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A method for determining the circumsolar sky function

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ABSTRACT

In 1980 a new pyrheliometric scale (World Radiometric Reference) will be introduced. This scale will be realized with higher accuracy than the previous one, therefore the accuracy of the transfer of the new scale could also be increased. In this paper a method is described for determining the circumsolar sky function which could be used to significantly decrease the systematic error caused by the circumsolar radiation during calibration of geometrically different pyrheliometers.

1. Introduction

In May 1979 the Eighth Congress of the World Meteorological Organization adopted some amendments to its Technical Regulations. The amended version will come into force on July 1 1980. The following sentence has also been adopted: "Pyrheliometric measurements shall be expressed with respect to the World Radiometric Reference (WRR)."

The WRR will be realized by a group of absolute pyrheliometers of a new type in the World Radiation Centre, Davos. This group will represent the true physical scale with an estimated accuracy of greater than $\pm 0.3\%$. Since this accuracy is significantly higher than that of the previous scale, the accuracy of the transfer of the pyrheliometric scale could also be increased.

Most of the regional and national standard pyrheliometers are geometrically different from the new absolute pyrheliometers. This difference results in a systematic error when they are calibrated in Davos (1650 m above sea level) and are used in other places. In this paper a method of significantly decreasing this error is presented.

2. State of art

Since the 1930s it has been known that the geometrically different pyrheliometers receive different amounts of scattered radiation from the circumsolar sky together with the direct radiation. The attempts made to treat the pyrheliometers from this point of view are synthetized in Pastiels' theory (Pastiels, 1959). According to this theory (if the radiance of the circumsolar sky is circularly symmetric around the sun) the output of a pyrheliometer should be written in the following form:

$$V = U\pi \int_0^{z_h} D(z)F(z) \sin 2z \, dz$$

where U is the mean sensitivity of the receiver surface

z is the angle measured from the centre of the sun

z_h is the limit-angle of the pyrheliometer, that is the largest angle from which radiation could fall on the receiver

$D(z)$ is the circumsolar sky function; it describes the distribution of the spectrally integrated radiance around the sun

$F(z)$ is the effective penumbra function of the pyrheliometer, it contains both the geometrical properties of the pyrheliometer and sensitivity distribution along the surface of the receiver.

It should be noted that, in using a computer, the approximations of Pastiels' theory can be avoided. However, in the case of circular symmetry, the application of this theory is more simple.

The first successful measurements of the circumsolar sky were made by Linke and Ulmitz (1940). Their differential actinometer measured the difference between radiations obtained across different entrance diaphragms, that is:

$$V_i - V_{i-1} = \Delta V_i = U\pi \int D(z)[F_i(z) - F_{i-1}(z)] \times \sin 2z \, dz \quad i = 1, 2, 3, 4$$

The above system of equations is an integral equation for $D(z)$. Linke and Ulmitz could not properly solve the integral equation. Bossy and Pastiels (1948) improved the previous solution but even they could not withdraw most of the information contained in the measurements. The numerical methods for solving such integral equations were developed in the 1960s; all of them require some kind of *a priori* information about the solution. This information could be obtained from direct (non-integrated) measurements or calculations.

Calculations were made by Schüepp (1967) and Fröhlich and Quenzel (1972) for obtaining the circumsolar sky function and the scattered circumsolar radiation entering into different pyrheliometers. Since the sky functions are not published, they cannot be used as *a priori* information.

In 1968 a circumsolar meter similar to that of Linke and Ulmitz was built (Major, 1970) and an improved version was constructed by Mersich as part of his diploma work in 1973 (Major, 1975). Several measurements were made in Budapest but the evaluation was not quite successful due to the lack of proper *a priori* information. On the basis of statistics, a mean sky function and the variance around it have been derived for Budapest from indirect measurements. Since the circumsolar sky function varies within wide limits, even in one place, and all numerical solution methods of the above integral equation require such *a priori* information which corresponds to the conditions of the measurements, then this *a priori* information will give good results only when the scattering properties of the atmosphere are close to the Budapest means. The derivation of similar *a priori* information for very different scattering conditions would require approximately 1000 measurements made in different places. The different conditions have been simulated by calculations instead of measurements.

3. The construction of the *a priori* information

The spectrally integrated circumsolar sky functions have been calculated for three aerosol models and for Deirmendjian's C3 thin cloud model. The aerosol size distributions have been derived from measurements (Mészáros, 1979) therefore they describe the real conditions better than the theoretical size distributions. The increase of the particle radius with relative humidity and the complex refractive index have been determined from data published by Hänel and Bullrich (1978). The necessary Mie values have been taken from the tables compiled by Shifrin and Zelmanovich (1968), the spectral composition of the solar constant has been used as suggested by Smith and Gottlieb (1974). Single Mie and Rayleigh scattering has been taken into account.

The model atmospheres are spatially homogenous with the following characteristics.

(i) *Mountain*

Surface pressure, 911 mb; water vapour content, 0.6 cm; relative humidity, 40%. The radius of the humid particles is 1.02 times larger than that of the dry, the complex refractive index is $1.48–0.01 \cdot i$. The total number of particles in the atmospheric column of 1 μm^2 of cross section is 0.52. The size distribution of dry particles (Fig. 1):

$$\frac{1}{N}\frac{dN}{dr} = 53 \exp[-12 (\log r + 1.8)^2] + 16 \exp[-44 (\log r + 1.25)^2] + 5 \exp[-18 (\log r + 1)^2] + 0.05 \exp[-4.5 (\log r + 0.7)^2]$$

(ii) *Continental background*

Surface pressure, 1013 mb; water vapour content, 0.8 cm; relative humidity, 65%. The radius of the humid particles is 1.066 times larger than that of the dry, the complex refractive index is $1.46–0.04 \cdot i$. The total number of particles in the atmospheric column of 1 μm^2 of cross section is 1.9. The size distribution of dry particles (Fig. 1):

$$\frac{1}{N}\frac{dN}{dr} = 24 \exp[-9.1 (\log r + 1.8)^2] + 9.5 \exp[-17 (\log r + 1.25)^2] + 1.9 \exp[-50 (\log r + 0.83)^2] + 0.13 \exp[-3.7 (\log r + 1)^2]$$

(iii) *Urban atmosphere*

Surface pressure, 1013 mb; water vapour content, 0.8 cm; relative humidity, 56%. The radius of the humid particles is 1.16 times larger than that of the dry, the complex refractive index is 1.46–

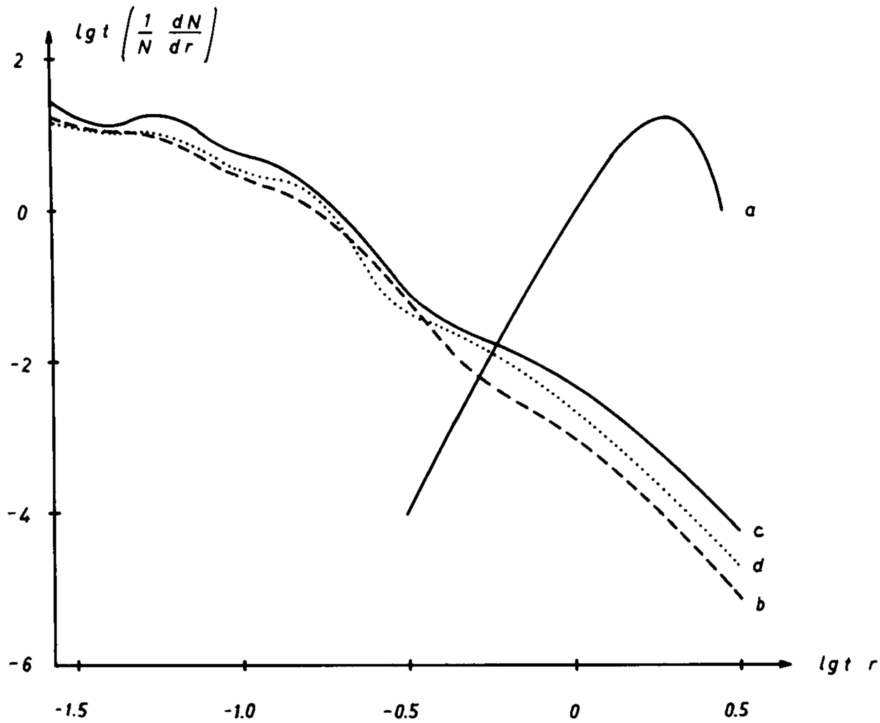

Fig. 1. The size distribution functions. a: thin cloud; b: urban aerosol; c: mountain aerosol; d: background aerosol.

$0.08 \cdot i$. The total number of particles in the atmospheric column of 1 μm^2 of cross section is 6.5. The size distribution of dry particles (Fig. 1):

$$\frac{1}{N}\frac{dN}{dr} = 36 \exp[-9.7(\log r + 1.8)^2] + 7.9 \exp[-27(\log r + 1.25)^2] + 2.2 \exp[-14(\log r + 0.96)^2] + 0.015 \exp[-4.8(\log r + 0.7)^2]$$

(iv) *Thin cloud*

Surface pressure, 1013 mb-water vapour content, 1 cm. The complex refractive index is $1.34 - 0.001 \cdot i$. The size distribution of particles (Fig. 1):

$$\frac{1}{N}\frac{dN}{dr} = r^8 \exp[-0.333 r^3]$$

The total number of particles in the atmospheric column of 1 μm^2 of cross section is 0.001.

The results are shown in Table 1. It should be noted that in the case of the thin cloud the optical depth of the atmosphere is so high that multiple scattering ought to be applied. Inclusion of the multiple scattering would reduce radiance values in the last column of Table 1.

It is quite clear that there is a non-linear dependence between the direct radiation and the circumsolar radiances. To obtain versatile *a priori* information, the following equation has been fitted to the mountain, background and urban data:

$$\pi \cdot D(z) = \exp[A \cdot I^2 + B \cdot I + C] \quad (1)$$

where A, B and C are coefficients depending on the scattering angle z (Table 2). I is the direct radiation.

Formula (1) gives smaller values of the circumsolar for the thin cloud model than those in the last column of Table 2, thereby correcting somewhat the deviation between the single and multiple scattering calculations. The validity of formula (1) is supposed to extend from 550 W/m² to 1050 W/m² of direct radiation. This interval covers all the practical cases which occur during pyrheliometeric comparisons held in continental places.

To solve numerically, the integral equation can

Table 1. *The direct radiation and the circumsolar sky function ($\pi \cdot D(z)$) for different model atmospheres*

	Models			
	Mountain	Background	Urban	Cloud
Direct radiation (W/m²)	960	845	743	438
Scattering angle (deg.)	Sky function (W/m²)			
0.9	4581	6998	8318	9972
1.2	4046	6353	7854	9185
2.0	3270	5253	6587	7871
2.5	2672	4231	5305	6389
3.0	1993	3001	3673	4602
3.5	1412	2018	2372	3188
4.0	1212	1751	2105	2733
5.0	981	1336	1600	2126
6.0	834	1122	1265	1768
7.0	706	922	1015	1471
8.0	561	725	832	1221
10.0	438	544	604	917

Table 2. *The values of the coefficients A, B and C of formula (1)*

z	*A*	*B*	*C*
0.9	−6.10680E-06	7.47630E-03	6.8698
1.2	−6.24910E-06	7.51720E-03	6.8419
2	−6.36970E-06	7.52790E-03	6.7319
2.5	−5.92360E-06	6.89130E-03	6.7374
3	−5.53100E-06	6.42650E-03	6.5066
3.5	−4.24310E-06	4.76590E-03	6.5853
4	−4.26430E-06	4.78170E-03	6.4428
5	−3.49970E-06	3.79420E-03	6.4629
6	−3.33410E-06	3.57770E-03	6.3601
7	−2.70410E-06	2.68200E-03	6.4746
8	−2.06990E-06	1.70830E-03	6.6006
10	−1.41810E-06	8.78140E-03	6.5489

be rewritten into the following matrix formalism:

$$\mathbf{g} = \mathbf{A}\mathbf{f}, \qquad (2)$$

where **g** is the vector of measured circumsolar differences

A is the matrix of equations; its elements are calculated from the instrumental characteristics

f is the vector of the circumsolar sky values at some scattering angles.

For solving equation (2), in the case of temperature sounding from satellite-measured radiation data, Karpov (1972) suggested the following iteration:

$$\mathbf{f}_{i+1} = \mathbf{f}_i - \frac{2}{m + M} \mathbf{A}^{\mathrm{T}}\mathbf{S}\,[\mathbf{A}\mathbf{f}_i - \mathbf{g}]$$

where $\mathbf{A}^{\mathrm{T}}$ is the transposed matrix of **A**

S accelerates the convergence, it could be determined by knowing **A** (Major and Németh, 1978)

m and M are the smallest and largest eigenvalues of the matrix $\mathbf{A}\mathbf{A}^{\mathrm{T}}\mathbf{S}$.

The process of iteration could be terminated if

$$|[\mathbf{A}\mathbf{f}_i - \mathbf{g}]_j|$$

becomes smaller than the error of measurements g_j.

The first **f** vector of the iteration should be calculated by formula (1).

4. The instrument and the error of measurements

The circumsolar meter consists of two actinometers. One of them has seven different entrance diaphragms (see Fig. 2). The output of the two actinometers are read simultaneously by using two galvanometers. If 1 symbolizes the smallest entrance diaphragm and 7 symbolizes the largest one, the sequence of readings is as follows: 1, 2, 3,

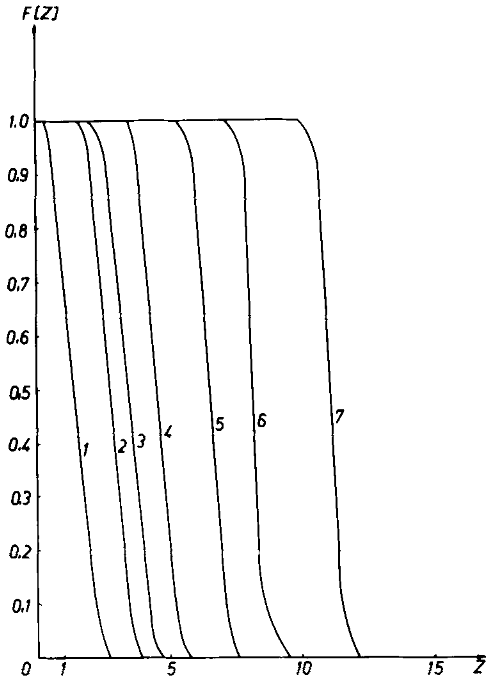

Fig. 2. The penumbra functions ($F(z)$) belonging to the different entrance diaphragms of the circumsolar meter. z is the angle measured from the optical axis (degree).

4, 5, 6, 7, 1, 7, 6, 5, 4, 3, 2, 1. Such a set of readings takes about 10 min, which is the same time as is required for a set of readings of pyrheliometers (e.g. one run) during pyrheliometric comparisons.

Each reading of the circumsolar radiation measuring actinometer is divided by the simultaneous readings of the fixed actinometer. Afterwards the proper ratios are averaged and multiplied by the instrument constant as well by the averaged reading of the fixed actinometer. A set of fluxes is then obtained: I_1, I_2, I_3, I_4, I_5, I_6, I_7, from which the **g** vector calculated:

$$g_i = I_{i+1} - I_i$$

The random error of measurements consist of the error of readings, the uncertainty of pointing to the sun and of some minor effects. The error of readings is one-tenth of the scale division, this is approximately 0.2% of the reading. The dividing increases, the averaging decreases this error by $\sqrt{2}$ times, that is, the error of the fluxes is 0.2%. During continental conditions this error rarely exceeds 2 W/m^2, so the flux difference has less error than 3 W/m^2 from the uncertainty of the readings. Taking into account the other effects the error of g_i vector components is estimated to be less than 4 W/m^2. This last value is used as the criterion for ceasing the iteration process.

5. Check of the method

The circumsolar meter took part in the International Pyrheliometer Comparisons held in Davos in October 1975, and in the sub-regional comparisons held in Budapest in September 1977, with the following Ångström pyrheliometers: A_{46} (Austria), A_{140} (German Democratic Republic), A_{212} (U.S.S.R.) and A_{596} (Hungary). The data obtained during these comparisons could serve as a basis for checking the usefulness of the above-described method for evaluating the effect of the circumsolar radiation on the comparisons of geometrically different pyrheliometers under different circumsolar conditions.

The A_{46} is a very short-tube Ångström pyrheliometer, the A_{140} and the A_{212} are almost identical short-tube pyrheliometers, while the A_{596} is a representative of the modern long-tube Ångström pyrheliometers. Using the proper penumbra functions, the circumsolar differences between the above-mentioned pyrheliometers and the PAC-RAD-III pyrheliometer (as standard geometry) have been calculated for each run of the comparisons.

On the receiver of the pyrheliometer, numbered by i, the following radiation flux falls:

$$I_0 + C_i'$$

here I_0 symbolizes the really direct radiation, C_i' symbolizes the circumsolar radiation. Generally the calibration of the pyrheliometers is not quite correct, therefore their measurements, S_i, differ from the incident flux. Neglecting the random error, the relation could be written:

$$S_i = k_i(I_0 + C_i)$$

here k_i shows the systematic error of the calibration, C_i is the effect of circumsolar radiation in the measurement.

The report on the Davos pyrheliometric comparison contains the ratios to PACRAD, that is the S_i/S_P quantities (WCRD, 1976). Since the PACRAD is one of the basic instruments of WRR,

Table 3. *Mean values obtained from the Davos comparison*

Pyrheliometers	A_{46}	A_{140}	A_{212}	A_{596}
Number of runs	56	55	41	55
Mean S_i/S_P	0.9766	0.9796	0.9818	0.9965
Mean k_i	0.9694	0.9723	0.9741	0.9955

therefore $K_P = 1$. Then the k_i factors could be calculated for each run:

$$k_i = \frac{S_i}{S_P} \frac{1}{1 + \dfrac{C_i - C_P}{S_P}}$$

In Table 3 the mean ratios to PACRAD and the mean k_i factors are shown. The difference between them is due to the circumsolar radiation. When using the k factors instead of the ratios, the receivers are calibrated to that of the PACRAD and not the receiver-tube systems to that of the PACRAD in the Davos circumsolar conditions.

In Budapest, artificial PACRAD values could be calculated from the measurements of the calibrated pyrheliometers:

$$\text{PAC}_i = \frac{S_i}{k_i} - (C_i - C_P)$$

If these values agree well with each other, then the circumsolar corrections between the four pyrheliometers are correct.

The best reproduction of the PACRAD measurement should be the mean of the four PAC values. For each run the standard deviation has been calculated around the mean. Fifty-six runs were collected in Budapest and the mean standard deviation was 1.53 W/m^2. This is somewhat less than 0.2% of the mean PAC value. In Davos the standard deviation of the ratios (calculated from the mean ratios of the runs) was about 0.001, that is 0.1%. Taking into account this last value, the indirect reproduction in Budapest could be regarded as good enough.

The k_i/k_j values could be determined for both places, Davos and Budapest:

$$\frac{k_i}{k_j} = \frac{S_i}{S_j} \frac{S_P + C_j - C_P}{S_P + C_i - C_P}$$

For Davos the S_P values are at hand, while for Budapest the mean of the four PAC values could be used.

In Table 4 two kinds of k_i/k_j ratios are shown. In the first row the ratios of the mean k values (taken from Table 3) could be seen, while in the second and third rows are the means of the ratios of the runs. The difference between the Davos and the Budapest values is less than 0.1%.

In Budapest the mean of the measurements of A_{140} and of A_{212} was taken as reference radiation. Supposing that only these two instruments had been calibrated to the PACRAD pyrheliometer beforehand, the calibration of A_{46} and of A_{596} could be checked. The k factor derived from the Budapest measurements for A_{46} is 0.9702 and for A_{596} is 0.9963, somewhat higher in both cases than the proper value in Table 3. In the case of A_{46}, where the circumsolar correction to PACRAD is large, the difference between the direct and indirect calibration is not significant. But in the case of A_{596}, where the circumsolar correction is only 0.1%, the indirect calibration is not so good.

The result of pyrheliometric comparison depends on factors other than the circumsolar effect. Therefore it cannot be expected that the circumsolar corrections exclude all uncertainty from the comparisons. It seems from the above checkings, however, that almost the total circumsolar effect could be handled by using this method.

6. Application to Linke–Ulmitz measurements

Linke and Ulmitz (1940) published the circumsolar differences measured on ten occasions. From

Table 4. *The k_i/k_j ratios in Davos and Budapest*

	k_{46}/k_{140}	k_{46}/k_{212}	k_{46}/k_{596}	k_{140}/k_{212}	k_{140}/k_{596}	k_{212}/k_{596}
Ratio of means, Davos	0.9970	0.9952	0.9738	0.9982	0.9767	0.9785
Mean of ratios, Davos	0.9970	0.9951	0.9738	0.9976	0.9767	0.9787
Mean of ratios, Budapest	0.9975	0.9958	0.9735	0.9988	0.9764	0.9782

Table 5. *The circumsolar sky functions (in W/m^2) derived from the measurements made by Linke and Ulmitz*

			Sky (degrees)										
Date 1940	Time	Direct W/m^2	0.9	1.2	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0
June 23	10^{38}	856	6594	5983	4951	4003	2847	1910	1653	1269	1074	887	696
June 26	9^{57}	740	8590	7965	6733	5396	3766	2413	2093	1563	1316	1074	838
June 26	10^{56}	848	6757	6142	5091	4112	2919	1950	1688	1292	1093	902	707
Jule 3	7^{29}	660	9323	8692	7120	5607	3621	1734	1222	425	819	1287	932
July 3	10^{33}	878	6164	5568	4589	3721	2658	1806	1562	1208	1024	850	668
July 15	6^{59}	408	7357	7105	6266	5234	3669	2498	2173	1683	1429	1235	1046
July 15	8^{16}	567	9373	8912	7726	6250	4325	2761	2400	1788	1505	1244	996
July 19	8^{17}	768	8175	7542	6344	5090	3565	2303	1996	1498	1262	1031	805
July 19	10^{44}	910	5520	4952	4054	3305	2379	1650	1426	1116	948	793	627
July 20	8^{05}	563	9320	8807	7345	5849	3801	1895	1334	285	225	827	961

a table in their article, the direct radiation registered at approximately the same time can be taken out. Since the circumsolar radiation frequently shows pulsations at a length of some seconds, which were not compensated by simultaneous measurements of a fixed actinometer, the error of the circumsolar differences was taken as 5 W/m^2. The penumbra functions are tabulated by Bossy and Pastiels (1948). The application of the evaluation method explained in this paper gave circumsolar sky functions for each measurement, as seen in Table 5.

The column of direct radiations shows that the measurements were made during very different turbidity conditions. If the evaluation techniques were appropriate in the 1940s, then it would be obvious that the circumsolar sky could vary within wide limits, and should be measured together with the pyrheliometric measurements.

It should be noted that the minimums at 5 or 6 degrees, appearing in two sky functions, were also pointed out by Linke and Ulmitz.

7. Conclusions

(i) The circumsolar measurements by Linke and Ulmitz can be evaluated using flexible *a priori* information and modern computing techniques for solving the integral equation.

(ii) Proper *a priori* information can be obtained by theoretical calculation of the circumsolar radiation using models for mountain, urban and background aerosols and thin cloud.

(iii) Using the above-mentioned *a priori* information for evaluating the measurements of our simple circumsolar meter, the results give the possibility of estimating the effect of circumsolar radiation upon pyrheliometric comparisons.

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МЕТОД ДЛЯ ОПРЕДЕЛЕНИЯ ФУНКЦИИ ОКОЛОСОЛНЕЧНОЙ РАДИАЦИИ

В 1980 г. новая пиргелиометрическая шкала (World Radiometric Reference) вступит в силу. Эта шкала реализируется с большей точностю чем настоящая, поэтому целесообразно повысшчть точность переноса новой шкалы. В Статье описан метод определения функции околосолнечной радиации, которая используется для устранения (или значительного уменьшения) систематической ошибки появляющейся при калибрации пиргелиометров разных по геометрии.