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Vortex cloud street during AMTEX 75

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ABSTRACT

Strong northerly flow across Cheju Island, Korea, during the 1975 Air Mass Transformation Experiment (AMTEX 75) resulted in a pronounced vortex cloud street to the lee of the island on February 17 1975. This pattern has been studied and explained in terms of classical von Kármán eddies shed in laminar free flow moving past a cylindrical obstacle under subcritical Reynolds number conditions.

DMSP satellite imagery and AMTEX aerological data have given the shedding frequency of the vortices that are contained in the mixed boundary layer below the capping inversion to be one per 3 h. The island extends to nearly 2 km above sea level and penetrates the base of the inversion layer at a height of approximately 600 m. At this height the island diameter is about 20 km, a figure in good agreement with the independently inferred vortex shedding diameter.

1. The von Kármán vortex street

Rhythmic vortex shedding is a very common fluid dynamic phenomenon, experienced, for example, through the aeolian sounds of a car roof rack which is set into coupled vibrations by the air stream, or visually observed in the lee water behind pillars standing in a river. The phenomenon is seen as an alley of alternately clockwise and counterclockwise rotating vortices (von Kármán, 1911) moving downstream with a velocity slightly smaller than the velocity of the free fluid as measured relative to the body causing the phenomenon. Qualitatively, as fluid encounters an obstacle, the elements on collision course must deflect and the fluid close to the cross-stream crests of the body must move faster than in the free stream. This speed-up is accompanied by a drop in pressure along the sides of the obstacle as given by the equation of motion in its simplest form, the Bernoulli equation. On the front side fluid is stagnant and, thus, accordingly a relatively high pressure prevails. Hence, it is seen that the motion along the front side of the obstacle is facilitated by a positive pressure gradient.

On the rear side, the fluid is allowed to diverge again; the velocity goes down and, consequently, the pressure goes up. In the case of a circular cylinder, inviscid theory tells us that the conditions on the front and rear sides are symmetrical: pressure is converted into kinetic energy per unit volume, and subsequently again converted into pressure. In reality, due to the viscous friction on the cylinder wall, some of the kinetic energy is drained out during this process. Thus a fluid element travelling in a layer close to the wall does not have sufficient remaining kinetic energy to overcome the adverse pressure gradient on the rear side: it is brought to rest and, thereafter, to move in the direction of the pressure, i.e. backwards (Schlichting, 1968).

Behind each of the two crests of the body, increasing amounts of fluid are thus converged. By the frictional influence of the external flow, these lumps are set into rotation, creating two vortices opposite to each other. This gradual development, of course, influences the original pressure distribution: since the flow no longer has to divert immediately around the rear side of the body, the adverse pressure is no longer maintained and the vortices are liberated. Why this sequence of events takes place in alternating order on the two sides of the body is because any random event causing the one vortex to grow slightly larger than the other causes an asymmetry of the rear inflow to the wake of the body, tending to destroy the weaker of the vortices.

Whether the above-mentioned series of events will continue to occur in a regular manner, and in particular whether the vortices created are stable with respect to each other, such that a Kármán vortex street is formed, depends on the Reynolds number

$$Re = ud/\nu, \tag{1}$$

where u is the free stream velocity, d is the obstacle diameter, and ν is the kinematic viscosity of the fluid. A regular Kármán street is observed only in the range of Re numbers from about 50 to 5000. Below 50 any wake is simply absent and the flow field resembles that of the classical hydrodynamic solution, above 5000 the pattern starts to become unstable, chaotic and turbulent (Schlichting, 1968). Above the onset of (supercritical) turbulent flow at Re 10^6 rhythmic shedding continues, but the vortex street is not stable.

It has been shown experimentally that the frequency, n, by which the vortices are shed, written in non-dimensional form as

$$S = n\frac{d}{u} \tag{2}$$

and known as the Strouhal number, is a unique function of the Reynolds number (Fig. 1). Above $Re = 500$, but less than critical, the function even attains the simple form $S = 0.21$. An interesting application of this simple relation is its use in measuring very small flow speeds, simply by counting vortices behind wires of known diameter.

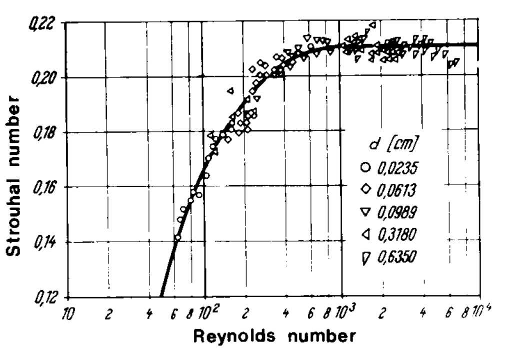

Fig. 1. The Strouhal number versus the Reynolds number for flow past a right circular cylinder. Taken from Roshko (1954).

The measurements in Fig. 1 relate to laminar free flow conditions. In case the free flow is turbulent, regular vortex streets are also observed. This, for example, has some importance for the dynamic wind loading of structures. It is expected though that the Strouhal number, in addition to the dependence on the Reynolds number, also becomes dependent on certain measures of free-stream turbulence, e.g. its intensity and, maybe, also its scale compared to d.

2. Atmospheric mesoscale vortices

Since the advent of the meteorological satellite program in 1961, many cloud pictures with patterns very much resembling the classical von Kármán vortex shedding phenomenon have been reported in the literature. Typically, the cloud patterns are seen downstream of islands of regular shape in the presence of a capping inversion at a height which is low enough to let the top of the island protrude through it. Studies of this phenomenon have been made by Hubert and Krueger (1962), Bowley et al. (1962), Chopra and Hubert (1965), Zimmerman (1969) and Agee (1975). It has been found that the distance between subsequent vortices relates to the relevant dimension (in this case the island size) in the same way that Kármán vortices appear in the wake of circular cylinders in wind tunnel and fluid tank experiments, that is, the Strouhal number is observed to be close to 0.21.

One such example is shown in the DMSP satellite imagery in Fig. 2 with an exceptionally well-developed vortex street. This photo was taken over the AMTEX (Air Mass Transformation Experiment) region in the East China Sea on February 17 1975, 1136 JST, during a period of cold air outbreak. The trigger of the vortex street is the island Cheju (22°31′N, 126°30′E) south of the Korean coast. The island is visible in the satellite imagery of the preceding day (see Fig. 3) as the vortex street is being formed. It has an elliptical shape with a major axis of approximately 70 km and a nicely centered mountain top reaching an altitude of 1950 m. The major axis is approximately perpendicular to the vortex street.

The distance, l, between subsequent vortices is approximately 90 km (Fig. 2). Assuming that the travelling speed of the vortices is insignificantly

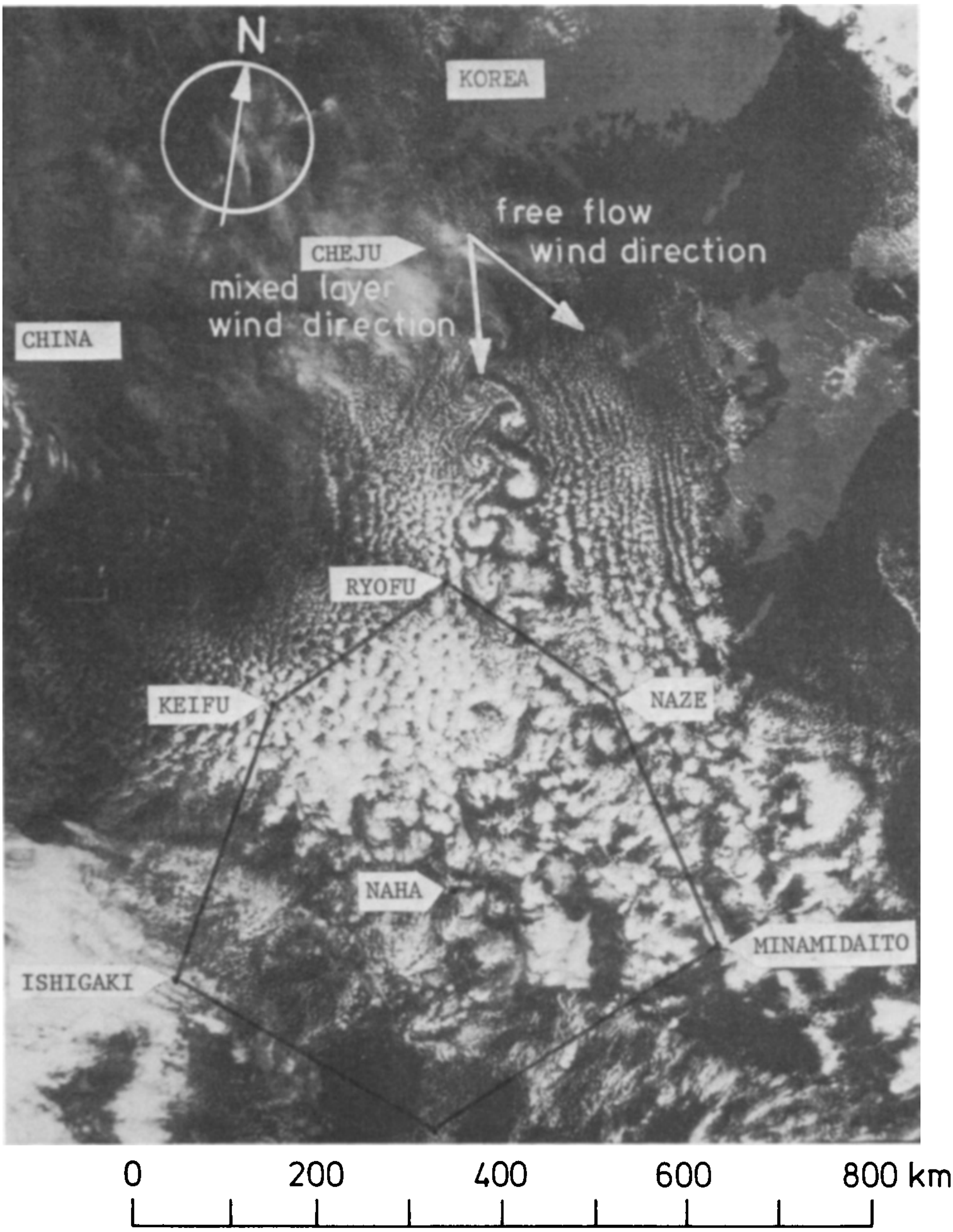

Fig. 2. DMSP satellite imagery at 1136 JST, February 17 1975, revealing well-developed Kármán vortex street extending downwind of Cheju Island into the region of the East China Sea and the AMTEX hexagonal network. The *in situ* mixed layer wind direction, as well as the wind direction immediately above the capping inversion (at Keifu), is indicated.

smaller than the mixed layer speed, which at Keifu was measured to be 8 m/s, the time lapse from shedding of one vortex to the next is a little over 3 h. The assumption about the travelling speed also implies $S \simeq d/l$, or a ratio of the effective shedding diameter to the length l of approximately $\frac{1}{5}$. Thus d is close to 20 km, and, appropriately, less than the 70-km surface dimension of the island.

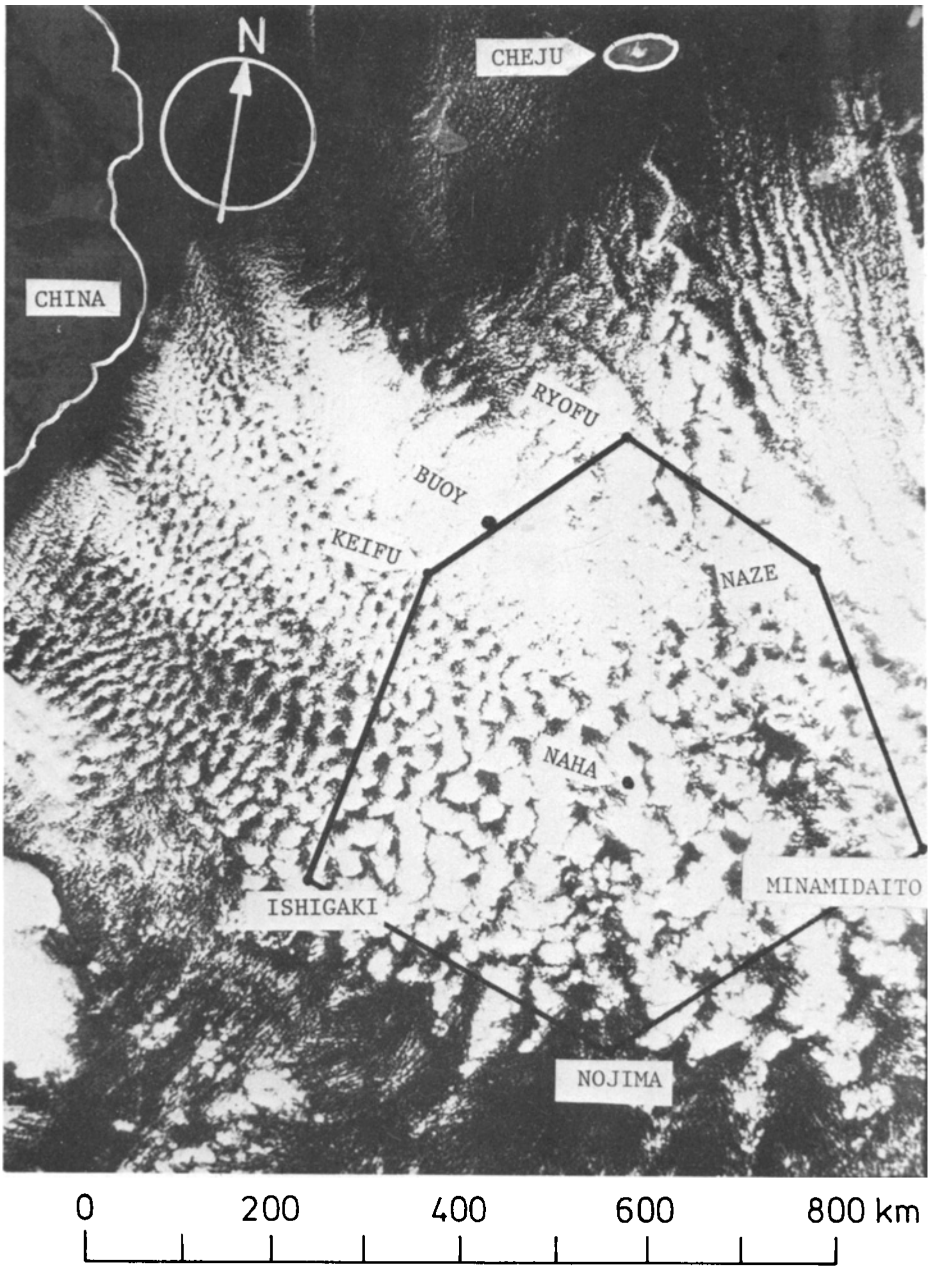

Fig. 3. DMSP satellite imagery at 1155 JST, February 16 1975, showing a clear view of Cheju Island, not discernible in Fig. 2 due to high level cloudiness. The coastlines of Cheju and China have been marked in white outline.

3. Further observations

Cheju has an aerological station but it was not part of the intensive AMTEX network. Though data from this station are available, unfortunately it did not report on February 17 1975, consequently the height of the capping inversion in the Cheju region was not available.

To compare the DMSP imagery with the AMTEX aerological data (closest available) some indirect methods are required. In the case of mixed layer developments, however, this is not all that bad as close agreement is usually obtained from even simple models. One such is given by Tennekes (1973) where constant surface heat flux and free flow stratification are assumed. In a coordinate system moving with the mean flow it takes the form $h = Cx^{1/2}$, where h is the height of the mixed layer, x is the downwind distance from the heat flux discontinuity, i.e. the coastline, and the value of the constant C depends on surface heat flux, free flow stratification, and wind speed. The mixed layer heights at two stations with different distance to shore are thus expected to relate as

$$h_1/h_2 = (x_1/x_2)^{1/2} \tag{3}$$

In this study the rawinsonde data collected at Keifu are used, as only radiosonde data (no wind data) were collected at Ryofu, the nearest station (see Fig. 2). At Keifu the mixed layer wind direction was measured to be between 10° and 20° east of north, in good agreement with the cloud streets direction at this station. Measuring the distance x off Korea in this direction, to Cheju and Keifu, respectively, one obtains from eq. (3) that the ratio of the mixed layer heights for these two stations should be $\sim(\frac{1}{3})^{1/2}$.

According to this scaling, the sounding actually measured at Keifu has been plotted in Fig. 4, adjacent to the cross-wind profile of Cheju. Also shown is the height at which the width of the island is equal to the effective shedding diameter of 20 km, previously estimated. It is seen that this height approximately agrees with the expected height of the capping inversion.

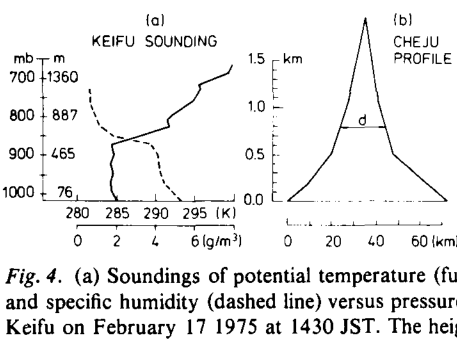

Fig. 4. (a) Soundings of potential temperature (full line) and specific humidity (dashed line) versus pressure from Keifu on February 17 1975 at 1430 JST. The heights in meters refer to the Keifu pressures converted to height above sea level and then scaled according to eq. (3). (b) The profile of Cheju normal to the direction of the vortex street. The effective shedding diameter d (~20 km) is noted.

That the observed vortex shedding actually takes place below the capping inversion layer is also indicated by the humidity sounding (Fig. 4) which limits cloud tops to be below the inversion, hence the vortices, since clouds are the tracer in this context. Furthermore, in the free flow just above the capping inversion, the wind direction as obtained from the sounding at Keifu is ~300°, indicating a sizeable wind direction shear within the capping inversion. The free flow wind direction is indicated in Fig. 2.

4. Discussion

It seems that the observed phenomenon is well explained by classical von Kármán theory. What is not in accordance, though, is the Reynolds number which in the atmosphere is $0(10^{10})$. This is well outside the range at which regular vortex streets are formed in wind tunnel experiments, as a matter of fact well outside the range at which wind tunnel experiments are performed. However, if the Reynolds number is defined on the basis of the eddy viscosity rather than molecular viscosity, it becomes $0(10^3)$ or less.

In a mixed layer the eddy viscosity is $0(w_* h)$, where w_* is the convective velocity scale (Deardorff, 1973), which is given by

$$w_* = \left(\frac{g}{T} Q_0 h\right)^{1/3} \tag{4}$$

In eq. (4) g is gravity and T is mean temperature. A typical value of the surface heat flux, Q_0, is 0.01 K ms^{-1} (average of February 16 and February 18 AMTEX 1975 flights reported by Wyngaard et al., 1978) which with the estimated mixed layer height for Cheju gives $w_* = 1.3$ m/s. With $u = 8$ m/s and $d = 20$ km the corresponding Reynolds number is about 200.

According to Fig. 1 the corresponding Strouhal number is 0.19 only, so the effective shedding diameter would have to resume a smaller value. On the other hand, the vortices move with a velocity smaller than the wind speed, which when taken into account would make the estimate on d larger. Hence we shall keep the original estimate of $d =$

20 km. Comparing the vortex cloud street in Fig. 1 with the vortex street ($Re = 225$) shown in Fig. 1.6 of Schlichting (1968), resulting from oil streaming around a small cylinder, actually shows a great deal of similarity. The similarity also holds for the value of D/l, where D (in the present case ~50 km) is the lateral separation of vortices: after the same number of vortices downstream, the ratio in both cases is about 0.5, a value about twice the value of 0.28 required for the streets to be stable (Kármán, 1912).

The concept of a "turbulent Reynolds number" or "atmospheric Reynolds number" being relevant is not so strange since the von Kármán street development relates to the fluid's ability to transport. In this case, momentum on relevant scales is transported to make up for the frictional loss of kinetic energy in the internal boundary layer of the shedding body. For laminar free flow, this ability is described by molecular viscosity, but for turbulent flows by the eddy viscosity. Since we are now dealing with a mixed layer, there is no need to distinguish between transport properties in the various directions, because the mid and upper parts of such a layer are fairly homogeneous.

One notable deficiency is that the von Kármán vortex street is a purely two-dimensional phenomenon, while the observations reported here are three dimensional. Since the observed phenomenon occurs below a relatively strong capping inversion, this may act to suppress the three dimensionality of the disturbance. To obtain an order of magnitude estimate of the vortex shedding related drag on Cheju, we again apply the classical von Kármán theory. Per unit height of cylinder the drag is given by (Schlichting, 1968)

$$F_v \simeq 1.5\rho u^2 D \quad (5)$$

On the other hand, the frictional drag per unit area of a plane surface can be expressed as

$$F_f = c_D \rho u^2 \quad (6)$$

For a sea surface a typical value of c_D is $1.2 \cdot 10^{-3}$. By comparing the two drag expressions we obtain that for every 5 m of vortex height one gets a drag equivalent to the frictional surface drag on an area corresponding to the actual shedding diameter. Since it is conceivable that the vortex height is at least several hundred meters, it is realized that the shedding related drag is of significant magnitude. This drag occurs in addition to the surface and form drag on the remaining part of the island. Also, invisible Kármán streets could be shed from the higher parts of the island.

5. Acknowledgements

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ОБЛАЧНЫЕ ВИХРЕВЫЕ ДОРОЖКИ В ПЕРИОД АМТЕХ 75

Сильный поток воздуха с севера в течение Эксперимента по трансформации воздушных масс (АМТЕХ 75) привел к появлению хорошо выраженной облачной вихревой дорожки за подветренной стороной острова Чедзюдо (Корея), как это наблюдалось 17 февраля 1975 г. Эта картина была изучена и объяснена в терминах классической вихревой дорожки Кармана в свободном ламинарном потоке за цилиндрическим препятствием в условиях докритических чисел Рейнольдса.

Изображения со спутника и аэрологические данные показывают, что частота излучения вихрей, которые находятся в пограничном слое под инверсией, равна единице за три часа. Остров выступает над уровнем моря до высоты почти 2 км и проникает высоту основания инверсии на уровне около 600 м. На этом уровне диаметр острова около 20 км, что согласуется с независимо определенным диаметром излучаемых вихрей.