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Climate modeling using an equivalent meridional circulation

By ERIK ELIASEN, *Institute of Theoretical Meteorology, University of Copenhagen, Haraldsgade 6, DK-2200 Copenhagen N, Denmark*

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ABSTRACT

Numerical integrations of a simple two-layer general circulation model are used to determine the climate of the model, represented by long-term mean values. The response of the model to changes of the solar constant and the angular velocity of the earth is considered. Concerning the effect of the large-scale motions on the mean temperature field, it is shown that the total heating has clearly a more simple meridional variation than any of the parts into which the heating is normally divided. Furthermore it is found that the total heating due to the large-scale motions is reproduced very well by a certain hypothetical meridional circulation, determined from the mean temperature field. By using this equivalent meridional circulation in the thermodynamic equation, a climate model is formulated as an analogue to the general circulation model. The mean temperature fields at the two levels are the only dependent variables, and numerical integrations of the model are carried out very quickly. The results of the integrations show that the steady state solutions obtained simulate very well the long-term mean temperature fields found from the general circulation model.

1. Introduction

Since the development by Budyko (1969) and Sellers (1969) of energy balance global models, such simple climate models have been considered an important tool in the study of climate problems. From these simple models there is a wide gap to the large and complicated general circulation models, which are also used today for climate studies. In a general circulation model it is the intention to describe numerically every step in the development of the atmospheric (and eventually the oceanic) motion systems in great detail. This certainly involves an enormous amount of computational work, which seriously limits the use of the general circulation models for climate studies. So it seems quite appropriate to continue the development of the more simple climate models, which with present-day computers may be easily integrated through very long periods. It also seems likely that studies of simplified climate models may contribute basically to the development of a comprehensive theory of climate.

The global energy balance models of the type formulated by Budyko (1969) and by Sellers (1969) are indeed very simple. They contain only one variable quantity, namely the zonally averaged temperature of the atmosphere at the surface, and all physical processes represented in the model have to be parameterized in terms of this temperature field. The parameterizations are mostly based upon simple relationships with empirically determined coefficients. The temperature field itself is determined from the thermodynamic energy equation, and the model as a whole is generally tuned to describe as accurately as possible the mean temperature field of the present climate.

It seems natural to try to generalize the very simple energy balance models by representing the temperature field in a model with more than one

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level. This should make possible a more accurate treatment of the physical processes, like the short- and long-wave radiation and the convective processes. Furthermore it should also make it possible to develop a more advanced parameterization of the very important large-scale heat transport in the atmosphere carried out by the baroclinic waves as well as by the meridional circulations. In the present work we intend to focus on this basic aspect of the climate modelling problem by adapting a simple model with the temperature field and the heat transport represented at two levels in the troposphere.

The parameterization of the large-scale heat transport is developed by examining the corresponding heat transport obtained from long-term integrations of the quite analogous general circulation model. From such an examination it is found that especially the total heat transport has a quite regular appearance, which might be efficiently simulated by the heat transport of an hypothetical meridional circulation determined from the temperature field. The use of this parameterization based upon the concept of an equivalent meridional circulation in the thermodynamic equation leads to a rather simple mathematical model determining the zonally averaged temperature at the two levels considered. Time-integrations of the model are easily carried out and converge towards steady-state temperature fields, which are comparable with the mean temperature fields obtained from long-term integrations of the analogous general circulation model. As a test—judged from the general circulation model—the response of the models to rather large changes in the incoming solar radiation is determined, and it turns out that the changes of the temperature fields obtained with the parameterized model are very nearly the same as those obtained with the general circulation model.

2. The general circulation model

The general circulation model used is a rather conventional two-layer spectral model with a highly simplified treatment of heating and friction. Basically, the model determines the temperature and horizontal wind velocity at two tropospheric levels p_1 = 400 mb and p_3 = 800 mb. Using pressure as the vertical coordinate the horizontal components of the equation of motion at the two levels may be written

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{a \cos \phi \partial \lambda} + v_i \frac{\partial u_i}{a \partial \phi} - \frac{\operatorname{tg} \phi}{a} u_i v_i + \omega_i \frac{u_3 - u_1}{\Delta p} - f v_i + \frac{\partial \Phi_i}{a \cos \phi \partial \lambda} = (F_\lambda)_i,$$

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{a \cos \phi \partial \lambda} + v_i \frac{\partial v_i}{a \partial \phi} + \frac{\operatorname{tg} \phi}{a} u_i^2 + \omega_i \frac{v_3 - v_1}{\Delta p} + f u_i + \frac{\partial \Phi_i}{a \partial \phi} = (F_\phi)_i, \tag{1}$$

with index i = 1 and 3 referring to the pressure levels p_1 and p_3, respectively. The notation is quite conventional with λ and ϕ denoting the longitude and latitude, a being the radius of the earth and f the Coriolis parameter. **v** is the horizontal velocity vector with the eastward component u and the northward component v. Φ is the geopotential and F_λ and F_ϕ denote the eastward and the northward components of the horizontal frictional force per unit mass, **F** respectively. The finite difference approximation has been used to express $\partial \mathbf{v}/\partial p$ as $(\mathbf{v}_3 - \mathbf{v}_1)/\Delta p$, where $\Delta p = 400$ mb.

As usual ω denotes the individual time derivative of pressure, dp/dt. In consideration of the general crudeness of the model, it seems reasonable to use the condition

$$\omega = 0 \quad \text{at} \quad p = 1000 \text{ mb},$$

as an approximate lower boundary condition. Similarly it is assumed that

$$\omega = 0 \quad \text{at} \quad p = 200 \text{ mb},$$

which may be considered as a reflection of the stable stratification of the lower stratosphere. Together the two boundary conditions imply that the vertically integrated divergence of the flow is equal to zero, which means that external gravity waves are excluded. Using again the finite difference approximation in the vertical direction, the equation of continuity at the two levels p_1 and p_3 may be written

$$\frac{\omega_2}{\Delta p} = -\nabla \cdot \mathbf{v}_1 = \nabla \cdot \mathbf{v}_3 = -\nabla^2 \chi, \tag{2}$$

where ω_2 is the value of ω at the level $p_2 = 600$ mb and χ denotes the velocity potential at level p_1. Inside the framework of the two-level description, the values of ω at the levels p_1, p_2 and p_3 must be somehow related and in view of the applied boundary conditions for ω, it is simply assumed that

$$\omega_1 = \omega_3 = \tfrac{1}{2}\omega_2.$$

In order to have a consistent energy equation for the two-level model, it is necessary to coordinate the two-level formulation of the thermodynamic equation with the formulation of the hydrostatic equation. The thermodynamic equation may be expressed in the following general form

$$\frac{d\theta}{dt} = \frac{\theta}{T}\frac{Q+D}{c_p} \tag{3}$$

where D and Q denote the frictional and the non-frictional heating rates per unit mass, respectively. The potential temperature θ is connected with the temperature T by the relation

$$T = r\theta, \qquad r = \left(\frac{p}{p_*}\right)^{\kappa},$$

where $p_* = 1000$ mb and $\kappa = R/c_p$, with R being the gas constant and c_p the specific heat at constant pressure. With p used as an independent variable, eq. (3) may be written at the two levels p_1 and p_3:

$$\frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{a\cos\phi\,\partial\lambda} + v_i \frac{\partial T_i}{a\,\partial\phi} - r_i \sigma \frac{\omega_2}{\Delta p} = \frac{Q_i + D_i}{c_p}, \tag{4}$$

with $\sigma = \tfrac{1}{2}(\theta_1 - \theta_3)$. In view of the continuity eq. (2), eq. (4) may also be written in the form

$$\frac{\partial T_1}{\partial t} + \frac{\partial(u_1 T_1)}{a\cos\phi\,\partial\lambda} + \frac{\partial(v_1 T_1 \cos\phi)}{a\cos\phi\,\partial\phi} + r_1 \frac{\theta_2\omega_2}{\Delta p} = \frac{Q_1 + D_1}{c_p},$$

$$\frac{\partial T_3}{\partial t} + \frac{\partial(u_3 T_3)}{a\cos\phi\,\partial\lambda} + \frac{\partial(v_3 T_3 \cos\phi)}{a\cos\phi\,\partial\phi} - r_3 \frac{\theta_2\omega_2}{\Delta p} = \frac{Q_3 + D_3}{c_p}, \tag{5}$$

using $\theta_2 = \tfrac{1}{2}(\theta_1 + \theta_3)$. The hydrostatic equation

$$\frac{\partial\Phi}{\partial p} = -\frac{RT}{p} \quad \text{or} \quad \frac{\partial\Phi}{\partial(p^{\kappa})} = -c_p\, p_*^{-\kappa}\,\theta$$

may for the layer from p_1 to p_3 be written, consistent with eq. (5), in the following finite difference form

$$\Phi_1 - \Phi_3 = c_p(r_3 - r_1)\,\theta_2. \tag{6}$$

For the mean value, with respect to the total mass of the atmosphere, of the kinetic energy it seems natural to use the expression

$$K = \tfrac{1}{4}[u_1^2 + v_1^2] + \tfrac{1}{4}[u_3^2 + v_3^2], \tag{7}$$

where the meaning of the bracket is defined by

$$[A] = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} A\cos\phi\, d\phi\, d\lambda.$$

Similarly the mean value of the potential energy is expressed as

$$P = \tfrac{1}{2}c_p[T_1 + T_3]. \tag{8}$$

From the eqs. (1) and (2) we have

$$\frac{dK}{dt} = -\frac{1}{2}\left[(\phi_1 - \phi_3)\frac{\omega_2}{\Delta p}\right] - \frac{1}{2}[\mathbf{v}_1\cdot\mathbf{F}_1 + \mathbf{v}_3\cdot\mathbf{F}_3], \tag{9}$$

and from eqs. (5)

$$\frac{dP}{dt} = \frac{1}{2}(r_3 - r_1)c_p\left[\theta_2\frac{\omega_2}{\Delta p}\right] + \frac{1}{2}[Q_1 + Q_3] + \frac{1}{2}[D_1 + D_3]. \tag{10}$$

Considering these two equations together with the hydrostatic eq. (6), we may define the transformation of potential into kinetic energy as

$$C(P,K) = -\frac{1}{2}\left[(\phi_1 - \phi_3)\frac{\omega_2}{\Delta p}\right] = -\frac{1}{2}(r_3 - r_1)c_p\left[\theta_2\frac{\omega_2}{\Delta p}\right]. \tag{11}$$

For the long-term mean values we have from (9) and (10) the usual basic conditions

$$C(P,K) = \tfrac{1}{2}[D_1 + D_3] = \tfrac{1}{2}[\mathbf{v}_1\cdot\mathbf{F}_1 + \mathbf{v}_3\cdot\mathbf{F}_3] \tag{12}$$

and

$$[Q_1 + Q_3] = 0, \tag{13}$$

expressing that the total dissipation of kinetic energy is balanced in the steady state by the total production of kinetic energy from potential energy, and that the total non-frictional heating of the atmosphere is zero in the steady state.

The kinetic energy dissipation is represented in the model by an internal friction and surface friction. The internal friction is assumed to be proportional to the vertical wind shear, and at level 1 the frictional force per unit mass is written

$$\mathbf{F}_1 = -k_d(\mathbf{v}_1 - \mathbf{v}_3), \tag{14}$$

where k_d is treated as a constant. At the lower level the frictional force is given by the expression

$$\mathbf{F}_3 = k_d(\mathbf{v}_1 - \mathbf{v}_3) - k_s|\mathbf{v}_s|\mathbf{v}_s, \tag{15}$$

where the first term is equal and opposite to the internal frictional force at the upper level and the second term represents the surface frictional force. The wind vector $\mathbf{v}_s$ is introduced as $\mathbf{v}_s = \mathbf{v}_3 - \frac{1}{2}\mathbf{v}_1$ or approximately $\frac{3}{4}$ times the extrapolated surface wind and k_s is a constant. Integrations with different values of k_d and k_s have been carried out, but the values $k_d = 2 \cdot 10^{-6}\ s^{-1}$ and $k_s = 8 \cdot 10^{-7}$ m^{-1} have been used in all the cases discussed in the following section.

From the expressions (14) and (15) for the frictional force it follows that the total dissipation becomes

$$D_1 + D_3 = k_d(\mathbf{v}_1 - \mathbf{v}_3)^2 + k_s|\mathbf{v}_s|\mathbf{v}_s \cdot \mathbf{v}_3. \tag{16}$$

How this amount is distributed as frictional heating of the atmosphere is unknown, but in the model we have assumed that in the thermodynamic equation

$$D_1 = \tfrac{4}{5}k_d(\mathbf{v}_1 - \mathbf{v}_3)^2,$$

$$D_3 = \tfrac{1}{5}k_d(\mathbf{v}_1 - \mathbf{v}_3)^2 + k_s|\mathbf{v}_s|\mathbf{v}_s \cdot \mathbf{v}_3, \tag{17}$$

which means that all of the heating due to the surface friction is kept in the lower layer, whereas most of the heating due to the internal friction is placed in the upper layer.

The non-frictional heating Q can be separated into three components, namely the heating resulting from absorption of solar and long-wave radiation, and the heating due to the small-scale vertical heat transport. For the incident solar flux, or solar forcing of the model, we have the annual mean values given by

$$S = E \cdot s(\mu), \tag{18}$$

where E is the solar constant divided by 4 and $s(\mu)$ the annual average fraction of E received at the latitude ϕ for which $\mu = \sin\phi$. For $s(\mu)$ we use the following expression given by Coakley (1979)

$$s(\mu) = P_0 - 0.2133\,P_2 - 0.0150\,P_4 + 0.0022\,P_6 + 0.0034\,P_8, \tag{19}$$

where $P_n(\mu)$ is the Legendre polynomial of order n normalized by the condition

$$\tfrac{1}{2}\int_1^1 P_n(\mu)^2\,\mathrm{d}\mu = 1.$$

The value of $s(\mu)$ at the equator is 1.223 and at the poles 0.500.

The absorption of solar radiation by the atmosphere–earth system is treated in the most simple way by assuming that the fraction ε_0 is absorbed in the atmosphere above the 200 mb level, the fraction ε_1 in the layer between the 200 and 600 mb levels, the fraction ε_3 in the atmosphere below the 600 mb level, and the fraction ε_s in the surface layer of the earth. For the planetary albedo α we then have

$$1 - \alpha = \varepsilon_0 + \varepsilon_1 + \varepsilon_3 + \varepsilon_s, \tag{20}$$

with ε_0 being about 4%, and ε_1 and ε_2 each about 5–7%. The emission and absorption of long-wave radiation is described in a similar crude way by the following simple expressions for the heating of the different layers. For the surface of the earth we assume that a long-term energy balance may be expressed by the equation

$$\varepsilon_s S + a_3\sigma_B T_3^4 - a_s\sigma_B T_s^4 - H_s = 0, \tag{21}$$

where the second term is the long-wave radiation received from the lower layer of the atmosphere, the third term is the radiation emitted from the surface, and where finally H_s denotes the non-radiative energy transfer from the surface to the atmosphere. The coefficients a_3 and a_s are treated as constants, generally somewhat smaller than 1, and σ_B is the Stefan–Boltzman constant. The heating rate per unit mass in the lower layer is described by the form

$$Q_3 = \frac{g}{\Delta p}\{\varepsilon_3 S + a_1\sigma_B T_1^4 + a_4\sigma_B T_s^4 - 2a_3\sigma_B T_3^4 + H_s - H_t\}, \tag{22}$$

where the second term is the absorbed radiation from the upper layer, the third term is the absorbed radiation from the ground, and the fourth term is the total radiation emitted from the layer. H_l represents the small-scale vertical heat transport from the lower to the upper level. By the corresponding formulation for layer 1 we have

$$Q_1 = \frac{g}{\Delta p} \{\varepsilon_1 S + a_3 \sigma_B T_3^4 - 2a_1 \sigma_B T_1^4 + a_0 \sigma_B T_0^4 + H_l\}, \tag{23}$$

where the term $2a_1 \sigma_B T_1^4$ represents the total radiation from the layer and $a_0 \sigma_B T_0^4$ the long-wave radiation received from the atmosphere above the 200 mb level. As the model contains no description of the dynamics of the layer above the 200 mb level, it is assumed that the temperature of this layer is determined by a long-term radiative equilibrium of the form

$$\varepsilon_0 S + e_0 \sigma_B T_1^4 - 2a_0 \sigma_B T_0^4 = 0, \tag{24}$$

where the second term is that part of the radiation from the layer below which is absorbed in the layer, and the last term represents the total radiation emitted from the layer.

By using (21) we may eliminate H_s from the expression for Q_3 and by using (24) we may eliminate $a_0 T_0^4$ from Q_1. Assuming further that $a_1 = a_3 = 0.85$ and $e_0 = 0.20$ and approximating $(a_s - a_4)T_s^4$ by $0.20T_3^4$, we finally get

$$\frac{Q_1}{c_p} = L\{\varepsilon C s(\mu) + a_3 T_3^4 - b_1 T_1^4\} + B, \tag{25}$$

$$\frac{Q_3}{c_p} = L\{(1 - \alpha - e)\, C s(\mu) - b_3 T_3^4 + a_1 T_1^4\} - B,$$

with

$$b_1 = 1.60, \quad b_3 = 1.05, \quad e = 0.10, \quad \varepsilon = 0.07,$$

$$L = \frac{g\sigma_B}{c_p \cdot \Delta p} = 1.2 \cdot 10^{-9}\ \mathrm{K^{-3}\ day^{-1}},$$

$$C = \frac{E}{\sigma} = 6.0 \cdot 10^9\ \mathrm{K^4}.$$

The term $B = (g/c_p \cdot \Delta p)H_l$ represents a heat transfer between the two layers, and we shall assume that this is mainly connected with the convective adjustment process. As a simple parameterization of this process we shall use

$$B = \begin{cases} k_T(T_3 - T_1 - \Gamma) & \text{if } T_3 - T_1 > \Gamma, \\ 0 & \text{if } T_3 - T_1 < \Gamma, \end{cases} \tag{26}$$

which means that if the temperature difference $T_3 - T_1$ is larger than Γ, an adjustment towards that value will occur. For Γ the value of 33 K has been adopted, corresponding to a mean lapse-rate of magnitude 6.3 K/km. From the numerical time integration of the model discussed in Section 3, it turns out that $k_T = 5.0 \cdot 10^{-6}\ \mathrm{s^{-1}}$ leads to an appropriate rate of adjustment, avoiding any convective instability.

Since the studies by Budyko (1969) and Sellers (1969), the ice-albedo feedback has been considered as one of the important factors, which may easily be incorporated in the simple energy balance models. In several recent papers, for instance by Oerlemans and Van den Dool (1978), the parameterization of the planetary albedo $\alpha(\mu)$ has been discussed in detail and especially considered in relation to satellite observations. In the present study the following simple albedo parameterization has been used.

$$\alpha = (A_0 + A_T)\, Z(\mu),$$
$$Z(\mu) = 1 + 0.045\, P_2(\mu) + 0.013\, P_4(\mu), \tag{27}$$

$$A_T = \begin{cases} 0 & T_3 > T_F \\ G(T_F - T_3) & T_F > T_3 > T_G \\ 0.18 & T_3 < T_G \end{cases}$$

where

$$A_0 = 0.29, \qquad G = 0.009\ \mathrm{K^{-1}},$$

$$T_F = 273\ \mathrm{K}, \qquad T_G = 253\ \mathrm{K}.$$

The quantity A_T expresses the contribution from the ice-albedo, and the latitude where $T_3 = T_F$ is the poleward boundary of the area without any snow or ice, whereas $T_3 = T_G$ delineates the equatorward extent of permanent snow and ice. The factor $Z(\mu)$ describes a general increase of albedo at high latitudes. At the equator the value of the albedo given by (27) becomes 0.28, at $\phi = 30°$ the same value, and at the pole 0.54 (provided that $T_3 \leqslant 253$ K), values that are in quite good agreement with observations.

3. Numerical simulations of the general circulation

The model described rather briefly in Section 2 was integrated numerically by using the spectral method in the form given by Bourke (1974) and by Hoskins and Simmons (1975). In this formulation the horizontal velocity vector is expressed by the stream function ψ and the velocity potential χ, and the equations of motion (1) are replaced by the vorticity and divergence equations. With the special form of the continuity eq. (2) the model contains only 5 basic variables, namely the 5 scalar fields ψ_1, ψ_3, χ, T_1, and T_3. For the time derivatives of these five quantities we have the two vorticity equations derived from eqs. (1), the two temperature eqs. (4), and finally the prognostic equation for χ, obtained as the difference between the divergence equation at the levels 1 and 3. This last equation also contains the geopotential, but only as the difference $\Phi_1 - \Phi_3$, which is simply expressed by the two temperature fields, using the hydrostatic eq. (6).

By the spectral representation in the horizontal, each of the basic variables X are represented by truncated series of the spherical harmonics

$$X(\mu, \lambda) = \sum_{m,n} X_{m,n} P_{m,n}(\mu) \cdot e^{im\lambda},$$

where $P_{m,n}(\mu)$ is the associated Legendre polynomial of the first kind normalized to unity over the sphere, and $X_{m,n}$ denotes the time dependent expansion coefficients. The integrations considered in the following are restricted to the symmetrical case by using heating functions of the form (25) with (19) and (27). This implies that T_1, T_3, and χ are even functions about the equator, whereas ψ_1 and ψ_3 are odd functions about the equator. For the spectral representation we use the triangular truncation $|m| \leqslant M$ with $n \leqslant M$ for variables which are even about the equator and $n \leqslant M + 1$ for variables which are odd about the equator. With this truncation, each of the variables have a horizontal variation with the same number of degrees of freedom. Most of the integrations have been done with $M = 24$, in which case the number of degrees of freedom is 325, but truncations with $M = 12$, 18 and 30 have also been used.

It should be noted that the model as it is presented by eqs. (1), (2), (4) and (6) contains no kind of horizontal diffusion. In most numerical integration models horizontal diffusion terms of more or less complicated form are used in order to avoid an unrealistic accumulation of kinetic energy on the small-scale components—spectral blocking. Considering mean values of long-term integrations of the present model, it turns out that for the crude horizontal resolutions $M = 12$ and 18, there is a pronounced blocking in the kinetic energy spectrum, which may be removed rather arbitrarily by introducing a simple linear diffusion term into the vorticity and divergence equations. For the resolutions $M = 24$ and $M = 30$, however, the blocking effect in the long-term mean spectra becomes very modest, if any, and moreover the long-term mean values of the temperature and velocity fields are quite similar for the two resolutions. So, from such results it was decided to carry through the numerical experiments discussed below, with the horizontal resolution $M = 24$ and without any horizontal diffusion.

The time integration was performed using the semi-implicit method introduced in connection with the spectral representation by Bourke (1974) and Hoskins and Simmons (1975). For the present model and with the horizontal resolution $M = 24$, it turned out that it was possible to use a time-step of 1 h.

The purpose of the numerical experiments with the general circulation model was to obtain the relevant long-term statistics describing the climate of the model for different values of the external parameters. Each of the experiments was done by following a procedure similar to that used in the first numerical simulation of the general circulation by Phillips (1956). The numerical integration was started from a state of rest and without any horizontal temperature variation. More precisely T_1 was put equal to 250 K and T_3 to 275 K. Due to the heating terms (25), certain temperature gradients will develop and motions will begin. From the solar heating given by (19) it follows that the temperature fields will be symmetrical with respect to the axis of the earth and the motion will consist of a zonal flow u and a meridional motion v, independent of longitude. After a certain time, variations with longitude were introduced by imposing small random perturbations of the temperature field. Actually the perturbation was imposed after 30 days by giving each of the spectral components with $0 \leqslant m \leqslant n \leqslant 18$ a random phase angle and a random amplitude smaller than or equal to 1 K. Due to the baroclinic instability of the zonal flow a system of organized waves will develop and gradually increase in intensity. The kinetic energy of this eddy motion increases over a period of about 20 days, and after that it has a relatively constant value. Also the sum of the kinetic energy of the zonal mean motion and the kinetic energy of the eddies is found to be rather constant after a total integration period of about 50 days, and so it should be possible that the motion as a whole approaches a statistically steady state shortly after that time. In order to examine this possibility, mean values with respect to time for different time intervals have been computed for a number of quantities: first of all the zonal velocity, the meridional velocity and temperature as a function of latitude, but also quantities like the spectral distribution of the kinetic energy and the temperature variance. Generally it is found, that the differences between the mean values for the period from day 90 to day 180 and for the period from day 180 to day 270 are insignificant. So, in the following the mean values for the period from day 180 to day 270 after the start of the integration are used to represent the climate of the model.

Fig. 1 shows the mean values of the temperature fields $\overline{T}_1(\phi)$ and $\overline{T}_3(\phi)$ resulting from the integration with values of the different parameters as stated in Section 2—considered as the standard case. Fig. 1 also shows the temperature fields $T_1^*(\phi)$ and $T_3^*(\phi)$ approximately representing the temperature in the case of radiative-convective equilibrium. The values of $T_1^*(\phi)$ and $T_3^*(\phi)$ are obtained as the result of a long term—300 days—integration of the system of equations

$$\frac{\partial T_1}{\partial t}=\frac{Q_1}{c_p},\qquad \frac{\partial T_3}{\partial t}=\frac{Q_3}{c_p},\tag{28}$$

where T_1 and T_3 are still represented by the truncated series of the zonal spherical harmonics, and where the heating functions Q_1 and Q_3 are given by the expressions (25). As in the case with motions, the integration is started from the constant values T_1 = 250 K and T_3 = 275 K, and after the integration over the 300 days, the values of Q_1 and Q_3 are quite small. The further convergence of Q_1 and Q_3 towards zero is quite doubtful, however, which is due to the form of the

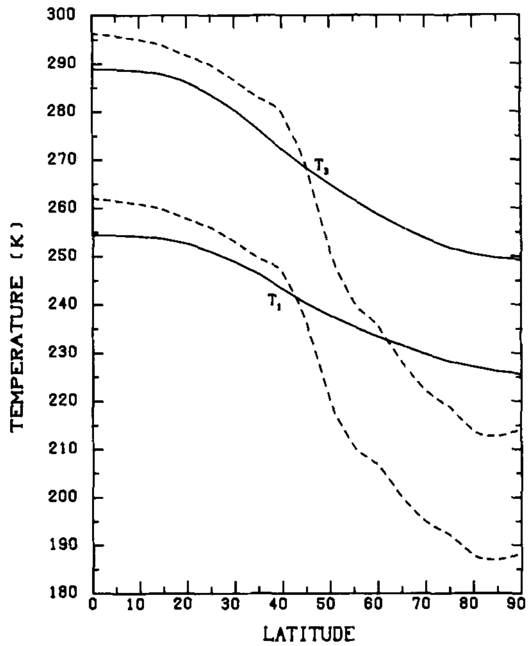

Fig. 1. Time averaged temperature fields $\overline{T}_1(\phi)$ and $\overline{T}_3(\phi)$ resulting from the general circulation model (solid curves), and the corresponding radiative-convective equilibrium temperature fields (dashed curves).

albedo α given by expression (27). With this albedo the equations

$$Q_1=0 \quad \text{and} \quad Q_3=0$$

have an infinite number of exact solutions for T_1 and T_3, each having a discontinuity somewhere in the interval of ϕ, where $T_F > T_3^* > T_G$. By numerical integration of (28), where T_1 and T_3 are represented by a finite number of spherical harmonics, it will not be possible to reach any of the stationary, exact solutions.

It is seen from Fig. 1 that the very important effect of the large-scale motions is to reduce the temperature difference between the equator and the pole to less than half its value in the case of no motions. The mean temperature fields $\overline{T}_1(\phi)$ and $\overline{T}_3(\phi)$ resulting from the numerical integration of the general circulation model agree with the actual yearly mean and zonal average temperature values of the northern hemisphere. Also the 90-day mean values of the zonal winds obtained from the model agree reasonably well with the observed atmospheric data. At the 800 mb level the mean zonal wind is westerly at all latitudes. From 0° to 10° and from 70° to 90° the values are quite small (~1 m s^{-1}), and the maximum value is 9.5 m s^{-1} at 39°. At the 400 mb level the picture is the same with a single maximum value of 23.8 m s^{-1} at 37°.

The mean values of the meridional wind component, $\bar{v}$, are of course essentially smaller. Due to the imposed symmetry condition, $\bar{v}$ is zero at the equator. At the 400 mb level, $\bar{v}$ increases from zero at the equator to a maximum value of 0.72 m s^{-1} at 22° and then decreases to small negative values around 40°. Poleward of this area, $\bar{v}$ is again positive with values of the order of 0.1 m s^{-1}. These mean values of $\bar{v}$ correspond to a well-developed Hadley cell from the equator to nearly 40°, a weak and narrow indirect meridional circulation around 40° and then a weak meridional circulation with upward motion from about 45° to about 55°, and downward motion in the area from about 55° to the pole.

It should be mentioned that the spectral distribution of kinetic energy is in accordance with what is known from the real atmosphere. The total kinetic energy is distributed with about 17.5% in the lower layer and about 82.5% in the upper layer. In the lower layer the zonal kinetic energy makes up only 32% of the total kinetic energy, whereas in the upper layer the zonal kinetic energy is slightly (4%) larger than the eddy kinetic energy.

Considering the long-term mean values as representing the statistically stationary state it is of course important to make sure that the basic energy balance conditions (12) and (13) are fulfilled. From the numerical integration considered, the mean values for the period from day 180 to day 270 become (in units of K/day)

$$\frac{[Q_1]}{c_p} = -0.5394 + [B],$$

$$\frac{[Q_3]}{c_p} = 0.5391 - [B],$$

$$[B] = 0.2398.$$

From these numbers it is seen, that the condition (13) is fulfilled to a high degree. For the transformation of total potential energy into kinetic energy the expression (11) leads to the following mean value

$$\frac{1}{c_p} C(P, K) = 0.05865 \text{ K/day}.$$

In accordance with this value, the mean value of the total dissipation given by (16) becomes

$$\frac{1}{2c_p} [D_1 + D_3] = 0.05861 \text{ K/day},$$

where, with the distribution given by (17), $[D_3]$ is nearly 10% larger than $[D_1]$. The values stated above are values per unit mass. For a vertical column from 1000 mb to 200 mb, the mean value of the total dissipation per unit area becomes

$$\frac{\Delta p}{g} [D_1 + D_3] = 6.4 \text{ W m}^{-2},$$

which is at least of the same order of magnitude as the different, rather uncertain estimates of the dissipation in the real atmosphere (cf. Lorenz, 1967, pp. 100–101).

A basic application of a climate model is the performance of response studies, i.e. studies of the changes of the long-term statistics caused by changes of one or more of the parameters of the model. Several such response studies have been carried out with the general circulation model presented in the foregoing. One series of response experiments has been carried out by varying the most important climate factor, namely the solar constant. For the case considered as the normal, the value $6.0 \cdot 10^9$ K^4 was used for the parameter C, corresponding to the value 1360 W m^{-2} for the solar constant. Table 1 shows the response of the temperature field to a decrease as well as to an

Table 1. *The horizontal mean temperature,* $[T]$, *and the temperature difference between the equator and the pole,* ΔT, *for different values of the solar constant and the angular velocity of the earth. The values are mean values obtained from the general circulation model. Temperature values in deg. K*

Ω	C (10^9 K^4)	$[T_1]$	ΔT_1	$[T_3]$	ΔT_3
Ω_E	5.76	242.2	29.1	271.9	40.4
	6.00	245.7	28.8	276.2	39.6
	6.24	249.0	27.6	280.1	37.6
$\frac{1}{2}\Omega_E$	6.00	246.5	23.4	277.1	35.5

increase of 4% of this value. As it must be expected, the horizontal mean value $[T]$ is increased in both layers with increasing value of the solar constant. It is seen, that the increase is somewhat larger in the lower than in the upper layer, and that the increase is not quite linear with C. ΔT denotes the temperature difference between the equator and the pole and this difference decreases with increasing value of the solar constant. Concerning the motions, it is found, as seen from Table 2, that the kinetic energy is generally decreased by about 9% when C is increased from $5.76 \cdot 10^9$ K^4 to $6.24 \cdot 10^9$ K^4. It should be mentioned that these results concerning the response of the model to changes of the solar constant are qualitatively quite similar to the results obtained by Wetherald and Manabe (1975) from a much more detailed general circulation model.

Another series of response experiments has been carried out by varying the value of the angular velocity of the earth, Ω. Certainly it would be of very great importance for the theoretical understanding of the general circulation of the atmosphere, if reliable numerical experiments with varying values of Ω could be accomplished. This, however, is not a straightforward task, as the formulation of the model itself may depend upon the value of Ω. At least it must be expected that the height and the structure of the boundary layer will change considerably with changing values of Ω, and unfortunately in a way about which very little is known. In spite of this shortcoming a series of experiments has been carried out with different values of Ω, but with fixed values of the frictional coefficients k_d and k_s, and with the unchanged expression (17) for the frictional heating. The results obtained are in general agreement with our theoretical knowledge concerning the influence of the earth's rotation upon the general circulation of the atmosphere, and they also agree quite well with the much more detailed results obtained by Hunt (1979) from similar experiments with a more comprehensive general circulation model.

In the case considered as the normal one, the usual value $\Omega_E = 7.292 \cdot 10^{-5}$ s^{-1} has been used. If Ω is reduced to half of this value, the long-term mean flow is changed considerably. It turns out that with the lower value of Ω, the direct Hadley circulation becomes more intense and is extended further towards the pole, the maximum value of $\bar{v}_1$ being 1.23 m s^{-1} at 31°. Simultaneously with this change of the meridional circulation, the maximum of the zonal flow is displaced from about 38° to about 58° and increased at the upper level from 25.3 m s^{-1} to 30.5 m s^{-1}. Also the maximum intensity of the eddy motion is displaced towards the pole for the smaller values of Ω, and as is seen from the values given in Table 2, the eddy kinetic energy becomes relatively much smaller in the case $\Omega = \frac{1}{2}\Omega_E$ than in the case $\Omega = \Omega_E$. Concerning the temperature field it is seen from Table 1 that the change of the horizontal mean temperature from the case $\Omega = \Omega_E$ to the case $\Omega = \frac{1}{2}\Omega_E$ is rather modest, smaller than 1K. The temperature difference between the equator and the pole, however, is reduced significantly, indicating that the broader and more intense Hadley circulation is accompanied by a more effective heat transport from the equator towards the pole.

Table 2. *Zonal kinetic (ZK) and eddy kinetic (EK) energy in each of the two layers for different values of the solar constant and the angular velocity of the earth. The values are mean values with respect to the total mass of the atmosphere in* $m^2\ s^{-2}$, *as obtained from the general circulation model*

Ω	C (10^9 K^4)	$(ZK)_1$	$(EK)_1$	$(ZK)_3$	$(EK)_3$
Ω_E	5.76	59.5	54.8	7.9	15.8
	6.00	55.1	52.8	7.4	15.3
	6.24	53.4	50.3	7.2	14.6
$\frac{1}{2}\Omega_E$	6.00	94.6	24.3	14.2	5.9

4. The analogous climate model

In a general circulation model, as the model described in the foregoing, the motion field and the temperature field are the basic variables, mutually influencing each other. Certainly, the temperature field is the most fundamental climate quantity, and as well from a practical as from a theoretical point of view it seems very important to study the possibilities of determining the mean temperature field directly from the parameters of the climate system without going through the cumbersome calculations of the rapidly changing motion field. It is the intention in this section to develop a formulation of the effect of the motions upon the temperature field, leading to such a simplified climate model having the same physical content as the general circulation model.

The zonal average of the thermodynamic equation in the form (5) becomes

$$\frac{\partial \overline{T}_1}{\partial t} + \frac{\partial(\overline{v_1 T_1}\cos\phi)}{a\cos\phi\,\partial\phi} + r_1\frac{\overline{\theta_2\omega_2}}{\Delta p} = \frac{\overline{Q}_1+\overline{D}_1}{c_p},$$

$$\frac{\partial \overline{T}_3}{\partial t} + \frac{\partial(\overline{v_3 T_3}\cos\phi)}{a\cos\phi\,\partial\phi} - r_3\frac{\overline{\theta_2\omega_2}}{\Delta p} = \frac{\overline{Q}_3+\overline{D}_3}{c_p}, \tag{29}$$

where the bar denotes a zonally averaged value. Introducing

$$T=\overline{T}+T', \qquad v=\bar{v}+v' \qquad \omega=\overline{\omega}+\omega',$$

the eqs. (29) may be written

$$\frac{\partial \overline{T}_1}{\partial t} + \bar{v}_1\frac{\partial \overline{T}_1}{a\partial\phi} - r_1\bar{\sigma}\frac{\overline{\omega}_2}{\Delta p} + \frac{\partial(\overline{v_1' T_1'}\cos\phi)}{a\cos\phi\,\partial\phi} + r_1\frac{\overline{\theta_2'\omega_2'}}{\Delta p} = \frac{\overline{Q}_1+\overline{D}_1}{c_p},$$

$$\frac{\partial \overline{T}_3}{\partial t} + \bar{v}_3\frac{\partial \overline{T}_3}{a\partial\phi} - r_3\bar{\sigma}\frac{\overline{\omega}_2}{\Delta p} + \frac{\partial(\overline{v_3' T_3'}\cos\phi)}{a\cos\phi\,\partial\phi} - r_3\frac{\overline{\theta_2'\omega_2'}}{\Delta p} = \frac{\overline{Q}_3+\overline{D}_3}{c_p}, \tag{30}$$

by using the continuity eq. (2) in the zonally averaged form

$$\frac{\partial(\bar{v}_1\cos\phi)}{a\cos\phi\,\partial\phi} = -\frac{\overline{\omega}_2}{\Delta p}, \qquad \bar{v}_3 = -\bar{v}_1. \tag{31}$$

Besides the mean temperature fields $\overline{T}_1$ and $\overline{T}_3$, eqs. (30) also contain the four unknown quantities

$$\overline{v_1' T_1'}, \qquad \overline{v_3' T_3'}, \qquad \overline{\theta_2'\omega_2'}, \qquad \bar{v}_1.$$

The first two of these quantities are—apart from a constant factor—the zonal mean values of the poleward eddy transport of sensible heat at level 1 and level 3, respectively, and the third term represents the vertical eddy heat transport. In order to use (30) as a closed system of equations, the four quantities mentioned must be determined from $\overline{T}_1$ and $\overline{T}_3$ in one way or another. From the numerical integrations of the general circulation model presented in Section 3 the long-term mean values of the quantities are easily computed. For the normal case integration, the mean value of $\overline{v_3' T_3'}$ as a

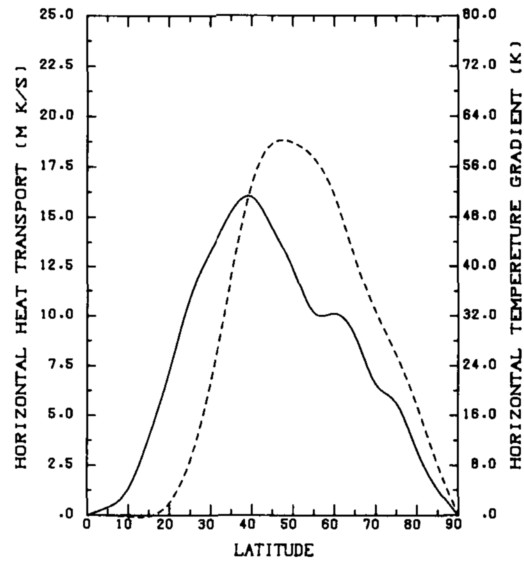

Fig. 2. Time averaged horizontal eddy heat transport $\overline{v_3' T_3'}$ as a function of latitude determined from the general circulation model (dashed curve, scale on left). The solid curve shows the corresponding values of the gradient, $-\partial\overline{T}_3/\partial\phi$ (scale on right).

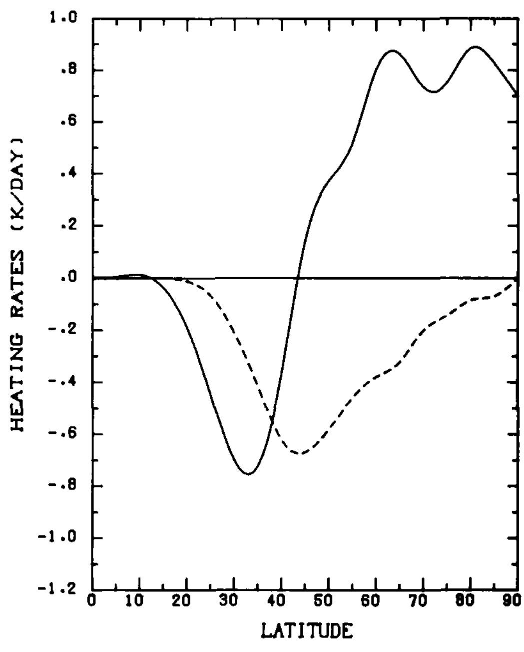

Fig. 3. Heating rates at the lower level due to the horizontal eddy heat transport (solid curve) and to the vertical eddy heat transport (dashed curve). The values are time averaged values obtained from the general circulation model.

function of latitude is shown in Fig. 2 by the dashed curve. The resulting heating rates are shown by the solid curve in Fig. 3. As seen from this figure, the horizontal eddy heat transport produces a maximum cooling of 0.75 K day^{-1} at about 32° and a heating with values between 0.70 and 0.89 K day^{-1} from 60° to the pole. In the upper layer the horizontal eddy heat transport, as well as the resulting heating rates, is of largely the same shape as in the lower layer, but with values which are about 40% smaller.

The dashed curve in Fig. 3 shows the values of the heating rates at the lower level due to the vertical eddy heat transport, and it is seen from the curve that there is a general cooling at all latitudes poleward of 20° with a maximum value of 0.67 K day^{-1} about 43°. The vertical eddy transport results in the upper layer in a heating which is the same as the cooling in the lower layer, but only smaller in magnitude by the factor $r_1/r_3 = 0.820$.

As a simple and frequently applied parameterization of the horizontal eddy heat transport we may assume, that this transport is determined from the mean temperature field by the diffusion relation

$$\overline{v'T'} = -K\frac{\partial \overline{T}}{a\partial\phi}, \tag{32}$$

where K is a constant, possibly varying with ϕ.

Together with the values of $\overline{v_3'T_3'}$, Fig. 2 also shows the corresponding values of $-\partial\overline{T}_3/\partial\phi$, given by the solid curve. It is seen that there is a certain similarity between the two curves, but also that there are significant differences, especially in the tropics. Thus, a relation more complicated than (32) must be constructed, presumably containing more than one constant. Concerning the vertical eddy transport $\overline{\theta_2'\omega_2'}/\Delta p$ and the mean meridional velocity $\overline{v}$, the situation is quite the same. Considering the mean values obtained from the general circulation model it should be possible to parameterize these quantities in terms of $\overline{T}_1$ and $\overline{T}_3$, but not in a simple and convincing way and not without introducing several independent constants.

Guided, however, by the results from the general circulation model, it seems much more efficient to try to parameterize the effect of the motions as a whole upon the mean temperature field, considering neither the mean meridional circulation and the eddy motion, nor the horizontal and the vertical motions separately. Fig. 4 shows the long-term mean values from the normal case integration of the heating rates

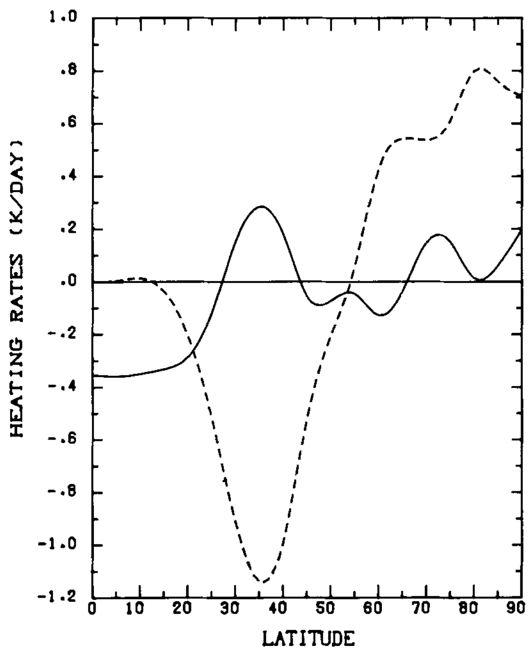

Fig. 4. Heating rates at the lower level due to the mean meridional circulation (solid curve) and to the eddy motion (dashed curve). The values are time averaged values obtained from the general circulation model.

$$\overline{\Sigma}_3 = \overline{v}_3\frac{\partial\overline{T}_3}{a\partial\phi} - r_3\overline{\sigma}\frac{\overline{\omega}_2}{\Delta p}, \tag{33}$$

$$\Sigma_3' = \frac{\partial(\overline{v_3'T_3'}\cos\phi)}{a\cos\phi\,\partial\phi} - r_3\frac{\overline{\theta_2'\omega_2'}}{\Delta p}, \tag{34}$$

where $\overline{\Sigma}_3$ given by the solid curve represents the total heating rate at the lower level due to the mean meridional circulation, and Σ_3' given by the dashed curve represents the total heating rate at the lower level due to the eddy motion. In Fig. 5 the same quantities are shown at the upper level, $\overline{\Sigma}_1$ and Σ_1'.

It is seen, that the curves make several bends, and furthermore that there is a certain tendency for the bends of $\overline{\Sigma}$ and Σ' to be in the opposite direction. This is seen clearly in Fig. 6 where the sum $\Sigma_3 = \overline{\Sigma}_3 + \Sigma_3'$ is shown by the solid curve and the sum $\Sigma_1 = \overline{\Sigma}_1 + \Sigma_1'$ by the dashed curve, i.e. the heating rates for each of the two layers due to the total motions. The two curves demon-

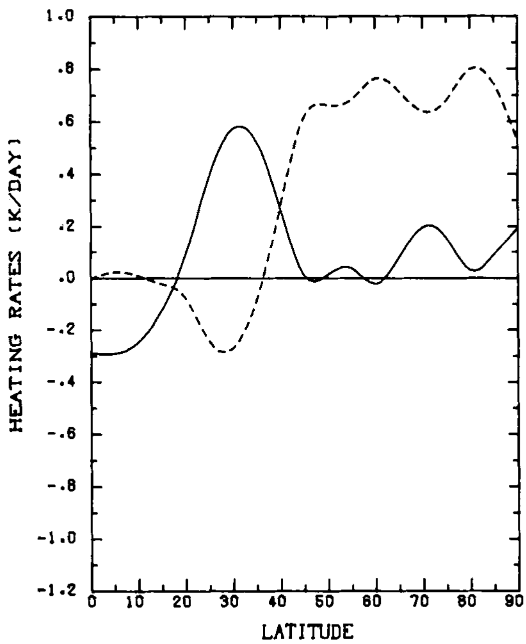

Fig. 5. As in Fig. 4 except for the upper level.

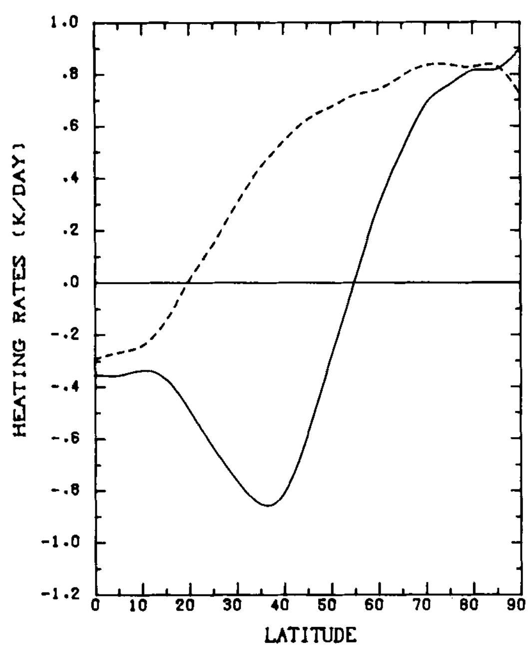

Fig. 6. Heating rates due to the total large-scale motions. Solid curve for the lower level and dashed curve for the upper level. The values are time averaged values obtained from the general circulation model.

strate that this total heating has a very regular and smooth meridional variation. The figure shows the general result that the effect of the large-scale motions is a relatively large cooling at the lower level from the equator to 54° with a pronounced maximum at about 36°, and a heating of the area from 54° to the pole with maximum heating at the pole. At the upper level there is a smaller cooling from the equator to 20° and a heating of the whole area from 20° to the pole with maximum value at about 75°. When we consider this heating due to the large-scale motions, it should be remembered that there is also a certain heating from the crude representation of a convective effect by the term B in (25). More precisely this term means a cooling at the lower layer and an equal heating at the upper layer with a maximum mean value 0.61 K day^{-1} at the equator, and with values decreasing to zero at about 40° latitude.

From the numerical experiments with changed values of the solar constant, mentioned in Section 3, it turns out, that the different heating quantities are changed by only a little. In the case where the angular velocity of the earth is reduced to half its normal value, the same principal result is obtained, namely that the heating due to the mean meridional circulation as well as the heating due to the eddy motion have a somewhat irregular meridional shape, whereas the meridional distribution of the heating due to the total motions has a very regular and smooth shape. Furthermore it is found, that the decrease of the angular velocity of the earth is accompanied by a poleward displacement of the maximum cooling due to the eddy motion, but also that this change is to a large extent compensated by the change of the cooling and heating, connected with the intensification and extension of the Hadley circulation.

Such considerations of the heating due to the large-scale motions simulated by the general circulation model, seem to advance the idea, that it should be adequate to parameterize the heating effect of the total motion as a whole instead of trying to deal with the heating of each part of the motion. This idea about a unified parameterization of the large-scale heat transport may be given a simple concrete form by assuming that a hypothetical meridional circulation may take over the whole heat transport necessary for maintaining the local long-term heat balance. In this case the mean temperature field is determined by the equations

$$\frac{\partial \bar{T}_1}{\partial t} + v_1^* \frac{\partial \bar{T}_1}{a \partial \phi} - r_1 \bar{\sigma} \frac{\omega_2^*}{\Delta p} = \frac{\bar{Q}_1 + \bar{D}_1}{c_p},$$
$$\frac{\partial \bar{T}_3}{\partial t} + v_3^* \frac{\partial \bar{T}_3}{a \partial \phi} - r_3 \bar{\sigma} \frac{\omega_2^*}{\Delta p} = \frac{\bar{Q}_3 + \bar{D}_3}{c_p}, \quad (35)$$

with

$$\nabla^2 \chi^* = \frac{\partial (v_1^* \cos \phi)}{a \cos \phi \, \partial \phi} = -\frac{\omega_2^*}{\Delta p},$$
$$v_3^* = -v_1^*, \quad (36)$$

where the star indicates the hypothetical meridional motion. The continuity eq. (36) for this hypothetical motion is written in accordance with the vertical formulation used in the general circulation model. Now the crucial point is to find a way in which χ^* may be determined from the mean temperature field. From the numerical integration of the general circulation model, the long-term mean values of $\partial \bar{T}_1 / a\partial\phi$, $\partial \bar{T}_3 / a\partial\phi$ and $\bar{\sigma}$ as well as the right-hand side of eq. (35) are given. Furthermore, as the long-term mean values of $\partial \bar{T} / \partial t$ and $\partial \bar{T}_3 / \partial t$ are nearly zero, it is possible to solve each of the eqs. (35) with respect to χ^* by applying a numerical mean square procedure to obtain that the two sides of the equation are very nearly equal. From such a computation it turns out that eqs. (35) actually indicate the same simple hypothetical meridional circulation. It is found that this meridional circulation has a vertical velocity which at the vertical mean level p_2 is closely proportional to the deviation of the temperature field from its horizontal mean value. More precisely it is found, that the relation

$$\frac{\omega_2^*}{\Delta p} = -A(\bar{\theta}_2 - [\theta_2]) \quad (37)$$

is satisfied very well with A being a constant. A field $\beta = \beta(\phi, t)$ may be introduced through the relation

$$a^2 \nabla^2 \beta = [\theta_2] - \bar{\theta}_2, \qquad [\beta] = 0, \quad (38)$$

or in the spectral representation

$$\beta = \sum_{n=1}^{N} \beta_n P_n(\mu), \qquad \bar{\theta}_2 = \sum_{n=0}^{N} (\bar{\theta}_2)_n P_n(\mu),$$
$$\beta_n = \frac{(\bar{\theta}_2)_n}{n(n+1)}. \quad (39)$$

Thus, eqs. (35) may be written in the form

$$\frac{\partial \bar{T}_1}{\partial t} + A\left\{ r_1 \bar{\sigma}(\bar{\theta}_2 - [\theta_2]) - \frac{\partial \beta}{\partial \phi} \frac{\partial \bar{T}_1}{\partial \phi} \right\} = \frac{\bar{Q}_1 + \bar{D}_1}{c_p},$$
$$\frac{\partial \bar{T}_3}{\partial t} + A\left\{ r_3 \bar{\sigma}(\bar{\theta}_2 - [\theta_2]) + \frac{\partial \beta}{\partial \phi} \frac{\partial \bar{T}_3}{\partial \phi} \right\} = \frac{\bar{Q}_3 + \bar{D}_3}{c_p}. \quad (40)$$

With β given by (38), these two equations determine the mean temperature fields $\bar{T}_1$ and $\bar{T}_3$, provided that the non-frictional heating and the dissipation on the right-hand side of (40) are expressed in terms of the mean temperature. Concerning the heating functions Q_1 and Q_3, this is simply done by using the approximation

$$\overline{T^4} \simeq \bar{T}^4,$$

based on the fact that the relative variation of T around the zonal mean value is rather small. From the long-term integration of the general circulation model, it turns out that the long-term mean values of B are nearly equal to the values obtained from (26) by using the zonal mean temperature, but reducing k_T to $2 \cdot 10^{-6}$ s^{-1} and Γ to 31 K. In the following all quantities are considered as independent of longitude and the indication with a bar is omitted.

To determine the mean value of the dissipation from the temperature field seems to be a quite difficult task. From the first of eqs. (40) we have in the steady state case for the horizontal mean value

$$-r_1 A \left[\frac{\partial \beta}{\partial \phi} \frac{\partial \theta_2}{\partial \phi} \right] = \frac{[Q_1] + [D_1]}{c_p}, \quad (41)$$

by utilizing the relation

$$[\sigma(\theta_2 - [\theta_2])] = \left[\frac{\partial \beta}{\partial \phi} \frac{\partial \sigma}{\partial \phi} \right].$$

Similarly we get from eq. (40) for the lower level

$$r_3 A \left[\frac{\partial \beta}{\partial \phi} \frac{\partial \theta_2}{\partial \phi} \right] = \frac{[Q_3] + [D_3]}{c_p}. \quad (42)$$

As the total non-frictional heating must be zero, cf. eq. (13), eqs. (41) and (42) may be added to give for the total dissipation

$$\frac{[D_1] + [D_3]}{c_p} = (r_3 - r_1) A \left[\frac{\partial \beta}{\partial \phi} \frac{\partial \theta_2}{\partial \phi} \right]. \tag{43}$$

From this relation it follows that A, which is the only constant entering eqs. (40), is fixed, if the total dissipation is given, either as a prescribed number or in terms of the temperature field. On the other hand, eq. (43) may also be used as a guide in making the choice of a reasonable expression for D_1 and D_3. Obviously D_1 and D_3 should be positive everywhere and it seems appropriate to assume that the dissipation is large in areas with large values of the temperature gradient, or in other words to assume that D is proportional to

$$\left(\frac{\partial T}{\partial \phi} \right)^2,$$

an assumption which is quite well justified by the long-term mean values obtained from the general circulation model. Equally justified, but in view of eqs. (40) even more simple, is the assumption

$$\frac{D_1}{c_p} = \frac{r_3 - r_1}{2r_1} A \frac{\partial \beta}{\partial \phi} \frac{\partial T_1}{\partial \phi},$$
$$\frac{D_3}{c_p} = \frac{r_3 - r_1}{2r_3} A \frac{\partial \beta}{\partial \phi} \frac{\partial T_3}{\partial \phi}, \tag{44}$$

in accordance with the claim that the relation (43) must be fulfilled. With this expression eqs. (40) become

$$\frac{\partial T_1}{\partial t} + A \left\{ r_1 \sigma(\theta_2 - |\theta_2|) - q_1 \frac{\partial \beta}{\partial \phi} \frac{\partial T_1}{\partial \phi} \right\} = \frac{Q_1}{c_p},$$
$$\frac{\partial T_3}{\partial t} + A \left\{ r_3 \sigma(\theta_2 - |\theta_2|) + q_3 \frac{\partial \beta}{\partial \phi} \frac{\partial T_3}{\partial \phi} \right\} = \frac{Q_3}{c_p}, \tag{45}$$

where

$$q_1 = 1 + \frac{r_3 - r_1}{2r_1} = 1.1097,$$

and

$$q_3 = 1 - \frac{r_3 - r_1}{2r_3} = 0.9100.$$

From these values of q_1 and q_3 it is seen that the inclusion of the dissipation on the left-hand side by the expression (44) means a minor change of the heating, quite well in accordance with the fact that the heating effect of the dissipation is relatively small.

Inserting into (45) the mean values of the temperature field resulting from the long-term integration of the general circulation model, and assuming that $\partial T_1/\partial t = \partial T_3/\partial t = 0$, it is found that the two sides of the equations are in good agreement. A numerical analysis indicates that the correlation coefficient between the two sides is as high as 0.99, and that in the case with the normal value of the external parameters, the appropriate value of A is

$$A = 3.1 \cdot 10^{-8}\,\mathrm{K}^{-1}\,\mathrm{s}^{-1}.$$

For the mean temperature fields obtained with changed values of the solar constant, it is found that the steady state form of eqs. (45) is satisfied to a very high degree with a constant value of A. Moreover it is found that the 4% changes of the solar constant, mentioned above, imply only very small changes of A. However, using the mean temperature fields from the numerical experiment with Ω reduced to $\frac{1}{2}\Omega_E$, it turns out that the steady state form of eqs. (45) is still usable, but in this case the constant value of A must be changed to a value of about $4.5 \cdot 10^{-8}\,\mathrm{K}^{-1}\,\mathrm{s}^{-1}$.

An important quality, arising through the proposed parameterization of the heating due to the total large-scale transport of sensible heat, is that only one single parameter, A, is involved. Certainly it would be very satisfactory if A could be determined in an elementary way from the external parameters of the model and some basic large-scale temperature quantities. Actually it seems possible to determine the correct magnitude of A and its possible variations with the external parameters of the system by assuming that A is connected in a very simple way with the thermal Rossby number of the model. By the definition of A given by the relation (37) it follows that a dimensionless quantity is obtained by multiplying A with a representative temperature and a representative time. From the way in which the vertical stability (expressed by σ) enters eqs. (45), it seems natural to consider $4[\sigma]\,\Omega^{-1}\,A$ as the adequate dimensionless quantity, where $4[\sigma]$ represents the difference between $[\theta]$ at the top and the bottom of the model and where Ω^{-1} is the time unit most immediately related to the dynamics of the large-scale motions. Equalizing this dimensionless quantity with the thermal Rossby number, R_0, we have

$$A = \frac{\Omega}{4[\sigma]} R_0. \tag{46}$$

The thermal Rossby number may be derived from the usual Rossby number

$$R_0 = \frac{U}{a\Omega}, \tag{47}$$

where the radius of the earth is used as the natural length scale and where U is a representative magnitude of the horizontal velocity. Determining the magnitude of U at the middle level from the thermal wind relation (cf. Lorenz, 1967, p. 118), the Rossby number (47) may be written

$$R_0 = \frac{1}{2}\frac{gh}{a^2\Omega^2}\frac{1}{[\theta_2]}\left(\frac{\partial\theta_2}{\partial\phi}\right), \tag{48}$$

where $(\partial\theta_2/\partial\phi)$ is the representative value for only one hemisphere, and where h is the total depth of the model. The magnitude of h may be determined by extending the hydrostatic relation (6) to the whole thickness of the model, which gives

$$gh = c_p(1 - r_0)[\theta_2],$$

where $r_0 = (0.2)^\kappa = 0.6314$. With this expression for h we finally get

$$A = \frac{(1 - r_0)c_p}{4\pi a^2 \Omega}\frac{\Delta\theta_2}{[\sigma]}. \tag{49}$$

By the relation (49), A is expressed in terms of the external parameters a and Ω and in terms of the internal large-scale parameters $\Delta\theta_2$ and $[\sigma]$ determined from the basic variables of the model, $T_1(\phi)$ and $T_3(\phi)$. With the normal values of a and Ω for the earth, the expression (49) becomes

$$A = 1.00 \cdot 10^{-8}\,\frac{\Delta\theta_2}{[\sigma]}\,\mathrm{K^{-1}\,s^{-1}},$$

which with the mean temperature fields from the long-term integrations of the general circulation model gives values of A around $3.1 \cdot 10^{-8}\ \mathrm{K^{-1}\,s^{-1}}$, in good agreement with the value mentioned above. In the case with the reduced angular velocity, $\Omega = \frac{1}{2}\Omega_E$, the agreement is not quite so convincing. As mentioned above the most appropriate value of A in relation to the results from the general circulation model is in this case $4.5 \cdot 10^{-8}\ \mathrm{K^{-1}\,s^{-1}}$, whereas the expression (49) yields a value about $5.0 \cdot 10^{-8}\ \mathrm{K^{-1}\,s^{-1}}$.

With A given by the expression (49) (or with a constant value for A) eqs. (45) may be integrated numerically in time from some initial temperature fields $T_1(\phi)$ and $T_3(\phi)$. This has been done by using a spectral representation for T_1 and T_3 in terms of the zonal spherical harmonics, i.e.

$$T = \sum_{n=0}^{N} T_n P_n(\mu).$$

The integrations have been carried out for temperature fields symmetrical with respect to the equator in consequence of the symmetrical solar heating given by (19) and the homogeneity of the surface. Starting from the constant values $T_1 = 250$ K and $T_3 = 275$ K, the two fields converge towards a steady state quite rapidly, such that after 500 days of integration the changes of the temperature are smaller than $10^{-4}\ \mathrm{K\ day^{-1}}$. The representation of the temperature is truncated by $N = 24$ and it seems to be quite insignificant to use a larger value of N. The transform method was used with the number of transform latitudes on the one hemisphere equal to 19. It is possible to use a time step as large as 24 h, and actually the integration is very fast—about 6000 times faster than the time-integration of the general circulation model.

With the normal Ω, the values for the temperature fields listed in Table 1 are reproduced very well by the steady state solutions of (45) and (49) for all three values of C, as seen by comparing Table 1 with Table 3. In all three cases the resulting value of A is $3.1 \cdot 10^{-8}\ \mathrm{K^{-1}\,s^{-1}}$. With $\Omega = \frac{1}{2}\Omega_E$ the steady state solution of (45) and (49) gives horizontal mean values for the temperature fields which are somewhat larger than the corresponding values in Table 1, and for the equator to pole temperature difference the values become a good deal smaller than those listed in Table 1. The resulting value of A is $4.7 \cdot 10^{-8}\ \mathrm{K^{-1}\,s^{-1}}$. It should be men-

Table 3. *The horizontal mean temperature [T], and the temperature difference between the equator and the pole, ΔT, for different values of the solar constant and the angular velocity of the earth. The values are obtained from the steady state solutions of (45) and (49). Temperature values in deg. K*

Ω	C (10^9 K^4)	$[T_1]$	ΔT_1	$[T_3]$	ΔT_3
Ω_E	5.76	242.0	28.8	271.4	39.5
	6.00	245.7	28.7	275.8	39.6
	6.24	249.1	28.3	279.8	39.0
$\frac{1}{2}\Omega_E$	6.00	247.1	20.5	278.1	30.1

tioned that the use of a constant and somewhat smaller value of A (a value around $4.4 \cdot 10^{-8}$ K^{-1} s^{-1}) will lead to temperature fields closer to those derived from the general circulation model. This, of course, could indicate that the way in which A depends upon Ω according to (49) is not the proper choice; but on the other hand it seems possible to modify the general circulation model with respect to the effect of Ω (for example on the friction layer) and so obtain a substantial change of the resulting temperature fields.

It is not only concerning the horizontal mean value and the equator to pole difference that the steady state solutions to (45) and (49) are in accordance with the results from the general circulation model. For the more detailed variation of the temperature there is also quite good agreement. For the experiments with the normal value of Ω, the deviation is in no case more than 1.2 K at the upper level and 1.7 K at the lower level. This seems to prove a sufficient degree of agreement in view of the fact that the mean temperature fields obtained from the general circulation model may quite well be changed to the same degree, for example by changing the horizontal resolution, changing the time integration scheme or changing the representation of the frictional effects.

5. Conclusions

The purpose of this study was to try to formulate on an elementary basis a climate model determining the same climate in terms of the mean temperature fields as the analogous general circulation model. Intentionally the general circulation model is kept relatively simple not only in consideration of the restricted computer capacity but also in order to have the relevant statistical results in a clear form. From the results obtained it seems possible to simulate the long-term heating effect of the large-scale heat transport in the general circulation model by using the concept of a hypothetical equivalent meridional circulation. This equivalent meridional circulation has the same basic effect upon the mean temperature field as the large-scale motions, i.e. a substantial reduction of the temperature difference between the equator and the pole and moreover a general cooling of the lower layer and a similar heating of the upper layer, with the largest values in both layers at the middle latitudes. From a theoretical point of view a justification of the proposed parameterization of the large-scale heat transport is given not only by the simple nature of the concept of an equivalent meridional circulation but mainly by the very simple way in which this meridional circulation is determined from the mean temperature field. This simplicity, of course, may be connected with the simple structure of the analogous general circulation model, and it should therefore be desirable to test the principle by using a more detailed general circulation model.

It should also be important to test the application of the equivalent meridional circulation by using it for more detailed modelling of the observed climate. It is therefore planned to apply the proposed parameterization of the large-scale heat transport to a climate model made more realistic, essentially by using a more detailed treatment of the short- as well as the long-wave radiation and by using a separate equation for the temperature changes of the surface. Certainly the concept of the equivalent meridional circulation has been considered only in relation to a statistically steady state, but nevertheless it seems worthwhile to also try to apply it to a relatively slow development of a quasi-stationary state. In this way it should be possible to include the seasonal variation, which is certainly one of the most important elements of the climate system.

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