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SHORT CONTRIBUTION

A diagnostic study of global energy and enstrophy fluxes and spectra

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ABSTRACT

A diagnostic study of energy and enstrophy fluxes and spectra in terms of the two-dimensional wavenumber was carried out using data from the FGGE III-a global data set for January 1979. The study provides statistics on the non-linear exchanges of energy and enstrophy which are useful for studying atmospheric energetics and for evaluating global numerical models. The possible existence of an intertial subrange as allowed for in two-dimensional and quasi-geostrophic turbulence theory is investigated. Evidence of an inertial subrange beginning near wavenumber 22 is found.

1. Introduction

Advances in numerical techniques for spectral atmospheric models, for example, the transform method of Eliasen et al. (1970), have greatly facilitated diagnostic studies of the non-linear exchanges of energy and enstrophy among the various spatial scales. Theoretical work on such exchanges is found in the two-dimensional turbulence theories of Kraichnan (1967) and Leith (1968). These theories predict the character of the inertial subrange lying between the large scale generation region of the spectrum and the small-scale dissipation region. The inertial subrange is characterized by a -3 power law in the kinetic energy (KE) spectrum, a zero flux of KE, and a constant flux of enstrophy. Charney (1971) showed that the character of the inertial subrange of two-dimensional theory would also hold for quasi-geostrophic flows and in addition, the available potential energy (APE) would have a -3 power law.

Leith (1972) has applied turbulence theory to develop a sub grid scale parameterization scheme for use with large-scale atmospheric numerical models. One of the parameters of this scheme is the magnitude of the enstrophy flux through the inertial subrange.

An observational study of energy and enstrophy fluxes and spectra was carried out by Chen and Wiin-Nielsen (1978) (henceforth referred to as CWN) in terms of the two-dimensional wavenumber or the degree of an associated Legendre polynomial and employed a winter hemispheric data set. Although they found evidence of an inertial subrange, they were unable to draw a firm conclusion concerning its existence.

This study which is an extension of the work of CWN was undertaken to determine if a global data set would offer stronger evidence of the existence of an inertial subrange. Since the non-linear exchanges are important quantities in the atmospheric energy cycle, global observations of these quantities are useful for study of atmosphere energetics and for evaluating the ability of numerical models in simulating the energy cycle. In addition this study is able to provide an observation of the enstrophy flux required by Leith's parameterization scheme.

2. Data

The data set used was the FGGE level III-a WMC Washington Operational Analyses for the period January 1, 1979, 00 GMT to January 31, 1979, 12 GMT. This data set contains grid point

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values of the horizontal wind field, temperature, and geopotential heights with a horizontal resolution of 2.5 degrees in both latitude and longitude and contains 12 pressure levels in the vertical. Only the lowest 10 levels were used in this study, i.e., 100, 150, 200, 250, 300, 400, 500, 700, 850, and 1000 mb. The analysis follows that of CWN except that a triangular resolution of 32 waves was used.

3. Results

The vertically and temporally averaged spectra for the energies and the enstrophy in terms of the two dimensional wavenumber are displayed in Fig. 1. The kinetic energy flux function F_K, is given in Fig. 2, the enstrophy flux function F_E in Fig. 3, and

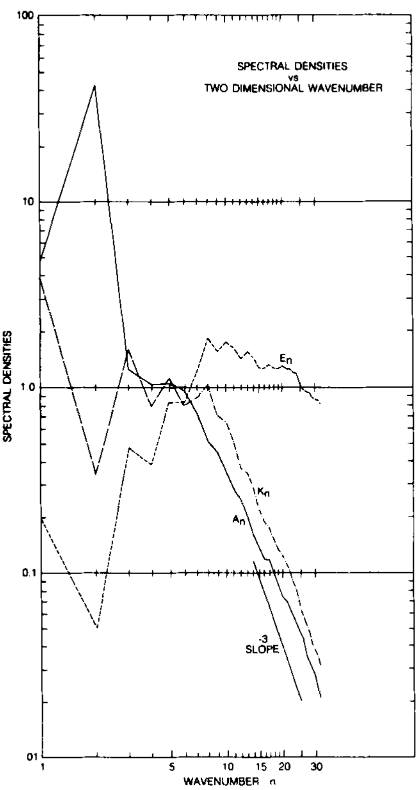

Fig. 1. Time averaged and vertically integrated spectral densities as a function of the two-dimensional wavenumber n for the kinetic energy, K_n, the available potential energy, A_n, and the time and vertically averaged enstrophy, E_n. The units of A_n and K_n are 10^5 J m^{-2} and of E_n are 10^{-11} s^{-2}.

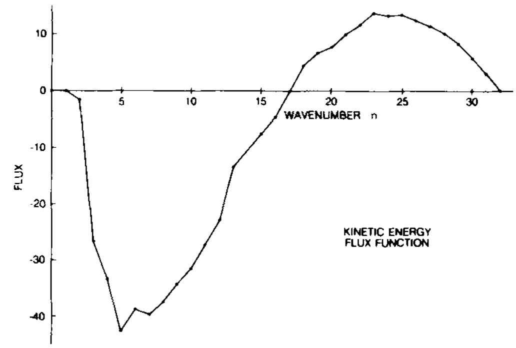

Fig. 2. Time averaged and vertically integrated kinetic energy flux function, $F_K(n)$ as a function of the two-dimensional wavenumber n. The units of $F_K(n)$ are 10^{-2} W m^{-2}.

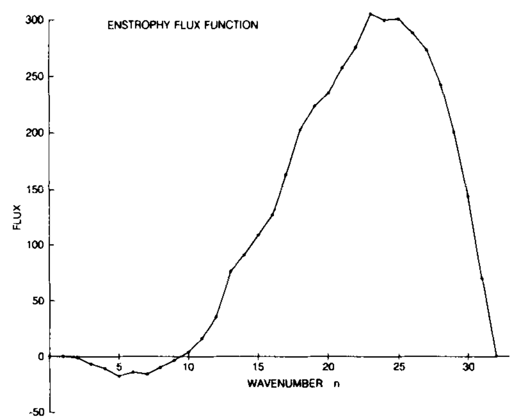

Fig. 3. Time and vertically averaged enstrophy flux function, $F_E(n)$, as a function of the two-dimensional wavenumber n. The units of $F_E(n)$ are 10^{-18} s^{-3}.

the available potential energy flux function in Fig. 4.

Based on a stability analysis of the quasi-geostrophic vorticity equation, Baer (1972) argued that baroclinic activity was confined to scales with n less than fourteen. Baer also suggested that current data could not resolve scales with n greater than 25 and as a result, the most appropriate scale range to look for an inertial subrange is $14 \leq n \leq 25$. In this spectral region, CWN observed a large flux of enstrophy from small n to large n and a relatively weak flux of kinetic energy. The spectral slopes of the kinetic energy, enstrophy, and the available potential energy were −2.6, −0.6, and −3.0 respectively. As a result of the finite truncation used, the enstrophy flux function cannot be

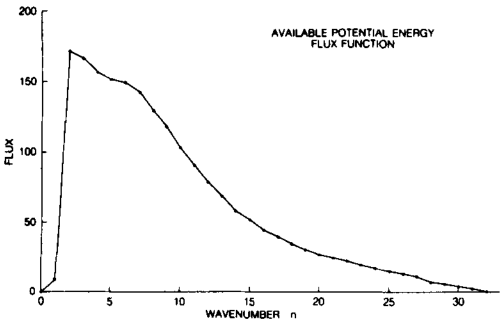

Fig. 4. Time averaged and vertically integrated available potential energy flux function, $F_A(n)$, as a function of the two-dimensional wavenumber n. The units of $F_A(n)$ are 10^{-2} W m^{-2}.

expected to show a region of constant flux. Hence it will be assumed that a region of constant flux begins near the function's maximum and that the magnitude of the flux is given by averaging over wavenumbers near the maximum.

In the same spectral range the present study shows a strongly increasing enstrophy flux and spectral slopes of −2.2, −0.3, and −2.1 for the kinetic energy, enstrophy, and available potential energy. Hence it appears that an inertial subrange does not exist in the region suggested by Baer.

The enstrophy flux function, Fig. 3, suggests that a possible constant flux region is in the interval $22 \leq n \leq 27$ and the spectral slopes for the KE and the APE are very close to −3 at these wavenumbers. It seems plausible that an inertial subrange begins near n equal to 22. The region of constant enstrophy flux is close to the truncation limit of $n = 32$ and is also in the spectral region where it is increasingly more difficult for the data to resolve the small scales. Therefore, caution must be exercised in attributing the observed fluxes and spectral slopes to the existence of an inertial subrange.

The shift of the beginning of the inertial subrange from approximately $n = 14$ found by CWN to near $n = 22$ of the present study could have several causes. Although the summer hemisphere is baroclinically less active than the winter hemisphere, it is possible that significant energy sources may occur at scales with n greater than 14 in summer and be absent in winter. Hence, the incorporation of the southern hemisphere into the analysis might shift the inertial subrange to scales with small n than those found by CWN. Alternately, if the data set used by CWN was unable to resolve the small scales, the peak of the enstrophy flux function would be shifted to smaller n and the spectral slopes could also be altered giving the false impression that the inertial subrange existed at larger n scales than was actually the case. Finally, normal atmospheric variability could also explain the existence of an inertial subrange at different parts of the spectrum when different data sets are used.

The three flux functions show that the addition of the less active summer hemisphere has reduced the magnitudes of all the fluxes. For example, the average enstrophy flux (which is required by Leith's (1972) sub grid scale parameterization scheme) through the possible inertial subrange, $22 \leq n \leq 27$, for the global data is 290×10^{-18} s^{-3} while the average enstrophy flux through the possible inertial subrange, $14 \leq n \leq 25$, of CWN was 385×10^{-18} s^{-3}.

4. Concluding remarks

The spectral densities and the fluxes of the non-linear interactions among different scales were computed for KE, enstrophy, and APE in terms of the two-dimensional wavenumber of the degree of an associated Legendre polynomial using the FGGE level III-a global data set for January 1979.

Evidence is found of the existence of an inertial subrange beginning near wavenumber 22. However, because of data reliability, it is not certain that the observed character of the non-linear fluxes and the spectra result from an inertial subrange.

Global statistics of non-linear exchanges and spectra should be useful in evaluating the performance of numerical models in simulating these important components of the atmospheric energy cycle.

5. Acknowledgements

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