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Lagrangian and Eulerian measurements of horizontal mixing in the Baltic

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ABSTRACT

Six dye diffusion experiments and simultaneous moored current-meter measurements with vector-averaging current meters (VACM) were carried out in the Baltic surface-mixed layer on four days with different wind and surface wave conditions. These measurements are used to test the Hay-Pasquill (1959) method of calculating Lagrangian diffusion coefficients from Eulerian fluctuation measurements which has been frequently applied to meteorological but not yet to oceanographic measurements.

For the fairly wide range of our experimental conditions it is found that the Lagrangian dye diffusion coefficients and the product $\overline{u_E^2} \cdot T_E$ of variances and integral over the autocorrelation function of the lateral Eulerian current fluctuations are significantly correlated. Due to the special circumstances of our moored current measurements the factor relating them is not exactly the β of Hay-Pasquill. This factor, for the definition of the commonly used apparent diffusion coefficient, was determined as 1.4 $\pm$0.4, but for diffusion coefficients more specifically related to the diffusion velocity model used in our analysis it would be about similar to those determined in earlier meteorological work.

The calculation was also done for the diffusion experiment which Kullenberg (1977) carried out in the pycnocline at about 45 m depth and there the factor was only one-tenth of that in the surface-mixed layer. This small value is due to a larger correlation time scale in the stratified part of the water column which was most likely caused by internal wave effects.

1. Introduction

The study of oceanic mixing involves a complicated experimental technique. The usual way to measure mixing processes is to introduce dye and to tow a fluorometer through the growing patch. A standard evaluation procedure is to plot the variances of the concentration distributions against time and then compare their time dependence with point source mixing models (e.g., Okubo, 1971). It is quite laborious to get a number of these experiments done under different environmental conditions.

On the other hand, self-contained automatically recording instruments have been used for many years in current-meter moorings to obtain continuous records of the horizontal current fluctuations. It might be of interest to find out whether it is possible to calculate mixing coefficients similar to those determined in the dye experiments also from these more easily obtainable current records. Measurements which might be useful for that purpose were done during the experiment "BALTIC '75" east of Bornholm, where small-scale dye diffusion experiments were carried out simultaneously with measurement of the small-scale current field by an array of moored current-meters.

One way to determine horizontal eddy coefficients from moored instruments is to calculate Eulerian momentum transfer coefficients as the ratio of the current covariances and horizontal mean current shears. Unfortunately, in our moored current measurements the mean shears for time scales of some hours were not significantly different

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from zero over the horizontal separations of some hundred metres between our moorings (Quadfasel, 1977). In addition, momentum transfer coefficients appear to be larger than mass transfer coefficients and therefore could not be used as a substitute for dye diffusion measurements.

The problem in the determination of Lagrangian eddy coefficients from moored current records is the transfer from Eulerian to Lagrangian quantities. A possible procedure was proposed by Hay and Pasquill (1959). They postulated, for a homogeneous and stationary environment, that Lagrangian variances can be derived from Eulerian fluctuation measurements by adjusting the time scale in these measurements for the advection which makes the Eulerian fluctuations appear to occur more rapidly than the Lagrangian ones. This method was tested in wind tunnel experiments and with smoke diffusion from chimneys for the lateral component and found to work reasonably well (reviews are given, e.g., in Monin and Yaglom, 1971; Csanady, 1973; Pasquill, 1974).

During BALTIC '75, dye diffusion experiments of two different kinds were carried out simultaneously. One set of experiments was done in the surface mixed layer (Schott et al., 1978). The spreading of the dye was measured by aerial photographs which were analyzed by a computerized processing device and calibrated by towed fluorometer measurements. The second kind of dye-mixing measurements were made at about 45 m depth within the thermo- and halocline by Kullenberg (1977). The dye was injected through a pipe and the three-dimensional distribution was measured by a fluorometer towed at varying depths. The Eulerian current fluctuations at the depths of both sets of dye experiments were recorded by vector-averaging current-meters (VACM).

The near-surface measurements should be a reasonable data set to test the applicability of the Hay-Pasquill method for oceanographic diffusion calculations. We will also apply it to the data from the stratified layer, where internal waves can be expected to dominate, just to see whether the result will differ significantly or not.

2. Dye-mixing experiments during BALTIC '75 and calculation of diffusion coefficients

Dye patches were released on May 2, 3, 5, and 8, 1975, using a solution of Rhodamine B and acetic acid which was adjusted to the density of the surface mixed layer. During two of these experiments, on May 3 and 5, two patches were released simultaneously.

The dye was introduced into the surface layer in the morning before near-surface stratification due to diurnal heating had been built up. In about 1 hour a quasi-homogeneous dye layer developed which did not penetrate further in the vertical but spread only horizontally. The mixing depths (Table 1) ranged from 10 m at May 3, when the average wind speed was only 0.8 m/s, to 35 m on May 5, with wind speed 8.2 m/s. Aerial photographs were

Table 1. *Conditions and results of near-surface dye experiments during May 2–8, 1975*

Day of experiment	Dye release	Time of last aerial photograph	Mixing depth (m)	No. of photographs used	No. of concentr. contours used for model fit	Diffusion velocity P (cm/s)	Wind speed at 1/3 m (m/s)	R.M.S. wave height (cm)
May 2	10.23 h	17.31 h	25	5	34	0.55 ± 0.03	6.6	31.7
May 3, A	8.15 h	13.58 h		3	26	0.26 ± 0.03		
			10				0.8	8.3
May 3, B	8.25 h	15.58 h		4	33	0.28 ± 0.04		
May 5, A	7.28 h	14.03 h		3	19	0.52 ± 0.07		
			35				8.2	44.9
May 5, B	7.58 h	14.00 h		3	23	0.63 ± 0.7		
May 8	10.39 h	15.58 h	20	3	26	0.28 ± 0.02	7.1	19.5

Table 2. *Determination of β' from dye diffusion coefficients and lateral current fluctuations at 9 m depth*

		May 3		May 5		
Experiment	May 2	A	B	A	B	May 8
$\overline{u_E^2} \cdot T_E$	3050	1080		3440		1440
"Apparent" dye diffusion coefficient along one axis (cm^2/s)	5830	1040	1600	4800	6470	1130
β'	1.9	1.0	1.5	1.4	1.9	0.80
i^{-1}	2.1	7.7		4.2		3.9

$V/(\overline{u'^2})^{1/2}$

taken at different times after dye release, mostly from an altitude of 1000 m, and a fluorometer was towed horizontally through the patches by R.C. "Alkor" during the times of the plane passes. In a computerized processing method the horizontal distributions of up to 10 intensity bands were determined in the red frequency band; then the areas enclosed by contours separating different intensity bands (equidensities) were plotted and from these the dye concentration contours were calculated using the fluorometer calibration tows. The total number of concentration contours used in the analysis ranged from 19 to 34 (Table 1) for each patch.

The areas enclosed by the concentration isolines were calculated, transformed into equivalent circles and the equivalent radii were used for the interpretation in terms of radially symmetric diffusion models. Schott et al. (1978) tested different classes of diffusion models based on different assumptions on the scale dependence of the eddy diffusion coefficient. In their analysis, a model fitting technique (Schott and Willebrand, 1973) was used to calculate a *unified* fit of mixing models to all the contour data of the patches as they developed in time and space. It was found that for the patch scales of less than a few hundred metres and the time scales of only a few hours of our experiments no satisfactory discrimination was possible between the Fickian diffusion model, i.e., that with a constant eddy diffusion coefficient, and the diffusion velocity model (Joseph and Sendner, 1958) where the diffusion coefficient varies linearly with the horizontal scale of the mixing process; both models yielded about equally small best-fit residuals and our error estimates imposed through the aerial photography method on the concentration data were not good enough to statistically distinguish between them.

Therefore the question arises which diffusion coefficient to use in the following. The variances of the Eulerian current fluctuations increase with the averaging time interval and this time dependence has to be matched by the Lagrangian diffusion coefficient for the Hay-Pasquill method to work. Hence, we will use a diffusion coefficient based on the Joseph-Sendner diffusion velocity model because it also increases with the time scale of the process. From the variances of this model

$$\sigma_r^2 = 6P^2 t^2$$

with P the best-fit diffusion velocity (Table 2) and t the time elapsed between dye release and the last aerial photograph of the patch used in the analysis we calculate the so-called apparent diffusion coefficient

$$K_a = \sigma_r^2/4t = \tfrac{3}{2}P^2 t$$

Our mixing experiments showed two interesting results. Firstly, the diffusion coefficients for patches A, B, which were generated simultaneously in the same environment, are very much the same although the patch shapes were quite different, suggesting that the limitations introduced by the use of radially symmetric models were not too severe. Secondly, there was a significant difference among diffusion coefficients for different experiment days and a positive correlation between the eddy coefficients and atmospheric forcing was found which was more pronounced with surface wave height than it was with wind stress.

3. The relation between the Lagrangian diffusion coefficient and Eulerian current fluctuations

Taylor (1921) has shown that a relation exists between the Lagrangian eddy diffusion coefficient and the Lagrangian autocorrelation function for current fluctuations in a stationary and homogeneous environment. The Lagrangian turbulent particle displacements $\xi(t)$ along one axis, say the x-axis, are related to the Lagrangian current fluctuations, $u_L(t)$, along this axis by

$$\xi(t) = \int_0^t u_L(t')\, dt'$$

The eddy diffusion coefficient, K, is obtained from the variance of $\xi(t)$:

$$K = \frac{1}{2}\frac{\overline{d\xi^2}}{dt} = \overline{\xi\frac{d\xi}{dt}} = \int_0^t \overline{u_L(t)\, u_L(t')}\, dt'$$

where the overbar denotes ensemble averaging.

Introduction of the Lagrangian autocorrelation function yields

$$K = \overline{u_L^2} \cdot \int_0^t R_L(\tau)\, d\tau \quad (1)$$

In a field of random fluctuations with no periodic waves $R_L(\tau)$ will drop off from 1 to small values beyond some time lag $\tau = t_L$, the Lagrangian correlation time scale. Hence, for times $t > t_L$ the integral will approach a constant T_L, the Lagrangian integral time scale yielding

$$K = \overline{u_L^2} \cdot T_L \quad (2)$$

From the dye-mixing experiments, K can be calculated, but for $\overline{u_L^2}$ and T_L we only know the corresponding Eulerian values. For the Lagrangian variance, $\overline{u_L^2}$, we use the hypothesis that it is equal to the variance of the Eulerian lateral velocity, $\overline{u_E^2}$, which should be justified for small scales, provided the assumptions of stationarity and homogeneity are satisfied (Lumley and Panowsky, 1964; Haugen, 1966). We will come back to this assumption later with respect to our measurements.

Hay and Pasquill (1959) introduced a simple argument for the relation between Lagrangian and Eulerian current autocorrelation functions. They stated that the Eulerian fluctuations will appear more quickly than the Lagrangian ones because the eddies are advected past the fixed point, and that it must be possible to calculate Lagrangian particle displacements from the apparent Eulerian displacements just by slowing down the time scale by an appropriate factor. They assumed that R_L and R_E have about the same shape but differ only by a factor on the time axis:

$$R_E(\tau) = R_L(\beta\tau)$$

where β is an empirical constant, and $\beta > 1$ should be expected. If fluctuations are measured lateral to the mean flow, β must depend on the spectrum of the lateral fluctuations.

Introducing this and the above-mentioned assumption about the variances into the formula for the diffusion coefficient yields

$$K = \beta\overline{u_E^2} \int_0^t R_E(\tau)\, d\tau \quad (3)$$

Hay and Pasquill (1959) and subsequent authors have used the formula for the variance, $\overline{\xi^2}$, derived by further integration of the diffusion formula above:

$$\overline{\xi^2} = 2\beta\overline{u_E^2} \int_0^t \int_0^{t'} R_E(\tau)\, d\tau\, dt'$$

They applied this method to the variances measured in smoke diffusions in the horizontal direction perpendicular to the mean wind direction. Hay and Pasquill (1959) reported a value of $\beta \approx 4$ for a wide range of experimental conditions. Thompson (1965) used the method to determine *vertical* diffusion and found β to vary systematically with stability from 2 at slightly stable to more than 15 in quite stable cases. Haugen (1966), again for the horizontal lateral component, found values of β around four with an indication to increase with the inverse of the intensity of lateral turbulence

$$i = (\overline{u^2})^{1/2}/V$$

Later, more evidence was provided (Pasquill, 1974) that a relation $\beta \sim i^{-1}$ should be appropriate. In those experiments, i was always small, typically less than 0.1.

4. Findings on β and discussion

Current records were obtained at a number of depths by vector-averaging current-meters (VACM) with a time interval of the vector-averaging current components of 112 s. We use one record from 9 m depth to compare with the near-surface mixing experiments and two records

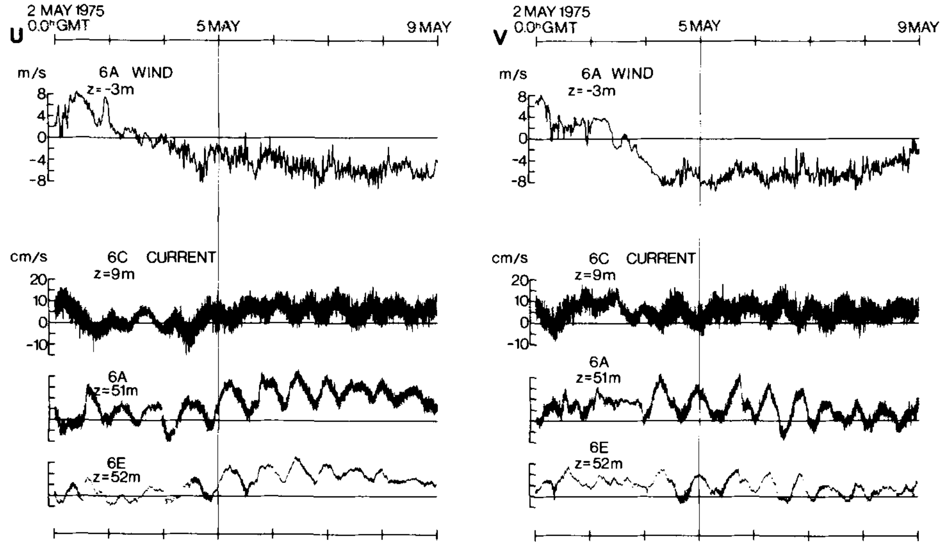

Fig. 1. Time-series of east (u) and north (v) components of wind and currents at closely spaced mooring positions 6A, C, E. Moorings 6A, 6C are surface moorings, mooring 6E is a sub-surface mooring.

from 51 m and 52 m depth at two stations 500 m apart to compare with the deep mixing experiments.

Samples of these records are shown in Fig. 1 for May 2–9. It is at once obvious that one condition mentioned above is not satisfied: The current components contain dominant inertia periodic fluctuations of period 14.6 h. This means that a straightforward calculation of the correlation function would not yield a convergent integral. We have therefore removed the inertia wave from the records, and have then calculated the variances and autocorrelation functions for the current-component normal to the mean flow during the time of the mixing experiments. We found that the integrals over the autocorrelation functions at 9 m depth converged already after less than 2 h.

A particular problem in the analysis of near-surface current-measurements is the influence of mooring motions on the current records. The mooring motions are induced by surface wave action on the floats which carry the moorings causing the whole mooring to oscillate vertically and this effect leads to aliasing of the whole current spectrum from all the instruments by the high-frequency noise. The effect is almost depth-independent.

We have analysed this effect in some detail for the measurements during BALTIC '75 (Quadfasel and Schott, 1978). There were four closely spaced moorings with VACM instruments near 51 m depth with a surface flotation and one (station 6E) with subsurface flotation. In Fig. 1 it is obvious that the high-frequency variance on the surface-mooring data (station 6A) is larger than that on station 6E. However, we found for these two time series that the product of variance and integral over autocorrelation function, $\overline{u_E^2}\, T_E$, did not differ more than 20% between the data from both moorings, i.e. the surface wave effect was greatly reduced. We assume now that this relation is also valid for the 9 m instrument where the variance is much larger than at depth, but there is an uncertainty factor in this assumption.

That we can only calculate the product $\overline{u_E^2} \cdot T_E$ from our current measurements means that we will not be able to determine the proper β from the Hay-Pasquill method. But this is not really necessary as we can see from (2) and (3): all we need to know is how the Lagrangian products $\overline{u_L^2} \cdot T_L$ and the Eulerian products $\overline{u_E^2} \cdot T_E$ relate.

In Table 2 the values of $\overline{u_E^2} \cdot T_E$ are given for May 2, 3, 5 and 8, calculated from current-meter records at 9 m depth for the corresponding time of the dye diffusion experiments. To compare the lateral diffusion coefficient K_x with the dye diffusion coefficient K_a determined from an isotropic model, the latter has to be divided by 2. In line 3 of Table 2 the ratio $K_x/(\overline{u_E^2} \cdot T_E)$ is presented; we call it β' because of the above-mentioned problems.

There is a significant correlation between $\overline{u_E^2} \cdot T_E$ and K_x, indicating that for the range of mixing situations studied a constant value of β' is justified. The inverse of the intensity of lateral turbulence is also given in Table 2. These values are somewhat smaller than those in the meteorological experiments referred to and there is also no systematic trend of β' with i^{-1}.

The mean value of $\beta' = 1.4 \pm 0.4$. This value is based on our use of the commonly used "apparent" diffusion coefficient $\sigma_r^2/4t$. But there are other definitions possible, for example that of the time derivative of σ_r^2 which yields $K = 6P^2t$. Then β' would be four times larger and quite close to the value $\beta \approx 4$ determined in earlier work. A second possible definition would be $K = P \cdot l$ where l has to be defined as the scale of the diffusion process, for example $l = 3\sigma$, which would yield a similar result, $K = 3\sqrt{6}\ P^2t$.

The horizontal diffusion on top of the pycnocline at about 44–46 m depth was studied on May 9 by Kullenberg (1977). Using the Joseph/Sendner model he calculated a diffusion velocity of 0.12 cm/s which yields an apparent diffusion coefficient of 470 cm²/s for a time scale of 6 h. We did not have a current-meter in exactly the same depth but used instead the instruments at 51 m and 52 m, where the stratification was stronger than at 45 m.

We have not evaluated the data from these instruments for May 9 but have calculated the lateral diffusion coefficients for the same time intervals as for the surface dye experiments. Averaging over $\overline{u_E^2} \cdot T_E$ for the four time intervals and using this to calculate β' from $K = 470$ cm/s one gets $\beta = 0.1$ only. The reason is the longer correlation time scale at this depth. A plausible explanation for what happens here in the stratified part of the water column is that the current fluctuations are dominated by internal waves, and in fact, internal-wave consistency tests applied to the coherences between the current components show that the fluctuations do agree with what isotropic internal wave models predict.

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ЭКСПЕРИМЕНТЫ ПО ПЕРЕМЕШИВАНИЮ НА БАЛТИКЕ И МЕТОД ХЭЯ-ПАСКУИЛЛА

В верхнем слое перемешивания и пикноклине на Балтике к востоку от о Борнхольм проводились эксперименты по диффузии красителя с одновременными измерениями течений на заякоренных буях. Из коэффициентов лагранжевой диффузии и эйлеровой дисперсии поперечных флуктуаций скорости и интегралов от автокорреляционных функций этих флуктуаций определялась константа в метода Хэя-Паскуилла, которая найдена равной 0.97 ± 0.26 для верхнего слоя перемешивания с тенденцией к росту при падении интенсивности турбулентности. Для экспериментов Кулленберга (1977) по диффузии в пикноклине на глубине около 45 м найдено, что β только около 0.1. Эта малая величина β вызывается гораздо большим временным масштабом корреляции в стратифицированной части столба воды, что, наиболее вероятно, имеет место благодаря действию внутренних волн.