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Computations of the flow of dense water into the Baltic Sea from hydrographical measurements in the Arkona Basin

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ABSTRACT

The dense water flowing through the Danish Sounds into the Baltic Sea creates a pool of dense water in the deepest parts of the Arkona Basin, just inside the sounds. The pool is usually thin since this basin has no sill in the east. The pool leaks and the leakage is assumed to be controlled by the vertical stratification in the Arkona Basin together with the rotation of the earth (rotational baroclinic control). For each instance of known vertical stratification in the Arkona Basin, the accompanying leakage may be estimated.

182 historical vertical hydrographical profiles taken in the Arkona Basin during a 25-year long period are utilized. The leakage of dense water is computed from each single profile. The estimated mean salt flux by the flow (from the Arkona dense pool) into the Baltic satisfies the continuity requirements for a stationary Baltic Sea. There is a positive correlation between the flow rate and the salinity of the dense water.

For a system where the salinity determines the density (brackish water), a dynamically equivalent, vertically homogeneous pool of dense water may be computed requiring conservation of profile salinity surplus Φ and profile potential energy PE. In this paper, it is demonstrated that Φ and PE completely determine the flows of volume and buoyancy (salt excess). Regardless of the actual shape of the vertical stratification, the volume flow is $Q = \mathrm{PE}/f$ and the flow of excess salt is $QS = g \cdot \beta \cdot \Phi^2/(2 \cdot f)$ where β is the salt expansion coefficient of sea water.

1. Introduction

The Baltic Sea (see the map in Fig. 1) is a huge estuary. The surface area is $\sim 350{,}000$ km^2 and the mean depth is ~ 60 m. The connections to the sea, through the Danish Straits, are rather narrow and shallow with a sill depth of only 18 m. Seaward of the Danish Sounds is the shallow Kattegat with a mean depth of ~ 23 m. The narrow and shallow connections to the sea together with the positive freshwater balance of the Baltic, the mean surplus is $\sim 15{,}000$ m^3/s, are the main reasons for the low salinity of the surface layer (salinity $\sim 7‰$, thickness ~ 60 m). Below the surface layer there is an ~ 10 m thick halocline on top of the lower layer of salinity 10–13‰. For most of the year, heating/cooling processes dominate the buoyancy flux through the sea surface and during a large part of the year, a thermocline exists above the pycnocline. During a few months in winter the thermocline and the halocline coalesce and in this period denser water is entrained, through the halocline, into the surface layer; see Stigebrandt (1985) for a description and a model for the seasonal behaviour of the Baltic.

Surface water exits the Baltic through the Danish Straits and sustains surface layers of relatively low salinity in the shallow Belt Sea and Kattegat. However, due to variations in the wind speed and in the supply of Baltic surface water the depths and salinities of the surface layers in these seas are quite variable. A vertical cross section, showing mean salinity in June, through the Baltic Sea, the Belt Sea and the Kattegat is shown in Fig. 2.

The classical, steady two-layer type of flow in the mouth of an estuary is atypical for the Danish

Sounds. Instead the flow is for most of the time dominated by the barotropic flow induced by the sealevel difference between the Kattegat and the Baltic. Thus, in periods when the Kattegat sealevel stands appreciably higher than the Baltic sealevel the flow on all levels is directed towards the Baltic and vice versa, see Stigebrandt (1980, 1983).

Owing to its dependence upon the sealevel difference between the Kattegat and the Baltic Sea, the inflow of denser "sea water" to the Baltic has a pulsating character. Because of the highly fluctuating currents the Danish Straits are not particularly well suited for measurements of the inflow of dense water. Walin (1981) chose a section between Sweden and the Bornholm Island, where he expected the flow of dense water to be rectified, for a large field program aimed at measuring the inflow of dense water to the Baltic. The "Bornholm Section" is shown in Fig. 1.

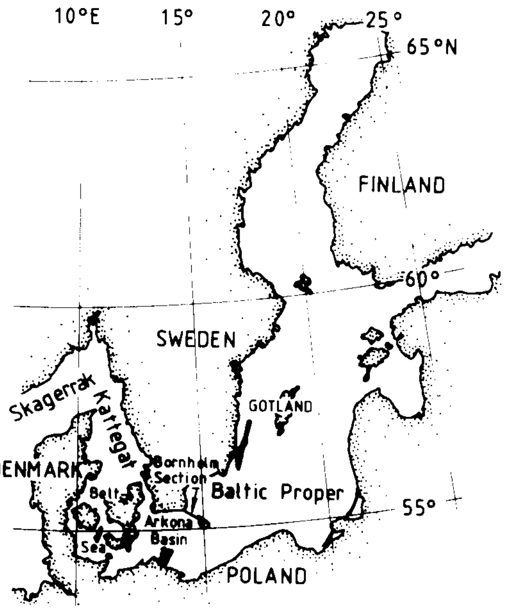

Fig. 1. Map of the Baltic Sea.

The dense water flowing into the Arkona Basin has the character of a bottom current. Baltic surface water should be entrained into the dense bottom current. Thereby the density decreases while the volume flow increases in the downstream direction. To the best of the present author's knowledge, there is no detailed description of the dense bottom current between the Danish Sounds and the Bornholm Section. The dense flow may take any route between two extreme routes.

In one extreme route, there is a coherent current along the (southern) slopes downwards to the Bornholm Section. In this case the rotation of the earth is important for the dynamics of the flow. The current should be almost parallel to the depth contours and there should be a direct response in the Bornholm Section to changes of the inflow through the Danish Sounds. Since inflow through the Danish Sounds occurs only about half of the time, and may be absent for periods of weeks, one would in this case expect that also the dense current in the Bornholm

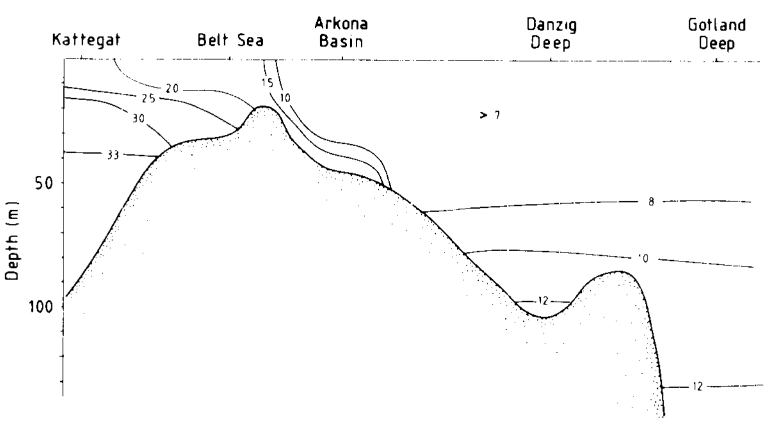

Fig. 2. The mean distribution of salinity (in June) in a vertical cross section from the Kattegat to the East Gotland Basin.

Section should be absent in a substantial number of the observations. However, Walin (1981) always found a dense bottom current. This fact strongly points to the other extreme route, namely that the dense water takes a shorter pass (thus makes a greater angle with the isobaths) down the slope. Thus frictional effects are quite important for the dynamics of the flow. Well down the current contributes to the filling of a dense pool in the deep parts of the Arkona Basin. Hydrographical measurements in the Arkona Basin show that there always are at least traces of dense water at the bottom. Thus it appears that the dense currents from the Sounds make a relatively large angle with the isobaths and create a dense pool in the Arkona Basin.

The pool of dense water in the Arkona Basin lacks a physical wall (a sill) in the east. By continuity reasons the pool must leak and dense water leaves the pool, mainly through the Bornholm Section (the passage west of Bornholm Island is rather shallow). The leakage of dense water from the pool should be controlled by the vertical stratification in the Arkona Basin and the rotation of the earth in much the same manner as the leakage of light surface water from the Arctic Ocean or the Kattegat, see Stigebrandt (1981, 1983), respectively. If this is the case it should be possible to compute the flow of dense water through the Bornholm Section corresponding to each hydrographical vertical profile available from the Arkona Basin.

In Section 2, we discuss different ways to compute the leakage from a dense, rotating bottom pool. In Section 3, we make use of historical hydrographical measurements from the Arkona Basin and compute the actual leakage from the pool. A statistical description of the flow of dense water is presented. The paper is brought to an end by a short discussion.

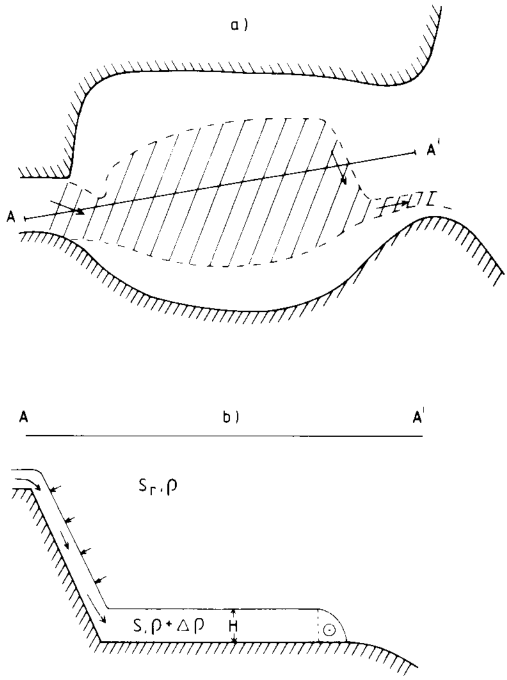

Fig. 3. A sketch of the anticipated distribution of dense water in a basin of the Arkona type; (a) seen from above and (b) vertical cross section.

2. The leakage from dense rotating bottom pools

A sketch of the anticipated hydrographical situation in the Arkona Basin is shown in Fig. 3. Dense water flows from the Danish Sound into the Arkona Basin as a dense bottom current. On its way down the slope the dense current entrains surrounding, lighter water. The dense current sustains the pool of dense water in the Arkona Basin. The pool has an open boundary (a front) in the east. Along the open boundary a current should run. The boundary current hits the wall in the south-east. The current is deflected eastwards by the wall and eventually the water crosses the Bornholm Section. We assume that the leakage from the pool is equal to the transport in the boundary current connected to the front. This hydrographical situation should be completely analogous to that in, e.g., the surface layer of the Kattegat where a pool of light water has an open boundary in the north, between Denmark and Sweden. Along the front, separating the light Kattegat surface water from the denser Skagerrak water, a current runs towards the east or north-east. The current hits the Swedish coast which it thereafter follows like a coastal current, see Stigebrandt (1983).

2.1. *The leakage from a two-layer system*

The transport carried by a boundary current in a two-layer system where the surrounding fluid is at rest may be computed from the integrated Margueles equation:

$$Q = g \cdot \Delta\rho/\rho \cdot H^2/(2 \cdot f), \tag{1}$$

where g is the acceleration of gravity, $\Delta\rho/\rho$ is the relative density difference between the fluids, H is the upstream thickness of the dense fluid and f is the Coriolis parameter.

Since density variations of the dense water in the Arkona Basin are largely determined by salinity variations we can use the simple equation of state for brackish water

$$\rho = \rho_f \cdot (1 + \beta \cdot S), \tag{2}$$

where $\beta \approx 8 \cdot 10^{-4}$ $(‰)^{-1}$, S is the salinity (‰) and ρ_f is the density of freshwater.

The dense water in the Arkona Basin is not homogeneous but vertically stratified. In order to make use of the simple transport equation, eq. (1), we have to transform the observed stratification to a dynamically "equivalent" two-layer stratification.

What is the salinity S and thickness H of the lower layer in a two-layer system which is dynamically "equivalent" to the real system? (cf. Fig. 4). One reasonable condition to put on the two-layer stratification is that the salt content should be the same as in the original profile. If the salinity of the upper layer is S_r (the water surrounding the dense water in the Arkona Basin) then *the profile salinity surplus* Φ is

$$\Phi = \int_0^D (S(z) - S_r)\,dz. \tag{3}$$

The lower layer in the equivalent two-layer system should have the same profile salinity surplus, thus

$$H \cdot (S - S_r) = \Phi. \tag{4}$$

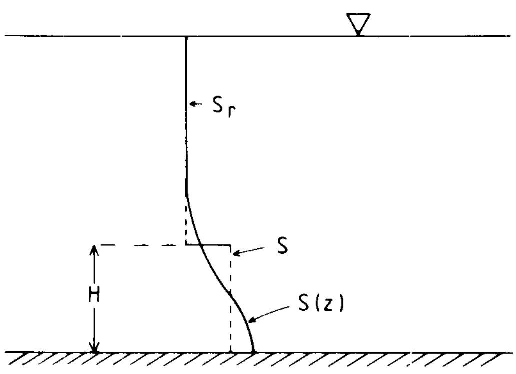

Fig. 4. Transformation of the real stratification to an equivalent two-layer stratification.

Since our two-layer stratification is characterized by two parameters the condition of conservation of profile salinity surplus is not sufficient to determine H and S. We need one more condition. The baroclinic transport in a boundary current is assumed to be proportional to the potential energy of the dense layer, cf. eq. (1). This observation leads us to choose to apply the condition of conservation of *profile potential energy PE*.

Thus

$$\mathrm{PE} = \int_0^D g \cdot \beta \cdot (S(z) - S_r) \cdot z\,dz \tag{5}$$

and for the profile potential energy of the two-layer stratification we require

$$g \cdot \beta \cdot (S - S_r) \cdot H^2/2 = \mathrm{PE}. \tag{6}$$

Combining eqs. (4) and (6) we obtain

$$S = S_r + g \cdot \beta \cdot \Phi^2/(2 \cdot \mathrm{PE}), \tag{7}$$

$$H = (2 \cdot \mathrm{PE})/(g \cdot \beta \cdot \Phi). \tag{8}$$

Thus eqs. (7) and (8) give the equivalent two-layer stratification under the conditions of conservation of profile salt surplus and profile potential energy. It may be interesting to note that, combining eqs. (1) and (6) and utilizing $\Delta\rho/\rho \approx \beta \cdot (S - S_r)$,

$$Q = \mathrm{PE}/f. \tag{9}$$

Thus the transport in the boundary current is proportional to the profile potential energy of the dense pool. The transport of excess salt, QS, is

$$QS = Q \cdot (S - S_r) = g \cdot \beta \cdot \Phi^2/(2 \cdot f). \tag{10}$$

Eqs. (9) and (10) applies at least to two-layer systems. However, it appears that these expressions have general validity, see subsection 2.2 below, thus being independent of the shape of the profile.

2.2. The leakage from a continuous system

Here we will make a "traditional" estimate of the geostrophic transport in the frontal current of the pool utilizing a set of salinity (density) measurements along a vertical in the pool. The measurements are from discrete levels along the vertical denoted by A in Fig. 5. The stratification

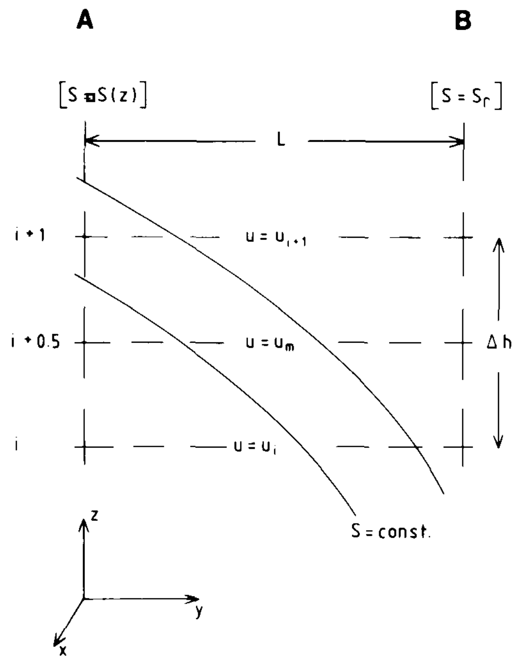

Fig. 5. Definition sketch for computations of transports from actual measurements.

in A represents the "upstream" stratification in the pool. The vertical B lies outside the pool and here the salinity is S_r = constant. In a right-handed coordinate system with the x-axis parallel to the front, as in Fig. 5, the relevant thermal wind equation reads

$$\delta u/\delta z = (g \cdot \beta/f) \cdot \delta S(y,z)/\delta y. \tag{11}$$

We integrate eq. (11) vertically from level i to level i + 1, obtaining

$$u_{i+1} = u_i + (g \cdot \beta/f) \cdot \delta/\delta y(\textstyle\int_i^{i+1} S(y,z)\mathrm{d}z) \tag{12}$$

since the levels are parallel to the y-axis. The value of the integral is approximated using the mid-interval value of S, thus

$$u_{i+1} = u_i + g \cdot \beta \cdot \Delta h/f \cdot \delta S(y, i + .5)/\delta y, \tag{13}$$

where Δh is the vertical distance between the levels i and $i + 1$.

The mean speed u_m of the layer between the levels i and $i + 1$ is

$$u_m = u_i + g \cdot \beta \cdot \Delta h/(2 \cdot f) \cdot \delta S(y, i + .5)/\delta y. \tag{14}$$

The volume transport q of the layer between the levels i and $i + 1$ may be obtained by integration of the speed over the surface generated by the intersection of the layer with a vertical plane parallel to the y-axis. Thus

$$q = u_i \cdot L \cdot \Delta h - g \cdot \beta \cdot \Delta h^2/(2 \cdot f) \cdot (S(A, i + .5) - S_r). \tag{15}$$

We will use eq. (15) together with eq. (13) to make an estimate of the transport of the frontal current connected to the pool. We then assume that the speed is zero where the salinity is equal to S_r, i.e., in the water surrounding the dense pool. This assumption also underlies eq. (1). Using the same reference salinity S_r it turns out that the transports of volume and salt computed from two-layer (Subsection 2.1) and "continuous" (Subsection 2.2) methods are identical. Thus it appears that eqs. (9) and (10) are of general validity.

3. Computations of the inflow from hydrographical measurements in Arkona Basin

Hydrographical measurements at station BY-1 in the Arkona Basin from the period 1958–1982 have been obtained from ICES. The stratification at BY-1 is assumed to represent the upstream stratification of the dense water pool in the Arkona Basin. In total we have used 182 vertical profiles extending from the surface to the bottom at about 44 meters depth. Each vertical profile contains observations from at least 5 depths. The reference salinity S_r is taken as the salinity at 15 m depth, if this is not measured S_r is the salinity at 10 m depth.

As can be seen from, e.g., eq. (1) the computed transport of the frontal current is sensitive to the choice of the bottom depth. There is no obvious way to determine the "effective" depth of the Arkona Basin. Therefore we decided to vary the bottom depth in the computations in order to find that depth which gives the correct salt transport into the Baltic. The mean flow of dense water is V_m ($\sum Q/N$, where $\sum$ denotes summation and N is the number of profiles) and the mean salinity is $S_m = \sum QS/\sum Q$. The mean outflow in the surface layer through the Bornholm Section is $V_m + F$, where F is the net mean freshwater supply to the Baltic inside this section. The salinity of the outflowing water is taken to be equal to S_r

Assuming that the Baltic is in a steady state continuity of salt requires

$$(V_m + F)\cdot S_r = V_m \cdot S_m,$$

or

$$V_m \cdot (S_m - S_r) = F \cdot S_r \qquad (16)$$

Since $F \sim 15{,}000$ m^3/s and $S_r \sim 8{\cdot}3‰$ one obtains $F \cdot S_r = 124{\cdot}5$ (ton/s). We find that our computed transport of excess salt, $V_m \cdot (S_m - S_r)$, becomes correct if we choose the bottom depth of the pool to be 41 m. This is thus the "effective" depth of the Arkona Basin.

We have used the two seemingly different approaches described in Subsections 2.1 and 2.2, respectively, to compute the fluxes of volume and salt, caused by the frontal current, associated with the 182 observed vertical stratifications. As already mentioned, in Subsection 2.2, it turned out that the two approaches gave identical results for each single profile.

It may be noted that in periods when the Arkona pool of dense water has a large vertical extension the pool may have double outlets because in addition to the leakage through the Bornholm Section dense water may in these periods leave the pool also through the shallow passage west of Bornholm Island. For a system with double, independent outlets the leakage may be larger than that computed from eq. (1) (see Stigebrandt, 1981). Computations show that as a long term average this second outlet may contribute only about 10% of the total flow from the dense pool if the sill depth is 30 m.

From the 182 profiles we estimated the ensemble mean flow rate from the dense pool, V_m, to be equal to 23,650 m^3/s. The mean salinity of this water, S_m, is 13.59‰. In order to show how the inflow is distributed among different salinity classes (of width 1‰) we have summed the observed inflows for each class. The final sum is divided by N (182). The result is shown in Fig. 6. The sum of the flows of all the salinity classes is of course equal to V_m.

In Fig. 7, we show the observed frequency of volume flows in different classes of flow-rates. From that figure one finds that there is a maximum of observations for the interval 10,000–15,000 m^3/s. One can also see that inflows greater than 100,000 m^3/s are extremely rare. For, e.g.,

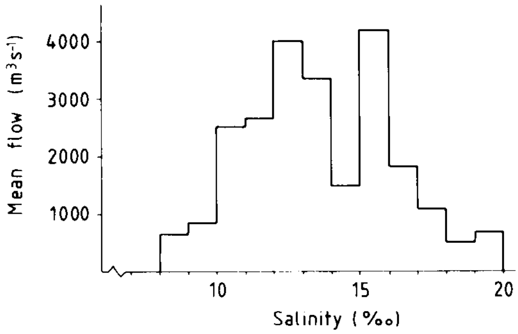

Fig. 6. The distribution of mean inflow rates among different salinity classes.

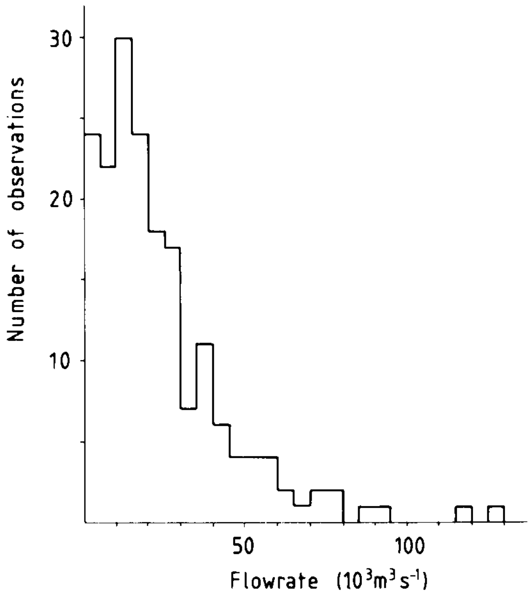

Fig. 7. The number of observations in different classes of flowrate. Total number of observations: 182.

vertical circulation models of the Baltic, it is of interest to know the probability for a certain rate of inflow and for the accompanying salinity of the inflowing water. To this end we have computed Fig. 8 which shows the cumulative frequency distribution of volume flows below a certain flowrate. Also shown is the observed mean salinity of water for different classes of volume flows. From the cumulative frequency distribution we find, for instance, that the inflow is less than 30,000 m^3/s in 73% of all observations. From Fig. 8 one finds that there is an evident correlation between

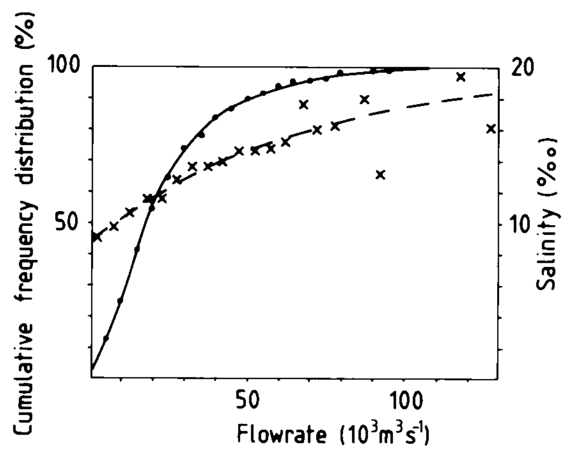

Fig. 8. The cumulative frequency distribution of volume flows (solid line) and the observed mean salinity of inflows (crosses — broken line fitted by eye) for different classes of volume flows.

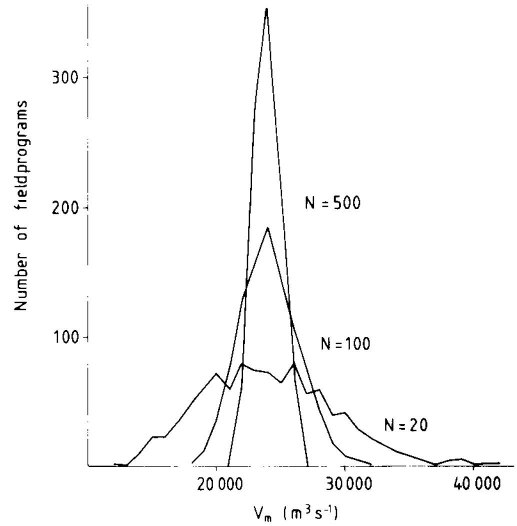

Fig. 9. Theoretical distributions of the results of field programs aimed at measuring the inflow (V_m) to the Baltic Sea if the inflow is described by the distribution shown in Fig. 8. *N* is the number of independent measurements in each experiment. The total number of experiments for each value of *N* is equal to 1000.

the salinity of the inflowing water and the flow rate such that the greatest inflows have the highest salinities. However from the individual estimates, one finds that there is much scatter around the suggested mean curve.

The deepest deep water in the Baltic Proper (below, say, 150 m) is known to be exchanged in more or less discrete events, see, e.g. Kullenberg (1981). There may be several years between such events. From the reasoning that follows below this appears to be in harmony with the information in Fig. 8. If we assume that an observed state in the Arkona Basin is representative for a 2-week period we can estimate the probability for inflows of magnitude greater than say 75,000 m^3/s. Such inflows occur only in 3% of all observations, i.e., with a mean interval of 2/0.03 (66) weeks. Because of the possible variation of the salinity of the inflowing water, one may expect that only some of these large inflows are dense enough to replace the resident deep water in the Baltic Proper. This should give an expected periodicity for the exchange of the deepest deep water of several years.

The estimate of the leakage from the dense pool in the Arkona Basin performed above may be compared with other estimates of the inflow of dense water into the Baltic Sea. Walin (1981) measured the flow of dense water through the Bornholm Section, see Fig. 1. From the measurements and a discussion he estimated $q(S)$ to be constant ($= 1,550$ $m^3/s/(‰)$) in the interval $8‰ < S < 18.5‰$ and zero for $S > 18.5‰$. From these figures we compute $V_m = 16,275$ m^3/s and $S_m = 13.25‰$, cf. eq. (16). Since Walin used $S_r = 8‰$, one would have expected $V_m \sim 23,000$ m^3/s (since $S_m = 13.25‰$) and hence $q(S) \sim 2,200$ $m^3/s/(‰)$ in order to fulfil the continuity requirements for salt and volume. Walin attributed the difference between these flow rates and those estimated by him to be due to (i) flow through the passage west of Bornholm, (ii) diffusive salt flux in the surface layer and (iii) other causes such as error in the estimate of the net freshwater supply. We note that the range of salinities for which inflows occur and the value of S_m obtained from Walins measurements are in accordance with the estimate previously made in the present paper.

We may utilize the cumulative inflow distribution in Fig. 8 to estimate the expected outcome of a field program aimed at measuring the inflow. By the means of random selection from the cumulative inflow distribution we estimate the mean flow-rate (V_m) for a field program of *N* (independent) measurements. We repeat this 1,000 times (i.e., for 1,000 field programs) to study the statistical distribution of V_m for a certain value of N. The results are shown in Fig. 9 for $N = 20$, 100 and 500. We can see that one field program with $N = 20$ may give an estimate of V_m in the range 12,000–42,000 m^3/s. Thus Walins measurements fell within the expected range for $N = 20$ and the deviation between the measured and the expected V_m may simply be explained by the fact that the number of measurements were few. In order to decrease the uncertainty of the estimate to $\pm 10\%$ of the true value of V_m the field program has to include about 500 independent measurements, see Fig. 9.

4. Discussion

For many purposes it would be valuable to have a continuous record of the deep inflow to the Baltic. The results obtained in this paper suggest that the actual inflow may be computed from the observed vertical stratification in the Arkona Basin. The organization of hydrographical observations of high frequency, for instance 1 vertical profile/week at BY 1, would be of great value for future Baltic research.

It would be useful to have a simple model for the volume of the pool of dense water in the Arkona Basin. However, there is one major uncertainty involved in the construction of such a model and that is concerned with the position of the open, eastern boundary of the pool. The analogous problem for the northern boundary of the Kattegat surface layer was discussed by the present author (Stigebrandt, 1983). A field program aiming at finding and describing the open boundary of the Arkona Basin dense pool has been initiated.

The properties of the inflow to the Baltic, estimated in this paper, are utilized in a model for the vertical circulation of the Baltic which is under development by the present author (see Stigebrandt, 1985). It should be noted that, due to entrainment of surrounding water, the inflow to the Baltic increases in the downstream direction. Preliminary computations show that the flow rate of the dense current almost triples between the Bornholm Section and the final interleaving at or below the Baltic halocline. Thus as an average about 65,000 m^3/s should be interleaved into the Baltic halocline and deep water. Some of this water is entrained from the Baltic deep water (below the halocline). An analysis of the vertical circulation in and below the Baltic halocline will be published elsewhere.

Finally we note that the relationships between the flows of volume and excess salt and the profile potential energy PE and the profile salinity surplus Φ of the pool (eqs. (9) and (10)) have been found to be of general validity for systems where salinity determines the density. It is easily realized that this has great implications for two-layer modeling of salt-stratified, geophysical systems. Eq. (9) is probably valid also for systems where the density is determined by two parameters (e.g., salt and temperature). However the analogue to eq. (10) is, if it exists in that case, probably more complicated.

5. Acknowledgement

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REFERENCES

Kullenberg, G. 1981. Physical oceanography. In *The Baltic sea* (ed. A. Voipio). Elsevier, 135–181.

Stigebrandt, A. 1980. Barotropic and baroclinic response of a semi-enclosed basin to barotropic forcing from the sea. In *Fjord oceanography* (eds. H. J. Freeland, D. M. Farmer and C. D. Levings). Plenum, 141–164.

Stigebrandt, A. 1981. A model for the thickness and salinity of the upper layer in the Arctic Ocean and the relationship between the ice thickness and some external parameters. *J. Phys. Oceanogr. 11*, 1407–1422.

Stigebrandt, A. 1983. A model for the exchange of salt and water between the Baltic and the Skagerrak. *J. Phys. Oceanogr. 13*, 411–427.

Stigebrandt, A. 1985. A model for the seasonal pycnocline in rotating systems with application to the Baltic Sea. *J. Phys. Oceanogr. 15*, 1392–1404.

Walin, G. 1981. On the deep water flow into the Baltic. *Geophysica 17*, 75–93.