Tellus (1979), *31*, 246–253

Acceleration of a diffluent jet stream by horizontal sub-grid scale processes—an example of a scale interaction study employing a horizontal filtering technique

By EERO HOLOPAINEN and PERTTI NURMI, *Department of Meteorology, University of Helsinki, Hallituskatu 11-13, 00100 Helsinki 10, Finland*

(Manuscript received October 5; in final form December 19, 1978)

ABSTRACT

A horizontal filtering technique is used to decompose the flow into a smoothed (larger-scale) component and a sub-grid scale component. The equation of motion for smoothed flow contains a force which may be interpreted in terms of a forcing effect derived from the motion systems which disappear in the smoothing process. The expression for this forcing (by smoothing out details smaller than about 400 km in the horizontal scale) is evaluated in a synoptic situation with a strong diffluent jet stream over northern Europe. It is found that the force in this kind of flow, which is relatively common over Europe, accelerates the smoothed flow. It is further shown that this force cannot be parametrized with a horizontal frictional force of the Fickian type, or with the more complicated formulations suggested by Smagorinsky (1963) and Leith (1969); in this particular case these three parametrizations all produce decelerating effects that are largely similar.

1. Introduction

No matter how good the spatial resolution of an observational system is, there are always certain scales of flow that are not observed. Thus, we follow only the behaviour of those phenomena whose horizontal scale is larger than the "grid scale" of the observation system concerned, e.g. the typical distance between the fixed observation stations or the resolution of the satellite-based measurements. The unresolved scales exert an influence on the flow of the resolved scales. When equations of motions are derived for the resolved flow, this effect represents a kind of frictional force. There is no general, exact way of parametrizing this force in terms of the resolved flow and in that sense these equations are empirical (Robinson, 1978).

In principle, if high-resolution data are available, the total flow can be split using the appropriate means into large-scale and small-scale components allowing empirical study of the forcing effect of the small scales on the large-scale flow. In practice this approach has not been used very much, one reason perhaps being that the normal method of flow decomposition, Fourier analysis, requires hemispheric data and is not appropriate for picking out the effect of local small-scale phenomena, even if the data density is some areas is good enough for their (local) description.

In this study we employ the horizontal filtering technique (the use of which for scale interaction studies has been suggested by Fortak (see Lange, 1974)) to decompose the flow into resolved and unresolved scales, the truncation wavelength being about 400 km. This corresponds roughly to the resolution with which the satellite-based observation network is likely to produce data for description of the global circulation (JOC, 1973). Our high-resolution data are simply the ordinary routine wind observations from Europe, where we have the densest network of aerological stations in the world. We are thus also able to depict details of the atmospheric flow with a scale smaller than 400 km.

The forcing effect of unresolved scales is here considered in only one particular synoptic situation. The case selected for investigation is one with a strong diffluent jet in the upper troposphere. This kind of synoptic situation is rather common over Europe.

0040-2826/79/030246-08$02.50/0

In Section 2 a general expression for the sub-grid scale forcing is derived when horizontal filtering is used for scale separation. In the next section this expression is applied in the chosen synoptic situation to evaluate the frictional force due to horizontal sub-grid scale processes. In Section 4 we compare the results obtained with the force fields obtained by applying the currently used expressions for the parametrization of the horizontal friction. In the final section we discuss the implications of the results obtained.

2. Sub-grid scale forcing when using the horizontal filtering techniques for the scale separation

Let s be an arbitrary property per unit mass. The formal equation for the changes in s can be written as

$$\frac{ds}{dt} = S \tag{1}$$

where S denotes the intensity of the source of this property per unit mass. Using pressure as the vertical coordinate and employing the equation for continuity of mass ($\nabla_p \cdot \mathbb{V} + \partial\omega/\partial p = 0$, $\omega = dp/dt$) we can write (1) in the form

$$\frac{\partial s}{\partial t} + \nabla_p \cdot s\mathbb{V} + \frac{\partial}{\partial p} s\omega = S \tag{2}$$

Let us define a smoothed field $\hat{s}$ as

$$\hat{s}(x, y, p, t, L) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x', y') \times s(x + x', y + y', p, t)\, dx'\, dy' \tag{3}$$

where the weighting function w ($\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x', y')\, dx'\, dy' = 1$) determines the degree of smoothing and thus the approximate truncation wavelength L, separating the resolved scales (those of the $\hat{s}$ field) from the unresolved ones (those of the $s - \hat{s}$ field). The smoothing operation (3) commutes with space and time differentiation. Applying (3) to (2), we get therefore (cf. Lilly, 1973, pp. 387–388)

$$\frac{\partial \hat{s}}{\partial t} + \nabla_p \cdot \widehat{s\mathbb{V}} + \frac{\partial}{\partial p} \widehat{s\omega} = \hat{S} \tag{4}$$

which can be written in the form

$$\frac{\partial \hat{s}}{\partial t} + \nabla_p \cdot \hat{s}\hat{\mathbb{V}} + \frac{\partial}{\partial p} \hat{s}\hat{\omega} = \hat{S} + A \tag{5}$$

where

$$A = -\nabla_p \cdot (\widehat{s\mathbb{V}} - \hat{s}\hat{\mathbb{V}}) - \frac{\partial}{\partial p} (\widehat{s\omega} - \hat{s}\hat{\omega}) \tag{6}$$

We notice from (5) that the smoothed flow satisfies basically the same equation as the unsmoothed flow (eq. (2)), except that all the quantities are replaced by the corresponding smoothed quantities and the source term contains a new term, A, which represents the effect of sub-grid scales on $\hat{s}$. It should be noted that A cannot generally be represented in terms of covariance of fluctuations, and the classical Reynold's postulates do not necessarily apply (e.g. in general $\hat{\hat{s}} \neq \hat{s}$).

Application of the above principles to the horizontal equations of motion gives for the smoothed flow a "frictional" force, the expression for which, on the basis of (6), is

$$\mathbb{A} = \mathbb{A}^H + \mathbb{A}^V \tag{7}$$

where

$$\mathbb{A}^H = A_x^H \mathbf{i} + A_y^H \mathbf{j}$$

$$A_x^H = -\frac{\partial}{\partial x} (\widehat{uu} - \hat{u}\hat{u}) - \frac{\partial}{\partial y} (\widehat{uv} - \hat{u}\hat{v}) \tag{8a}$$

$$A_y^H = -\frac{\partial}{\partial x} (\widehat{uv} - \hat{u}\hat{v}) - \frac{\partial}{\partial y} (\widehat{vv} - \hat{v}\hat{v}) \tag{8b}$$

and

$$\mathbb{A}^V = -\frac{\partial}{\partial p} (\widehat{\mathbb{V}\omega} - \hat{\mathbb{V}}\hat{\omega}) \tag{9}$$

Our purpose in this paper is not to conduct a general discussion concerning the properties of the "stresses" $\widehat{uu} - \hat{u}\hat{u}$, $\widehat{uv} - \hat{u}\hat{u}$ etc. and the associated forces $\mathbb{A}^H$ and $\mathbb{A}^V$, but rather to present an interesting example.

3. Evaluation of the horizontal sub-grid scale force $\mathbb{A}^H$ for a diffluent jet stream

Let us consider the case of a well-defined jet stream over northern Europe on April 9, 1974. This jet, which occurred between 300 and 400 mb, has previously been studied for other purposes by Shapiro (1976). Fig. 1 shows his analyses of the

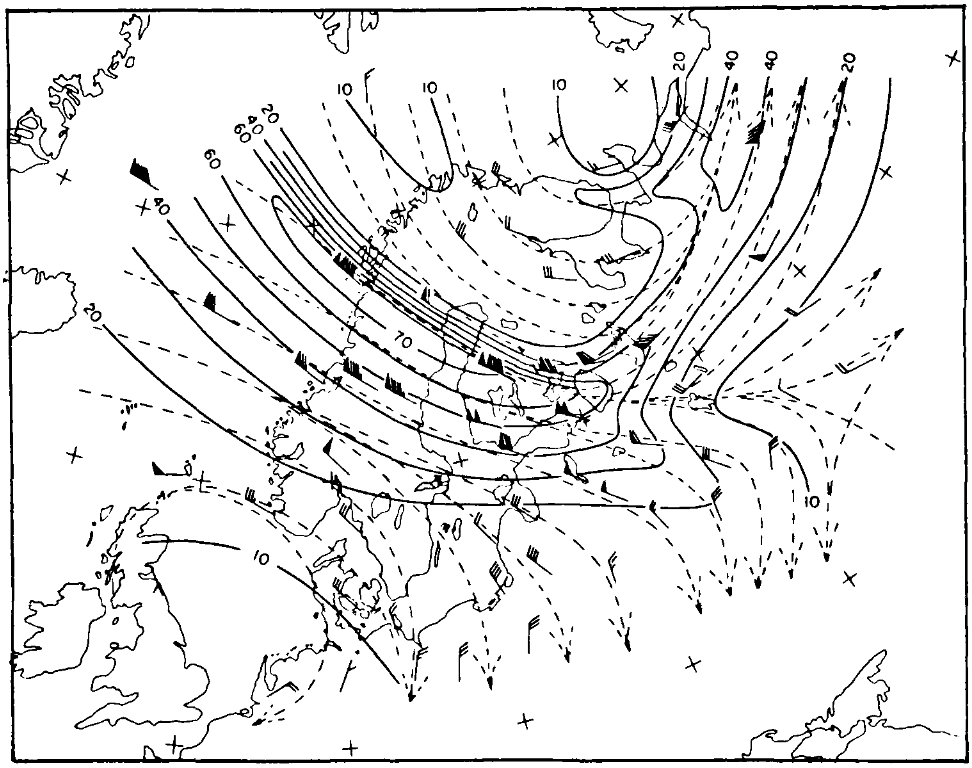

Fig. 1. Wind distribution over northern Europe on the 305 K isentropic surface for 00 GMT 9 April 1974. Unit for the isotachs: ms^{-1}. (From Shapiro, 1976, by permission of the American Meteorological Society.)

wind field on the 305 K isentropic surface, on which the jet stream had its highest intensity.

Calculations of $\mathbb{A}^H$ (see eq. (8)) were performed for the 300 mb surface. First, manual analyses were made of the wind components u and v. Values of u and v were then read off for the points of a rectangular grid having $\Delta x = \Delta y = 75$ km. These fields were then filtered by applying the weights shown in Fig. 2a (Holloway, 1958). The wave number response of this filter for a one-dimensional case is shown in Fig. 2b. It is seen that a 50% reduction in the wave amplitude occurs at a wavelength of about 400 km; shorter waves are effectively smoothed out in the filtering process, whereas longer waves are less affected. The resolution of the smoothed field obtained by applying this quasi-isotropic filter thus roughly corresponds to that of the analyses in the data-sparse areas (JOC, 1973) and to the grid size of many models used in the numerical prediction of the large-scale behaviour of the atmosphere.

The choice of the shape of the filter is by no means unique. For example, one may ask would it not be better, in the case of a jet stream, to apply a different weighting in the directions parallel to the flow and normal to the flow, with results probably different from those presented here. However, the resolution of, e.g. temperature measurements from satellites, which are likely to be important data for future global analyses, is the same in both directions. Therefore, the use of the simple, direction-independent filter applied here seems both justified and logical.

Fig. 3 shows the patterns of $\mathbb{V}$, $\hat{\mathbb{V}}$ and $\mathbb{V} - \hat{\mathbb{V}}$ for the exit area of the jet stream at 300 mb. The effect of filtering is seen essentially as a spreading out of the jet stream with the result that the "eddy" field $\mathbb{V} - \hat{\mathbb{V}}$ roughly has a character of vector bands parallel to the jet stream.

Fig. 4a shows the force $\mathbb{A}^H$ calculated on the basis of eq. (8). It can be seen that this force acts predominantly in the direction of the flow with a a)

$\frac{1}{256}$	$\frac{4}{256}$	$\frac{6}{256}$	$\frac{4}{256}$	$\frac{1}{256}$
$\frac{4}{256}$	$\frac{16}{256}$	$\frac{24}{256}$	$\frac{16}{256}$	$\frac{4}{256}$
$\frac{6}{256}$	$\frac{24}{256}$	$\frac{36}{256}$	$\frac{24}{256}$	$\frac{6}{256}$
$\frac{4}{256}$	$\frac{16}{256}$	$\frac{24}{256}$	$\frac{16}{256}$	$\frac{4}{256}$
$\frac{1}{256}$	$\frac{4}{256}$	$\frac{6}{256}$	$\frac{4}{256}$	$\frac{1}{256}$

b)

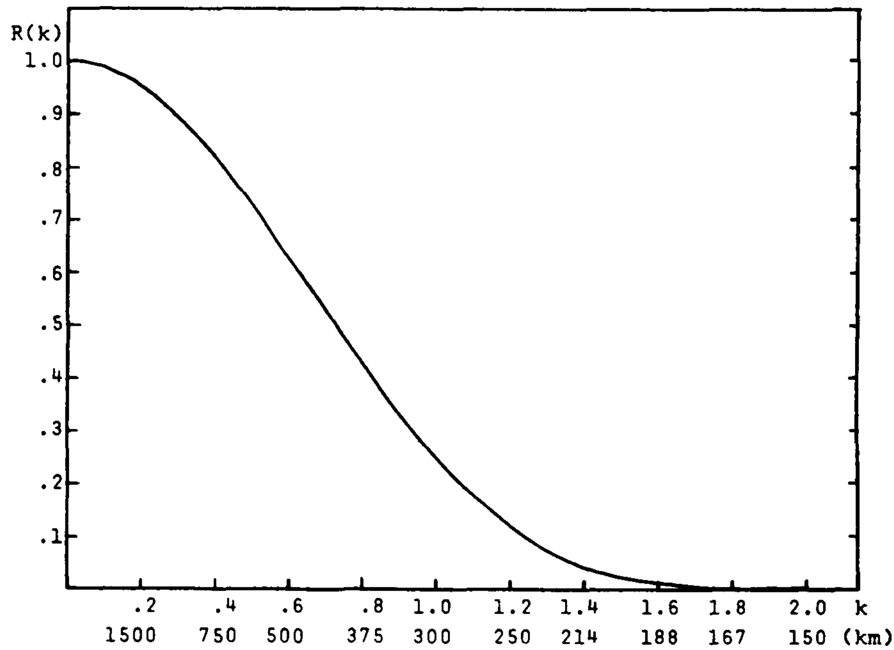

Fig. 2. (a) The weights used in the filtering. (b) The wave number response $R(k)$ of the filter used in one-dimensional case. $R(k) = w_0 + 2w_1 \cos(2\pi k\Delta x) + 2w_2 \cos(4\pi k\Delta x)$, where $w_0 = 6/16$, $w_1 = 4/16$, $w_2 = 1/16$.

maximum magnitude of about 5×10^{-4} ms^{-2}. Thus, the sub-grid scale processes tend to increase the velocity of the large-scale flow in this case of a diffluent jet. This forcing can be shown to be due mainly to the convergence of momentum flux in the longitudinal (parallel to the flow) rather than the lateral direction.

4. Parametrization problem

Three main ways of dealing with the effects of horizontal sub-grid scale systems have been used in the numerical models of atmospheric large-scale flow.

4.1. *The classical Fickian type of parametrization* $\mathscr{A}_F^H$

When applied to the $\hat{V}$ field, this parametrization gives

$$A_{xF}^H = K_F \nabla^2 \hat{u} \tag{10a}$$

$$A_{yF}^H = K_F \nabla^2 \hat{v} \tag{10b}$$

where $K_F = \text{constant}$.

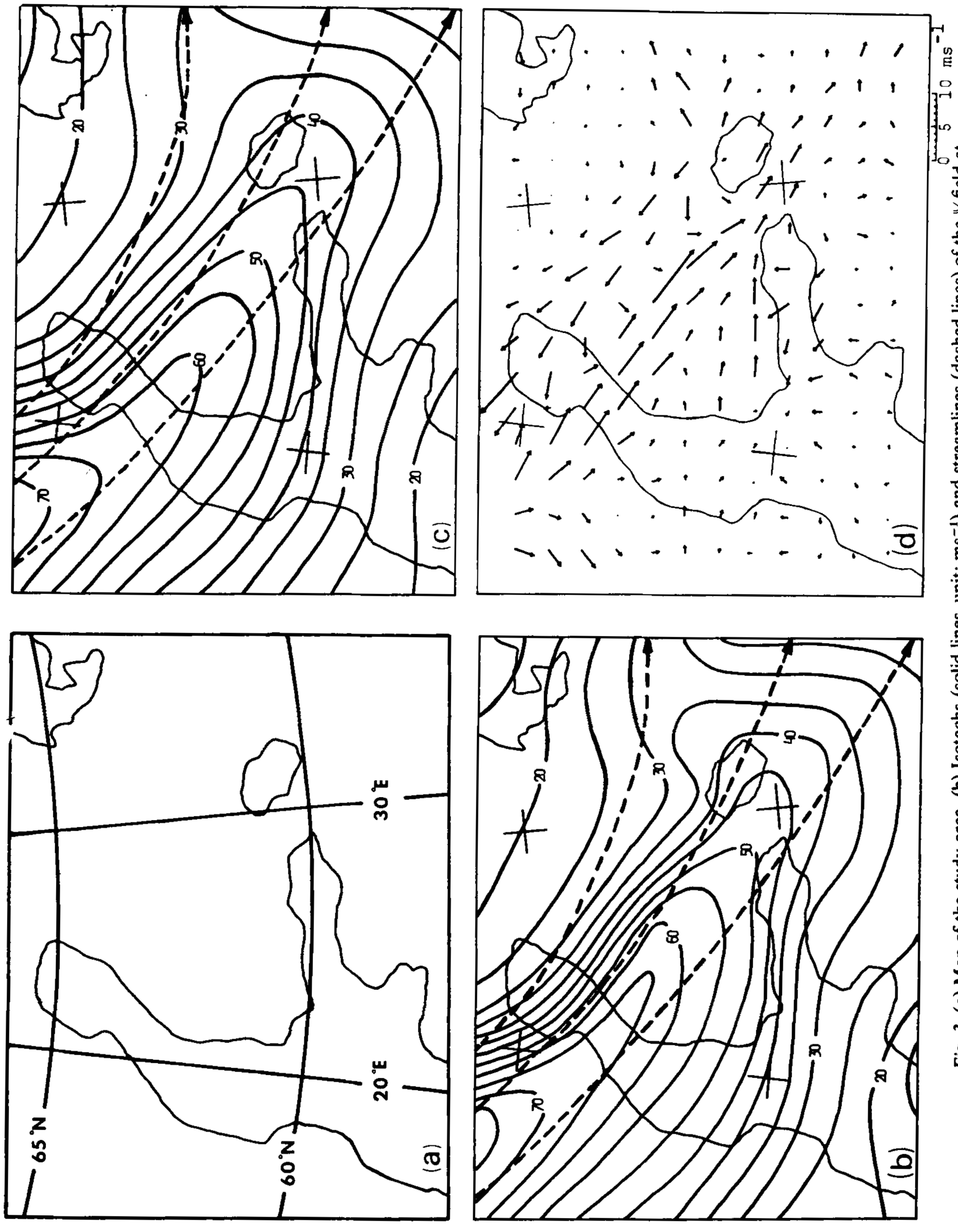

Fig. 3. (a) Map of the study area. (b) Isotachs (solid lines, unit: ms^{-1}) and streamlines (dashed lines) of the $\mathbb{V}$ field at 300 mb in the exit area of the jet stream in Fig. 1. (c) As in 3b but for the $\hat{\mathbb{V}}$ field. (d) The "eddy" velocity field $\mathbb{V} - \hat{\mathbb{V}}$ obtained from the fields seen in Figs. 3b and 3c.

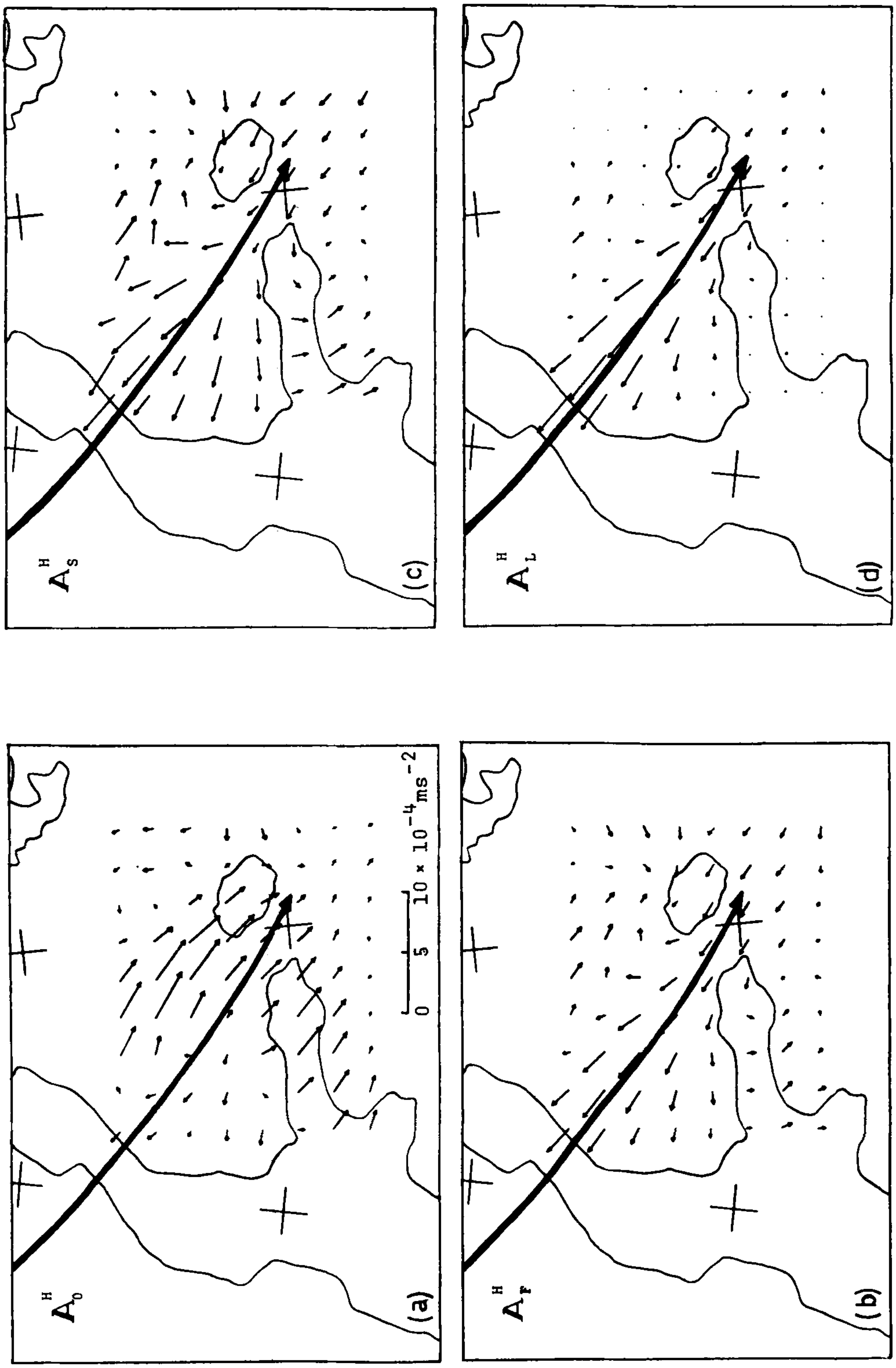

Fig. 4. Jet axis location (heavy arrow) and the distribution of the horizontal sub-grid scale force acting on the smoothed flow seen in Fig. 3c.
(a) A^H_O: "observed" force, determined from expression (8).
(b) A^H_F: "Fickian" formulation (see eq. (10)).
(c) A^H_S: "Smagorinsky" formulation (see eq. (11)).
(d) A^H_L: "Leith" formulation (see eq. (12)).

4.2. *Smagorinsky's (1963) formulation* $\mathbb{A}_S^H$

In this parametrization the frictional force depends upon the horizontal deformation, and we have

$$A_{xS}^H = \frac{\partial}{\partial x}(K_S D_T) + \frac{\partial}{\partial y}(K_S D_S) \tag{11a}$$

$$A_{yS}^H = \frac{\partial}{\partial x}(K_S D_S) - \frac{\partial}{\partial y}(K_S D_T) \tag{11b}$$

where

$$D_T = \frac{\partial \hat{u}}{\partial x} - \frac{\partial \hat{v}}{\partial y}$$

$$D_S = \frac{\partial \hat{v}}{\partial x} + \frac{\partial \hat{u}}{\partial y}$$

and

$$K_S = l_H^2 |D|$$

when $|D| = (D_T^2 + D_S^2)^{1/2}$ and l_H is proportional to the horizontal grid length.

4.3. *Leith's (1969) formulation* $\mathbb{A}_L^H$

In this parametrization, which assumes that the grid-scale falls in the enstrophy cascade range of two-dimensional turbulence, we have

$$A_{xL}^H = K_L \nabla^2 \hat{u} \tag{12a}$$

$$A_{yL}^H = K_L \nabla^2 \hat{v} \tag{12b}$$

where

$$K_L \sim h^3 |\nabla \hat{\xi}|$$

where h is the grid interval and $\hat{\xi}$ the vorticity ($= \partial \hat{v}/\partial x - \partial \hat{u}/\partial y$).

Figs. 4b–d show the vector fields $\mathbb{A}_F^H$, $\mathbb{A}_S^H$ and $\mathbb{A}_L^H$. The scaling is arbitrary (the constants in eqs. (10)–(12) have not been specified) but we are here interested mainly in the extent to which these parametrizations qualitatively simulate the "observed" force field $\mathbb{A}_O^H$ shown in Fig. 4a. It is seen that the force fields $\mathbb{A}_F^H$, $\mathbb{A}_S^H$ and $\mathbb{A}_L^H$ are very similar to each other and that they all act in almost the opposite direction to $\mathbb{A}_O^H$. Thus, to get even a rough representation of $\mathbb{A}_O^H$ by either (10), (11) or (12), negative eddy viscosity coefficients should be employed. We may thus conclude that, at least in the context of this particular situation and technique used, none of the classical formulations of horizontal friction work.

5. Discussion

The fundamental problem, highlighted by the example presented in this paper, concerns the dynamical forcing effect of unresolved scales on resolved scales. Some comments on both the method of approach and the results obtained are called for.

Scale interaction studies have normally been carried out using spectral representation of the horizontal hemispheric fields (e.g. Tsay and Kao, 1978). This approach is, however, purely mathematical and is not always well suited to a description of local atmospheric phenomena. For example, a large range of wave numbers, including the longest waves, is required to represent a hurricane or some other local phenomenon of relatively small scale. Although the exchange processes between different wave numbers in the wave number domain can be concisely formulated and investigated when the whole latitudinal belt is considered, this technique does not provide a picture of what kind of local effect certain small-scale phenomena have on the larger-scale flow upon which they develop. For this reason the literature does not provide any comparisons of the spatial distribution of $\mathbb{A}^H$, as determined from the different parametrization formulas, against the corresponding observed values.

In this paper we have used a horizontal filtering technique to split the fields into "resolved" and "unresolved" scales. This technique does not have the orthogonality of the spectral technique, however it does allow us to study the local effects of smaller scales on larger ones.

We have considered only a single case (a diffluent jet) and found that the smoothed flow is accelerated by horizontal sub-grid scale processes. The diffluent jet situation, however, is very common over western Europe. The processes described in the present article may therefore be one reason why, for example, diagnostic studies of the long-term kinetic energy balance in the upper troposphere over the British Isles give for the energy dissipation (when calculated as the residual term) values which mean energy input from the sub-grid scales into the larger scales (e.g. Holopainen, 1973). In the case of a confluent jet (a typical situation over North America) use of the technique presented here would be expected to produce a classical situation with $\mathbb{A}^H$ opposing the flow.

No general conclusions concerning the validity of the different parametrizations of the horizontal sub-grid scale forcing can be drawn on the basis of a single case study. One of the main points to emerge from this article is perhaps a warning: forcing due to unresolved horizontal scales may need more attention than is often believed (e.g. JOC, 1972).

It should be emphasized that we have not considered the forcing effect of all sub-grid scales, but only of those ranging roughly from 75 to 400 km. It may be that the forcing effect of the scales smaller than 75 km (on the scales larger than 400 km) can be parametrized in terms of a diffusivity coefficient. This question cannot, however, be studied only by the aid of ordinary synoptic data.

In addition to real data, a good basis for scale interaction studies employing the horizontal filtering technique would be the "data" from high resolution numerical models. Whatever the data source, both case studies with interesting situations and statistical studies with a large data sample should be performed.

REFERENCES

Holloway, J. L. 1958. Smoothing and filtering of time series and space fields. *Adv. in Geoph. 4*, 351–389.

Holopainen, E. O. 1973. An attempt to determine the effects of turbulent friction in the upper troposphere from the balance requirements of the large-scale flow: A frustrating experiment. *Geophysica 12*, 151–176.

JOC (Joint Organizing Committee) 1972. *Parametrization of sub-grid scale processes.* GARP publications series no. 8.

JOC (Joint Organizing Committee) 1973. *The first GARP global experiment—objectives and plans.* GARP publications series no. 11.

Lange, H. J. 1974. Übersicht über Methoden zur Untersuchung der energetischen Scale-Wechselwirkungen in der Atmosphäre. *Met. Abhandlungen der Freien Universität Berlin 109*, Heft 4.

Leith, C. E. 1969. Two dimensional eddy viscosity coefficients. *Proc. of the WMO/IUGG symposium on numerical weather prediction in Tokyo Nov. 26–Dec. 4, 1968.* Japan Met. Agency, Tokyo.

Lilly, D. K. 1973. Lectures in sub-synoptic scales of motion and two-dimensional turbulence. In *Dynamic meteorology* (P., Morel, ed.) D. Reidel Publishing Company, 353–418.

Robinson, G. D. 1978. Weather and climate forecasting as problems in hydrodynamics. *Mon. Wea. Rev. 106*, 448–457.

Smagorinsky, J. 1963. General circulation experiments with the primitive equations, I. The basic experiment. *Mon. Wea. Rev. 91*, 99–164.

Shapiro, M. A. 1976. The role of turbulent heat flux in the generation of potential vorticity in the vicinity of upper-level jet stream systems. *Mon. Wea. Rev. 104*, 892–906.

Tsay, C. Y. and Kao, S. K. 1978. Linear and nonlinear contributions to the growth and decay of the large-scale atmospheric waves and jet stream. *Tellus 30*, 1–14.

УСКОРЕНИЕ РАСХОДЯЩЕГОСЯ СТРУЙНОГО ТЕЧЕНИЯ ГОРИЗОНТАЛЬНЫМИ ПРОЦЕССАМИ ПОДСЕТОЧНОГО МАСШТАБА—ПРИМЕР ИЗУЧЕНИЯ ВЗАИМОДЕЙСТВИЯ МАСШТАБОВ С ИСПОЛЬЗОВАНИЕМ ГОРИЗОНТАЛЬНОЙ ФИЛЬТРУЮЩЕЙ ТЕХНИКИ

Горизонтальная фильтрующая техника используется для разделения течения на сглаженную (крупномасштабную) компоненту и подсеточную компоненту. Уравнение движения для сглаженного течения содержит силу, которая может бытъ интерпретирована в терминах возбуждающего эффекта, производимого движениями, которые исчезают в процессе сглаживания. Выражение для такого возбуждения (вызванного сглаживанием деталей, меньших чем 400 км в горизонтальном масштабе) оцениваются для синоптической ситуации с сильной расходящейся струей над северной Европой. Найдено, что сила для течения такого типа, который является обычным для Европы, ускоряет сглаженное течение. Далее показано, что такая сила не может быть параметризована с помощью горизонтальной силы трения типа Фика или с помощью более сложных выражений, предложенных Смагоринским (1963) и Лейсом (1969). В нашем конкретном случае все три параметризации приводят к почти одинаковым эффектам замедления.