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Rossby waves in a fluctuating zonal mean flow

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ABSTRACT

The effects of the zonal mean wind variability on the energy propagation of a stationary Rossby wave in a barotropic non-divergent atmosphere are studied. It is shown that the random nature of the zonal wind fluctuations do not allow Rossby wave energy to propagate from its energy source. The mechanism for this effect is strongly dependent on the spatial resolution at which the zonal mean flow is assumed to be known. Some speculations are offered on the relation between this mechanism and the systematic error of some low-resolution climate models.

1. Introduction

Presumably, the observed planetary scale climatological eddies owe their existence to the interactions between the free atmosphere and the asymmetries of the lower boundary forcings, i.e., the earth's topography and the large scale distribution of heat sources and sinks. Since the pioneering works of Charney and Eliassen (1949) and Smagorinsky (1953), the mechanisms involved in these interactions have been studied extensively by using models of increasing complexity.

Recently, some authors (notably Grose and Hoskins, 1979; Hoskins and Karoly, 1981; Held, 1983 and James, 1988) have stressed the importance of spherical geometry in determining the response of a model's atmosphere to a stationary forcing. In particular, it has been argued that beta-channel models overestimate the actual response to the topographical forcing because, by imposing artificial lateral boundaries, they allow for a resonant behavior. In spherical geometry, the disturbances are instead free to propagate away from their midlatitude sources and, unless reflection occurs, they will be absorbed poleward and, therefore, substantially influence the observed eddy climatology and low frequency variability. This mechanism may be important, for example, to account both for the teleconnection patterns described by Wallace and Gutzler (1981) and their statistical relationship with the anomalous low level thermal field as documented in Horel and Wallace (1981) (see Simmons et al., 1983).

The study of Rossby waves propagating in a dispersive medium by the method of geometrical optics (Lighthill, 1978) provides a powerful framework to describe the role of the earth's sphericity in the structure of large scale disturbances. In this framework, it has been possible to gain insight into the relative role of topography and diabatic heating in determining the behavior of atmospheric models, which, in turn, has helped explain the observed statistics. Geometrical optics theory is applicable to meteorological problems when the governing equations are linearized around a given mean flow (not necessarily zonally symmetric). Customarily, the observed climatological average is used for the mean flow. In this case, the low frequency dependence of the planetary scale disturbances upon the basic flow can be studied by calculating, with high spatial resolution, the system's response to different mean flow configurations. For the purpose of the present paper, in accordance with Hoskins and Karoly (1981), we assume that no interactions occur between the instantaneous basic flow and large scale eddies. The small portion of planetary scale eddy variability explained by the zonal mean flow daily fluctuations (Hansen and Sutera, 1987) provides evidence supporting this assumption. Equivalent observational evidence for the case in which the basic state is zonally varying is not yet available. Therefore, in the present paper we will restrict ourselves to the study of perturbations of a zonally symmetric flow. To keep our work in the same conceptual framework as Hoskins and Karoly (1981) we also assume that no planetary scale wave-wave interactions are allowed. Of course, the assumptions inherent in the above mentioned linear theories strongly limit their applicability to atmospheric studies, although many interesting results have been obtained along this line.

Given the time scales (infinite for a stationary wave) considered here, the basic state variability should be taken into account. For this purpose, unless the time dependent problem of the resulting linear model is solved (e.g., by taking the zonal wind time behavior from observations) an assumption on the nature of the flow is needed. Therefore, we will account for the effect of the basic state fluctuations by assuming that they are independent realizations of a random field and identify the eddy response as the mean of responses in the ensemble of the basic state realizations. We shall see how some of the results presented in this paper can be interpreted by simply extending the geometrical optics theory in the case of wave propagating in a random medium. Furthermore, we will show that wave dispersion is strongly affected by the zonal wind fluctuations.

2. The problem

Given the illustrative nature of the present study, we shall restrict ourselves to the study of the latitudinal energy propagation of a stationary Rossby wave, forced by a localized pulse, in a barotropic non-divergent atmosphere. By linearizing the potential vorticity equation around a zonal mean flow $U(y)$ the equation for the latitudinal structure ψ of a Rossby wave of zonal wavenumber n in a Mercator projection is (see Appendix):

$$\psi_{yy} + v^2(y; U, n)\psi = \delta(y + y_0) - \delta(y - y_0), \tag{1}$$

with boundary conditions

$$\psi(|y| \to \infty) \to 0, \tag{2}$$

where v is the index of refraction defined in the Appendix and δ is a Dirac delta distribution. Given the latitudinal dependence of v the problem is to determine the solution of (1). If the solution is a meridionally localized function, the energy associated with the stationary wave remains near its source; otherwise, propagation occurs. In general, the solution (a Green's function) will depend on the location of the source. We consider two cases:

(A) *The forcing is located in the tropics, say 10°N.* With this condition, we simulate some heating disturbance which, captured by the westerlies, may provide enough poleward energy flux to significantly affect the eddies structure in midlatitudes.

(B) *The source is localized in the extratropics.* In this circumstance we wish to determine whether the Rossby wave energy flux associated with this forcing will decay away from the source. If this occurs, the situation is similar to the beta-channel approximation and it is likely that a resonant response may be excited.

The solution of (1) requires the knowledge of v^2 as a function of the zonal mean wind U, which as discussed in the introduction is assumed to be a fluctuating field. A way to fully account for the randomness of the wind fluctuations is to solve (1) for each individual realization of U but to consider the ensemble average of the solution to be representative of the actual latitudinal behavior. We will contrast this case with the one in which the wind fluctuations are disregarded and (1) is solved by using v^2 computed as a function of the climatological mean wind $\langle U \rangle$.

3. The data

The data used for the computation of the refraction index v are National Meteorological Center's (NMC) analyses available on the NMC-UNIDATA compact disk. It consists of Northern Hemisphere winter (here defined as the months of December through February) analyses of the 200 mb and 500 mb geopotential heights. Because from 1963 to 1974 this data set has only 0.7% of the days missing (compared with 9% in the remaining period) we decided to consider this as the period covered by our basic data set. The zonal velocities were calculated geostrophically from the geopotential heights. Since one of our problems is to calculate the response to a forcing situated at low latitudes but in a westerly environment we decided to consider 200 mb fields. We notice that, by choosing this pressure level, we have the advantage that at this height no zonal mean easterlies are encountered at low latitudes for the whole data set here considered. Thus our solutions do not reflect any critical line effect.

The equation that we wish to solve requires the knowledge of v^2 through the whole latitudinal domain. Unfortunately the data here considered did not allow us to calculate v^2 to any degree of realism for latitudes equatorward of 20° N, both because of the lack of data and the actual quality of the analyses in this region. Thus, we assumed that from 20° N to the equator v^2 has the same constant value.

4. Results

In this section, results are presented for the case in which the 200 mb height data are assumed to have 1° × 1° resolution. Let us consider the case in which

$$v = v(\phi; \langle U \rangle, n) \tag{3}$$

where ϕ is latitude and $\langle\ \rangle$ denotes the ensemble average in our data set.

The geometrical optics approximation (forward scattering assumption) consists in calculating the WKB solution. In this approximation the propagating or decaying nature of solutions of (1) is determined by considering the separatrix $v = 0$. In regions where v is a real-valued function, ψ describes an energy propagating wave; otherwise it describes a decaying (trapped) solution (e.g., Held, 1983). For a given zonal wavenumber, the latitudinal location where $v = 0$ represents a turning point for the wave. In Fig. 1 $v = 0$ is plotted as a function of the latitude ϕ and the zonal wavenumber n. At variance with other studies, here zonal wavenumber 3 may be trapped in midlatitudes while zonal wavenumber 2 may be trapped in the region poleward of 70° N. In agreement with other studies, zonal wavenumber 1 can propagate through the whole hemisphere. The partial disagreement of the present results with the ones presented in other studies is partly due to our choice of calculating the wind at 200 mb. The purpose of the present paper is not to make a claim whether a stationary Rossby wave of a particular zonal wavenumber in the real atmosphere is likely to have a latitudinal structure which allows for energy propagation, but rather to shed some light upon the effects that a fluctuating wind has on a wave which propagates energy when these fluctuations are suppressed. Therefore, in what follows, we will concentrate on the behavior of zonal wavenumber 1 which manifestly possesses the required (propagative) structure. We present in Fig. 2 the numerical solutions of (1) for a forcing located at each of the three latitudes 10° N, 70° N

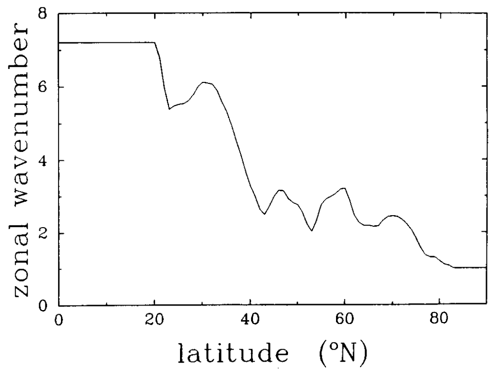

Fig. 1. The curve separating propagating from trapped (upper plane) solutions as a function of the zonal wavenumber and latitude.

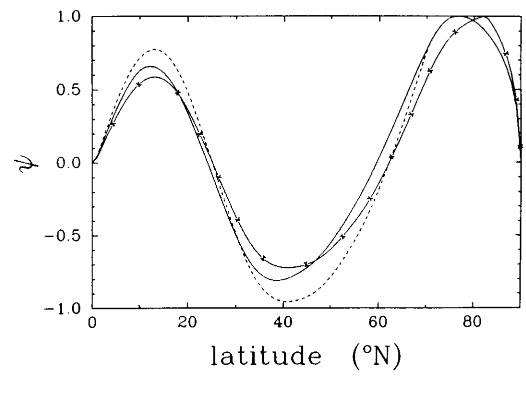

Fig. 2. ψ as a function of latitude for the case of v^2 computed by considering $\langle U \rangle$ and by having the forcing located at 10° N (solid), 70° N (dashed) and 82° N (crosses).

and 82° N. For the sake of comparison, all the curves have been normalized to unity. We emphasize the poleward increase of the amplitude, which is consistent with a WKB solution (Hoskins and Karoly, 1981), is entirely reproduced by our numerical approach. We recall also that it is this peculiar latitudinal behavior of the amplitude which allows us both to interpret atmospheric teleconnections and to discuss whether resonant amplification of waves in midlatitudes as property of propagating Rossby waves can be dismissed. For a stationary wave, energy is propagated. A good proxy for the perturbation's energy is ψ^2. For future comparison in Fig. 3 we plotted ψ^2 as a function of the latitude.

Next we study the case in which the zonal wind fluctuations are considered. We solve (1) for each day in our data set, normalize to unity each individual realization and then take the ensemble average. In Fig. 4 we show $\langle\psi^2\rangle$ as a function of latitude for the same forcing locations previously discussed. A considerable difference between these solutions and the ones previously discussed is readily seen. In particular, for disturbances generated in the tropical region the Rossby wave energy can only propagate up to 35° N and then, toward higher latitudes, is significantly attenuated. On the other hand, in response to a high latitude forcing the energy tends to be trapped in that region with less penetration to middle or lower latitudes. Similar conclusions can be drawn if we consider the forcing to be located at any other latitude. For example, the solution (not shown) for a forcing located at 40° N is undistinguishable from the one obtained with the forcing location at 10° N.

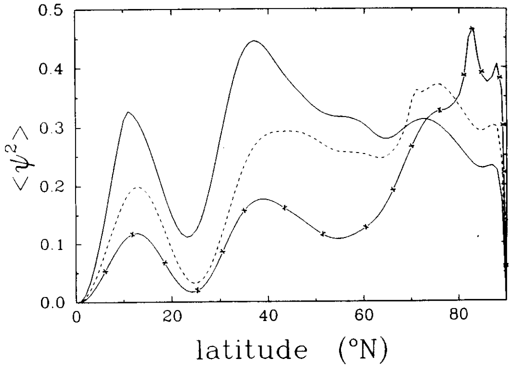

Fig. 4. $\langle\psi^2\rangle$ as a function of latitude for the case of v^2 computed by considering daily values of U and by having the forcing located at 10° N (solid), 70° N (dashed) and 82° N (crosses).

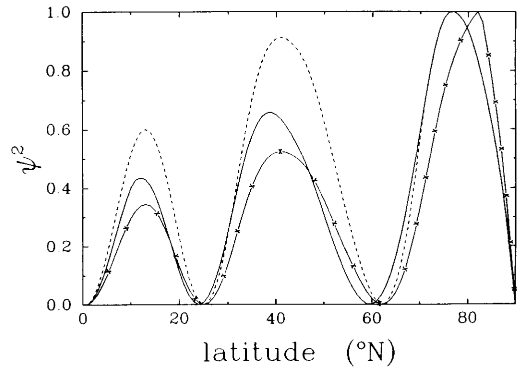

Fig. 3. ψ^2 as a function of latitude for the case of v^2 computed by considering $\langle U\rangle$ and by having the forcing located at 10° N (solid), 70° N (dashed) and 82° N (crosses).

Because the refraction index is inversely proportional to U it may be thought that the above result may be caused by a small number of days during which the wind achieves very small values, thus generating very high barriers to be tunnelled by the solution. In the sample set here considered about 10% of the cases (days) have some locations ϕ_i in midlatitudes (to the north of the jet axis) where the daily wind $U(\phi_i)$ is close to 0 m/s. To exclude this as the cause for the observed decay we eliminated the above mentioned 10% of days and computed the new ensemble average. The corresponding $\langle\psi^2\rangle$ (not shown) reproduces essentially the same behavior shown in Fig. 4.

Another possibility is that the above result may be a function of the interannual variability of the data. This dependence can be excluded since we have repeated the calculation for a single month without noticing a substantial change in the nature of the solution (see Section 5). Thus we can conclude that the novel features found here are due essentially to the random nature of the wind fluctuations. Therefore we should interpret our results as saying that perturbations originating in midlatitudes remain confined there and hence conditions for resonance are possible. Analogously, perturbations generated in the tropics are likely to remain in these regions with a limited effect on higher latitudes.

5. Interpretation

In order to understand the nature of our results, let us consider the case where $v = v(y; \langle U \rangle, n)$. Here the refraction index is a smooth function of space; therefore, the agreement between the numerical solution and the WKB approximation implies that the medium is transparent to Rossby wave energy propagation. Only forward scattering processes occur and the geometrical optics limit fully applies.

The situation is sharply different when the fluctuations are considered. The irregularities induced by the zonal mean wind fluctuations in the profiles of v^2 act as scattering centers which strongly affect the energy propagation. The presence of a scattering center increases the effective distance that the signal has to travel decreasing the effective group velocity. This effect is clearly demonstrated by the comparison of the behavior of $\langle \psi^2 \rangle$ (Fig. 4) with ψ^2 (Fig. 3). Moreover, the scattered fluid parcel is advected by the zonal mean flow in regions of different ambient potential vorticity which may or may not increase the effectiveness of the Coriolis force in acting as a restoring mechanism (Keller and Veronis, 1969).

The situation is reminiscent of the one encountered in solid state physics when a wave travels across a material doped with impurities (see for example, Anderson, 1958; Lifshitz and Kirpichenkov, 1979). For these systems, it has been proved that waves travelling through the medium are damped (i.e., localized) by the randomness of the refraction index, regardless of the occurrence in the domain of any turning point. In order to establish this analogy on a firmer ground, let us examine in more detail the nature of the refraction index for a typical realization of the zonal wind. The function v^2 for the day of December 4, 1963 is displayed as a function of latitude in Fig. 5. We notice that northward of the jet maximum v^2 varies erratically and the amplitude of the variations are large (compare with Fig. 1). In this environment, WKB theory fails and other means of analysis are required. A complete mathematical treatment (see Mysak, 1978 for a review) of such a problem is not the purpose of the present paper. However, we can illustrate further the source of the attenuation that we observed by considering the following argument. From Fig. 5 we notice that v^2 is not a positive valued function. Hence, we might argue that the overall decay can be caused by the occurrence of turning points. To exclude this interpretation we have added to v^2 a constant value such that the new v^2 is a strictly positive valued function in the entire domain and we again integrated (1) for a forcing located at 10° N. The behavior of ψ^2 (Fig. 6) shows a decay which is inconsistent with a WKB solution, but is consistent with the general behavior of $\langle \psi^2 \rangle$. Thus, we can conclude that the averaged behavior shown in Section 4 is the result of the spatially random nature of v^2.

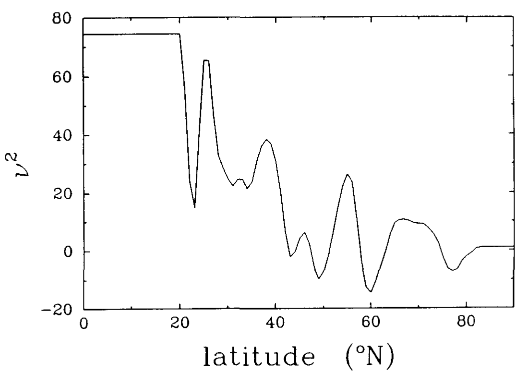

Fig. 5. The index of refraction squared for the day of 4 December 1963.

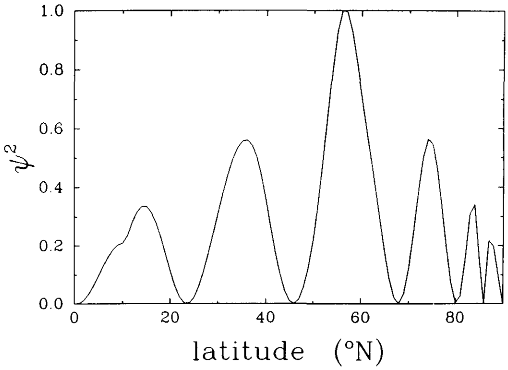

Fig. 6. Latitudinal behavior of ψ^2 for the day of 4 December 1963. The forcing is located at 10° N.

6. The sensitivity of wave propagation to model resolution

To further emphasize the role of the spatial fluctuations in determining the structure of the solutions discussed above, we extract the data at a different, 4°, resolution. In this case, we expect that ν^2 will be a rather smooth function of ϕ. Here again we consider the cases:

(1) index of refraction $\nu(\phi; \langle U \rangle, n)$ calculated from the climatological mean, zonally averaged flow;

(2) ensemble average solution $\langle \psi^2(\phi) \rangle$ obtained by averaging the normalized daily solutions $\psi^2(\phi)$ determined as response to the daily, zonally averaged flow U.

In this section, we consider solutions for the forcing located at 10°N and 82°N only. For case 1, as can be seen by comparing Fig. 1 and Fig. 7, the effect of lower resolution on the index of refraction seems to have been to smooth out any sharp irregularities. As a consequence, the numerical integration (Fig. 8) is well approximated by the WKB solution, which was to be expected on the ground that this solution was already a good fit to the numerical solution for the integration at 1° resolution.

For case 2, the behavior of $\langle \psi^2 \rangle$ compares rather well (Fig. 10) with the one obtained with the average wind (Fig. 9) when the forcing is at 82°N and it appears that any signature of a decaying behavior (localization) has been lost. More

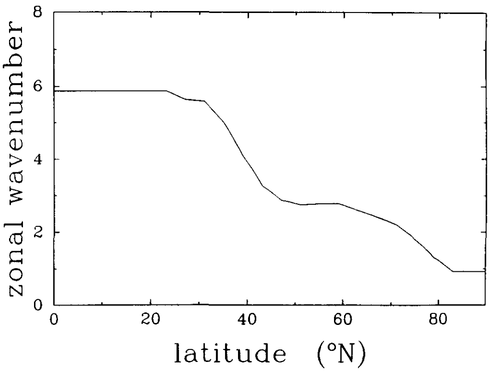

Fig. 7. The curve separating propagating from trapped (upper plane) solutions as a function of the zonal wavenumber and latitude, for the 4° case.

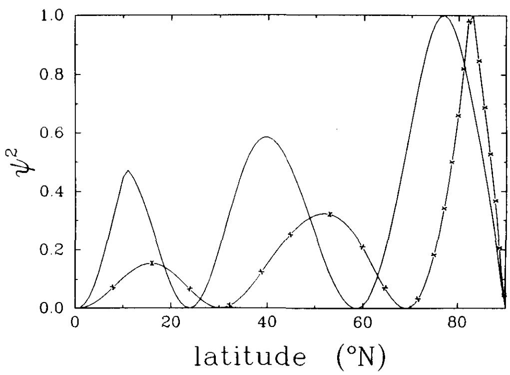

Fig. 9. ψ^2 as a function of latitude for the case of ν^2 computed by considering $\langle U \rangle$ and by having the forcing located at 10°N (solid) and 82°N (crosses), for the 4° case.

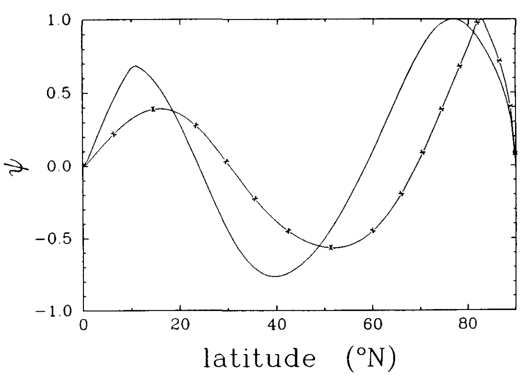

Fig. 8. ψ as a function of latitude for the case of ν^2 computed by considering $\langle U \rangle$ and by having the forcing located at 10°N (solid) and 82°N (crosses), for the 4° case.

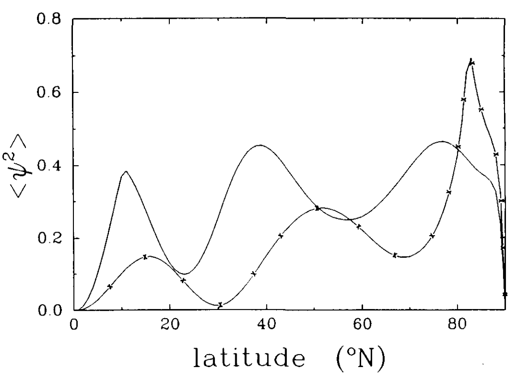

Fig. 10. $\langle \psi^2 \rangle$ as a function of latitude for the case of ν^2 computed by considering daily values of U and by having the forcing located at 10°N (solid) and 82°N (crosses), for the 4° case.

precisely, if any location occurs, it is confined in high latitudes (which can be seen by comparing the solid curves in Figs. 9 and 10 corresponding to a forcing located at 10° N). This can be argued on the ground that, compared with the behavior obtained considering $\langle U \rangle$, a somewhat slower increase is observed northward of 65° N when the forcing is at 10° N. This peculiar behavior seems to suggest that if we consider models with low spatial resolution we should expect a poleward shift of the planetary scale eddy variance with an overall weaker amplitude. This is indeed the case for some general circulation models, as has been documented by Hansen and Sutera (1990). Hence we are led to the speculation that the confinement mechanism discussed in the present paper may play a role in explaining some features of the systematic error of general circulation models with low spatial resolution.

7. Conclusions

We have analyzed the role of the zonal mean flow fluctuations in determining the nature of the energy propagation of a stationary Rossby wave in spherical geometry. It has been shown that the irregularities of the background flow significantly affect the latitudinal propagation of the wave's energy. In contrast with the case of a smoother background flow, the wave amplitude and energy are attenuated by a mechanism which is reminiscent of the stochastic damping occurring when waves propagate in a random medium.

The localized nature of the averaged solution obtained suggests that the wind fluctuations may provide a viable energy trapping mechanism which may establish favorable conditions for perturbations to resonantly amplify by interacting with the lower zonally asymmetric boundary forcing. On the other hand, the same attenuation process does not particularly favor the propagation of tropical disturbances to higher latitudes. The localization mechanism is not very efficient when the zonal mean flow is computed at low resolution. We have suggested that this inefficiency may play a role in explaining the deficit of variance observed in some low resolution climate models.

The work presented here can be expanded in several directions. In particular, we intend to consider the case of perturbations which propagate on background flows which are not zonally symmetric.

8. Acknowledgments

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9. Appendix

Derivation of the meridional structure equation

Spherical coordinates

Our investigations are based on the non-divergent barotropic vorticity equation

$$\frac{\partial}{\partial t}\Delta\Psi + \mathscr{J}(\Psi, \Delta\Psi + f) = 0, \tag{A1}$$

which in spherical coordinates reads

$$\frac{\partial}{\partial t}\Delta\Psi - \frac{\partial\Psi}{a\,\partial\phi}\frac{\partial\,\Delta\Psi}{a\cos\phi\,\partial\lambda} + \frac{\partial\Psi}{a\cos\phi\,\partial\lambda} \times\left(\frac{\partial\,\Delta\Psi}{a\,\partial\phi} + \frac{2\Omega}{a}\cos\phi\right) = 0, \tag{A2}$$

with

$$\Delta\Psi = \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\Psi}{\partial\phi}\right) + \frac{1}{a^2\cos^2\phi}\frac{\partial^2\Psi}{\partial\lambda^2}, \tag{A3}$$

$$u = \frac{-1}{a}\frac{\partial\Psi}{\partial\phi}, \tag{A4}$$

$$v = \frac{\partial\Psi}{a\cos\phi\,\partial\lambda}, \tag{A5}$$

and where Ψ is the horizontal streamfunction, $\mathscr{J}$ the Jacobian operator:

$$\mathscr{J}(A, B)=\frac{\partial A}{\partial x}\frac{\partial B}{\partial y}-\frac{\partial A}{\partial y}\frac{\partial B}{\partial x},$$

$\Delta\Psi$ the vertical component of vorticity, $f=2\Omega\sin\phi$, λ represents longitude and ϕ latitude. Since we are interested in studying the effects of daily wind fluctuations on the simplest form of (A2) we linearize (A2) about a zonally symmetric basic state with zonal flow $U(\phi, t)$. We then write

$$\Psi(\lambda, \phi, t)=-a\int U(\phi, t)\,\mathrm{d}\phi+\psi'(\lambda, \phi, t), \qquad \text{(A6)}$$

with ψ' denoting the perturbation part of Ψ. Because of the assumption of no correlation between the basic state zonally averaged flow and wave perturbations described by (A2) we take the zonally averaged daily winds to be independent realizations of a stochastic field. Therefore (A6) becomes

$$\Psi(\lambda, \phi, t)=-a\int U(\phi)\,\mathrm{d}\phi+\psi'(\lambda, \phi, t), \qquad \text{(A7)}$$

with $U(\phi)$ time-independent. Now, to study the latitudinal structure of the wave perturbations ψ' we concentrate on a given zonal wavenumber n and frequency ω and look for a solution of the form

$$\psi'(\lambda, \phi, t)=\psi(\phi)\exp[i(n\lambda-\omega t)]. \qquad \text{(A8)}$$

Substituting (A7) and (A8) into (A2), linearizing and multiplying by $a^2\cos\phi/in$ yields

$$-\frac{\omega}{n}\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\cos\phi\frac{\mathrm{d}\psi}{\mathrm{d}\phi}\right)+\frac{\omega n}{\cos\phi}\psi+\frac{U}{a\cos\phi}\times\left[\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\cos\phi\frac{\mathrm{d}\psi}{\mathrm{d}\phi}\right)-\frac{n^2}{\cos\phi}\psi\right]+\psi\left[2\Omega\cos\phi-\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\frac{1}{\cos\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}(U\cos\phi)\right]\right]=0, \qquad \text{(A9)}$$

which for stationary waves ($\omega=0$) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\cos\phi\frac{\mathrm{d}\psi}{\mathrm{d}\phi}\right)-\left[\frac{\cos\phi}{U}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\frac{1}{\cos\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}(U\cos\phi)\right]+\frac{n^2}{\cos\phi}-\frac{2\Omega a\cos^2\phi}{U}\right]\psi=0. \qquad \text{(A10)}$$

With the addition of suitable boundary conditions (A10) represents a Sturm-Liouville problem for ψ which can be written simply as

$$\mathscr{L}\psi=-F(\phi), \qquad \psi(|\phi|=\pi/2)=0, \qquad \text{(A11)}$$

where

$$\mathscr{L}=\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\cos\phi\frac{\mathrm{d}}{\mathrm{d}\phi}\right)+\frac{v^2(\phi; U, n)}{\cos\phi}, \qquad \text{(A12)}$$

$$v^2(\phi; U, n)=-\cos\phi\left[\frac{\cos\phi}{U}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\frac{1}{\cos\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}(U\cos\phi)\right]+\frac{n^2}{\cos\phi}-\frac{2\Omega a\cos^2\phi}{U}\right], \qquad \text{(A13)}$$

is the square of the index of refraction and F corresponds to some forcing function. This problem would be well posed over the whole interval $\phi\in[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ were it not for the limited latitudinal extent of the index of refraction v. Due to practical considerations (as discussed in Section 3) the available data (geopotential height fields at 200 mb and 500 mb) extend only from 19°N to 84°N. Solving (A11) in that meridional range would actually mean imposing arbitrary boundary conditions at the end points. Instead we define a new index of refraction v such that

$$\begin{aligned} v^2(\phi)&=v^2_{\text{data}}(20°\text{N}), && 0°\leqslant\phi<20°\text{N},\\ v^2(\phi)&=v^2_{\text{data}}(\phi), && 20°\text{N}\leqslant\phi\leqslant 83°\text{N},\\ v^2(\phi)&=v^2_{\text{data}}(83°\text{N}), && 83°\text{N}<\phi\leqslant 90°\text{N},\\ v^2(-\phi)&=v^2(\phi), && 0°<\phi\leqslant 90°, \end{aligned} \qquad \text{(A14)}$$

the last line implying that a Southern Hemisphere is present but that it is an identical replica of the Northern one (reflection symmetry with respect to

$\phi = 0°$). This will allow us to keep the correct boundary conditions, as given in (A11), for the end points. It is now possible to apply a localized forcing in the form of delta distributions ($F(\phi) = \delta(\phi - \phi_0) - \delta(\phi + \phi_0)$) to generate a wave perturbation, the latitudinal structure of which will be governed by (A11). The presence of two delta distributions in F (corresponding to a unit source at $\phi = \phi_0$ and a unit sink at $\phi = -\phi_0$) comes from the fact that we are solving the Sturm-Liouville equation for two hemispheres one of which is a mirror image of the other.

Mercator projection

Since many theoretical considerations in the text are made using (A11) projected onto the Mercator plane, we give below the transformed version of (A11). Using

$$y = a \ln\left[\frac{1 + \sin\phi}{\cos\phi}\right], \tag{A15}$$

which implies that:

$$\frac{\cos\phi}{a}\frac{d}{d\phi} = \frac{d}{dy}, \tag{A16}$$

the non-dimensional version of (A11) on the Mercator plane, reads

$$\frac{d^2\psi(y)}{dy^2} + v^2(y; U, n)\,\psi(y) = -F(y),$$
$$\psi(|y| \to \infty) \to 0. \tag{A17}$$

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