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Available energy and the maintenance of a moist circulation

By EDWARD N. LORENZ, *Department of Meteorology, Massachusetts Institute of Technology, Cambridge, Mass. 02139, U.S.A.*

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ABSTRACT

Moist available energy is defined as the amount by which the potential plus internal (including latent) energy of a given atmospheric mass field exceeds that of a hypothetical reference field, which can be constructed from the given field by rearranging the atmospheric mass, under reversible dry-adiabatic and moist-adiabatic processes, to minimize the potential plus internal energy. *Dry available energy* is equal to the amount of moist available energy which would be present in a dry atmosphere having the same temperature field as the given moist atmosphere, and is identical with available potential energy.

Graphical procedures are presented for determining the moist and dry reference fields, and evaluating the available energies. In general the moist available energy exceeds the dry available energy.

Both heating and cooling can produce and can also destroy moist and dry available energy. Evaporation can produce moist available energy, while precipitation can destroy it. Preliminary computations based upon averages indicate that the total production of moist available energy by evaporation-precipitation is at least as great as the production by heating-cooling, and possibly much greater.

1. Introduction

The atmosphere possesses energy in many forms. Since it is a circulating atmosphere, it possesses kinetic energy (KE). Other forms which are directly or indirectly transformed into KE in significant amounts are potential and internal energy; these we shall refer to collectively as nonkinetic energy (NKE). Internal energy includes thermal energy and the latent energy of condensation and fusion of water.

Much of this energy is originally supplied to the atmosphere by the oceans, through the evaporation of water. On a somewhat smaller scale, many of the more spectacular weather events, from tropical hurricanes to polar blizzards, owe their existence to the water in the atmosphere. Nevertheless, among those studies which have been widely recognized as competent works in dynamic meteorology, those which deal with a "dry" atmosphere of uniform composition probably make up a majority.

The ultimate source of atmospheric energy is of course the sun. Mere external heating, however, need not drive a circulation. An essential feature of solar radiation as received by the earth is that it is horizontally non-uniform. Heating of this sort will always produce a circulation, at least in a dry atmosphere enveloping a planet with a uniform surface. For in that case, in the absence of a horizontal temperature contrast, the outgoing radiation would be horizontally uniform, and the incoming radiation would produce the temperature contrast. Moreover, in the absence of motion, the unbalanced pressure forces demanded by the temperature contrast would produce motion. Qualitatively it is as though there existed still another form of energy, which is measured in terms of the horizontal temperature contrast, generated by horizontally non-uniform heating, converted into KE by reversible adiabatic processes, and virtually unaffected by the irreversible processes which dissipate KE.

In a previous paper (Lorenz, 1955), hereafter referred to as A, we identified available potential energy (APE) as such a form. This is not a form of energy to be added to KE and NKE to make up the total, but is simply a portion of the existing NKE available for conversion into KE under suitable conditions. We defined it in A as the amount by which the NKE of the existing atmospheric state exceeds that of a hypothetical state, which can be constructed from the existing state by rearranging the mass of the atmosphere, under reversible adiabatic processes, to minimize the NKE. It is equivalent to a quantity which Margules (1903) introduced as "available kinetic energy" in his famous paper on the energy of storms.

We showed in A that APE can be approximated in terms of the variance of temperature within horizontal layers, while the production of APE can be approximated by the covariance of heating and temperature within these layers. The conversion of APE into KE is the same thing as the conversion of NKE into KE, while almost no APE is produced when frictional dissipation of KE produces NKE. In short, APE provides a convenient framework for relating the production of KE to the distribution of external heating.

In A we implicitly assumed that "reversible adiabatic processes" meant reversible dry-adiabatic processes. This does not mean that we ignored the release of latent energy which occurs during condensation. We simply did not treat moist-adiabatic processes as being adiabatic, and did not look upon latent energy as a form of internal energy; instead we treated the release of latent energy as a form of heating.

This approach does not introduce any quantitatively incorrect results. Nevertheless, it is not altogether satisfactory in investigations where water plays an important role. From a logical point of view, moist-adiabatic processes are adiabatic, and the release of latent energy is something which has to accompany a moist-adiabatic temperature decrease, rather than a form of external heating which might or might not occur. From a practical point of view, evaluation of the production of APE requires a knowledge of the regions where latent energy is released. It is not at all obvious how these should be related to the regions of surface evaporation. Hence APE fails to provide the ideal framework for relating *all* of the external influences to the production of KE.

The principal purpose of this work is to reformulate the concept of available potential energy in a manner which is more in keeping with the importance of water. Our definition will be unaltered, except that reversible adiabatic processes will include moist-adiabatic as well as dry-adiabatic processes. We shall call the newly defined quantity *moist available energy* (MAE). We shall use the term *dry available energy* (DAE) interchangeably with APE, as previously defined, and refer simply to *available energy* in statements which apply equally well to MAE and DAE. We shall subsequently investigate the effects of evaporation and precipitation, as well as heating and cooling, upon the production of MAE.

2. Graphical determination of the reference field

The state of the atmosphere at a particular moment is determined by its field of mass and its field of motion. The former is given for practical purposes by the spatial distributions of density, pressure, temperature, and the various phases of water. The latter is given simply by the distributions of the wind components. Although the two fields are often in approximate geostrophic equilibrium, we shall recognize the superposition of any field of motion upon any field of mass as a physically possible atmospheric state.

It is convenient to think of the atmosphere as consisting of a large number of "parcels" of equal mass. A parcel is supposed to be so small that within it the spatial variations of the meteorological variables can be disregarded. These variables will acquire new values as time progresses, but each parcel will continue to contain the same matter. Changes in the state of the atmosphere may then be looked upon as rearrangements of the parcels.

We shall call two fields of mass *equivalent* if each parcel can pass from its state in one field to its state in the other by a thermodynamically reversible adiabatic process. No restriction will be placed on the mechanical forces needed to rearrange the parcels. Given a global mass field, we seek among all equivalent fields the one which possesses the least NKE. We shall call this the *reference field*. The available energy is the difference between the NKE of the given field and that of the reference field.

In practice we must often work with hemispheric instead of global mass fields. Sometimes we purposely work with fields occupying limited regions of the atmosphere.

We shall deal separately with a dry atmosphere and a moist atmosphere, the latter being a mixture of dry air, water vapor, and liquid water. The liquid water will occur as cloud droplets. We shall not consider the possible presence of ice, which in any event would not greatly alter our results. We may therefore disregard the latent energy of fusion. We shall make the common assumption that if the total water content of a parcel is insufficient to produce saturation, there will be no cloud, while, if it is more than sufficient, all of the excess water will appear as a cloud. We shall assume a state of approximate hydrostatic equilibrium, and disregard the presence of topography.

The internal energy of a parcel depends upon its thermodynamic state. The potential energy depends instead upon its height z. Under hydrostatic conditions z is determined by the thermodynamic states of all the parcels in the same column; the relationship is in fact used to evaluate heights from radiosonde ascents. As a consequence, the total NKE of the atmosphere is determined by the distributions of the thermodynamic variables, and is, moreover, equal to the total enthalpy. The reference field is therefore the equivalent field possessing the least enthalpy. For the dry atmosphere enthalpy is synonymous with sensible heat; for the moist atmosphere it equals sensible heat plus latent energy.

In the dry atmosphere two variables will determine the thermodynamic state of a parcel. Common choices are pressure p and specific volume α, or p and temperature T. By contrast, three variables are needed in the moist atmosphere. A natural choice is p, α, and the total mass of water $\bar{q}$ per unit mass of the mixture. An alternative choice is p, T, and the relative humidity r, provided that a value of r exceeding unity indicates the presence of clouds rather than supersaturation.

Fig. 1 represents a hypothetical field of mass. The ordinate is p, decreasing upward, while the abscissa x, which ranges from 0 to 1, represents fractional area of the earth's surface. Mass is represented by area in the diagram, while surfaces become curves.

The solid and dashed curves have been freely transcribed from Figs. 10 and 16 of a previous work (Lorenz, 1967); these give estimates of the normal northern-hemisphere winter distributions of T and r. In transcribing the curves, we have identified $1-x$ with the sine of the latitude. With slight alterations we have made $z=0$ everywhere on the 1000-mb surface, and $T = 220$ K and $r=0.55$ everywhere on the 200-mb surface.

The field of Fig. 1, being based on averages, is presumably different from any instantaneous mass field which has ever occurred. In particular it has no clouds. It nevertheless shares some features with real global mass fields. Notable among these, besides the obvious poleward and upward temperature decrease, are the conditional instability and high moisture content at low levels in the tropics.

We shall use this field to demonstrate the procedure for finding the reference field. Because T and r are constant at 200 mb, the parcels below the 200-mb level may be rearranged independently of those above, and we shall not consider the region above 200 mb at all.

Precise determination of the reference field would involve rather awkward formulas which are best handled numerically. Pending the development of a suitable computer program, we shall offer a graphical procedure. The computer will be replaced by the meteorological thermodynamic diagram, or adiabatic chart.

The equation for a reversible adiabatic process is then replaced by the state curve for a parcel. The dry adiabat and the moist adiabat which form the state curve can be identified by the condensation pressure p_c and the condensation temperature T_c; these are the values of p and T at their intersection. The dry adiabat can also be identified by the condensation potential temperature θ_c while the moist adiabat can be identified by the equivalent potential temperature θ_e; these are the values which the potential temperature θ of the parcel acquires or approaches as p reaches p_c or approaches 0. In the absence of clouds $\theta_c = \theta$; in the absence of water vapor $\theta_e = \theta$.

In passing from the given field to an equivalent field a parcel retains its state curve and hence its values of θ_c and θ_e. Its new state can therefore be identified by specifying its new value of p. It follows that, given a mass field, the reference field can be completely described by specifying the pressure p_r which each parcel will acquire. We shall call p_r the *reference pressure*.

The adiabatic chart is perhaps more familiar to earlier generations of meteorologists than today's

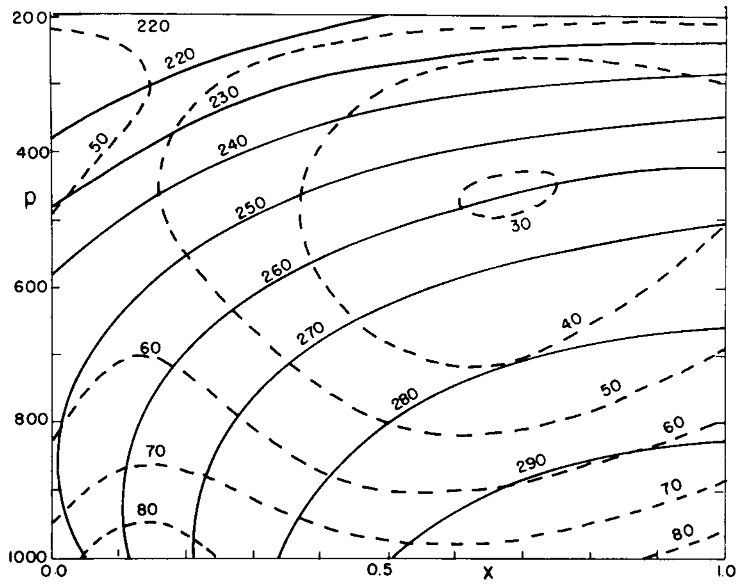

Fig. 1. A hypothetical mass field. Abscissa is fractional area of earth's surface. Ordinate is p (mb). Solid curves show T (K). Dashed curves show r (%).

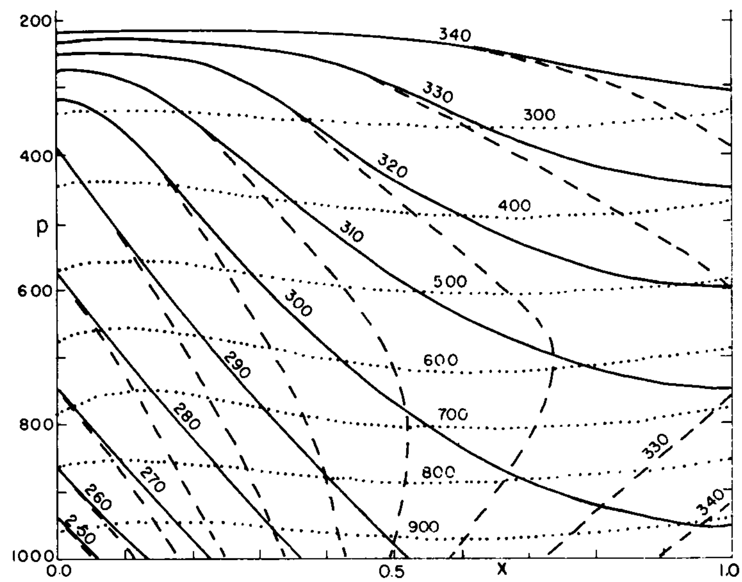

Fig. 2. Alternative representation of mass field of Fig. 1. Solid curves show θ_c (K). Dashed curves show θ_e (K); unlabeled θ_e-curves are asymptotic to θ_c-curves with same values. Dotted curves show p_c (mb).

computer-oriented generation. It is described in as much detail in earlier textbooks (Haurwitz, 1941; Petterssen, 1941) as in more recent ones. One feature should be noted.

The most important dynamic effects of moisture are those resulting from the release of latent heat during condensation, and the reverse process during evaporation. Secondary effects occur because the density of air containing water vapor or liquid water differs from that of dry air at the same pressure and temperature, and the effective specific heats are different. The secondary effects are disregarded in constructing the adiabatic chart. Without this simplification there would be an infinite number of dry adiabats and moist adiabats through each point on the chart, instead of just one of each.

Our graphical procedure must therefore be approximate. As an additional approximation we shall not distinguish between temperature and virtual temperature. This will not lead to any thermodynamic inconsistencies; it is possible, for example, to visualize a planet where "dry air" consists of a mixture of nitrogen, oxygen, and helium having the same effective molecular weight as water.

In Fig. 2 the mass field of Fig. 1 is represented by curves of constant θ_c and θ_e. Also shown are curves of constant p_c. All of the values have been determined from an adiabatic chart. We seek the curves of constant p_r.

Among the possible transformations of a given mass field into an equivalent field, certain ones can be realized by superposing an arbitrary motion field on the mass field, and then allowing the atmospheric state so obtained to vary according to a closed system of equations, including the reversible thermodynamic equation without heating, the equation of mass continuity, and the equations of motion without friction. The change in the NKE of the mass field during such a transformation is equal and opposite to the change in the KE of the accompanying motion field.

From these transformations we can readily deduce certain properties of the reference field. First of all, the isobaric surfaces must be horizontal. For otherwise we could superpose a zero motion field on the reference field, and obtain a state where the horizontal accelerations would produce KE. In particular, the surface pressure p_0 must be constant, and approximately equal to its average value $\bar{p}_0$ in any equivalent field.

Next, the field must be in exact hydrostatic equilibrium. For otherwise we could again superpose a zero motion field on the reference field, and the vertical accelerations would produce KE. Hence α must also be horizontally stratified, and so therefore must every function of α and p, including the (virtual) temperature.

Finally, the stratification must be stable. For otherwise we could superpose a weak random motion field on the reference field, and the KE would subsequently increase. The conditions for stability are precisely those appearing in classical atmospheric convection theory; in the dry atmosphere θ cannot decrease with increasing elevation, while in the moist atmosphere θ_c cannot decrease in unsaturated air and θ_e cannot decrease in saturated air. Furthermore there can be no real latent instability (see the textbooks previously cited).

In the dry atmosphere it follows that θ is horizontally stratified. This requirement, together with the upward increase of θ, immediately determines the reference field. A p_r-curve, say $p_r = p_1$, is a θ-curve, say $\theta = \theta_1$. Moreover, $p_1/\bar{p}_0$ is the fraction of the mass of the atmosphere where θ exceeds θ_1.

For the dry atmosphere with the same temperature field as the moist atmosphere shown in Fig. 1, the field of θ is the same as the field of θ_c in Fig. 2, since there is no saturation. The corresponding reference field is described by relabeling the θ-curves with the appropriate values of p_r; these appear in Fig. 3. The curve $p = p_r$, which separates the parcels which rise in passing to the reference field from those which sink, is also included.

In the moist atmosphere the horizontal stratifications of p and α do not require that any quantity which is constant along a state curve be horizontally stratified everywhere. We find instead that θ_c must be horizontally stratified within the unsaturated portion of the reference field while θ_e must be horizontally stratified within the saturated portion. A p_r-curve therefore consists of a part of a single θ_c-curve, together with a part of a single θ_e-curve. Moreover the intersection of these curves, if it exists, lies on the curve $p_r = p_c$.

To locate the p_r-curves in Fig. 2, consider a curve of constant p_c, say $p_c = 800$ (mb). Choose an arbitrary point C on this curve, say where $x = 0.4$. Through C we find the curves $\theta_c = 290$ (K) and $\theta_e = 300$. If C lies on the curve $p_r = 800$, and hence on $p_r = p_c$, the remainder of the curve $p_r = 800$ must consist of the portion of the curve $\theta_c = 290$ which is unsaturated in the reference field, and hence where $p_c < 800$, and the portion of the curve $\theta_e = 300$ which is saturated in the reference field, and hence where $p_c > 800$. That is, it must consist of the portions of the θ_c-curve extending upward and the θ_e-curve extending downward from C. A further condition, imposed by the upward increases

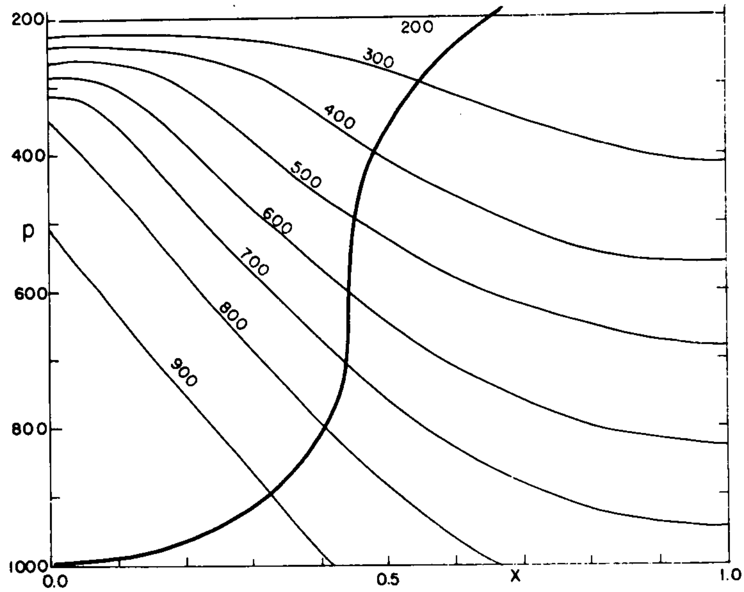

Fig. 3. Distribution of reference pressure p_r (mb) for dry atmosphere with same temperature distribution as moist atmosphere of Fig. 1. Heavy solid curve is curve $p_r = p$.

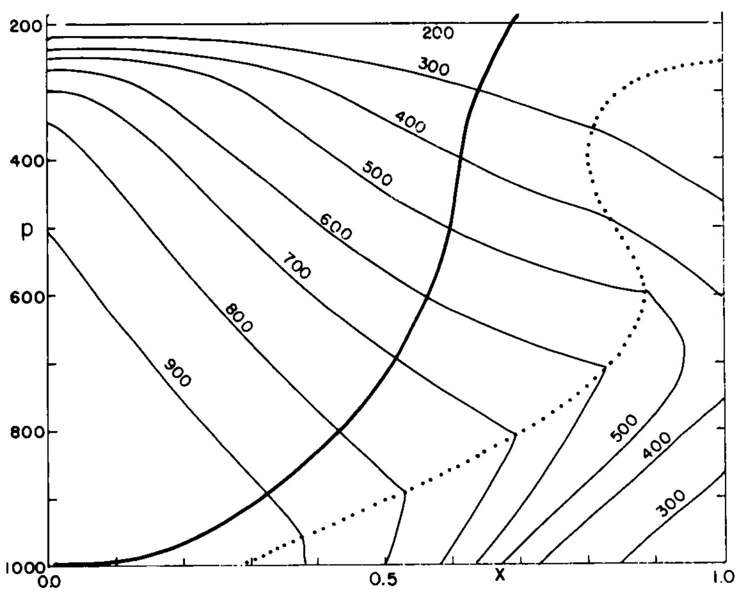

Fig. 4. Distribution of reference pressure p_r (mb) for moist atmosphere of Fig. 1. Heavy solid curve is curve $p_r = p$. Heavy dotted curve is curve $p_r = p_c$.

of θ_c and θ_e, is that 8/10 of the area of Fig. 2 (extended to $p = 0$) must lie above the curve $p_r = 800$, and 2/10 must lie below.

By measurement (counting squares on graph paper) we find that the area below is too small. If a second point farther to the right is chosen, the area becomes larger. By trial and error we find that the curve $p_r = 800$ intersects the curve $p_c = 800$ at x

$= 0.53$, and consists of portions of the curves $\theta_c = 295$ and $\theta_e = 311$.

Repeating this procedure for other values of p_r, we finally obtain Fig. 4, which is analogous to Fig. 3. The curve $p_r = p_c$, which passes through the kinks in the other curves, separates air which becomes saturated in the reference field from air which remains unsaturated. For the field of Fig. 1, the problem is solved.

Because this field is rather specialized, the solution of the general problem involves additional considerations. First of all, the atmosphere is three-dimensional, while Fig. 1 is clearly two-dimensional. We cannot expect in general that all parcels having a given pressure and temperature will have the same relative humidity. To handle the three-dimensionality, we note that we can replace a given mass field by any equivalent mass field without affecting the reference field.

Suppose that the given field is represented by the distributions of p, T, and r in a large number of vertical columns, obtained perhaps from individual radiosonde ascents. For any condensation pressure p_{c1} we can measure the fraction of the mass of each column, and hence of the atmosphere, for which $p_c < p_{c1}$; let $p_1/\bar{p}_0$ denote this fraction. We can then construct an intermediate field by placing all parcels with condensation pressure p_{c1} at the pressure level p_1, and arranging the parcels so that θ_c, and hence θ_e, increases with increasing x. This field will be two-dimensional, and may be treated in the same manner as the field in Fig. 1.

Next, although the p_r-curves are found by locating the curve $p_r = p_c$, there is always the possibility that, for some p_1, the curves $p_r = p_1$ and $p_c = p_1$ will not intersect at all. This would have happened near $p_1 = 500$ in the case just considered, for example, if the relative humidity had been somewhat lower. We can deal with this potentiality by appending to the right-hand edge of the diagram a strip of negligible width. Across the strip we let T retain the same value as at $x = 1$, but let r increase to values as large as desired. The curves $p_r = p_1$ and $p_c = p_1$ will then intersect within the strip. In effect we shall have added to the given field a single cumulonimbus cloud. Since real global mass fields invariably contain cumulonimbus clouds, we shall not have done violence to the given field, nor to the reference field.

A more serious problem can arise when a considerable portion of the given field is conditionally unstable. It may then occur that, as we determine the intersections of the curves $p_r = p_1$ with $p_c = p_1$ for successively higher values of p_1, we shall at some point encounter higher values of θ_e, even though we continue to encounter lower values of θ_c. The p_r-curves would then cross one another. Clearly this cannot happen, and the procedure is unacceptable.

A solution is as follows. We let the curve $p_r = p_c$ determined by the original procedure be a provisional curve. We accept this curve from the top of the atmosphere down to the point where it first becomes tangent to a θ_e-curve, say $\theta_e = \theta_{e1}$, where $p_c = p_{c1}$. Below the point where the provisional curve again intersects the curve $\theta_e = \theta_{e1}$, where $p_c = p_{c2}$, if such an intersection exists, we again accept the curve, unless it again becomes tangent to a θ_e-curve, in which case we repeat the process.

For values of p_r between p_{c1} and p_{c2}, however, the p_r-curves consist entirely of segments of θ_c-curves, originating at the left of the diagram and terminating on the curve $\theta_e = \theta_{e1}$. Within this interval the curve $p_r = p_c$ does not exist, and the curve which separates saturated air from unsaturated air in the reference field is the curve $\theta_e = \theta_{e1}$. Parcels which are close together but on opposite sides of this curve in the given field may be far apart in the reference field.

We suspect that this situation does not often arise in the course of determining a global or hemispheric reference field. We are more likely to encounter it in working with a limited region, which can easily be conditionally unstable everywhere.

3. Determination of the available energy

Having found the reference field, we can also evaluate the available energy graphically. Fig. 5 illustrates the method for the dry atmosphere. It is an adiabatic chart, without the background curves. The abscissa is θ and the ordinate is $(p/\bar{p}_0)^\kappa$, decreasing upward. Here κ, assumed equal to 2/7, is the ratio of the gas constant R to the specific heat c_p at constant pressure.

The chart is an equal-area transformation of a p–α diagram. It is more convenient for our purposes than the more familiar chart with coordinates T and $\ln p$; the dry adiabats are exactly vertical, and the ordinate at the top of atmosphere is finite.

The heavy curve is a "sounding" through the reference field, i.e., a graph of $(p_r/\bar{p}_0)^\kappa$ against θ,

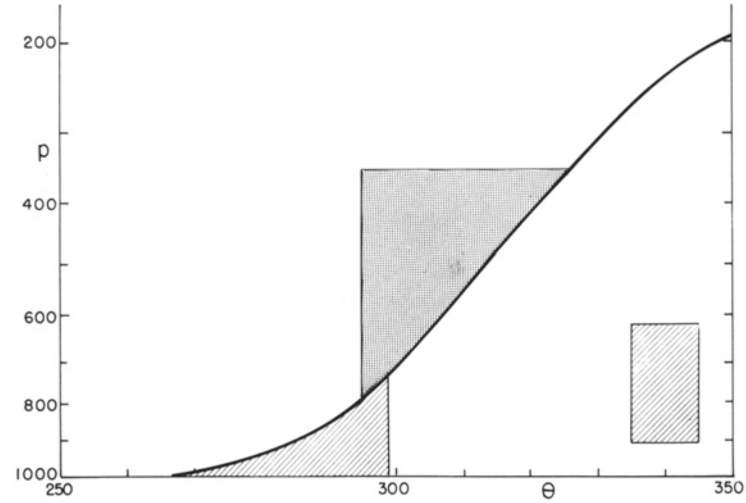

Fig. 5. Graphical procedure for evaluating dry available energy. See text for explanation.

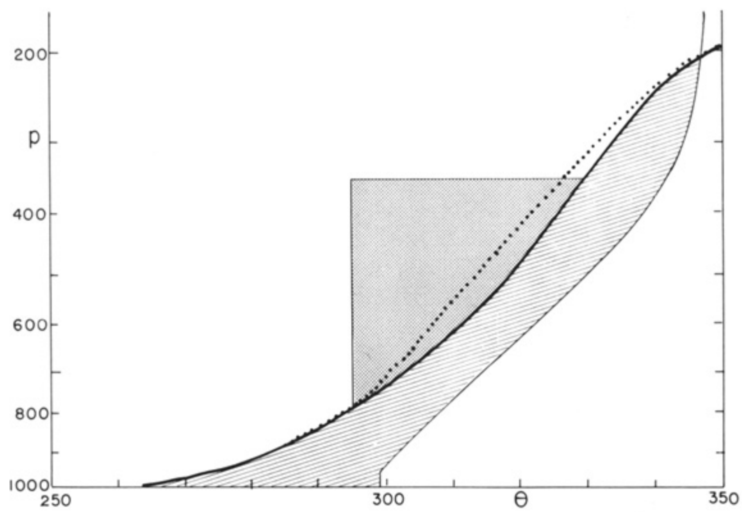

Fig. 6. Graphical procedure for evaluating moist available energy. See text for explanation.

determined from Fig. 3. The thin lines are state curves for two selected parcels, located in Fig. 1 at $x = 1$, $p = 1000$, where $\theta = 299$, and $x = 0$, $p = 350$, where $\theta = 295$. Since the atmosphere is dry the state curves are dry adiabats and are therefore vertical.

The decrease in the specific enthalpy h of a parcel as it rises through a segment of its state curve is proportional to the area on the chart (extended to $\theta = 0$) to the left of this segment. The factor of proportionality is c_p ($= 1000\ \mathrm{m^2\ s^{-2}\ K^{-1}}$). The shaded rectangle in the lower right of Fig. 5 is of unit area (in K), and so represents a specific enthalpy of $10^3\ \mathrm{J\ kg^{-1}}$. Thus, as the lower parcel ascends to its reference pressure, h drops by $26 \times 10^3\ \mathrm{J\ kg^{-1}}$, while, as the upper parcel descends, h rises by $57 \times 10^3\ \mathrm{J\ kg^{-1}}$. The total loss of enthalpy, and hence the available energy, may be obtained by summing the areas for all parcels. However, since many imperfectly estimated areas of opposing signs must be combined, the error could well be comparable to the result.

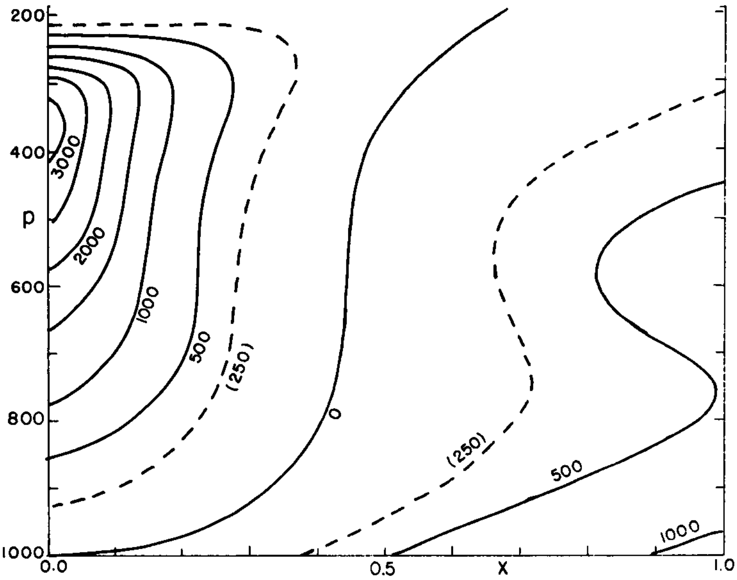

Fig. 7. Distribution of specific DAE a_d (J kg^{-1}) for dry atmosphere with same temperature distribution as moist atmosphere of Fig. 1.

Since as many parcels pass upward across each pressure level as downward, we do not alter the sum of the areas by first subtracting from each area to the left of the state curve the area to the left of the same portion of the reference sounding. Since the rising parcels are all to the right of the sounding while the sinking parcels are all to the left, we obtain a positive (or zero) net area for each parcel. These areas have been shaded in Fig. 5; their magnitudes are 1.1 K and 3.1 K, the estimates again being made by counting squares on graph paper.

Fig. 6 is like Fig. 5, but for the moist atmosphere. The heavy solid curve is the reference sounding determined from Fig. 4, while the dotted curve is the dry reference sounding copied from Fig. 5. The two soundings are identical near the ground and the tropopause, but in much of the middle troposphere the moist sounding is nearly 5 K warmer. The parcels are the same as before, but now the state curve for the lower parcel follows a dry adiabat only up to $p = 960$, and a moist adiabat from there up to $p = 210$ where it meets the reference sounding. As a consequence a much larger area (4.4 K) lies between the state curve and the sounding. The upper parcel also produces a larger area (3.9 K), mainly because the reference sounding is farther to the right.

Since a reference sounding is not a state curve, the area to its left does not directly represent enthalpy. However, the sounding can be approximated by a large number of short segments of state curves. Each shaded area therefore represents the net enthalpy loss which will occur if the corresponding parcel rises, or sinks, to its reference pressure, while the numerous parcels which it passes all sink, or rise, by minute amounts to fill the gap. We shall call such an enthalpy loss the *specific dry available energy* a_d or the *specific moist available energy* a_m of the parcel, according to whether the atmosphere is treated as dry or moist. For the parcels in Figs. 5 and 6, a_d equals 1100 and 3100 J kg^{-1}, while a_m equals 4400 and 3900 J kg^{-1}. To our knowledge, the specific available energy of a parcel has not previously been defined.

The parcels in Figs. 5 and 6 were chosen as extremes. Most parcels contribute much less to the total. Figs. 7 and 8 show the distributions of a_d and a_m for the dry and moist atmospheres. In either case the larger values occupy small portions at low levels in warm regions, and at high levels in cold regions. The big contribution in the moist case which has no counterpart in the dry case is due to the warm humid air near the surface in the tropics.

By again counting squares in Figs. 7 and 8 we have produced the curves in Fig. 9. The ordinate is

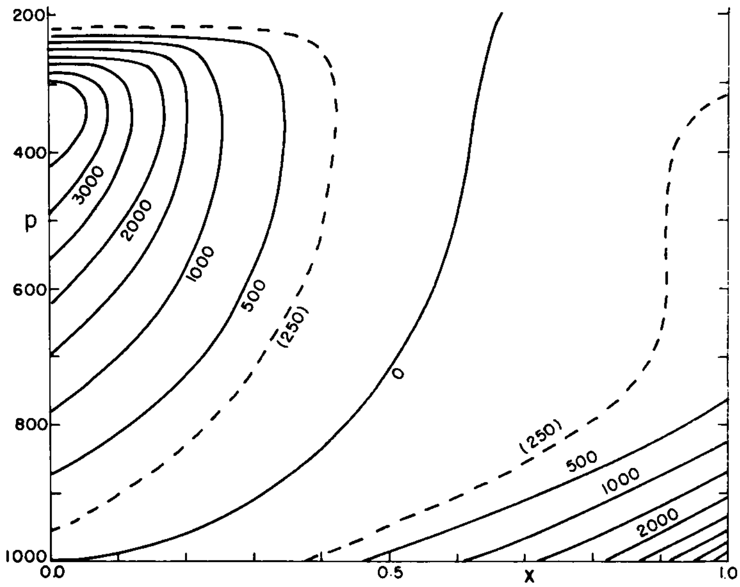

Fig. 8. Distribution of specific MAE a_m (J kg^{-1}) for moist atmosphere of Fig. 1.

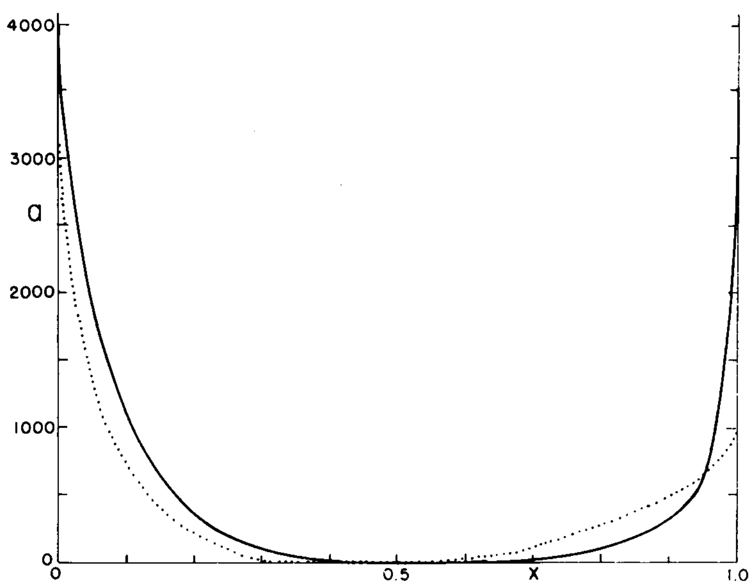

Fig. 9. Probability distributions of a_d (dotted curve) and a_m (solid curve) determined from Figs. 7 and 8. Ordinate is a_d or a_m (J kg^{-1}). Abscissa is fractional area to left of corresponding a_d- or a_m-curve in Fig. 7 or 8.

the label on a curve in Fig. 7 or Fig. 8, while the abscissa represents the area to the left of the same curve. Above 200 mb a_d and a_m are assumed to vanish. The average heights of the dotted and solid curves are proportional to the total DAE and MAE, respectively. These are found to be 330 and 400 joules per kg of the atmosphere's mass.

For the general case a preliminary step is needed, since p_0 is not ordinarily constant. We construct an intermediate equivalent field by

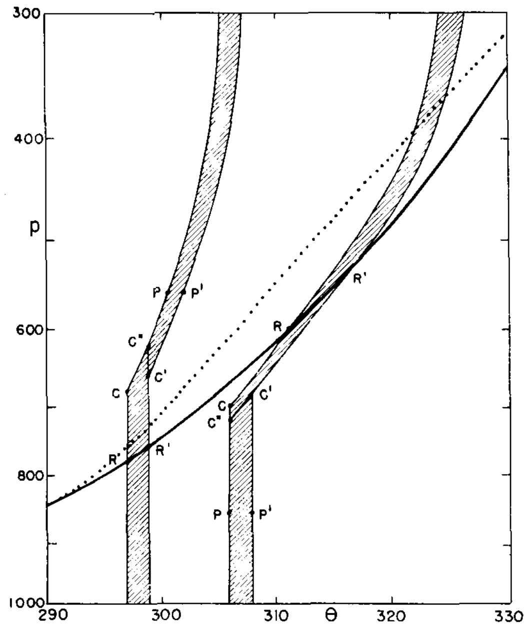

Fig. 10. Graphical procedure for evaluating efficiency factors N and N'. See text for explanation.

displacing each parcel whose pressure exceeds $\bar{p}_0$ along its state curve to a pressure less than $\bar{p}_0$, thus "filling in" the bottom of the region where $p_0 < \bar{p}_0$. The additional contribution of each of these parcels to the available energy is represented by the full area to the left of the segment of the state curve, and may therefore be substantial. However, since not too many parcels are involved, the total effect need not be too large. If, for example, p_0 varies linearly from 1040 to 960 mb, the net contribution will be about 20 J kg^{-1}.

The result that MAE exceeds DAE if there is any saturation is general. To establish this, we may construct an intermediate equivalent mass field by displacing each parcel along its moist state curve to the pressure which it would assume in the *dry* reference field. Again as many parcels pass upward across each pressure level as downward, so that the areas between the moist state curves and the dry reference sounding again represent enthalpy changes. For those parcels which are unsaturated in the given and intermediate fields, the areas are the same as in the dry case (Fig. 5); for those parcels which encounter saturation, the areas are greater. The loss of enthalpy in passing from the given to the intermediate field therefore exceeds the DAE, while, since the intermediate field is not the moist reference field, the MAE is still larger.

It is of interest that Margules (1903) concluded that water would have little effect on the amount of available energy. However, in his highly idealized treatment, he compared a dry air mass having a dry-adiabatic lapse rate with a moist air mass having a moist-adiabatic lapse rate and the same average temperature. Had he compared two masses with the same lapse rates as well as temperatures, he would have found more available energy in the moist case.

We may compare our numerical values with the composite estimate of 550 J kg^{-1} for (dry) APE, obtained by Oort (1964). Both of our values are smaller, but, since Fig. 1 is based on averages, our values should be better representations of zonal available energy, which Oort estimated to be 400 J kg^{-1}. Moreover, since the real contribution of parcels above 200 mb is not zero, our values are presumably underestimates. They are therefore in agreement with what we should have expected. Very likely we have underestimated the *ratio* of MAE to DAE by averaging the relative humidities.

4. Generation and destruction of available energy

Like available energy itself, the influence of heating and cooling, and also evaporation and precipitation, upon available energy can be evaluated graphically. The area to be measured for this purpose is the area between the state curve of a parcel and the state curve which the same parcel will subsequently possess.

Fig. 10 illustrates the evaluation procedure. It consists of an enlarged section of an adiabatic chart. The heavy solid curve is the moist reference sounding copied from Fig. 6. The two points labeled P represent the states of two parcels, chosen so that p_c lies between p and p_r in either case. For one parcel $p = 860$, $T = 293$, $r = 0.40$, while for the other $p = 560$, $T = 254$, $r = 2.00$ (meaning that the liquid water content equals the water vapor content). Parcels with these states do not actually appear in Fig. 1. The two curves PCR and their extensions are the state curves.

A suitable amount of heating will displace the points P to the points P'. The curves $P'C'R'$ and their extensions are the resulting state curves. Since

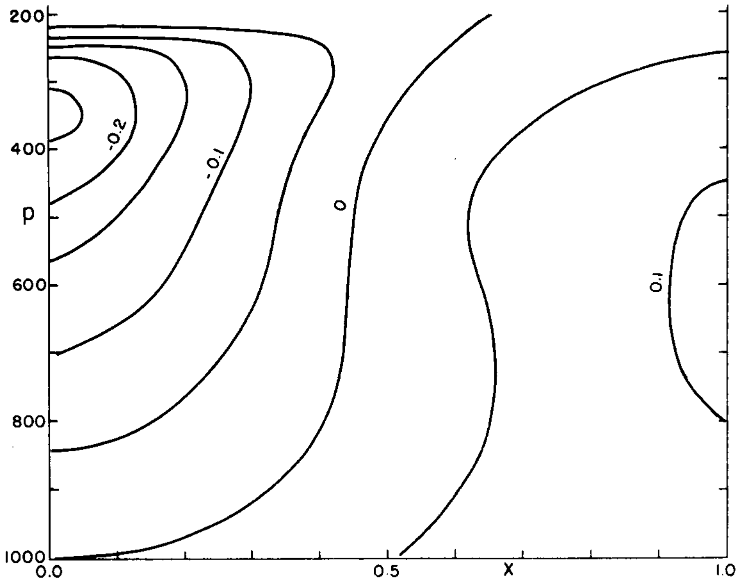

Fig. 11. Distribution of N (dimensionless) for dry atmosphere with same temperature distribution as moist atmosphere of Fig. 1.

the mass of a parcel is negligible compared to that of the atmosphere, changes in the reference field need not be considered. The heating raises the condensation levels; thus, as one moves upward past C, the curves PCR and $P'C'R'$ become suddenly much closer, then slowly diverge. The changes in MAE are represented by the areas $PCRR'C'P'$.

To determine the effect of an arbitrary amount of heating (or cooling) we can define an efficiency factor N as the ratio of the gain in available energy to the total heating. The latter amount equals the accompanying gain in enthalpy, and is represented by the portion of the shaded area extending from PP' to the top of the atmosphere. In measuring this area we note that Fig. 10 would need to have been drawn 3.44 times as tall to reach the zero pressure level. The extensions of the state curves are, for practical purposes, parallel above 300 mb, where $(p/p_0)^\kappa = 0.709$. The values of N are the limiting ratios of the former areas to the latter as P' approaches P; these are found to be +0.043 for the right-hand parcel and –0.062 for the left.

For the dry atmosphere the evaluation of N proceeds similarly, but the state curves are vertical throughout the atmosphere. As a consequence it is unnecessary to measure areas; lengths will suffice. The dotted curve is the dry reference sounding copied from Fig. 5; using this instead of the solid curve, we find that the values of N are +0.038 and –0.066.

Fig. 11 shows the distribution of N for the dry atmosphere. Heating in the tropics is at most 10% effective. Cooling in the upper polar troposphere seems to be the most efficient means of producing DAE.

The distribution of N for the moist atmosphere appears in Fig. 12. The extreme values are appreciably larger than in the dry case. Evidently the mere presence of water vapor can make heating in the tropics more efficient.

To determine the direct effect of evaporation and precipitation we return to Fig. 10. By evaporating a suitable amount of water into the right-hand parcel or precipitating a suitable amount from the left, without simultaneously changing the temperature, we can replace the state curves by curves passing through the points R'. The shaded areas then have zero width between P and the new condensation levels C'', and the changes in MAE are represented by the areas $CRR'C''$, which are smaller than the areas $PCRR'C'P'$.

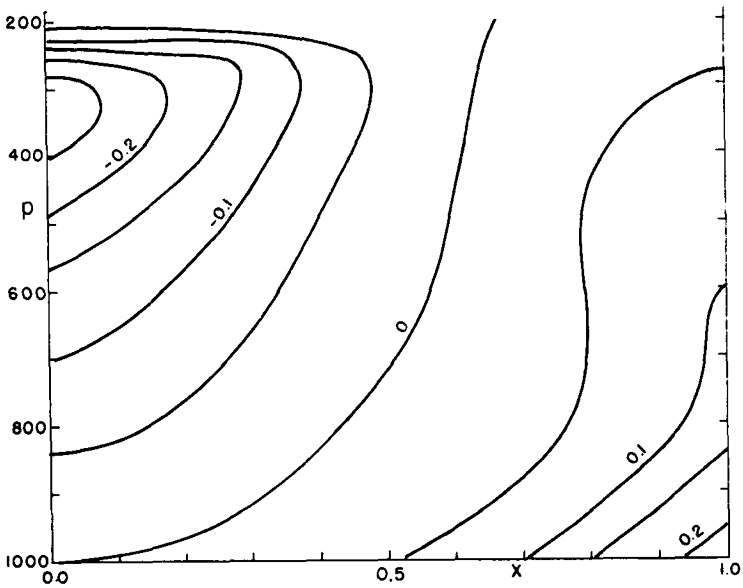

Fig. 12. Distribution of N (dimensionless) for moist atmosphere of Fig. 1.

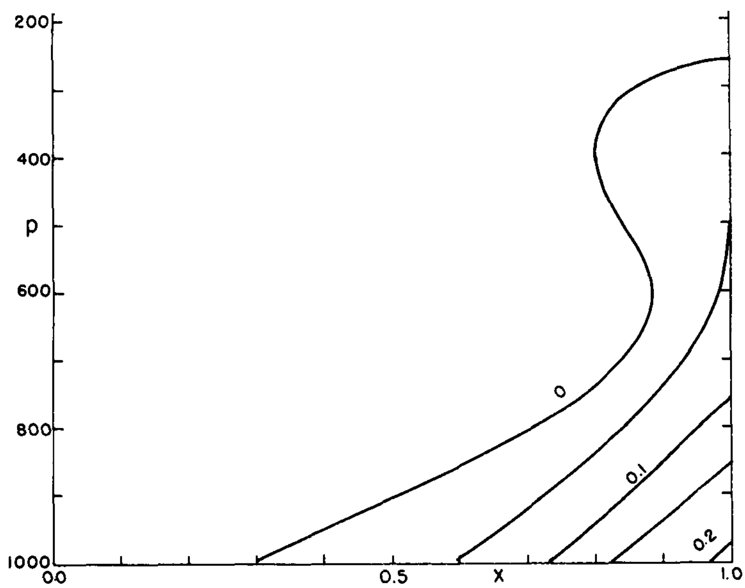

Fig. 13. Distribution of N' (dimensionless) for moist atmosphere of Fig. 1.

For evaporation we can define a second efficiency factor N' as the ratio of the gain of MAE to the latent energy added. The latter is equal to the gain in enthalpy, and is represented by the shaded area extending from CC'' to the top of the atmosphere. We find that $N' = +0.020$.

For precipitation there is no enthalpy change. We therefore define N' as the ratio of the (negative) gain in MAE to the latent energy which the precipitating water possessed before it condensed. To evaluate the latter amount graphically we note that if, following the precipitation, the parcel at P

should be lowered to C', approximately the desired amount of latent energy would be added by evaporating as much water as had precipitated, without changing the temperature. This would lower the condensation level from C'' back to C', and the accompanying gain in enthalpy would be represented by the shaded area extending from $C''C'$ to the top of the atmosphere. We find that $N' = -0.041$.

These results can be generalized. When a parcel is on the verge of saturation, points P and C coincide. Hence $N' = N$, and evaporation and precipitation are just as efficient as heating in producing MAE. When C is much closer to P than to R, as is typical, for example, in unsaturated air at low levels in the tropics, and in clouds at higher levels in the polar regions, evaporation is nearly as efficient as heating. When C is instead close to R, evaporation and precipitation are inefficient, and when C passes R, N' vanishes. Thus evaporation into air which remains unsaturated in passing to the reference field, e.g., into cold dry air masses moving over polar oceans, and precipitation from air which remains saturated in passing to the reference field, e.g., from cumulonimbus clouds in the tropics, do not affect MAE at all.

When the evaporation occurs from falling rain instead of from the surface, it is accompanied by cooling. According to our analysis, the cooling will have the greater influence on MAE, since N is numerically greater than N'.

Fig. 13 shows the distribution of N' for the moist atmosphere. Evaporation is necessarily effective only below the curve $p_r = p_c$, and most efficient in the tropics. No negative efficiencies (for precipitation) appear above the curve $p_r = p_c$, since no clouds were assumed in Fig. 1. With clouds, negative values generally smaller than those in Fig. 12 would appear in similar locations.

5. Maintenance of a moist circulation

We have shown that it is feasible to redefine available potential energy, basing the new definition upon dry-adiabatic and moist-adiabatic changes of state. To show that we have done something beyond finding a pretty solution to a pretty problem, we must demonstrate that the new concept can eventually add to our understanding of the general circulation.

In A we compared a typical temperature distribution with a typical distribution of heating, and obtained a crude estimate of 4 watts per m^2 of the earth's surface for the rate at which APE is generated. On the average this rate must equal the rate at which APE is converted into KE, thus maintaining the circulation, and also the rate at which KE is dissipated.

By considering the typical wind structure in the boundary layer, Brunt (1926) estimated the latter rate to be 5 W m^{-2}. Computation of the generation of APE provides an independent means of estimation. Subsequent estimates have been abundant, and have ranged from less than half to more than twice this amount. Yet despite the comparative wealth of data, there is still no clear indication that Brunt's value was too high or too low.

We can likewise estimate the rate of generation of MAE, by comparing the distributions of the efficiency factors N and N' with those of the generating processes—heating and cooling, evaporation and precipitation. Ideally we should obtain the same value as in the dry case, because both values should equal the rate at which KE is generated and dissipated. It is hardly to be expected that the answers will agree, especially in view of the difficulty in estimating the vertical distributions of the generating processes. What should be indicated more reliably is the relative importance of heating-cooling and evaporation-precipitation.

For the most meaningful results we should determine the generation on a day-by-day basis. It is doubtful that suitable data are available, and, in any event, the procedure would require laborious computations. Meanwhile we can make crude estimates, analogous to the one in A, by using averages.

We have done this by comparing the efficiency factors in Figs. 12 and 13 with northern-hemisphere annual average rates of the generating processes, as given by Newell et al. (1974). Using the values of net radiation originally obtained by Dopplick (1974), we find a contribution to the generation of MAE of -0.4 W m^{-2}. Boundary-layer heating, due mainly to small-scale turbulence, and assumed to be uniformly distributed between 1000 and 850 mb, contributes 0.9 W m^{-2}. The combined contribution of heating and cooling, 0.5 W m^{-2}, seems totally inadequate to maintain the supply of MAE.

Table 1. *Estimated rates of generation of moist available energy (MAE) and dry available energy (DAE) by various generating processes. Letters (D) or (R) indicate that radiation values of Dopplick (1974) or Rodgers (1967) were used. Units are W m^{-2}*

Generating process	MAE (D)	MAE (R)	DAE (D)	DAE (R)
Radiation (Ra)	−0.4	1.3	−1.3	−0.1
Boundary-layer heating (BL)	0.9	0.9	0.7	0.7
Evaporation (Ev)	4.3	4.3	—	—
Precipitation (Pr)	−1.3	−1.3	—	—
Condensation (Co)	—	—	2.4	2.4
Ra + BL	0.5	2.2	−0.6	0.6
Ev + Pr, or Co	3.0	3.0	2.4	2.4
Total	3.5	5.2	1.8	3.0

If we compare estimates of evaporation with values of N' at the surface, we obtain a generation of 6.0 W m^{-2}. We believe that this figure is unrealistically large. In order that evaporation into the surface layer may continue, the newly acquired water vapor must be continually carried upward by turbulent eddies. Much of this turbulence occurs on the scale of cumulus convection. Such motion would convert the available energy into *turbulent* kinetic energy, whose subsequent dissipation is not included in conventional estimates of KE dissipation.

We can estimate the combined effect of evaporation and turbulence, without considering turbulence explicitly, by assuming that the evaporation occurs uniformly in a somewhat deeper boundary layer. We have taken this layer to extend from 1000 to 850 mb, as in the Newell et al. treatment of boundary-layer heating. We then obtain a generation of 4.3 W m^{-2}.

Precipitation requires further assumptions, since Fig. 13 is based upon a completely cloudless mass field. For our computations we have taken N' to equal N, as given in Fig. 12, wherever N is negative, while $N' = 0$ where N is positive. We obtain a generation of -1.3 W m^{-2}, giving 3.0 W m^{-2} for the combined evaporation-precipitation effect. Our tentative conclusion is that the latter effect is the dominating one.

Dopplick presents a second set of values for net radiation, obtained by Rodgers (1967). Using these values instead of Dopplick's own, we obtain a radiative generation of $+1.3$ W m^{-2}. According to Dopplick, the differences in the two estimates result mainly from different assumptions regarding the distribution of clouds. If the revised estimate is acceptable, heating-cooling and evaporation-precipitation may be of comparable importance in generating MAE. In either event, it would seem that any description of the general circulation as a dry circulation is open to question.

Newell et al. encountered a similar situation when using both sets of radiation data to evaluate the generation of (dry) APE. As a check, we have made similar computations, using the values of N in Fig. 11, and our results differ only slightly from theirs. Table 1 compares our evaluations of the generation of both MAE and DAE by the various generating processes, using both sets of radiation data.

Perhaps more striking than the differences due to the uncertainty in radiation is the fact that, whichever set of data is used, the generation of MAE exceeds that of DAE by a substantial amount. In the long run this should not happen, since both rates should equal the rate of production of KE. An explanation is needed.

It is easy enough to say that our fabricated mass field and the data of Newell et al. are incompatible, and that no agreement should have been expected. For one thing, the moisture which was averaged to produce our relative humidities in Fig. 1 presumably did not include the cloud water. However, we believe that the excess generation of MAE over that of DAE by heating and cooling may be real.

The differences between the effects of heating arise mainly because the moist reference field is warmer than the dry reference field in the middle troposphere, whence the values of N at these levels in Fig. 12 are lower than those in Fig. 11. The net heating proves to be negative everywhere, except in the boundary layer, and as a result heating produces a smaller negative or greater positive amount of MAE than of DAE.

Evaporation and precipitation should therefore produce less MAE than the DAE produced by condensation, and the cause of the discrepancy should be sought here. Aside from noting that our assumed values of N' in the precipitating regions are highly questionable, we offer three suggestions.

First, we may have overlooked some processes which dissipate MAE. In tropical cumulonimbus convection, for example, the precipitation appears to be ineffective, but the mixing may be significant. To include its effects we should, in lower latitudes, extend the effective layer of evaporation to a still higher level, thereby further reducing the effective N', and lowering the generation.

Another possibility arises because our computations are based on averages. There may also be a generation of *eddy* DAE which is not accompanied by a similar generation of eddy MAE. Condensation which is positively correlated with N, perhaps in lower middle latitudes where precipitation has little or no effect on MAE, could, if present, produce the effect.

Finally the distribution of condensation, which has been assumed to be the same as that of precipitation, may be somewhat different. If water has condensed and moved poleward as clouds for some distance before precipitating, thereby reaching lower values of N, the generation of DAE by condensation will be underestimated. It seems, indeed, that by introducing the concept of moist available energy we have raised more questions than we have settled.

6. Concluding remarks

Available potential energy, originally defined in terms of dry-adiabatic processes, can readily be redefined in terms of dry and moist processes. Evaluation of the newly defined available energy is a tractable problem. It can be accomplished graphically with the aid of an adiabatic chart.

We have illustrated the evaluation procedure with a single hypothetical atmospheric state, based upon averages. Even this simple example is sufficient to reconfirm the importance of considering water in the atmosphere when dealing with atmospheric energetics.

For more definitive estimates of moist available energy we must evaluate it day by day. For this task the graphical method would be extremely tedious. Moreover, it is highly susceptible to human errors, such as misreading an adiabatic chart or misplacing a curve. Numerical procedures are therefore in order.

If we could obtain a mathematical expression or approximation for MAE, as we did in A for DAE, we should encounter no difficulty in preparing a program to evaluate MAE directly. However, a suitable expression has so far eluded us. We must therefore seek an alternative procedure, which presumably will begin by determining the reference field.

One possibility would be to have the computer duplicate as closely as possible the steps which we have performed graphically. Higher-order interpolation would probably be used. Discontinuities, such as those arising when much of the atmosphere is conditionally unstable, could pose problems.

We suspect that a procedure which simply arranges all the parcels in order of decreasing reference pressure, after choosing a set of equally spaced allowable reference pressures, would be preferable. The procedure could begin with an initial guess, obtained perhaps by arranging the parcels in order of increasing θ_c. Rearrangements, consisting of moving a single parcel up or down several rungs on the ladder, and moving each intervening parcel down or up one rung, could then be made, until the enthalpy could no longer be reduced. The "sounding" through the reference field would then have been determined, and the numerical equivalent of measuring areas on the adiabatic chart could be used to evaluate specific available energies.

Having proposed such a procedure, we may stop and ask whether we have really defined available energy in the most appropriate manner. In an earlier work (Lorenz, 1965) we introduced another quantity which we also called "moist available energy", and which we defined in terms of a less restricted reference field. In passing to this field from the given field, the parcels were required to retain their individual values of θ_e and their average value of $\bar{q}$, but not their individual values of $\bar{q}$; thus the water could be redistributed. With this or perhaps still another definition of moist available energy, our conclusions concerning the relative importance of evaporation-precipitation and heating-cooling could conceivably be reversed.

We did not continue with the earlier work because we were not altogether satisfied with the definition. Our reasoning had been that precipitation from clouds, followed by evaporation from the falling precipitation, would constitute a completely internal process which would redistribute the water, and that the reference field ought to be defined so as not to be appreciably altered by this process. Our present view is that the process, while internal, is irreversible, and should, like other irreversible processes, be permitted to alter the reference field.

Finally we may ask whether we have used the correct equation of state. The requirements of no supersaturation, and no clouds with subsaturation, are probably appropriate for small enough parcels, but, when we deal with the general circulation, our parcels may be many kilometers in length, and may contain both cloudy and cloudless regions. Some equation expressing a continuous relation between the average relative humidity of a parcel and its total liquid water may be more realistic. The corresponding state curves would then be smooth, perhaps approaching the dry adiabats and moist adiabats as asymptotes.

It appears that the graphical procedure would then no longer be feasible. The discontinuities in the slopes of the state curves, which we had initially feared might make the evaluation of available energy intractable, proved to be the feature which made it possible. On the other hand, when numerical methods are to be used, a new equation of state might simplify matters.

7. Acknowledgement

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ДОСТУПНАЯ ЭНЕРГИЯ И ПОДДЕРЖАНИЕ ЦИРКУЛЯЦИИ ВЛАЖНОГО ВОЗДУХА

Доступная энергия влажного воздуха определяется как величина, на которую потенциальная плюс внутренняя (включая скрытую) энергия данного поля атмосферных масс превосходит таковую для гипотетического отсчетного поля, которое может быть построено из данного поля перераспределением атмосферной массы при обратимых сухо- и влажноадиабатических процессах с целью минимизации потенциальной плюс внутренней энергии. *Доступная энергия сухого воздуха* равна величине доступной энергии влажного воздуха, которая присутствовала бы в сухой атмосфере с тем же самым распределением температуры, как и во влажной атмосфере, и идентична доступной потенциальной энергии.

Описаны графические процедуры для определения отсчетных полей для сухого и влажного случаев и для оценки доступных энергий. В общем случае доступная энергия влажного воздуха превосходит таковую для сухого воздуха.

Как нагревание, так и охлаждение оогут производить или разрушать доступные энергии влажного и сухого воздуха. Испарение может увеличивать доступную энергию влажного воздуха, в то время как осадки могут ее уменьшать. Предварительные вычисления, основанные на осредненных полях, показывают, что полное производство доступной энергии влажного воздуха в процессах испарения — осадков, по крайней мере, так же велико, как ее производство в процессах нагревания — охлаждения, и, вероятно, много больше.