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Reference pressure changes and available potential energy in isobaric coordinates

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ABSTRACT

Previous isobaric formulations of the exact available potential energy equation have included a term involving the rate of change of reference pressure (the areal mean pressure on an isentropic surface). A revised formulation in which this term is rewritten and combined with $\omega\alpha$ clarifies the role of mechanical work and boundary work in altering the available potential energy of an open region.

1. Introduction

Since Lorenz' 1955 reformulation of the concept of available potential energy (APE) based on Margules' (1903) earlier work, energetics studies have contributed significantly to our understanding of baroclinic and barotropic processes operative on global scales. Subsequent formulations of the equations for open systems in isobaric coordinates (Smith, 1969; Vincent and Chang, 1973; Smith et al., 1977; and Edmon, 1978) and in isentropic coordinates (Johnson, 1970) have permitted studies of individual synoptic events. Included in the more recent isobaric formulations is a term identified by Egger (1976) which involves the Lagrangian rate of change of reference pressure. This term has proven somewhat awkward to interpret physically. Johnson's isentropic formulation does not contain temporal changes of reference pressure, but apparently has not been utilized extensively because many investigators prefer to work in isobaric coordinates.

The purpose of this note is to present a formulation of the APE equation in isobaric coordinates which alleviates the need for computing temporal derivatives of reference pressure and provides a better physical description of how work done relates to changes in the APE of a limited region. This development results in an APE budget equation which possesses terms analogous to those in Johnson's (1970) isentropic version. The results should be of practical importance since both the isobaric and isentropic approaches are exact representations of APE in contrast to the approximate form used by Lorenz (1955). Dutton and Johnson (1967), Min and Horn (1982) and others have noted that exact and approximate forms of APE contents and generation by diabatic effects can differ substantially.

2. Work and changes of TPE

We consider first the relationship between work done in an open region and the release of total potential energy (TPE) through vertical overturning as given by

$$\int_m -\omega\alpha\,\mathrm{d}m = -\int_m \mathbf{V}\cdot\nabla_p\phi\,\mathrm{d}m + \int_m \nabla_p\cdot\phi\mathbf{V}\,\mathrm{d}m + \int_m \frac{\partial}{\partial p}\omega\phi\,\mathrm{d}m, \tag{1}$$

(see Appendix A for a list of symbols). The time rate of change of TPE is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{TPE} = \frac{\mathrm{d}}{\mathrm{d}t}\int_m c_p T\,\mathrm{d}m = \frac{\mathrm{d}}{\mathrm{d}t}\int_v \rho c_v T + \rho g z\,\mathrm{d}v. \tag{2}$$

Substituting from the first law of thermodynamics and using the mass continuity equation, we obtain

$$\int_m -\omega\alpha = \int_v p\nabla_3\cdot\mathbf{V} + pgw\,\mathrm{d}v, \qquad (3)$$

which states that the release of TPE is equivalent to the thermodynamic work of expansion minus the conversion of kinetic to potential energy[1]. Furthermore, thermodynamic work can be expanded as

$$p\nabla_3\cdot\mathbf{V} = \nabla_3\cdot p\mathbf{V} - \mathbf{V}\cdot\nabla_2 p - w\frac{\partial p}{\partial z}, \qquad (4)$$

showing the relationship between thermodynamic work, boundary pressure work ($\nabla_3\cdot pV$), and mechanical work ($\mathbf{V}\cdot\nabla_2 p + w\partial p/\partial z$). Assuming hydrostatic balance and substituting (4) into (3) yields the result

$$\int_m -\omega\alpha\,\mathrm{d}m = \int_v -\mathbf{V}\cdot\nabla_2 p + \nabla_3\cdot p\mathbf{V}\,\mathrm{d}v. \qquad (5)$$

Eq. (5) states that the release of TPE is manifest either as mechanical work (generation of kinetic energy by horizontal motions) or as pressure work. Since

$$\mathbf{V}\cdot\nabla_2 p = \rho\mathbf{V}\cdot\nabla_p\phi, \qquad (6)$$

Substitution of (1) and (6) into (5) yields

$$\int_m -\nabla_p\cdot\phi\mathbf{V} - \frac{\partial}{\partial p}\omega\phi\,\mathrm{d}m = \int_v -\nabla_3\cdot p\mathbf{V}\,\mathrm{d}v. \qquad (7)$$

Thus, the integrated three-dimensional flux divergence of gravitational potential energy in isobaric coordinates is equivalent to the integrated pressure work. We will refer to the left-hand side of (7) as boundary work since it arises because of normal velocity components on the boundary of the open domain.

It is important to recognize that, in general, (1) says nothing regarding how work alters *APE* since it does not address changes in the structure of the reference atmosphere (the existing atmosphere restratified adiabatically to a hydrostatic, horizontal, barotropic state). Only if the system is closed, e.g. the global domain, are the last two terms of (1) identically zero. In this case, the release of TPE, the release of APE and the generation of kinetic energy are all equivalent. However, in an open system the presence of the boundary work term implies the possibility for communicating at least some portion of this released TPE to the surrounding environment. In the next section we will see that reformulating the reference pressure change term in the APE equation permits an evaluation of boundary work effects on the reference state and suggests formulating a modified version of (1).

[1] References to TPE release and work are understood to mean rates of these quantities.

3. Reformulation of the DP term

We consider first the so-called "exact" APE equation in isobaric coordinates. Smith et al. (1977), hereafter referred to as SVE, have defined the available potential energy of an open region as

$$A = \frac{c_p}{\sigma}\int_m ET\,\mathrm{d}m, \qquad (8)$$

where

$$E = 1 - (p_r/p)^\kappa \qquad (9)$$

is the efficiency factor, and

$$p_r(\theta, t) = \frac{1}{\sigma}\int_\sigma\int_{\theta_T}^{\theta}\frac{\partial p}{\partial\theta}\,\mathrm{d}\theta\,\mathrm{d}\sigma \qquad (10)$$

is the reference pressure. SVE's Eulerian formulation of the APE budget equation is obtained by differentiating (8) with respect to time and substituting from the first law of thermodynamics and the continuity equation:

$$\frac{\partial A}{\partial t} = \frac{1}{\sigma}\int_m E\dot{Q} + \omega\alpha - \nabla_p\cdot c_p ET\mathbf{V} - \frac{\partial}{\partial p}c_p ET\omega - c_p\frac{T}{p^\kappa}\frac{\mathrm{d}p_r^\kappa}{\mathrm{d}t}\mathrm{d}m - \frac{c_p}{\sigma g}\int_\sigma (ET)_s\frac{\partial p_s}{\partial t}\mathrm{d}\sigma \qquad (11)$$

Using the Poisson equation for the reference atmosphere,

$$T_r = \theta\left(\frac{p_r}{p_0}\right)^\kappa, \qquad (12)$$

the last term in the first integrand of (11) can be written

$$-c_p\frac{T}{p^\kappa}\frac{\mathrm{d}p_r^\kappa}{\mathrm{d}t} = -\frac{RT_r}{p_r}\frac{\mathrm{d}p_r}{\mathrm{d}t} \qquad (13)$$

The area averaged mass integral of (13) is equivalent to (10) in SVE and for convenience is denoted as DP. By regarding $dp_r/dt = \omega_r$ as the counterpart of ω for the reference atmosphere, and noting that $RT_r/p_r = \alpha_r$ Lin (1980) rewrote the term as

$$DP = -\frac{1}{\sigma}\int_m \omega_r \alpha_r \, dm, \tag{14}$$

but did not relate the quantity to boundary work. We now demonstrate that this is possible. SVE have shown that when the area average is taken over the same area over which the reference pressure is determined,

$$DP = -\frac{R}{\sigma g}\int_\sigma \int_{\theta_T}^{\theta_s} \overline{\frac{\theta}{P_r}\left(\frac{p_r}{p_0}\right)^\kappa \times \int_{\theta_r}^{\theta} - \nabla_\theta \cdot \frac{\partial p}{\partial \theta} \mathbf{V} \, d\theta} \frac{\partial p}{\partial \theta} \, d\theta \, d\sigma, \tag{15}$$

where $\overline{(\;)} = \frac{1}{\sigma}\int_\sigma (\;)\, d\sigma$.

Since the reference atmosphere is by definition hydrostatic,

$$-\frac{R\theta}{P_r}\left(\frac{p_r}{p_0}\right)^\kappa = -\alpha_r = \frac{\partial \phi_r}{\partial p_r}. \tag{16}$$

Noting that α_r is constant on θ surfaces and that

$$\frac{1}{\sigma}\int_\sigma \frac{\partial p}{\partial \theta} \, d\sigma = \overline{\frac{\partial p}{\partial \theta}} = \frac{\partial p_r}{\partial \theta}, \tag{17}$$

(12), (16) and (17) can be used to rewrite (15) as

$$DP = \frac{1}{g}\overline{\int_{\theta_T}^{\theta_s} \frac{\partial \phi_r}{\partial \theta} \int_{\theta_T}^{\theta} - \nabla_\theta \cdot \frac{\partial p}{\partial \theta} \mathbf{V} \, d\theta \, d\theta}. \tag{18}$$

Through repeated integration by parts and changing from isentropic to isobaric coordinates (see Appendix B) we arrive at the more simplified form

$$DP = \frac{1}{g}\overline{\int_{p_T}^{p_s} \nabla_p \cdot \phi_r \mathbf{V} + \frac{\partial}{\partial p} \omega \phi_r \, dp}. \tag{19}$$

Eq. (19) states that as the mass and motion fields interact in the open system, APE is altered by boundary flux divergence of reference state gravitational potential energy. Substituting (19) into (11),

$$\frac{\partial A}{\partial t} = \frac{1}{\sigma}\int_m E\dot{Q} + \omega\alpha - \nabla_p \cdot c_p ET\mathbf{V} - \frac{\partial}{\partial p} c_p ET\omega + \nabla_p \cdot \phi_r \mathbf{V} + \frac{\partial}{\partial p}\omega\phi_r \, dm - \frac{c_p}{\sigma g}\int_\sigma (ET)_s \frac{\partial p_s}{\partial t} dm \tag{20}$$

By invoking (1), the terms $\omega\alpha$, $\nabla_p \cdot \phi_r$ $\mathbf{V}$ and $\partial/\partial p(\omega\phi_r)$ in (20) can be combined and expressed as

$$\frac{1}{\sigma}\int_m \omega\alpha + \nabla_p \cdot \phi_r \mathbf{V} + \frac{\partial}{\partial p}\phi_r \omega \, dm = \frac{1}{\sigma}\int_m \mathbf{V}\cdot\nabla_p \phi - \nabla_p \cdot (\phi - \phi_r)\mathbf{V} - \frac{\partial}{\partial p}(\phi - \phi_r)\,\omega \, dm. \tag{21}$$

Recalling Lin's (1980) equivalent formulation of DP in (14),

$$\frac{1}{\sigma}\int_m \omega\alpha - \omega_r \alpha_r dm = \frac{1}{\sigma}\int_m \mathbf{V}\cdot\nabla_p \phi - \nabla_p \cdot (\phi - \phi_r)\mathbf{V} - \frac{\partial}{\partial p}(\phi - \phi_r)\,\omega \, dm. \tag{22}$$

Finally, (21) is substituted into (20) yielding

$$\frac{\partial A}{\partial t} = \frac{1}{\sigma}\int_m E\dot{Q} + \mathbf{V}\cdot\nabla_p \phi - \nabla_p \cdot c_p ET\mathbf{V} - \frac{\partial}{\partial p} c_p ET\omega - \nabla_p \cdot (\phi - \phi_r)\mathbf{V} - \frac{\partial}{\partial p}(\phi - \phi_r)\,\omega \, dm - \frac{c_p}{\sigma g}\int_\sigma (ET)_s \frac{\partial p_s}{\partial t} d\sigma. \tag{23}$$

4. Discussion

Consider now some properties of (22) and (23). Terms in (23) corresponding to diabatic heating, $E\dot{Q}$; flux divergence of APE, $-\nabla_p \cdot c_p ETV - \partial/\partial p \times c_p ET\omega$ and surface pressure change, $c_p(ET)_s \times \partial p_s/\partial t$, are the same as in SVE's Eulerian equation. However, as evident in (22), the release of TPE and the DP term have now been combined to yield quantities referring explicitly to mechanical work and boundary work done in conjunction with relative vertical overturning of the existing and reference atmosphere. Each term in (23) also corresponds to an analogous term in Johnson's (1970) isentropic version (except the quantities involving vertical motion and surface pressure change which are implicit in isentropic coordinates). From (1) it was noted that the release of TPE through vertical overturning was manifest either in mechanical or boundary work in the existing atmosphere. Vertical overturning also alters the reference atmosphere TPE but only through the resulting boundary transport of ϕ_r as shown through the equivalence of (14) and (19). This is because reference state TPE is invariant under adiabatic mass redistribution. Collectively, the terms on the right-hand-side of (23), except differential heating, constitute the net effect of restratification on the APE of the open region. APE changes result from either mechanical work inside the domain or an exchange of energy via boundary processes with the surrounding environment. Of course, we can only discern how these boundary processes alter the energetics of the environment by explicit calculations of the budgets for that domain. The importance of boundary work and flux in open system energetics has been emphasized by Johnson (1970), Smith (1970) and Johnson and Downey (1982).

A modification of SVE's equations to obtain more physical understanding has also been given by Edmon (1978). His version includes a term involving $E\omega\alpha$ which is negative for rising (sinking) motion in relatively warm (cold) air. However, this approach does not explicitly identify the mechanical and boundary work. Eq. (23) seems more desirable than previous isobaric versions because it provides a direct statement of the specific processes (mechanical work, boundary work and boundary transport) which are involved in the restratification and which produce APE changes in an open volume.

Conceptually it is useful to consider boundary work and APE flux together since both arise because the limited region is not mechanically closed. Integrating the hydrostatic equation for the existing atmosphere with respect to pressure and changing to potential temperature as a vertical coordinate, the geopotential energy is expressed as

$$\phi(\theta, t) = \phi(\theta_s, t) - c_p \int_{\theta_s}^{\theta} \frac{\theta}{p_0^\kappa} \frac{\partial p^\kappa}{\partial \theta} \, d\theta. \tag{24}$$

The analogous expression for the reference atmosphere, derived by integrating (16) is

$$\phi_r(\theta, t) = \phi_r(\theta_s, t) - c_p \int_{\theta_s}^{\theta} \frac{\theta}{p_0^\kappa} \frac{\partial p_r^\kappa}{\partial \theta} \, d\theta. \tag{25}$$

Subtracting (25) from (24) and using (9), yields

$$\phi - \phi_r = (\phi - \phi_r)_s - c_p \int_{\theta_s}^{\theta} \frac{\theta}{p_0^\kappa} \frac{\partial E p^\kappa}{\partial \theta} \, d\theta. \tag{26}$$

Poisson's equation and an integration by parts can now be used to obtain

$$\phi - \phi_r + c_p ET = (\phi - \phi_r)_s + c_p (ET)_s + c_p \int_{\theta_s}^{\theta} \frac{ET}{\theta} \, d\theta,$$

or, in isobaric coordinates,

$$\phi - \phi_r + c_p ET = (\phi - \phi_r)_s + c_p (ET)_s - c_p \int_{p}^{p_s} \frac{ET}{\theta} \frac{\partial \theta}{\partial p} \, dp. \tag{27}$$

The flux divergence of (27) yields the net boundary effects in (23) due to flux of APE and work at the boundary of the open system. It is clear from (9) and (10) that for a given theta surface intersecting the ground p and ET will be close to their maximum value on that surface. Since ϕ_r is always positive, $\phi - \phi_r$ for that theta surface minimizes at the ground. Consequently, the first two terms of (27) will likely tend to offset each other. Furthermore, because of the factor $1/\theta \times \partial\theta/\partial p$, the third term in (27) may be approximately an order of magnitude smaller than $c_p ET$. It appears that because net boundary effects are determined by the flux divergence of two quantities that largely cancel, accurate explicit calculations of net boundary effects may be difficult with observational data. Obtaining boundary processes as a residual may therefore remain a practical alternative. An additional complicating factor arises where the influence of topography becomes important. Taylor (1979) has discussed this problem in depth. Since errors arise in computing p_r and ϕ_r for sloping terrain if $p(\theta) = p_s$ is assigned to underground theta surfaces, only the term $\mathbf{V} \cdot \nabla_p \phi$ in (23) will be free from this effect. This problem and the possible solutions proposed by Taylor should be considered in future studies.

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6. Appendix A

- $\dot{Q}$ diabatic heating rate per unit mass
- p pressure
- $\mathbf{V}$ horizontal wind vector
- ω vertical motion in pressure coordinates, dp/dt
- w dz/dt
- T temperature
- θ potential temperature
- ϕ gravitational potential energy
- α specific volume
- ρ density
- p_0 1000 mb
- R gas constant for air
- c_p specific heat capacity for air at constant pressure
- c_v specific heat capacity for air at constant volume
- κ R/c_p
- g gravity
- σ area of limited domain
- z height above mean sea level

Subscripts and Integral Limits

- r reference atmosphere quantity
- s surface
- T top of atmosphere

Operators and Integrals

$$\int_m (\quad)\,dm = \frac{1}{g}\int\int\int (\quad)\,dx\,dy\,dp$$

$$\int_v (\quad)\,dv = \int\int\int (\quad)\,\alpha^{-1}\,dx\,dy\,dz$$

- ∇ del operator

7. Appendix B

The reference pressure change term,

$$DP = \frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\frac{\partial \phi_r}{\partial \theta}\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,d\theta\,d\sigma}. \tag{B1}$$

can be expanded to yield

$$DP = \frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\left\{\frac{\partial}{\partial\theta}\left[\phi_r\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,d\theta\right] - \phi_r\frac{\partial}{\partial\theta}\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,d\theta\right\}}\,d\theta = \frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\times\left\{\frac{\partial}{\partial\theta}\left[\phi_r\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,d\theta\right] + \phi_r\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\right\}}\,d\theta, \tag{B2}$$

where we have assumed $\partial p/\partial\theta = 0$ a… …ne atmosphere. Using the identity

$$\nabla_\theta\cdot\xi\mathbf{V} = \nabla_p\cdot\xi\mathbf{V} - \frac{\partial p}{\partial\theta}\frac{\partial}{\partial p}(\xi\mathbf{V})\cdot\nabla_p\theta, \tag{B3}$$

the first half of the integral is written as

$$\frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\frac{\partial}{\partial\theta}\left[\phi_r\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,d\theta\right]}\,d\theta$$

$$= \frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\frac{\partial}{\partial\theta}\left[\phi_r\int_{p_T}^{p} -\frac{\partial\theta}{\partial p}\nabla_p\cdot\frac{\partial p}{\partial\theta}\mathbf{V} + \frac{\partial}{\partial p}\times\left(\frac{\partial p}{\partial\theta}\mathbf{V}\right)\cdot\nabla_p\theta\,dp\right]}\,d\theta = \frac{1}{g}\int_{\theta_T}^{\theta_s} \overline{\frac{\partial}{\partial\theta}\left[\phi_r\int_{p_T}^{p} -\nabla_p\cdot\mathbf{V} - \frac{\partial\theta}{\partial p}\mathbf{V}\cdot\nabla_p\frac{\partial p}{\partial\theta} + \frac{\partial}{\partial p}\left(\frac{\partial p}{\partial\theta}\mathbf{V}\cdot\nabla_p\theta\right) - \frac{\partial p}{\partial\theta}\mathbf{V}\cdot\nabla_p\frac{\partial\theta}{\partial p}\,dp\right]}\,d\theta. \tag{B4}$$

But

$$-\frac{\partial p}{\partial\theta}\mathbf{V}\cdot\nabla_p\frac{\partial\theta}{\partial p} = -\mathbf{V}\cdot\frac{\partial p}{\partial\theta}\nabla_p\frac{\partial\theta}{\partial p}$$

$$= \mathbf{V}\cdot\frac{\partial\theta}{\partial p}\nabla_p\frac{\partial p}{\partial\theta} = \frac{\partial\theta}{\partial p}\mathbf{V}\cdot\nabla_p\frac{\partial p}{\partial\theta}. \tag{B5}$$

Thus, using (B5) and the continuity equation, assuming $\omega = \partial p/\partial\theta = 0$ at $p = p_T$, and changing to isobaric coordinates, (B4) becomes

$$\overline{\frac{1}{g}\int_{\theta_t}^{\theta_s}\frac{\partial}{\partial\theta}\left[\phi_r\int_{\theta_T}^{\theta} -\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V}\,\mathrm{d}\theta\right]\mathrm{d}\theta}$$

$$= \overline{-\frac{1}{g}\int_{p_T}^{p_s}\frac{\partial}{\partial p}\left(\phi_r\,\omega + \phi_r\frac{\partial p}{\partial\theta}\mathbf{V}\cdot\nabla_p\theta\right)\mathrm{d}p}, \quad \text{(B6)}$$

The second right-hand-side term of (B2) is evaluated by first noting that since ϕ_r is constant on theta surfaces,

$$\phi_r\nabla_\theta\cdot\frac{\partial p}{\partial\theta}\mathbf{V} = \nabla_\theta\cdot\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}. \quad \text{(B7)}$$

Then, applying (B3)

$$\overline{\frac{1}{g}\int_{\theta_T}^{\theta_s}\nabla_\theta\cdot\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}\,\mathrm{d}\theta} = \overline{\frac{1}{g}\int_{p_T}^{p_s}\frac{\partial\theta}{\partial p}\nabla_p\cdot\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}}$$

$$\overline{-\frac{\partial}{\partial p}\left(\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}\right)\cdot\nabla_p\theta\,\mathrm{d}p} \quad \text{(B8)}$$

Using vector differentiation, the integrand on the right-hand side of (B8) is written as

$$\frac{\partial\theta}{\partial p}\nabla_p\cdot\phi_r\frac{\partial p}{\partial\theta}\mathbf{V} = \nabla_p\cdot\phi_r\mathbf{V} + \frac{\partial\theta}{\partial p}\phi_r\mathbf{V}\cdot\nabla_p\frac{\partial p}{\partial\theta}, \quad \text{(B9)}$$

and, with the use of (B5),

$$-\frac{\partial}{\partial p}\left(\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}\right)\cdot\nabla_p\theta = -\frac{\partial}{\partial p}\left(\phi_r\frac{\partial p}{\partial\theta}\mathbf{V}\cdot\nabla_p\theta\right)$$

$$-\phi_r\frac{\partial\theta}{\partial p}\mathbf{V}\cdot\nabla_p\frac{\partial p}{\partial\theta}. \quad \text{(B10)}$$

Substituting (B9) and (B10) into (B8) and substituting that result along with (B6) into (B2) yields (19),

$$DP = \overline{\frac{1}{g}\int_{p_T}^{p_s}\nabla_p\cdot\phi_r\mathbf{V} + \frac{\partial}{\partial p}\omega\phi_r\,\mathrm{d}p}. \quad \text{(B11)}$$

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