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A simple and efficient approach to the initialization of weather prediction models

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ABSTRACT

An iterative method is suggested, by which the gravity modes of a primitive shallow water equation model are damped very efficiently while the meteorological modes are preserved fairly well. The model equations are integrated in a forward/backward cycle. It is shown that information about the normal modes of the model can be used to increase the damping rate even if these modes are not known explicitly. The meteorological modes can be conserved by keeping the relevant terms constant during each iteration cycle. Experiments with a simple one-dimensional model show that this model reaches a balanced state after a couple of iterations.

1. Introduction

The equations that form the basis for numerical weather predictions (the primitive equations) describe a large variety of solutions, many of which are hardly represented in the real atmosphere. In particular we find a large group of wave types with phase speeds much larger than the speeds of usual meteorological systems. For simplicity we will refer to these waves as gravity waves, although inertial and acoustic effects are essential for some of them. The observations that are used to produce initial fields for numerical weather prediction models do not contain enough information to prevent the occurrence of these waves in the simulations, and it is common practice to try to extract supplementary information from the constraint that they should have small amplitudes. For this purpose a number of so-called initialization methods have been developed.

In this paper we will suggest some improvements to a classical method due to Nitta (1969) where a frequency-dependent damping is accomplished by applying an iterative procedure to the model's own equations. It will be demonstrated that the damping rate can be increased drastically by means of some simple modifications to the method, and that the response can be made much more selective so that meteorological modes can be preserved.

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2. Nitta's method

Let us assume that the equations of the model have been linearized about a basic state so that they can be written in the form:

$$\frac{\partial}{\partial t}\mathbf{Z} = -i\mathbf{\Omega}\mathbf{Z}, \tag{1}$$

where the vector $\mathbf{Z}$ describes the state of the model. If the matrix $\mathbf{\Omega}$ has a complete set of eigenvectors $\mathbf{Z}_m$ with eigenvalues ω_m, the state-vector can be expressed as a sum of normal modes:

$$\mathbf{Z} = \sum_m c_m(t)\,\mathbf{Z}_m, \tag{2}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t}c_m = -i\omega_m c_m. \tag{3}$$

Our aim now is to suppress gravity modes without destroying the meteorological modes. A well-known approach to this problem (Nitta, 1969) goes as follows:

(i) the model is integrated one step forwards; for each mode this gives

$$c_f^{(n)} = c^{(n)} - i\omega\Delta t c^{(n)} = (1 - i\omega\Delta t)c^{(n)} \quad (4)$$

(n refers to the number of iterations made);

(ii) then a backward step is made, giving

$$c_{fb}^{(n)} = c_f^{(n)} + i\omega\Delta t c_f^{(n)} = (1 + \omega^2\Delta t^2)\, c^{(n)} \quad (5)$$

(due to the truncation error all the modes have now been amplified);

(iii) if the iteration cycle is completed by

$$\mathbf{Z}^{(n+1)} = \mathbf{Z}^{(n)} - \gamma\,(\mathbf{Z}_{fb}^{(n)} - \mathbf{Z}^{(n)}), \quad (6)$$

which gives

$$c^{(n+1)} = (1 - \gamma\omega^2\Delta t^2)\, c^{(n)} \quad (7)$$

for each mode, damping will take place provided that $0 < \gamma\omega^2\Delta t^2 < 2$ (here we have assumed that ω is real).

To avoid divergent behaviour for certain modes, γ and Δt have to be chosen in such a way that $0 < \gamma\omega_{max}^2\Delta t^2 < 2$ where ω_{max} is the highest frequency of the system. In this way all the modes will be damped, but the damping rate will depend heavily on the frequency and will become small for slow meteorological modes. The method has, however, some severe shortcomings:

(i) Many of the modes which we want to damp have frequencies which are much smaller than ω_{max}. Obviously the method will converge very slowly for these modes.

(ii) The decision on whether a mode should be removed or retained does not depend on the frequency alone. A short meteorological wave can, for instance, have a higher frequency than a long gravity wave.

(iii) In the general case, the fields modified by this method will not fit the original observations.

A detailed study of these problems is presented in Økland (1972). Although he treats a different damping device (the Euler-backward scheme) his discussion applies to Nitta's scheme as well.

3. Faster convergence

Obviously we need a different γ for each mode. To provide this we shall use a linear operator $\mathbf{\Gamma}$ instead of γ in (6). The ideal choice for $\mathbf{\Gamma}$ is an operator which has the normal modes of the model as eigenvectors, i.e.:

$$\mathbf{\Gamma}\mathbf{Z}_m = \gamma_m \mathbf{Z}_m \quad (8)$$

with $\gamma_m = 0$ for the meteorological modes and with $\gamma_m = (\omega_m\Delta t)^{-2}$ for the rest. In recent years the normal modes of certain models have been found explicitly (see for instance the discussion and references in Temperton and Williamson, 1979), and in such cases this ideal approach can be carried out. For many models, however, the normal modes are very difficult to find. Fortunately the convergence rate can be improved considerably even if our information about the modes is incomplete and the operator $\mathbf{\Gamma}$ differs from the ideal choice.

To demonstrate these ideas we will study the shallow water equations in one dimension:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + fv - \frac{\partial \varphi}{\partial x},$$
$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - fu + fu_g, \quad (9)$$
$$\frac{\partial \varphi}{\partial t} = -u\frac{\partial}{\partial x}(\varphi - \varphi_s) - (\varphi - \varphi_s)\frac{\partial u}{\partial x}.$$

Here u and v are velocity components, φ is the geopotential, f the Coriolis parameter, u_g the background geostrophic velocity and φ_s a topographic forcing. For later reference we note that if (9) is linearized about the basic state $(u, v, \varphi) = (U, 0, \Phi)$ and perturbations of the form $\exp[i(kx - \omega t)]$ are introduced, we find three normal modes for each wavenumber k, one "meteorological" mode with $\omega = \omega_a = Uk$ and two inertia-gravity modes with $\omega = \omega_a + \omega_g$ where $\omega_g = \pm(\Phi k^2 + f^2)^{1/2}$.

Eqs. (9) were integrated by means of a finite difference model which is described in the Appendix. We used $\Delta x = 2 \cdot 10^5$ m, $\Delta t = 300$ s, $f = 10^{-4}$ s^{-1}, and cyclic boundary conditions with period $L = 20\Delta x$.

In this section we will study the initial state:

$$u = u_g = 20 \text{ m s}^{-1},$$
$$v = 10 \text{ m s}^{-1} \cos(2\pi x/L),$$
$$\varphi = 10^4 \text{ m}^2 \text{ s}^{-2},$$

and use $\varphi_s = 0$. The average depth was chosen in order to simulate internal gravity waves with phase speeds close to 100 m s^{-1}. Because we want to copy the usual conditions in numerical weather prediction models we will assume that we also have to pay attention to external waves with phase speeds faster than 300 m s^{-1}, and that we therefore have to choose $\gamma \leqslant 2$ in order to avoid amplification of the fastest mode. Here it is tempting to try to distinguish between modes with different vertical structures and different equivalent depths. This problem can usually be reduced to a matrix-eigenvalue problem of sufficiently low order so as to be solved by modern computers (see Økland, 1972). If our internal modes can be separated from the external modes, we can increase γ by one order of magnitude. The effect of increasing γ from 2 to 20 is demonstrated in Fig. 1 where the amplitude of v is shown as a function of the number of iterations.

The computation of the horizontal structure of the modes leads to formidable computational problems unless the two horizontal dimensions can be separated in some way (see the discussion in Temperton and Williamson, 1979). However, even if we do not have a detailed knowledge of the modes, we know that short gravity waves will have higher frequencies than longer waves with the same vertical structure. Therefore, if γ is replaced by an operator $\mathbf{\Gamma}$ which increases the amplitude of the larger scales relative to the smaller scales, we will be able to damp the long waves more efficiently without risking instability for the short waves. In our simple example we can for instance use the operator:

$$\mathbf{\Gamma} = 150\ \mathbf{F}^3 \tag{10}$$

where $\mathbf{F}$ represents the usual low-pass filter:

$$F\{\alpha\} = \alpha(x) + \tfrac{1}{4}(\alpha(x+\Delta x) + \alpha(x-\Delta x) - 2\alpha(x)) \tag{11}$$

With this device the longest waves will experience a γ which is increased by a factor 7.5 ($\gamma \approx 150$) with a corresponding increase of the damping rate (see Fig. 1).

Obviously the damping can be made even stronger if $\mathbf{\Gamma}$ is a Fourier-filter where the component with wavenumber k is multiplied by the response-function

$$R(k) = (\omega_k' \Delta t)^{-2}. \tag{12}$$

Here ω_k' is an estimate of the frequency of gravity waves with a characteristic scale corresponding to k. In our simple example the normal modes have a trigonometrical x dependence, and if the finite difference equations are linearized about the basic state $(u, v, \varphi) = (0, 0, \Phi)$, the appropriate frequency turns out to be

$$\omega_k' = \left[\Phi\left(\frac{\sin(k\Delta x/2)}{\Delta x/2}\right)^2 + f^2\right]^{1/2}. \tag{13}$$

With this choice for ω_k', the gravity waves of our test case are damped very quickly (Fig. 1). Although the operator described was constructed without any explicit use of the relationships between the u, v, and φ fields of the normal modes, it satisfies the ideal criterion (8) for the gravity modes. It seems reasonable to believe that this approach can give very efficient damping even if the spatial variation of the normal modes differs from that of the Fourier-components, and ω_k' has to be based on rough estimates.

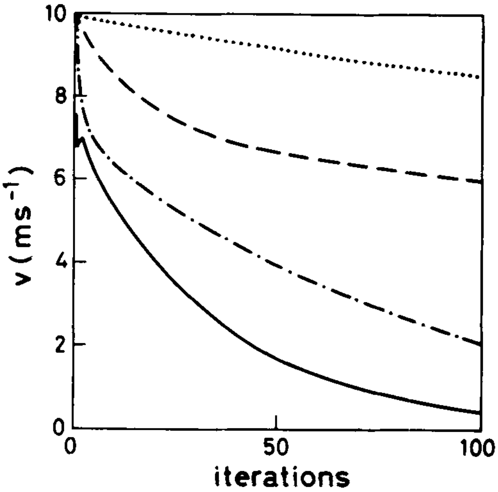

Fig. 1. The amplitude of v as a function of the number of iterations for $\gamma = 2$ (dotted line), $\gamma = 20$ (dashed line), $\mathbf{\Gamma} = 150\ \mathbf{F}^3$ (dashed-dotted line) and Fourier method (full line).

4. Conservation of meteorological modes

The techniques outlined above will give a damping rate which depends not only on the frequency, but also on the horizontal and vertical scales, and this makes it possible to damp long gravity waves more efficiently than short meteorological waves even if their frequencies are similar. Nevertheless the destruction of meteorological modes is still a problem, and we will now discuss some means by which it can be avoided.

A simple method is to restore some of the information from the initial fields after each iteration (or after a suitable number of iterations). This can be done by restoring one of the fields, for instance the rotational part of the wind-field, or by reinserting the original observations in some way.

Another possibility is to change the sign of γ from one iteration to the next. After two iterations we then have the fourth-order response

$$c^{(n+2)} = (1 - \gamma\omega^2 \Delta t^2)(1 + \gamma\omega^2 \Delta t^2) c^{(n)} = (1 - \gamma^2 \omega^4 \Delta t^4) c^{(n)}, \quad (14)$$

which is much more sensitive to the frequency than our previous second-order method. Unfortunately the response is also much more sensitive to the choice of γ, and the method should not be used unless very good γs can be provided.

A more sophisticated approach is to separate the terms of the model equations in groups which can be treated differently. Kurihara and Tripoli (1976) have used this technique in connection with the Euler-backward scheme. By integrating the advection terms by means of a relatively accurate modified Euler scheme, they were able to protect the meteorological modes from the damping effect of the Euler-backward scheme which was applied to the other terms. The accuracy of the modified Euler scheme can be utilized in connection with the Nitta scheme as well. However, it seems more natural to keep the advection terms and other "slow" terms constant during an iteration cycle. Let us assume that the normal modes are governed by equations of the form

$$\frac{\mathrm{d}}{\mathrm{d}t} c = -i\omega_g c + r, \quad (15)$$

where r represents the "slow" terms while $-i\omega_g c$ represents the terms which are responsible for the gravity modes. If r is equal to $r^{(n)}$ for the complete forward-backward cycle number n, eq. (7) is replaced by:

$$c^{(n+1)} = (1 - \gamma\omega_g^2 \Delta t^2) c^{(n)} - i\gamma\omega_g \Delta t^2 r^{(n)}. \quad (16)$$

In the linear case we can assume a relation of the form $r^{(n)} = -i\omega_a c^{(n)}$ which gives:

$$c^{(n+1)} = (1 - \gamma\omega_g(\omega_g + \omega_a)\Delta t^2) c^{(n)}. \quad (17)$$

If $|\omega_g| \gg |\omega_a|$ we get the same type of damping as before. For $|\omega_g| \ll |\omega_a|$, however, the response will become much weaker.

When the experiments of Section 3 were repeated with this trick included (see the Appendix for details), the amplitudes converged towards a constant level different from zero (Fig. 2). This behaviour is in perfect agreement with linear theory and simply shows that the meteorological modes are not damped at all because they have $\omega_g = 0$. In a realistic weather prediction model ω_g will not vanish, but for most of the meteorological modes it will be sufficiently small to make the method useful.

In the examples described so far, the final result was a state of geostrophic balance ($u = u_g, v = f^{-1}\partial\varphi/\partial x$). Certainly this is not a very hard test. In order to study a more complicated balance with non-linear effects and divergent winds, we shall introduce a topography of the form:

$$\varphi_s(x) = \hat{\varphi}_s \exp[-(x/a)^2],$$

where $\hat{\varphi}_s = 2000\ \mathrm{m^2\ s^{-2}}$ and $a = 3\Delta x$. The initial state was chosen to be $u = u_g = 20\ \mathrm{m\ s^{-1}}$, $v = 0$ and $\varphi = 10{,}000\ \mathrm{m^2\ s^{-2}}$. When the operator based on Fourier-decomposition was used, the fields converged very quickly towards a state composed of a stationary part forced by the topography and a transient part close to geostrophic equilibrium. The fast convergence is illustrated in Fig. 3 which shows the u-component of the wind-field after 0, 1, 2, and 20 iterations. In order to study the quality of

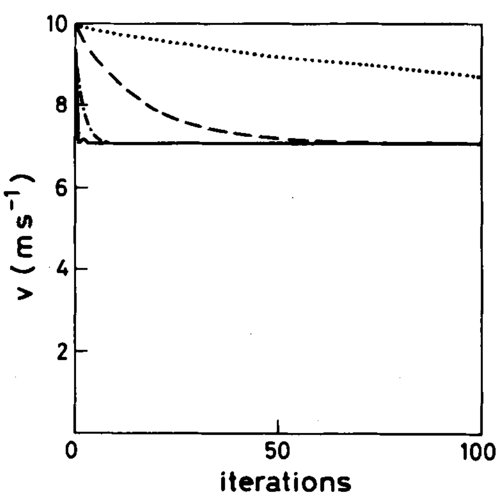

Fig. 2. Same as Fig. 1 but with conservation of meteorological modes.

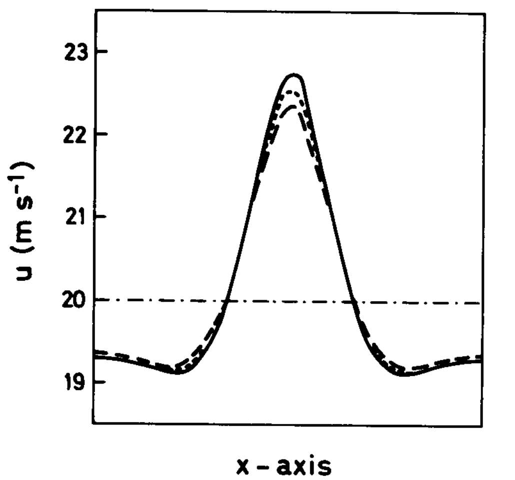

Fig. 3. Initialization with topography. u as a function of x, after 0 iterations (dashed-dotted line), 1 iteration (dashed line), 2 iterations (dotted line) and 20 iterations (full line).

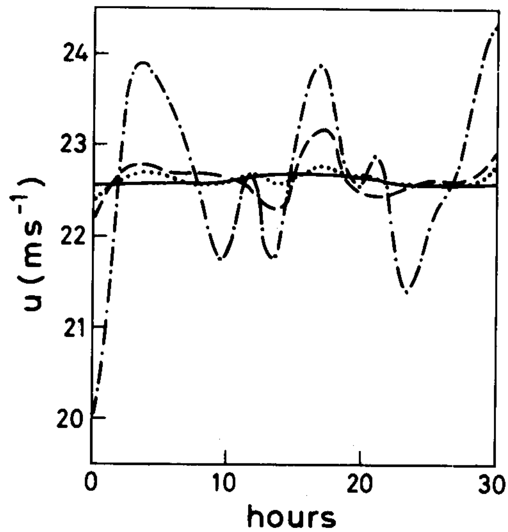

Fig. 4. Initialization with topography. u at a point as a function of time, without initialization (dashed-dotted line), 1 iteration (dashed line), 2 iterations (dotted line) and 20 iterations (full line).

these initial states, the model was integrated forwards by means of the leapfrog scheme (centred differences in time). Fig. 4 shows u at a point near the top of the mountain as a function of time. As we see, the balance of the fields is already quite good after two iterations.

At this point it is appropriate to ask what kind of balance we have obtained. In order to shed some light on this problem, we shall assume that we have found the normal modes of the model linearized relative to a state of rest. In the non-linear model, these modes are governed by equations of the form (15) where r is interpreted as the forcing from the non-linear terms. To simplify the discussion we shall assume that all these terms are treated as "slow" terms so that (16) can be used. If a stationary state is reached, (16) is reduced to

$$\omega_g^2 c + i\omega_g r = 0,$$

and for modes with $\omega_g \neq 0$ we have

$$-i\omega_g c + r = 0. \qquad (18)$$

This is essentially the same type of balance as that obtained by the non-linear normal mode initialization (see Machenhauer, 1977) where the meteorological modes are retained while the other modes are fixed by neglecting dc/dt in (15). It is important to note that the result is independent of γ.

For certain intertia-gravity modes, $|\omega_a|$ may become larger than $|\omega_g|$ and the expression $\omega_g(\omega_a + \omega_g)$ in (17) may become negative. To avoid amplification of such modes we may have to use $\gamma = 0$ for the horizontal and vertical scales where this can happen. In general it is very difficult to find a γ which gives efficient damping when $|\omega_a|$ is comparable to or larger than $|\omega_g|$. This problem may not represent a serious limitation because it is not obvious towards which state the process should converge. The non-linear normal mode balance where dc/dt is neglected in (15) is not justified in this case. In fact when the non-linear forcing r varies with a time scale which is comparable to that of the free gravity mode, the forcing may produce transients of significant amplitude. In such cases the free gravity waves must be considered as a natural part of the meteorological fields and can only be found by proper observations.

Physical processes like friction, eddy diffusion, condensation etc. should probably be included among the "slow" terms in (15). If serious convergence problems are encountered, we may have to keep some of these terms constant throughout the complete initialization procedure ($r^{(n)} = r^{(0)}$).

5. Concluding remarks

A properly initialized simulation should have the following properties:

(i) Modes that are not wanted should have small amplitudes.

(ii) The equilibrium state, about which these modes oscillate, should fit the observations reasonably well at the initial time.

It seems that the methods suggested in this paper can be used to solve the first problem quite efficiently. The approach is simple and does not require a complete knowledge of the normal modes. However, a one-dimensional shallow-water model is too simple to provide a realistic test, and a final evaluation of these ideas has to be postponed until they have been tested in a multi-layer weather prediction model.

The initialized fields will deviate from observations in a way which is outside our control, and the second criterion is therefore not satisfied. It seems natural to try to restore the observed data in some way, for instance by considering the initialized state as a first guess for a new analysis. The procedure can then be repeated several times.

6. Acknowledgement

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7. Appendix

Finite difference model

The velocities u and v were defined at the points

$$x_i = i\Delta x \quad i = 1, \ldots, I,$$

while φ and φ_s were defined at the intermediate points:

$$x_i = (i + \tfrac{1}{2})\,\Delta x \quad i = 1, \ldots, I.$$

By means of the definitions:

$$\delta_x \alpha = (\alpha(x + \tfrac{1}{2}\Delta x) - \alpha(x - \tfrac{1}{2}\Delta x))/\Delta x,$$

$$\bar{\alpha}^x = \tfrac{1}{2}[\alpha(x + \tfrac{1}{2}\Delta x) + \alpha(x - \tfrac{1}{2}\Delta x)],$$

the approximations for the "slow" terms in (9) can be written as:

$$A_u = -u\delta_x \bar{u}^x,$$

$$A_v = -u\delta_x \bar{v}^x,$$

$$A_\varphi = -\delta_x [\overline{(\varphi - \varphi_s - \Phi)}^x u].$$

Here Φ is an average value for $\varphi - \varphi_s$. The rest of the terms were computed as:

$$G_u = fv - \delta_x \varphi,$$

$$G_v = -f(u - u_g),$$

$$G_\varphi = -\Phi\, \delta_x u.$$

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