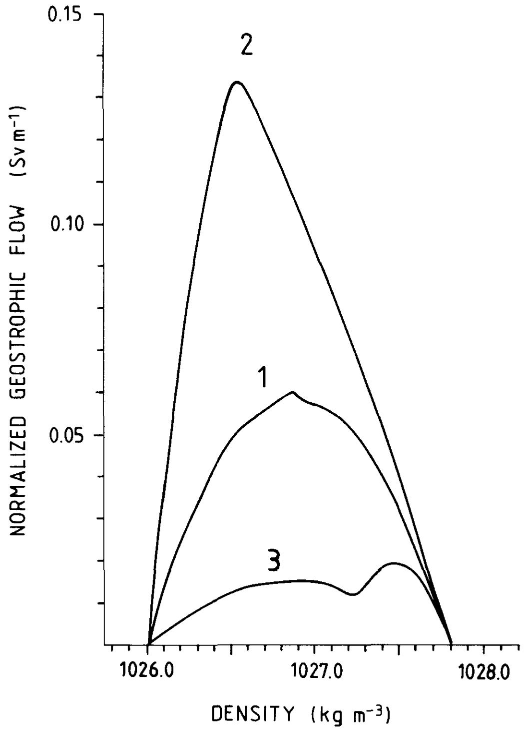

Fig. 7. The normalized geostrophic flow per unit density interval, $\mu(\varrho)$, in the surface water of the three different cases. The numbers in the figure correspond to the respective density profiles in Fig. 6. The total transport, M_0, differs, as expected, in the three cases.

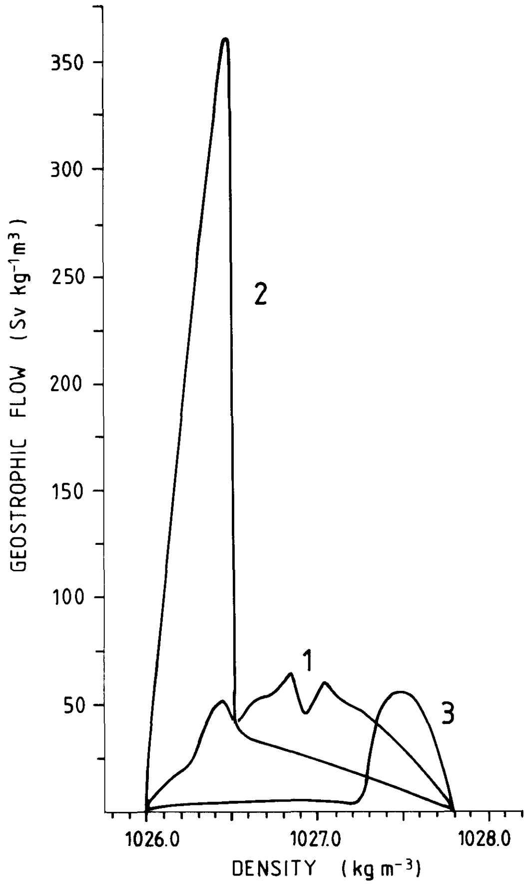

Fig. 8. The flow per unit density interval, $m(\varrho)$, corresponds to the density profiles in Fig. 6. The numbers in the figure correspond to the respective density profiles. Comparing Figs. 7 and 8, it is obvious that the shape of the normalized distribution functions is much closer to the standard shape (in Fig. 5) than the original distribution functions $m(\varrho)$.

case but change the manner in which the isolines approach the surface. Regrettably, such changes can be done in infinitely many ways and, in fact, we may with sufficiently extreme assumptions obtain any distribution of the flow.

Here we consider deviations from our standard case such that the isopycnals converge or diverge from each other on their way to the surface. The two cases are illustrated in Figs. 10 and 11. The density in the surface water at $y = 0$ is considered to increase linearly with depth as in the standard case. The total transport, M_0, in both these cases will therefore be the same as in the standard case.

The resulting distribution functions, $m(\varrho)$, for these cases are shown in Fig. 12. The $m(\varrho)$ belonging to the converging isopycnals is indicated by the number 1 and the other one is indicated by the number 2.

As can be seen in the figure, converging isopycnals move the flow to lower values of ϱ (i.e., to the uppermost part of the surface layer in physical space). Diverging isopycnals transfer the flow to higher values of ϱ (i.e., the lowest part of the surface layer).

4.4. *Some remarks on stability*

The main interest has so far been to investigate how a given front can distribute a certain volume flow, depending on the structure of its density

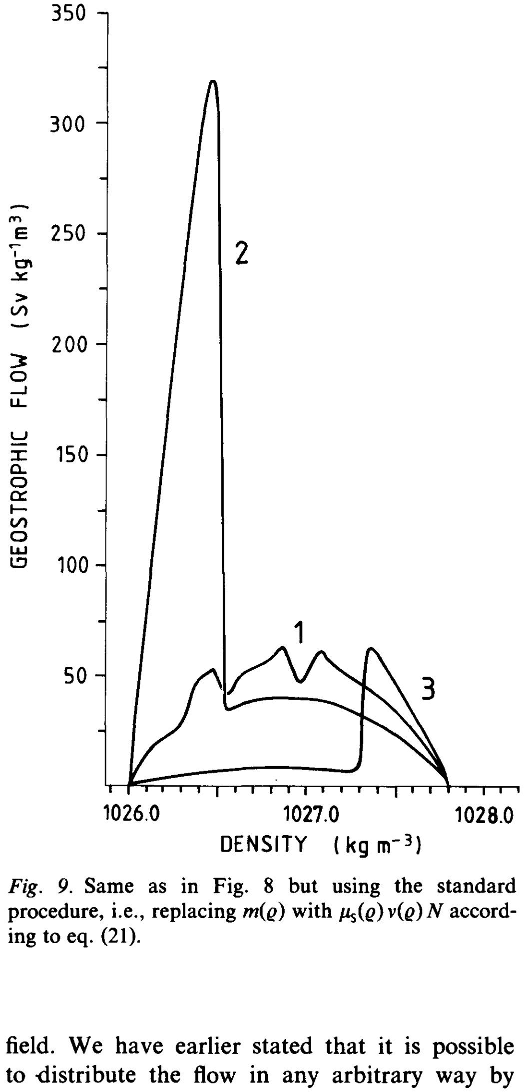

Fig. 9. Same as in Fig. 8 but using the standard procedure, i.e., replacing $m(\varrho)$ with $\mu_S(\varrho)\,v(\varrho)\,N$ according to eq. (21).

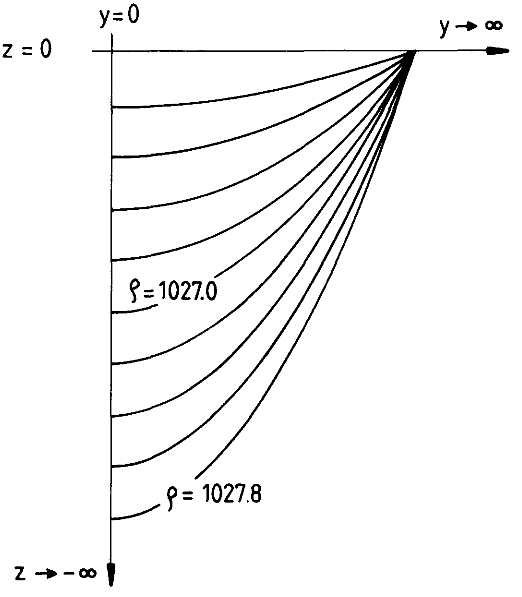

Fig. 10. Isopycnals between stations 1 and 2 in the case of convergent isolines. The density stratification at station 1 linearly increases with depth in the surface layer, as in the standard case. The form of the density field in Figs. 10 and 11 were chosen to investigate how changes in the distribution of the density between stations 1 and 2 would influence the distribution functions $M(\varrho)$ and $m(\varrho)$.

field. We have earlier stated that it is possible to distribute the flow in any arbitrary way by making sufficiently extreme assumptions about the density field. In reality however, it is probable that the front would become unstable under some density configurations, so that these distributions never show up.

A measure of the stability of a stratified fluid to shear induced turbulence is the gradient Richardson number, defined as

$$\mathrm{Ri} = -g\,\partial\varrho/\partial z/[\varrho(\partial u/\partial z)^2]. \tag{23}$$

A parallel, inviscid stratified flow will be stable to infinitesimal disturbances if the gradient Richardson number everywhere exceeds $\frac{1}{4}$ (Miles, 1961). Since we are discussing only geostrophic-

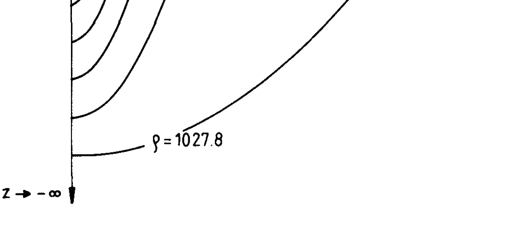

Fig. 11. Isopycnals between stations 1 and 2 in the case of divergent isolines. The density stratification in the surface layer at station 1 linearly increases with depth in the surface layer.

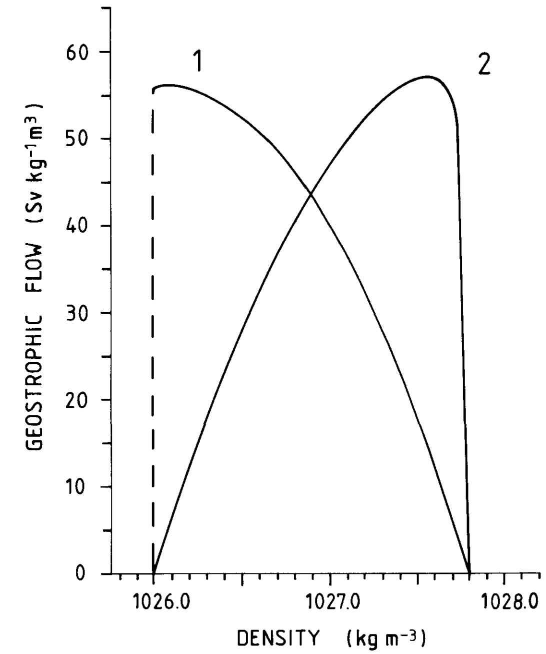

Fig. 12. The geostrophic flow per unit density interval, $m(\varrho)$, in the surface layer of case 1 (shown in Fig. 10) and case 2 (shown in Fig. 11). Note that when the isopycnals converge, the maximum transport moves upwards, to lower densities. When the isopycnals are divergent, the maximum transport moves downwards, to higher densities compared to the standard case shown in Figs. 4, 5.

ally balanced flow, we can make use of the geostrophic balance:

$$f\,\partial u/\partial z = g/\varrho(\partial\varrho/\partial y) \tag{24}$$

to rewrite the Richardson number as

$$\mathrm{Ri} = -f^2\varrho\,\partial\varrho/\partial z/[g(\partial\varrho/\partial y)^2]. \tag{25}$$

It is well documented that the width, L, of the frontal region is typically of the order the internal Rossby radius, or possibly larger (see, e.g., Gill (1982), Walin (1972)), i.e.,

$$L \approx (1/f)\sqrt{(gH\Delta\varrho/\varrho)}, \tag{26}$$

where $\Delta\varrho$ and H are the characteristic density variation and the depth of the front. Estimating $\partial\varrho/\partial y$ and $\partial\varrho/\partial z$ by $\partial\varrho/\partial y \sim \Delta\varrho/L$ and $\partial\varrho/\partial z \sim \Delta\varrho/H$ in eq. (25) we obtain

$$\mathrm{Ri} \approx 1. \tag{27}$$

Eq. (25) may be rewritten as

$$\mathrm{Ri} = -f^2\varrho(\partial\varrho/\partial z)^{-1}(\tan\varphi)^{-2}, \tag{28}$$

where φ is the local angle between the isopycnic surface and the horizontal plane. Now, eq. (27) represents a characteristic value of Ri in the frontal region.

It follows from eq. (28) that locally larger values of $\partial\varrho/\partial z$ and/or $\tan\varphi$ will decrease Ri locally, possibly below the critical limit $\mathrm{Ri} = \frac{1}{4}$. We thus expect that shear induced turbulence may limit the possibilities for the flow field to arrange itself in steep fronts with sharp density gradients underneath.

5. Discussion

An expression for the geostrophic flow as a function of density has been formulated. The equation was derived for a stratified surface layer on top of a homogeneous and stagnant bottom layer. Generally, this distribution function of the geostrophic flow was found to depend on the shape of the whole density field. If, however, the vertical distance between any two isolines is constant as the isolines approach the surface, the distribution function will depend only on the density stratification at $y = 0$, making eq. (18) valid. It was furthermore found that a horizontal stretching of the density field does not change the distribution functions describing the flow. We can thus conclude that the horizontal scale of the frontal region is not of crucial importance for the distribution function $M(\varrho)$. A very wide front may have identically the same transport properties as a narrow front.

If the density field is changed in other ways, the distribution of the flow will change accordingly. Our results consequently indicate that: (a) converging isolines imply a shift of the flow towards lower densities; (b) diverging isolines imply a shift towards higher densities.

If only the vertical density profile at $y = 0$ is disturbed, while keeping the shape of the isolines unchanged, the distribution function as well as the total transport will change, according to the nature of the disturbance. If the distribution function is normalized by $(\partial\varrho(0,z)/\partial z)^{-1}$, it turns out to be less sensitive to the disturbances of the density stratification at $y = 0$. It was also found that the distribution of the flow could be quite well reproduced in such cases, by assuming equidistant isopycnals, employing a normalized standard distribution function, and information on the density profile at $y = 0$.

We can anticipate the different kinds of disturbance, e.g., wind mixing, will act to redistribute the density in a front and possibly also in the basin behind the front. According to our calculations, this may bring about a change in the flux of, e.g., buoyancy through a section across the frontal region. Such a change may occur even without any change in the total volume flow.

There is a need in modelling to estimate $M(\varrho_*)$ from hydrographic observations, even if knowledge of the frontal structure is not available. We suggest that such estimates are possible using either of eqs. (18) or (21).

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