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A theory for propagation of an oceanic warm front with application to Sagami Bay

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ABSTRACT

The propagation of an oceanic warm front with a coastal jet is discussed from a viewpoint as a shock wave in a rotating stratified fluid. Postulating mass and potential vorticity conservation, the phase speed of the front and the amplitude of the coastal jet are derived and used to explain the phenomenon of "Kyucho" in Sagami Bay of Japan.

1. Introduction

More than 50 years ago, Miura (1927) reported the sudden warming and its duration of a few days with abnormally strong current along the coast of Sagami Bay (Fig. 1) in winter and spring. The phenomenon is called "Kyucho", which means swift currents in Japanese, and is well known to fishermen for providing a good catch of yellowtails, and sometimes for severe destruction of driftnet fishing. Kimura (1942) and Uda (1953) showed further evidence of the phenomenon, summarizing the major characteristics such as the cyclonic propagation (phase speed: 0.5–1.0 m s^{-1}) of a steep rise in temperature with strong alongshore current (0.5–1.0 m s^{-1}). Uda (1953), especially, suggested a relationship with the passage of the atmospheric cyclone, which might produce some kind of variability of the Kuroshio path and the major tide. He also conjectured ways of prediction.

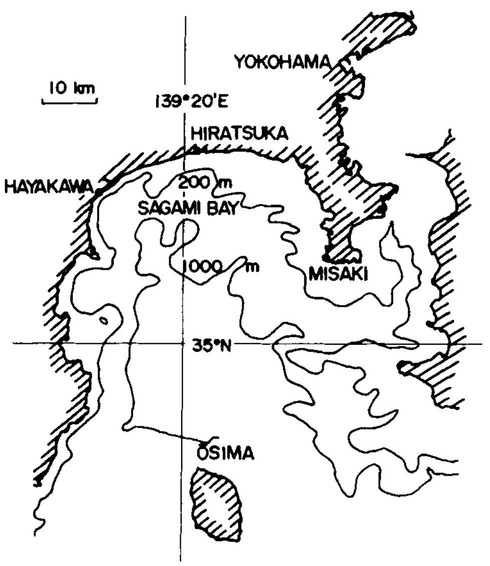

Fig. 1. Location and bathymetry of Sagami Bay.

Recently, Matsuyama and Iwata (1977) reported the sudden intrusion of the warm and more saline water mass facing the coast in the right-hand side for the typical event of "Kyucho" occurring during 22–24 April 1975 (Fig. 2). The water mass was found to be quite similar to the average one of the Kuroshio in spring. They also reported the small amount of density anomaly (δ_t) across the front, suggesting that the mass exchange occurs along the equal σ_t line.

Similar cyclonic propagation of a steep rise in temperature was observed by Mortimer (1963) along the coast of the Lake Michigan. Motivated by the observation, Bennett (1973) showed the steepening of wind-generated non-dispersive Kelvin waves due to non-linearity. Csanady (1977a), reviewing the phenomenon of coastal jets in shallow seas, pointed out the coastal jets confined

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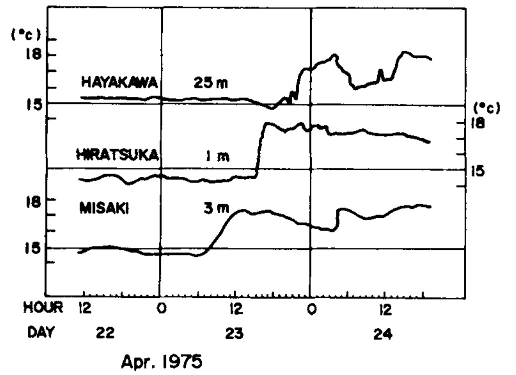

Fig. 2. Temperature observations during 22–24 April, 1975, from three different stations along the bay. Sensor depth is also represented.

to the warm upper layer which accompanies the coastal downwelling flow leaving the shore to the right in Lake Ontario. Csanady (1977b) also reported the westward movement (at a speed of 0.17 m s^{-1}) of a warm front along the north shore of Lake Ontario.

In a somewhat different context, Yoon and Suginohara (1977) performed numerical experiments concerning the intrusion of a warm current into the Japan Sea, aiming to simulate branching phenomenon of the Tsushima Current. They showed a warm front insensible to planetary β-effect progresses along the northern coast of Japan in a transient state, suggesting the similar dynamics for other narrow currents which flow leaving the coast to the right such as the Soya Current and Tsugaru Current. Endoh (1978) showed the cyclonic longshore progression of a thermal front with high alongshore velocity in the lee generated by the local heating in one side of a bay to clarify one aspect of the estuarine dynamics by using numerical and laboratory models.

Thus the cyclonic progression of a thermal front with high alongshore current will be one of the important aspects of coastal dynamics. Nevertheless we have no satisfactory explanation of such a front up to now. Here I derive Burgers equation in a rotating frame of reference after introducing the assumption about the form of the eddy viscosity. The shock solution of the equation is shown to possess main characteristics of the fronts described above. The phenomenon and dynamics presented here should be classified as the bore in a rotating stratified fluid. The phenomenon seems to be ubiquitous because of its simplicity.

2. Formulation

We adopt a two-layer ocean model of unform depth, shown in Fig. 3, and introduce the shallow-water approximation. Assuming the geostrophic alongshore current and the motion confined in the upper layer, we get the following governing equations:

$$
\begin{aligned}
&-fv = -g^* h_x \\
&v_t + uv_x + vv_y + fu = -g^* h_y + D \qquad (2.1)\\
&h_t + (uh)_x + (uh)_y = 0
\end{aligned}
$$

where f is the Coriolis parameter, g^* is the reduced gravity, (u, v) are velocities in the x (inshore coordinate) and y (alongshore coordinate) directions, respectively, h is the depth of the upper layer and D is the fictitious eddy friction whose form is proposed later. The boundary conditions are

$$
\begin{aligned}
&u = 0 \quad \text{at} \quad x = 0 \\
&(u, v) \to 0, \quad h \to h_i \quad \text{at} \quad x = -\infty \qquad (2.2)
\end{aligned}
$$

where h_i is the undisturbed depth of the upper layer.

3. Burgers' equation and its shock solution

Since the alongshore scale of the front is very small in comparison with the offshore scale, we assume that D is independent of x. We then obtain the potential vorticity conservation:

$$
\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{v_x + f}{h}\right) = 0 \qquad (3.1)
$$

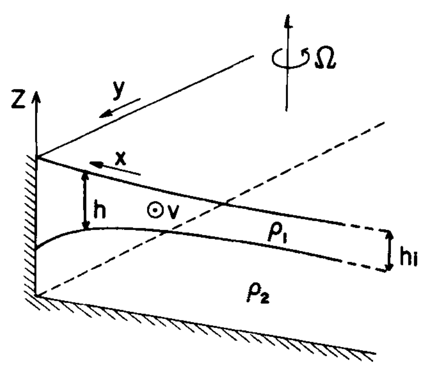

Fig. 3. Two-layer ocean model with a coast in the northern hemisphere.

from (2.1). At $t = 0$, we postulate that the sea is quiet and the upper layer has the depth h_l. We then get

$$\frac{v_x + f}{h} = \frac{f}{h_l} \tag{3.2}$$

The above relation is appropriate for discussing the coastal jet associated with the upwelling if the curl of the wind force acting on the upper layer vanishes (Csanady, 1977b). However, whether or not we can postulate the potential vorticity conservation to the "Kyucho" remains questionable.

Since the alongshore current is geostrophic, (3.2) reduces to

$$h_{xx} - \frac{f^2}{g^* h_l} h + \frac{f^2}{g^*} = 0 \tag{3.3}$$

with the boundary condition

$$h \to h_l \quad \text{at } x = -\infty \tag{3.4}$$

The solution is found as

$$h = h_l\{1 + c^{-1} Q(y, t)\, e^{fx/c}\} \tag{3.5}$$

where $c = \sqrt{g^* h_l}$. The alongshore current is then written as

$$v = Q(y, t)\, e^{fx/c} \tag{3.6}$$

(3.5) and (3.6) show that the offshore scale is given by the Rossby internal radius of deformation. Utilizing these expressions at $x = 0$, we find the governing equation of Q from the second equation of (2.1) as

$$Q_t + (Q + c)\, Q_y = D \tag{3.7}$$

Hereafter we assume that

$$D = \nu Q_{yy} \tag{3.8}$$

Then the eq. (3.7) is the well-known Burgers' equation for $Q + c$ (Burgers, 1948). The exact solution of shock type which depends only on $y - Vt$ is known as

$$Q = (V - c)\left[1 - \tanh\left\{\frac{V - c}{2\nu}(y - Vt)\right\}\right] \tag{3.9}$$

where V denotes the propagation speed of the front and $(V - c)$ must be positive. Now, (3.9) makes $Q = 2(V - c)$ at $y = -\infty$, so (3.5) gives

$$V = c[1 + (h_{-\infty} - h_l)/2h_l] \tag{3.10}$$

which is always positive, meaning the propagation leaving the shore to the right. For $\nu \to 0$, we obtain

$$Q = 2(V - c)\{1 - H(y - Vt)\} \tag{3.11}$$

where H means Heaviside function.

Assuming tentatively that

$$\left.\begin{aligned} h_l &= 10 \text{ m} \\ h_{-\infty} &= 50 \text{ m} \\ g^* &= 10^{-2} \text{ m s}^{-2} \\ f &= 10^{-4} \text{ s}^{-1} \end{aligned}\right\} \tag{3.12}$$

we obtain plausible values such as

$$\left.\begin{aligned} c &= 0.32 \text{ m s}^{-1} \\ V &= 0.95 \text{ m s}^{-1} \\ Q_{-\infty} &= 1.26 \text{ m s}^{-1} \\ c/f &= 3.16 \times 10^3 \text{ m} \end{aligned}\right\} \tag{3.13}$$

4. Discussion

Here we briefly point out the limitations and further scope of the problem. First, the model presented here has the basis on the assumption of mass and potential vorticity conservation. Actually, the mass conseration can be written as

$$\int_{-\infty}^{0} V h_l\, dx = \int_{-\infty}^{0} (V - v) h\, dx. \tag{4.1}$$

Using (4.1), (3.5) and (3.6), we can easily find (3.10) again.

As mentioned in Section 3, to appeal to the potential vorticity conservation remains questionable for the phenomenon of "Kyucho". Second, the temperature data in Fig. 2 show the undulation after a steep rise, especially at Hayakawa where the bottom slope is strong. This undulation might be explained by the dispersive effect due to the bottom slope. Our model, however, cannot explain this point because of the assumption of compensation. Third, the assumption of hydrostatic balance fails near the front. Fourth, the effect of the coastal viscous boundary layer will not be negligible in shallow seas.

In order to forecast "Kyucho", we should also clarify the generating mechanism. It is probable that one of the branches of the Kuroshio intrudes into the bay due to some kind of variability caused by the meteorological or other forcing mechanisms.

Unfortunately, our knowledge about such small-scale variability of the Kuroshio is very poor at this stage. Thus, it is hoped not only to perform systematic observations of temperature, salinity and velocity fields at several stations along and perpendicular to the coast, but also to grasp the atmospheric state, tides and the short-term behaviour of the Kuroshio at the same time.

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ТЕОРИЯ РАСПРОСТРАНЕНИЯ ОКЕАНИЧЕСКОГО ТЕПЛОГО ФРОНТА С ПРИМЕНЕНИЕМ К ЗАЛИВУ САГАМИ

Циклоническое распространение теплого фронта с прибрежной струей в залив обсуждается с точки зрения ударной волны во вращающейся стратифицированной жидкости. При постулировании сохранения массы и потенциальной завихренности находится фазовая скорость фронта и амплитуда прибрежной струи, которые используются для объяснения явления "Ключо" в заливе Сагами в Японии.