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A parameterization of large-scale heat transport in mid-latitudes. Part II. Stationary waves and the Ferrel cell

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ABSTRACT

Based on the recent studies by Andrews and McIntyre (1976) and Boyd (1976) a simple equation is formulated to calculate the mean meridional circulation forced by the stationary planetary waves and the latent heat release and its correlation with the stationary planetary waves. The westerly momentum equation and the thermodynamic equation are combined to form a single linear equation for the meridional circulation based on the assumption that the momentum dissipates exclusively in the lowest layer of the atmosphere and that the heat dissipates into the ocean and lithosphere. The ratio between the dissipation time constants for momentum and heat turns out to be one of the most important parameters governing the intensity of the meridional circulation. The mid-latitude Ferrel cell appears to be a composite circulation originated from two circulations: one from the stationary planetary waves (Charney–Drazin circulation) and the other from the transient eddies biased by the latent heat release. The adiabatic heating computed from the model is compared favorably with observations.

1. Introduction

In addition to the transient eddies discussed in Part I of this paper, the large-scale stationary waves existing in mid-latitudes also contribute to the horizontal heat transport (Fig. 1). The energy sources of the waves are primarily due to large-scale orography and ocean-continent heating contrasts. The wave energy transport in the meridional plane is due to the non-geostrophic wind component in the horizontal motions in the wave generated by the zonal mean wind. The interaction of the wave and zonal mean flow tilts the phase of the wave both in the vertical and meridional directions, thus producing the heat and momentum transports in the meridional plane. A simple relation was found by Eliassen and Palm (1961) that as waves transport the energy upward from a source near the surface, they transport sensible heat toward the pole if mean zonal motion is westerly. Also, when the waves transport energy meridionally toward the equator, they transport westerly momentum poleward. These relations have been confirmed by observations.

Using a quasi-geostrophic model with the beta-plane approximation Charney and Drazin (1961) first analyzed the effects of the heat and momentum transports upon the mean zonal wind and the mean temperature. It is concluded from this and later studies (Holton, 1974) that when the waves are free from transience, dissipation of heat and momentum, and diabatic heating, that the flux divergences of sensible heat and momentum are exactly compensated by the adiabatic heating and momentum advection of the secondary meridional circulation. This circulation is necessarily created by the differential heating and torque generated by

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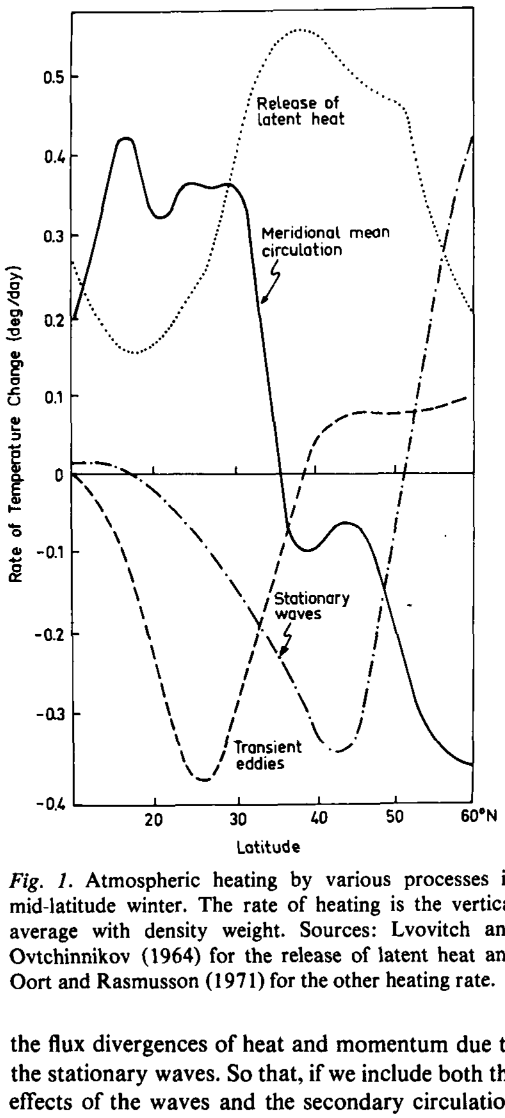

Fig. 1. Atmospheric heating by various processes in mid-latitude winter. The rate of heating is the vertical average with density weight. Sources: Lvovitch and Ovtchinnikov (1964) for the release of latent heat and Oort and Rasmusson (1971) for the other heating rate.

the flux divergences of heat and momentum due to the stationary waves. So that, if we include both the effects of the waves and the secondary circulation in our dynamical system, the presence of the stationary planetary waves has no effect on the mean field of the atmosphere (now referred to as Charney and Drazin's Theorem). This theorem was recently proved for general conditions by the independent studies of Andrews and McIntyre (1976) and Boyd (1976), where it is also proved that when the Charney and Drazin Theorem holds, the stream function for the secondary circulation is simply equal to the horizontal sensible heat flux of the waves divided by the vertical static stability squared. We will refer to this meridional circulation as C–D circulation.

In addition to the C–D circulation the latent heat released by condensation of water vapor may also be a driving force for the meridional circulation in the mid-latitudes. The zonal mean of the condensation heating is at a maximum at 40° N in winter (Fig. 1) due to the large variance in the vertical velocities primarily by the baroclinic unstable waves. The rate of heating decreases both toward the equator where the relative humidity is low owing to the downward motion in the equatorial Hadley cell, and toward the pole where the cold temperatures keep the absolute humidity low.

The general mathematical framework for the meridional circulation controlled thermally or frictionally in rotating atmospheres was given earlier by Eliassen (1951) using a mid-latitude Cartesian coordinate with constant rotation, and by Kuo (1956) for the spherical earth with variable Coriolis parameters[1]. In the Eliassen model the stream function of the meridional circulation is the solution of a Poisson equation when the basic state is prescribed by quasi-static equilibrium between the zonal wind and the zonal mean geopotential. The solutions with the point sources of heating and momentum (Green's function) were discussed in detail for various cases with different stabilities of the basic atmosphere in the horizontal and vertical directions. Although the mathematics developed by Eliassen are heuristic for finding the fundamental feature of the meridional circulation in mid-latitudes, the following differences are to be noted between the real meridional circulation and his results:

(i) The mid-latitude Ferrel cell is a mean of many circulations each of which is approximately governed by the Eliassen equation.
(ii) The quasi-static approximation adopted in his approach is valid only when heat sink and dissipation of momentum are given as entirely external forcing to the circulation system, but in the real atmosphere these are often passive processes in the sense that the rate of dissipation is a function of the mean-state variables.

In regard to the first point mentioned above one may be able to produce, in principle, the Ferrel cell

[1] A concise description for the dynamical constraints in maintaining the meridional circulation is also found in Holton (1972, p. 228) in the quasi-geostrophic hydrostatic context.

by performing a statistical average of many of Eliassen's individual solutions with their external sources prescribed statistically by a proper probability function. However, this does not seem practically feasible, since the Ferrel cell is a mean of many circulations driven by at least two physically different sources: almost random heating by condensation of water vapor in the free atmosphere and forcing from the characteristics of the lower surface whose distribution is well prescribed geographically. In Part I it was mentioned that the large-scale atmospheric motions in mid-latitudes are grouped separately into the stationary long waves and the transient eddies of which spatial scales are smaller than Rossby's radius of deformation. The latter form nearly isotropic and homogeneous turbulence when observed from Charney's stretched coordinates; there would be virtually no selective direction of the mean circulation averaged for many individual eddies if the atmosphere is dry. Since precipitation in mid-latitudes is mainly due to vertical motions associated with transient traveling storms, we may view the Ferrel cell as a composite mean circulation of the meridional circulations around the individual transient eddies biased, however, by the latent heat release, and the C–D circulation driven by the stationary long waves.

The purpose of this paper is to demonstrate how well this idea can produce the observed Ferrel cell in the mid-latitudes. To make the method of solution simple we will assume a stationary state of the atmosphere at the beginning. This assumption is not entirely valid because we know that there exists a slight seasonal variation in the zonal mean precipitation in mid-latitudes superimposed on the annual mean precipitation and that the characteristic time scale for heat dissipation is not very short compared to a year, as indicated by a time lag of mid-latitude temperature behind the phase of the seasonal change of insolation. Therefore, the results presented in this paper should be used with caution when applied to the seasonal change of the Ferrel cell.

Since pure C–D circulation is known to have no net effect on the zonal mean temperature field when it is combined with the stationary planetary waves, in this paper we will only deal with the meridional circulation which is thermally driven by the latent heat release, although the computed circulation will be superimposed on the theoretical C–D circulation calculated from the observed heat flux by the stationary waves, so that the total stream function can be compared with the observed Ferrel cell.

2. Formulation of the model

Most of the standard symbols have been defined in Part I, so that we do not repeat them here unless stated otherwise. We denote the earth's radius by a, latitude by θ, longitude by λ and the log-pressure vertical coordinate by $z = -H \ln (p/p_s)$. For any atmospheric variable X, we define the linear operators by

$$L_\theta X \equiv \frac{1}{a\cos\theta}\frac{\partial}{\partial\theta}(X\cos\theta)$$

$$L_\theta^1 X \equiv \frac{1}{a\cos^2\theta}\frac{\partial}{\partial\theta}(X\cos^2\theta)$$

$$L_z X \equiv \left(\frac{\partial}{\partial z} - \frac{1}{H}\right) X$$

$$\frac{\partial X}{\partial x} \equiv \frac{1}{a\cos\theta}\frac{\partial X}{\partial\lambda}$$

$$\frac{\partial X}{\partial y} \equiv \frac{\partial X}{a\,\partial\theta}.$$

The Coriolis parameters modified for the spherical coordinates are defined as

$$\hat{f} = f + (\bar{u}/a)\tan\theta - \frac{\partial\bar{u}}{a\,\partial\theta}$$

and

$$\hat{\hat{f}} = f + 2\tan\theta\bar{u}/a.$$

The basic state of the mid-latitude atmosphere is defined by the stationary zonal mean temperature $\bar{T}$, which is split into $\bar{T}_1(z)$, a function of z only and $\bar{T}_2(\theta, z)$, a function of both θ and z:

$$\bar{T} = \bar{T}_1(z) + \bar{T}_2(\theta, z).$$

The overbar denotes the statistical average over a long time interval and over a longitudinal line. The mean geopotential $\bar{\phi}$ is defined from the hydrostatic relation by

$$\frac{d\bar{\phi}_1}{dz} = \frac{R}{H}\bar{T}_1,$$

$$\frac{\partial\bar{\phi}_2}{\partial z} = \frac{R}{H}\bar{T}_2,$$

and $\bar{\phi} = \bar{\phi}_1(z) + \bar{\phi}_2(\theta, z)$.

We assume that the mean zonal wind is in geostrophic equilibrium with $\bar{T}_2$:

$$\hat{f}\frac{\partial\bar{u}}{\partial z} = -\frac{R}{H}\frac{\partial\bar{T}_2}{\partial y}. \quad (1)$$

Since the distribution of $\bar{T}$ in the mid-latitudes is a result of the whole global dynamical processes, including the mid-latitudes, tropics and high latitudes, we are not able to determine it merely from the dynamical processes in the mid-latitudes. Eq. (1) is, however, a necessary equation to solve the meridional circulation in mid-latitudes which is driven by forcing in the mid-latitudes.

The stationary equation for east–west momentum may be written as

$$-\hat{f}\bar{v} + \bar{w}\frac{\partial\bar{u}}{\partial z} = -L_\theta^{\frac{1}{2}}(\overline{u'v'} + \overline{u^*v^*}) - L_z(\overline{u'w'} + \overline{u^*w^*}) + \bar{D}, \quad (2)$$

where $\bar{D}$ is the rate of dissipation of the mean westerly momentum by subsynoptic processes including the drag due to the lower surface. The primed variables are transient deviations from the mean and those with the asterisk are stationary deviations from the mean. When the overbar is used for the correlation of the atmospheric variables, it will be understood that it means the statistical average integrated for all possible eddies and waves in the atmosphere. The reader is referred to Holton (1975) for the equation of motion on the globe with log-pressure coordinates.

The thermodynamic equation may be written

$$-\bar{v}\hat{f}\frac{\partial\bar{u}}{\partial z} + N^2\bar{w} = -L_\theta\left(\overline{v'\frac{\partial\phi'}{\partial z}} + \overline{v^*\frac{\partial\phi^*}{\partial z}}\right) - L_z\left(\overline{w'\frac{\partial\phi'}{\partial z}} + \overline{w^*\frac{\partial\phi^*}{\partial z}}\right) + \frac{\kappa}{H}\bar{Q}, \quad (3)$$

where

$$N^2 = \frac{R}{H}\left(\frac{d\bar{T}_1}{dz} + \frac{\kappa}{H}\bar{T}_1 + \frac{\partial\bar{T}_2}{\partial z}\right) \quad (4)$$

is the vertical static stability, $\bar{Q}$ the sum of the mean diabatic heating and cooling per unit mass of air and $\kappa = R/c_p$ with R the gas constant of the air and c_p its heat capacity at constant pressure.

The equation of continuity is given by

$$\bar{v} = L_z\chi \quad (5)$$

and

$$\bar{w} = -L_\theta\chi, \quad (6)$$

where χ is the stream function for the meridional circulation. Since both eq. (2) and (3) are linear in terms of χ, we assume that χ is decomposed into

$$\chi = \tilde{\chi} + \overline{\frac{v^*}{N^2}\frac{\partial\phi^*}{\partial z}} + \chi_H. \quad (7)$$

Here the second term on the right-hand side is the stream function of the C–D circulation (Boyd, 1976) and χ_H is the stream function of the Hadley cell which protrudes into the subtropics, and its downward motion is an important component to generate the sensible heat which tends to balance with the horizontal heat transport by transient eddies in mid-latitudes (see Fig. 1). $\tilde{\chi}$ is the stream function defined as a residue from the total stream function subtracted by the C–D circulation and the Hadley cell.

In order to eliminate from eq. (2) and (3) the fluxes of heat and momentum by stationary waves, we use a set of first-order linearized equations for a stationary wave:

$$\bar{u}\frac{\partial u^*}{\partial x} - \hat{f}v^* + w^*\frac{\partial\bar{u}}{\partial z} + \frac{\partial\phi^*}{\partial x} + M = 0 \quad (8)$$

$$\bar{u}\frac{\partial v^*}{\partial x} + \hat{f}u^* + \frac{\partial\phi^*}{\partial y} + P = 0 \quad (9)$$

$$\bar{u}\frac{\partial^2\phi^*}{\partial x\,\partial z} - \hat{f}\frac{\partial\bar{u}}{\partial z}v^* + N^2w^* - \frac{\kappa}{H}Q^* = 0, \quad (10)$$

where M and P are the rate of dissipation of wave momenta due to transient eddies acting upon the wave and the boundary layer friction, and Q^* is the heat source (or sink) for the wave.

Upon multiplying (10) by $\partial\phi^*/\partial z$ and then taking the statistical average of the result integrated for all waves in the system, we obtain

$$\overline{w^*\frac{\partial\phi^*}{\partial z}} = \frac{\hat{f}}{N^2}\frac{\partial\bar{u}}{\partial z}\overline{v^*\frac{\partial\phi^*}{\partial z}} + \frac{\kappa}{N^2H}\overline{Q^*\frac{\partial\phi^*}{\partial z}}. \quad (11)$$

We take the vertical differential of the above equation to get

$$L_z\left(\overline{w^* \frac{\partial \phi^*}{\partial z}}\right) = \hat{f}\frac{\partial \bar{u}}{\partial z} L_z\left(\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\right) + \overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\frac{\partial}{\partial z}\left(\hat{f}\frac{\partial \bar{u}}{\partial z}\right) + L_z\left(\frac{\kappa}{N^2 H}\overline{Q^* \frac{\partial \phi^*}{\partial z}}\right). \quad (12)$$

The divergence of horizontal heat flux is written as

$$L_\theta\left(\overline{v^* \frac{\partial \phi^*}{\partial z}}\right) = N^2 L_\theta\left(\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\right) + \overline{\frac{v^*}{aN^2}\frac{\partial \phi^*}{\partial z}}\frac{\partial N^2}{\partial \theta}. \quad (13)$$

By substituting (12) and (13) into (3) and using (1) and (4) we obtain, after some further rearrangement,

$$-\hat{f}\frac{\partial \bar{u}}{\partial z} L_z \tilde{\chi} - N^2 L_\theta \tilde{\chi} = \frac{\kappa}{H}\bar{Q} - L_z\left(\frac{\kappa}{N^2 H}\overline{Q^* \frac{\partial \phi^*}{\partial z}}\right) - L_\theta\left(\overline{v' \frac{\partial \phi'}{\partial z}}\right) - L_z\left(\overline{w' \frac{\partial \phi'}{\partial z}}\right) - N^2 \bar{w}_H + \hat{f}\bar{v}_H \frac{\partial \bar{u}}{\partial z} \quad (14)$$

where

$\bar{v}_H = L_z \chi_H$

and

$\bar{w}_H = -L_\theta \chi_H$

are the meridional and vertical velocities, respectively, of the Hadley cell.

In order to also write the momentum eq. (2) in terms of the new stream function $\tilde{\chi}$ we introduce the following quantities with the dimension of momentum flux (Boyd, 1976):

$$F_v = -\overline{u^* v^*} + \frac{\partial \bar{u}}{\partial z}\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}} \quad (15)$$

$$F_w = -\overline{u^* w^*} + \hat{f}\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}. \quad (16)$$

By operating L_θ^1 on (15) and L_z on (16) we obtain

$$L_\theta^1(\overline{u^* v^*}) = -L_\theta^1 F_v + \frac{\partial \bar{u}}{\partial z} L_\theta\left(\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\right) - \overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\left\{\frac{\tan\theta}{a}\frac{\partial \bar{u}}{\partial z} - \frac{\partial}{a\,\partial \theta}\frac{\partial \bar{u}}{\partial z}\right\} \quad (17)$$

and

$$L_z(\overline{u^* w^*}) = -L_z F_w + \hat{f} L_z\left(\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\right) + \left(\overline{\frac{v^*}{N^2}\frac{\partial \phi^*}{\partial z}}\right)\frac{\partial \hat{f}}{\partial z}. \quad (18)$$

The substitution of (17) and (18) into (2) yields

$$-\frac{\partial \bar{u}}{\partial z} L_\theta \tilde{\chi} - \hat{f} L_z \tilde{\chi} = L_\theta^1(F_v - \overline{u'v'}) + L_z(F_w - \overline{u'w'}) - \frac{\partial \bar{u}}{\partial z}\overline{w_H} + \hat{f}\overline{v_H} + \bar{D}. \quad (19)$$

More explicit expressions for F_v and F_w have been derived from (8)–(10) by Boyd (1976) in terms of the correlation of M, P and Q^* with the variables for the stationary waves. Also proved is that $L_\theta^1 F_v + L_z F_w = 0$ when the stationary wave is adiabatic and free from the dissipation of momentum. However, as will be shown later, (15) and (16) are sufficient definitions for the purpose of the present study, since we are able to estimate the magnitude of F_v and F_w based on the observed $\overline{u^* v^*}$, $\overline{u^* w^*}$ and $\overline{v^* \partial \phi^* / \partial z}$, so that their forcing upon $\tilde{\chi}$ will be compared with the thermal forcing.

$\tilde{\chi}$ is the stream function defined by (7) as a residue from the total stream function subtracted by the C–D circulation and the Hadley cell. Since the C–D circulation is the mean meridional circulation around the stationary long wave biased by the zonal mean wind and rotation of the earth, the stream function must be solved with spherical geometry and the variation of the Coriolis parameter.

In eq. (14), the meridional temperature advections by $\tilde{\chi}$ as well as χ_H may be negligible as compared to the temperature change due to vertical motion by $\tilde{\chi}$. Furthermore, in eq. (19), the vertical advection of the zonal momentum and the meridional advection of $\hat{f}$ by the Hadley cell may both be negligible as compared to $\hat{f}L_z\tilde{\chi}$. Finally, since $\tilde{\chi}$ involves few components of motion larger than the radius of deformation, we may approximate the linear horizontal operator by

$$L_\theta\tilde{\chi} = \partial\tilde{\chi}/\partial y,$$

and since $\bar{u}$ tan θ/a and $\partial\bar{u}/a\,\partial\theta$ are both smaller than f we approximate $\hat{f}$ by

$$\hat{f} = f,$$

where f is set to be a constant at mid-latitudes. Thus (19) and (14) are now reduced to

$$-fL_z\tilde{\chi} = L_\theta^1(F_v - \overline{u'v'}) + L_z(F_w - \overline{u'w'}) + \bar{D} \qquad (20)$$

$$-N^2\frac{\partial\tilde{\chi}}{\partial y} = \frac{\kappa}{H}\bar{Q} - L_z\left(\frac{\kappa}{N^2H}\overline{Q^*\frac{\partial\phi^*}{\partial z}}\right) - L_\theta\left(\overline{v'\frac{\partial\phi'}{\partial z}}\right) - L_z\left(\overline{w'\frac{\partial\phi'}{\partial z}}\right) - N^2\overline{w_H}. \qquad (21)$$

Up to this point the derivation of eq. (20) and (21) are essentially the same as Eliassen (1951), Leovy (1964) and Dickinson (1971). Since $\tilde{\chi}$ should satisfy both (20) and (21) at the same time we need to form a single equation by eliminating a common factor involved in both equations which, however, is not obvious in the present form. Eliassen combines these two equations based on the assumption of quasi-stationarity which makes it possible to eliminate the local time changes of the temperature and zonal mean wind through their geostrophic relation. In dealing with the meridional circulation in the stratosphere and mesosphere Leovy, on the other hand, combines these equations by linearizing first $\bar{D}$ and the radiative cooling in $\bar{Q}$ in terms of the zonal mean wind and the temperature and then by eliminating the wind and the temperature through their geostrophic relation. Dickinson (1971) follows Leovy's approach in his study of Hadley circulation. However, he ignored the transient eddy heat transport in the subtropics, which is apparently important in the troposphere. Although his solution produced results which compare favorably to observations, it leaves a question as to whether the linear damping term in his thermodynamic equation really represents the radiative dissipation or the dissipation due to transient eddies. It is to be noted that the way in which eqs. (20) and (21) are coupled is very much dependent on the regions of the atmosphere considered because individual thermal processes have a different influence in each region. After trying several possible combinations among the processes involved in $\bar{D}$ and $\bar{Q}$ we found that the following assumption produces results which compare most favorably with observations. We assume that the dissipation of momentum takes place exclusively in the lowest layer of the atmosphere where the dissipation rate is approximately proportional to the geostrophic wind near the surface:

$$\bar{D} = -\beta_m\bar{u}_s, \qquad (22)$$

where β_m is the dissipation constant with dimension of (time^{-1}) and $\bar{u}_s$ is the geostrophic wind near the surface, say, at the top of the planetary boundary layer.

In the thermodynamic equation $\bar{Q}$ may be separated into individual processes:

$$\bar{Q} = \bar{Q}_r + \bar{Q}_c + \bar{Q}_s + \bar{Q}_h,$$

where the terms on the right-hand side are the infrared cooling, the condensation heating, the solar heating and the sensible heating from the lower surface. For the last component we assume that

$$\bar{Q}_h = -c_p\beta_h(\bar{T}_s - T_0), \qquad (23)$$

where β_h is the relaxation time constant of the surface temperature toward a long-term mean of the temperature T_0, which is typically the temperature at a depth where the seasonal variation is zero. By substituting (22) into (20) and (23) into (21), respectively, and then eliminating $\bar{T}_s$ and $\bar{u}_s$ from the results through the geostrophic relation we obtain

$$\frac{\partial^2\tilde{\chi}}{\partial\hat{y}^2} + \beta\frac{\partial}{\partial\hat{z}}\left(\frac{\partial}{\partial\hat{z}} - 1\right)\tilde{\chi} = -\frac{H}{f}\beta\frac{\partial F}{\partial\hat{z}} - \frac{R_0}{g_0}\frac{\partial J}{\partial\hat{y}}, \qquad (24)$$

where

$$g_0 \equiv N^2H/\kappa,$$

$$\beta \equiv \beta_h/\beta_m,$$

$$F \equiv L_\theta^1(F_v - \overline{u'v'}) + L_z(F_w - \overline{u'w'}),$$

$$J \equiv \bar{Q}_c + \bar{Q}_s + \bar{Q}_r - L_z\left(\overline{\frac{Q^*}{N^2}\frac{\partial\phi^*}{\partial z}}\right) - \frac{H}{\kappa}\left\{L_\theta\left(\overline{v'\frac{\partial\phi'}{\partial z}}\right) + L_z\left(\overline{w'\frac{\partial\phi'}{\partial z}}\right) + N^2\overline{w_H}\right\}.$$

In the course of the derivation it was found convenient to use scaled coordinates defined by

$\hat{y} = y/R_0$

and

$\hat{z} = z/H,$

with

$R_0 = NH/f,$

Rossby's radius of deformation.

3. Prescription of forcing and parameter

$\partial J/\partial\hat{y}$ is the horizontal differential heating by various components. Detailed calculations of the differential heating are not within the scope of the present study, rather we simplify the calculations by neglecting the terms which are apparently unimportant. First of all, it is known from the energy budget analysis that the divergence of the heat flux by transient eddies is largely compensated in the subtropics by both adiabatic heating due to the downward motion of the Hadley cell and the solar heating, and that the convergence of the same flux in the high-latitudes is balanced by the net radiative cooling. Thus the most significant mean differential heating remaining in $\partial J/\partial\hat{y}$ is the condensation heating which has a distinct maximum at the center of the mid-latitudes.

Turning to the correlation of the diabatic heating with the stationary wave temperature we are able to only crudely estimate its mid-latitude distribution. For the winter northern hemisphere the correlation takes the sign for each component,

$\overline{Q_c^*\,\partial\phi^*/\partial z} > 0$	Möller (1951)
$\overline{Q_h^*\,\partial\phi^*/\partial z} > 0$	Budyko (1963)
$\overline{Q_s^*\,\partial\phi^*/\partial z} < 0$	Katayama (1967)
$\overline{Q_r^*\,\partial\phi^*/\partial z} < 0$	Katayama (1967).

Among these components solar heating seems to be the least important compared to the others. The infrared cooling and sensible heating both have significant correlations with temperature, but their vertical and horizontal distributions are not well known, and we will tentatively neglect them in the present calculation. Since both correlations seem to have a maximum near the center of the mid-latitudes and decrease with height (Budyko, 1963; Katayama, 1967), we suspect that the omission could lead to an underestimation of the $\tilde{\chi}$-circulation. Finally, the most important correlation is again the condensation heating. The time-averaged latent heat release is not uniform along longitudinal lines in the mid-latitudes. It is known from observations that precipitation takes place almost exclusively over the oceanic areas and that the large continents receive far less precipitation, particularly in winter. Since the ocean surface temperature is higher than the continental temperature in winter, there exists a strong correlation between latent heat release and temperature along the longitudinal circles in mid-latitudes. Thus, we will keep $\overline{Q_c^*\partial\phi^*/\partial z}$ in the forcing term J.

An important parameter governing the solution of (24) is the ratio between the coefficients for the sensible heat dissipation and the momentum dissipation near the surface. Dickinson (1971) has discussed in detail how this ratio affects the intensity of the circulation in his study of the equatorial Hadley cell. In general a small β_h enhances a stronger circulation because, if β_h is large, the heat added to the system will dissipate before it becomes available for the potential energy of the circulation. The geostrophic wind at the top of the boundary layer is on the order of 3–4 m s^{-1}. The kinetic energy dissipation rate estimated by Kung (1967) suggests that

$\beta_m^{-1} \approx$ 1–3 days.

On the other hand, from the time-lag of seasonal change of temperature (Lettau, 1951, for example) the coefficient β_h^{-1} is about 45 days in midlatitudes. Thus we set

$$\beta \approx 1/25. \qquad (25)$$

The difference of more than one order of magnitude between β_m and β_h is simply due to the zonal mean momentum dissipation which takes place only in the atmospheric boundary layer (including the surface) and heat dissipation which is associated with heat conduction into the ocean and lithosphere. The important implication of $\beta \ll 1$ will be apparent later when we deal with the explicit solution for (24) in the next section.

Finally, for this prescribed value of β the frictional forcing estimated from the observed F has turned out to be smaller than the forcing from J by two orders magnitude in the free atmosphere. However, condensation is minimal near the surface and the frictional dissipation of the waves tends to be at a maximum therefore the relative importance of heating versus friction may reverse within the lowest layer of the atmosphere. Although the neglect of friction upon waves in the entire atmosphere may lead to an unrealistic distribution of the meridional stream function in the lowest layer, the errors in the mean adiabatic heating by the downward motion would be small because the depth of the frictional layer is only a small fraction of the total tropospheric depth and the mean vertical velocity is also small in the lowest boundary layer.

4. Results

In this section an example calculation is shown for the northern hemisphere in winter. The heating distribution may be expressed by a product of the meridional and vertical structure functions:

$$\bar{Q}_c = Z_A(\hat{z})Y(\hat{y}) \tag{26}$$

and

$$-L_z\left(\overline{\frac{Q_c^*}{N^2}\frac{\partial\phi^*}{\partial z}}\right) = Z_B(\hat{z})Y(\hat{y}), \tag{27}$$

where we have assumed the same meridional structure function for both mean condensation heating and its correlation with the stationary waves. A symmetrical function is assumed for the meridional structure as shown in Fig. 2. Based on the mean cloudiness distribution (Telegadas and London, 1954) the vertical distribution is inferred for condensation heating as shown in Fig. 2. The analysis of the precipitation in winter (Möller, 1951) indicates that the oceans receive about 2/3 of the total precipitation and the atmospheric temperature over the oceans is higher than the temperature over the continents by about 15 °C in winter. From these data Z_A and Z_B are specified as shown in Fig. 3. For the symmetrical function $Y(\hat{y})$,

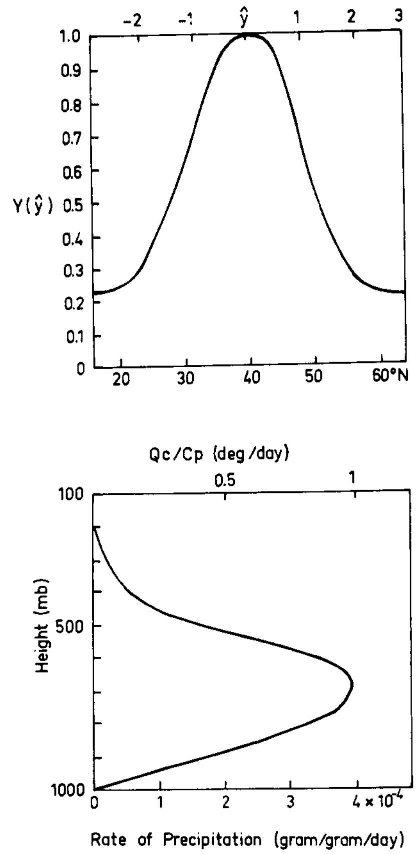

Fig. 2. Top: The meridional structure function for the condensation heating. Bottom: The rate of precipitation (gram of water/gram of air/day) as a function of height.

the differential heating is closely approximated by a single sine function:

$$-\frac{\partial J}{\partial \hat{y}} = (Z_A + Z_B)\sin(d\hat{y}) \tag{28}$$

where $d = 2\pi/4 = 1.571$. Thus, the solution may be assumed in the form

$$\tilde{\chi} = \psi(\hat{z})\sin(d\hat{y}). \tag{29}$$

By substituting (28) and (29) into (24) we obtain

$$\frac{d^2\psi}{d\hat{z}^2} - \frac{d\psi}{d\hat{z}} - \frac{d^2}{\beta}\psi = \frac{R_0 Z(\hat{z})}{g_0\beta}, \tag{30}$$

where $Z = Z_A + Z_B$. Note that the source function is now inversely proportional to β. Eq. (30) was solved numerically using the boundary condition

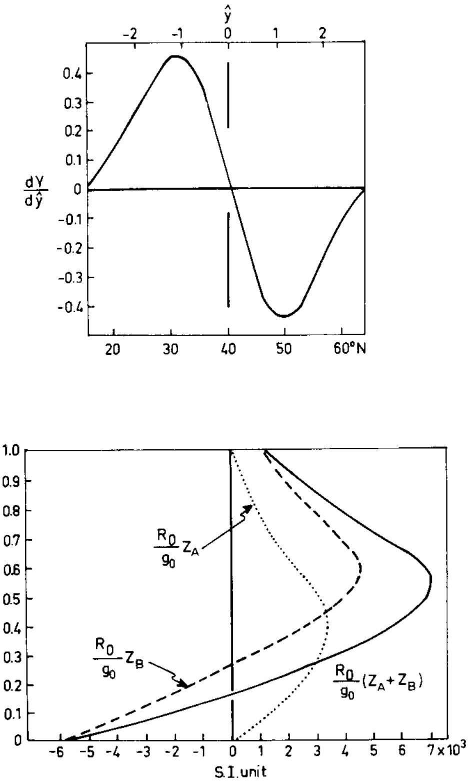

Fig. 3. Top: Meridional differential heating. Bottom: Vertical distribution of the heating functions. The dotted line represents zonal mean heating, the broken line represents heating due to the correlation of precipitation and stationary wave and the solid line represents the total heating.

$\psi = 0$ at $\hat{z} = 0$ and 1, using the method described by Richtmyer and Morton (1967, p. 198). The final solution for $\tilde{\chi}$ is shown in Fig. 4.

The C–D circulation is computed from the sensible heat flux transported by the stationary waves (Oort and Rasmusson, 1971, Table c2b) and is depicted in Fig. 5. The results shown in Fig. 4 and Fig. 5 are significant in the following points:

(i) The $\tilde{\chi}$-circulation is a two-cell circulation with the maximum upward motion where the condensation heating is at a maximum. The individual circulation has a meridional span on the order of two times the radius of deformation. This result is consistent with our assumption that the $\tilde{\chi}$-circulation is essentially a mean of many circulation systems around the transient eddies which are biased by the condensation heat release.

(ii) The C–D circulation is a single cell with maximum downward motion around 40° N and maximum upward motion around 65° N. The meridional span is about five times the radius of deformation. This is again compatible with the fact that the C–D circulation is a mean circulation around the stationary planetary waves. These waves are of a scale larger than the radius of deformation.

(iii) The upward motion by $\tilde{\chi}$-circulation tends to cancel the downward motion by the C–D circulation at the center of the mid-latitudes (40–45° N). However, this coincidence may not be just accidental because the C–D circulation is primarily driven by the ocean–continent temperature contrast, which may in turn relate with the meridional mean temperature gradient which governs the activity of the transient eddies, thus the $\tilde{\chi}$-circulation.

(iv) The mean rate of heating by downward motion for each individual circulation is shown in Fig. 6. The role of $\tilde{\chi}$-circulation is to equalize the mid-latitude temperature, which otherwise tends to increase at latitudes where the condensation heating is at a maximum. Because of this tendency the subtropical atmosphere receives sensible heat originating from condensation heating in the mid-latitudes, but nearly the same amount of the sensible heat is also received by the high-latitudes, so that the effect of the $\tilde{\chi}$-circulation on the meridional temperature gradient seems rather small. However, the presence of a double adiabatic heating maximum at 20° N and 30° N (Fig. 1), may be interpreted as follows: the first maximum in the lower latitude is due to the downward branch of the equatorial Hadley cell, while the second maximum at the higher latitude may be the result of the mid-latitude circulation discussed in this paper.

5. Summary and concluding remarks

From the analysis presented in this paper it appears that the mid-latitude Ferrel cell is a composite circulation originating from two circulation systems with different physical origins:

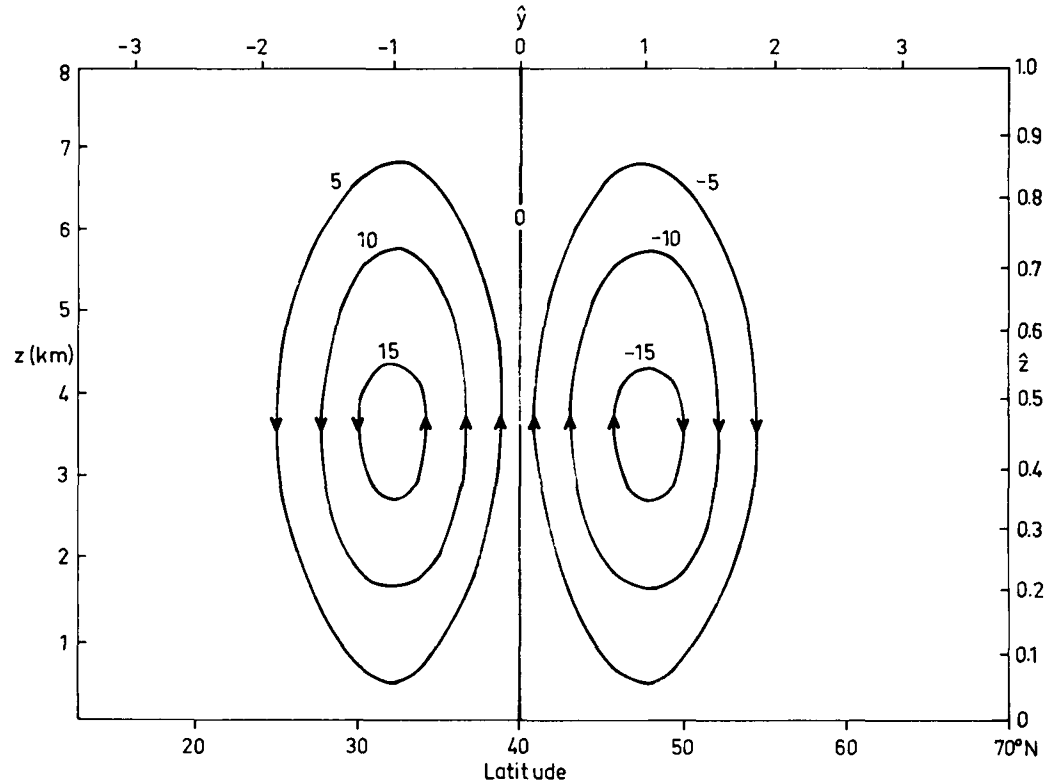

Fig. 4. The stream function $\tilde{\chi}e^{-z/H}$. Units are 10^2 SI.

one from the stationary Rossby waves whose scale is larger than the radius of deformation in both longitudinal and meridional directions (C–D circulation) and another from the transient eddies biased by the latent heat release. Since the C–D circulation was prescribed from the observed horizontal sensible heat flux by the stationary waves, the present study is not able to determine how these two circulations are interrelated although it seems plausible that both must be dependent on the mean meridional temperature gradient which simultaneously controls the activity of the transient eddies and the mean zonal geostrophic wind which in turn influences the amplitudes of the stationary planetary waves.

A complete solution for the C–D circulation needs equations written for global geometry with a variable Coriolis parameter as well as the global distribution of the forcing from the lower boundary. The problem has been only approximately solved using the beta-plane at mid-latitudes (for example, Hirota, 1971, among others). It is a matter of interest to know how many stationary Rossby waves are needed to express the total C–D circulation in the troposphere. It can be shown from the approximate solutions mentioned above that the horizontal heat transport is significant only for the external Rossby waves which penetrate through the troposphere into the stratosphere. Those internal Rossby waves trapped in the troposphere have negligible heat transport. The observations of the Rossby waves in the stratosphere indicate that only the very long waves with longitudinal wavenumber 1–4 have tropospheric roots. Also, the observed sensible heat flux is indeed negligible for waves with wavenumber larger than five (Van-Mieghem et al., 1959). Thus actual computations may be necessary only for the limited number of waves with longitudinal wavenumber 1–4. The relevant solutions for such large-

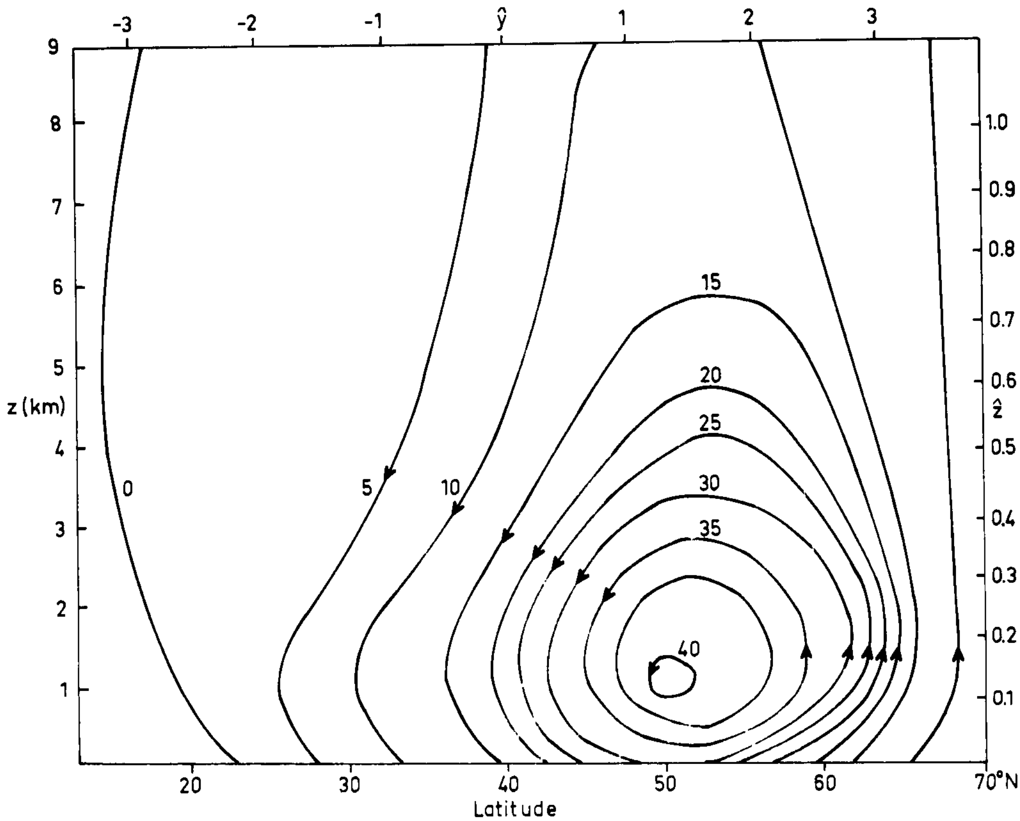

Fig. 5. The C–D circulation $\left(\overline{\frac{v^*}{N^2}\frac{\partial\phi^*}{\partial z}}\right) e^{-z/H}$, computed from the data by Oort and Rasmusson (1971). Units are 10^2 SI.

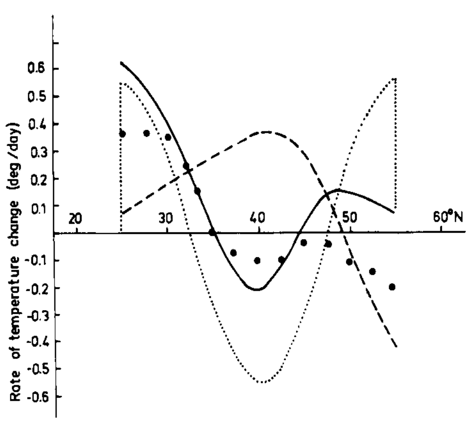

Fig. 6. Adiabatic heating by meridional circulation in mid-latitudes winter. The dotted line represents $\tilde{\chi}$ circulation, the broken line represents C–D circulation, the solid line represents the total circulation and the dots represent observations.

scale Rossby waves may be obtained most appropriately by using the Hough functions which have been discussed recently by Kasahara (1977).

Another prescription made in this analysis for the latent heat release may be removed fairly easily if we use the parameterization outlined in Sasamori (1975). The statistical model for the precipitation of water vapor needs the values of the mean vertical velocity and the variance around the mean, both being provided by the models presented in Parts I and II of this paper.

Finally, it is desirable to calculate the vertical velocity of the Hadley cell intruding into the subtropics. There are two basic problems yet to be solved in this regard. First the determination of the geographic center of the Hadley cell where the upward motion becomes maximum (i.e., the Intertropical Convergent Zone), and secondly the forcing upon the Hadley cell due to condensation heat release. The first problem may involve the global differential heating and the response of the global atmosphere to it, and the second may involve the convective instability in the moist atmosphere, which may be classified as entirely different from the baroclinic instability in the mid-latitudes.

After these problems are solved stepwise, the global atmosphere may be divided into tropics, mid-latitudes and high-latitudes and the proper parameterizations of the heat transport may be applied for the individual regions together with the processes coupling these regions. The present parameterization discussed in this paper will then find its relevant application in the study of the global and regional energy balance and the climate.

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ПАРАМЕТРИЗАЦИЯ КРУПНОМАСШТАБНОГО ПЕРЕНОСА ТЕПЛА В СРЕДНИХ ШИРОТАХ. ЧАСТЬ 2. СТАЦИОНАРНЫЕ ВОЛНЫ И ЯЧЕЙКА ФЕРРЕЛЯ

На основе последних работ Эндрюса и Макинтайра (1976) и Бойда (1976) формулируется простое уравнение для вычисления средней меридиональной циркуляции, возбуждаемой стационарными планетарными волнами и освобождением скрытого тепла. Находится корреляция меридиональной циркуляции со стационарными планетарными волнами. Уравнение для импульса и термодинамическое уравнение комбинируются в одно линейное уравнение для меридиональной циркуляции, основанное на предположении, что импульс диссипирует исключительно в нижнем слое атмосферы, а тепло диссипирует в океан и литосферу. Отношение между временами диссипации импульса и тепла оказывается наиболее важным параметром, определяющим интенсивность меридиональной циркуляции. Ячейка Ферреля в средних широтах, по-видимому, является комбинацией двух циркуляций: возбуждаемой стационарными планетарными волнами (циркуляция Чарни-Дразина) и циркуляции, порождаемой нестационарными вихрями, подверженными влиянию скрытого тепла. Вычисленное по модели адиабатическое нагревание хорошо согласуется с данными наблюдений.