Tellus (1977), 29, 48–53

On initialization for primitive equation models formulated for an alternating grid

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(Manuscript received April 5, 1976)

ABSTRACT

In a primitive equation model which uses an alternating grid for the integration, it is necessary to specify initial data for two sets of grid points. It is shown that improved results are obtained if the balance equation is solved for both sets, rather than for only one, and the other one being deduced by interpolation, although the improvement is modest. The requirement of zero initial divergence, imposed by the balance equation, causes gravity waves to appear. The behaviour of these waves is studied, and they are found to be of some importance for atmospheric processes governed by the vertical velocity.

1. Introduction

Forecasting with a primitive equation model requires a careful specification of initial data. Otherwise high-frequency oscillations are created as a result of lack of initial balance between mass and wind fields. To avoid this, the initial wind and geopotential fields have in many cases been related to each other by the balance equation.

Some primitive models are, mostly to reduce the computational demands, integrated using the so-called Eliassen grid or alternating grid, sometimes also called a staggered grid. Models utilizing an alternating grid require the values of the prognostic variables to be specified only in gridpoints where they are needed for the finite difference form of the forecast equations. For a grid staggered in time, the parameters have to be given in different points at two successive timesteps. In the following we shall call the grid used at even timesteps for the even grid, and the one used at odd timesteps the odd grid.

If a requirement of balance interrelating the mass and wind fields is satisfied initially for the even grid, it is desirable to find an odd grid that will minimize the creation of artificial gravity waves. For instance, if interpolation is used, the balance equation is not satisfied for the odd fields and the initial unbalance is probably more severe for the odd than for the even fields. Presumably, a better method would be to solve the balance equation for both sets of fields, and it must be expected that energy of the gravity waves resulting from an initial unbalance would be smaller in the latter case.

The purpose of this study is to investigate the possible improvement in a forecast when the balance equation is solved for both sets of initial fields. No attempts are made to introduce any kind of divergence into the initial velocity fields.

In this type of studies it is always a problem how to compare initialization techniques and determine which one is the best. In this study mainly the mean absolute value of surface pressure tendencies over the whole forecast area

$$P_t = \overline{\left|\frac{\partial p_s}{\partial t}\right|} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\partial p_s}{\partial t}\right|_i$$

(p_s is the surface pressure at the gridpoints) has been utilized to determine the possible presence of

[1] Contrib. no. 321.

This work has partly been supported by the Swedish Natural Science Research Council Contract No. G 0223-058.

artificial gravity waves. P_t is evaluated from two consecutive even (or odd) timesteps, and is thus a pressure change over two timesteps ($2\,\Delta t = 8$ min), but it may be converted into the unit millibars per 3 hours as used in synoptic meteorology. This pressure tendency results from meteorological waves as well as artificial gravity waves created by the initial unbalance. P_t is thus larger the more gravity waves present in the forecast. To get an idea of the value of P_t if high-frequency waves are absent, the pressure change as obtained from the model forecast over a period of 3 hours has been evaluated. Thereby P_t was evaluated utilizing a 12-hour forecast and a 9-hour forecast (i.e. $\Delta t =$ 3 hours). Since gravity waves have comparatively high frequencies, these waves would to a great extent be filtered out in this 3-hour pressure change, although gravity waves with longer periods still contribute to P_t. The difference between this average change and P_t obtained from two consecutive timesteps will thus give an estimate of the high-frequency gravity waves present in the forecast.

Closely connected to the surface pressure tendency in a point is the divergence at every level above the point. The temporal variation of the divergence at different levels reveals if the gravity waves are internal or external.

2. Forecasting utilizing different initialization techniques

The numerical model used for the experiments is a primitive equation model with staggered grid (both in time and space) and sigma ($\sigma = p/p_s$) as the vertical coordinate (Sundqvist, 1974). A leap-frog scheme is employed for the time-integration and the time-step was 4 minutes. Five equally spaced vertical levels were used and the forecast area, centred at the north pole, was octagonal and covered the Northern Hemisphere down to a latitude of about 30° N. The grid-distance was 300 km and the number of grid points about 2000. The balance equation employed for this sigma system model has been described by Sundqvist (1975).

The weather situation used as initial data for the present study is 1 November 1969 00Z.

2.1. *The balance equation applied to even fields only*

In the *first experiment* the balance equation was solved for the even grid with the non-linear terms neglected, i.e. the initial mass and wind fields were in *geostrophic balance.* The fields for the odd grid were obtained by interpolation in the sigma-levels from the even fields, using bilinear interpolation. The value of P_t as a function of time for the geostrophic case is seen as curve a in Fig. 1. The in-

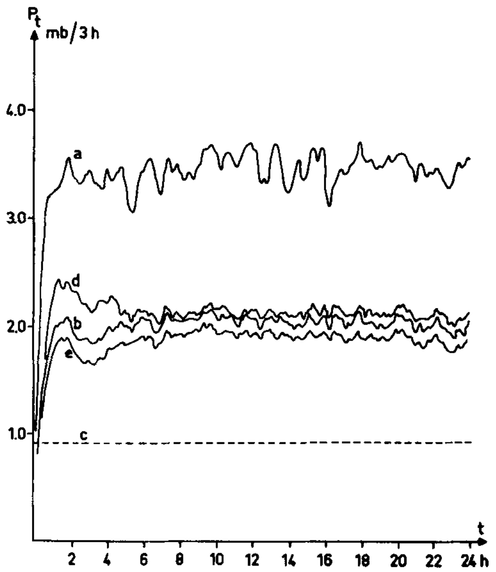

Fig. 1. Surface pressure tendencies as a function of time obtained during a 24-hour forecast from 1 November 1969 00Z without orography. Curve a: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from even timesteps with geostrophic balance initially in even timestep fields. Curve b: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from even timesteps with the balance equation solved for the even timestep fields. Curve c: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ as obtained from a 12-hour forecast minus a 9-hour forecast in the same experiment as curve b. Curve d: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from odd timestep fields in the same forecast as curves b and c. Curve e: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from even timesteps when the balance equation is solved for both even and odd timestep fields.

itialization procedure is based on the assumption of non-divergent mass flow at individual σ-levels and vanishing $\dot{\sigma}$ ($= d\sigma/dt$), and as a consequence the surface pressure tendency, P_t, is initially small. During the first hours of the forecast a divergent wind field develops and P_t increases. After about 2 hours P_t has reached a level of about 3 mb/3 hours around which it fluctuates during the remainder of the forecast.

In the *second experiment* the *balance equation* was solved for the even fields, and the odd fields were interpolated in the same way as before. Curve *b* in Fig. 1 shows P_t as a function of time for this experiment. P_t starts almost from zero and increases to a value where it levels off, but in this experiment the level is at about 2.0 mb/3 hours. The value for P_t as obtained from a 12-hour forecast minus a 9-hour forecast is shown by line *c* in Fig. 1. This mean 3-hour tendency is in the present forecast less than 1 mb/3 hours. Obviously the tendencies shown in curves *a* and *b* of Fig. 1 to a considerable extent are caused by high-frequency waves. Apparently, by including the non-linear terms in the balance equation, the gravity waves in the forecast have been reduced considerably but not completely. The magnitude of the tendencies remains the same throughout the forecast showing that the gravity waves do not disappear.

The curves *a* and *b* in Fig. 1 show P_t for the even timesteps, while curve *d* (corresponding to *b*) shows P_t for the odd timesteps. Except for the first 5 or 6 hours the difference between the values of the two curves *b* and *d* is of the order 0.1 mb/3 hours. The reason why the odd and even P_t-values do not differ more, might be the coupling between them during the forecast. In a leap-frog scheme the new values in an odd timestep are obtained using the values from an even timestep and vice versa. Thus gravity waves in the odd timestep fields will influence the fields of the even grid and likewise will the odd grid be influenced by gravity waves in the even grid.

To establish the character of gravity waves, the divergence is a suitable instrument, since these waves very significantly affect the divergence. The vertically integrated divergence is almost equivalent to the surface pressure tendency. The variations of the divergence $\partial u/\partial x + \partial v/\partial y$ at every sigma level, and the vertically integrated value in a point east of Greenland, are shown in Fig. 2. This point was chosen because it is situated in an area with weak synoptic activity. The behaviour was, however, found to be similar in other points.

The initial requirement of no divergence in the mass flow causes the divergence to start from zero at every sigma level. Certainly this is an artificial state, since the synoptic waves are connected with divergent wind fields. Accordingly the model tries

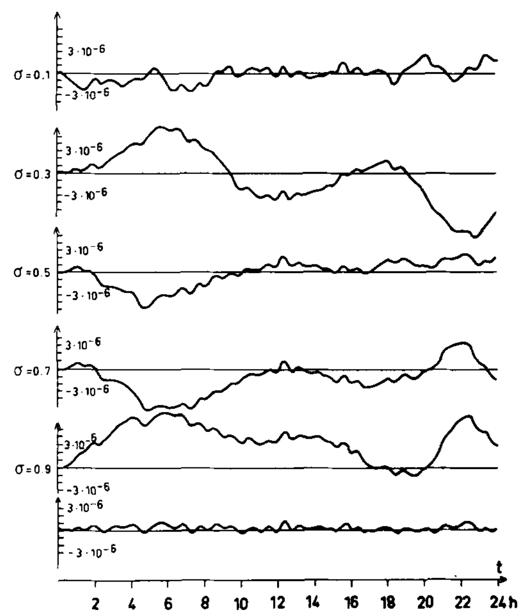

Fig. 2. The variation of the divergence $\partial u/\partial x + \partial v/\partial y$ in a point east of Greenland at every sigma-level and the vertically integrated value obtained during a 24-hour forecast from 1 November 1969 00Z with the balance equation solved initially for the even timesteps.

to establish a divergent mass flow suitable for those meteorological waves that are present. As seen in Fig. 2 the divergence initially increases (or decreases) and grows to a too large value, decreases (increases) and continues to oscillate around a mean value. Obviously gravity waves have developed, and they are partly of the internal type, since the amplitude is of different sign at different levels. Their amplitude is of the order $3 \cdot 10^{-6}\ s^{-1}$ and their period somewhat shorter than 12 hours. The frequency (ω) for gravity waves is given by

$$\omega^2 = f^2 + k^2 c_0^2$$

where f is the Coriolis parameter, $k = 2\pi/L$ is the wavenumber, L being the wavelength, and c_0 is the phase velocity of a wave on a non-rotating earth ($f = 0$).

Økland (1972) showed that c_0 for an internal gravity wave having the highest eigenvalue (i.e. having amplitudes of opposite sign at consecutive sigma levels) in a five-level model is of the order 15 $m \cdot s^{-1}$. For a 6000 km wavelength $k^2 c_0^2$ is approximately $2 \cdot 10^{-10}\,s^{-1}$. Thus, the internal gravity waves should have a period close to half a pendulum day, the period for inertial oscillations, which is a good agreement with what is observed (Fig. 2). The internal gravity waves are found to be weak at the uppermost level. This is also in accordance with Øklands results, which showed that the higher modes are small in the stratosphere (the uppermost level). The vertically integrated divergence, shown at the bottom of Fig. 2, reflects the external gravity waves. Obviously waves of this kind also exist. The external waves are also seen at every sigma level superimposed on the internal waves. For the external waves $k^2c_0^2 \gg f^2$, and thus is $\omega \sim kc_0$. Økland (1972) showed that when a finite difference formulation is used, the wavenumber k will take the form

$$k^2 = k_x^2 + k_y^2 = \frac{1}{(\Delta x)^2} \sin^2\left(\frac{2\pi}{L_x}\Delta x\right) + \frac{1}{(\Delta y)^2}\sin^2\left(\frac{2\pi}{L_y}\Delta y\right)$$

where Δx and Δy are the horizontal space increment in the x and y direction. Here centred differences have been used, and applying this to the staggered grid in our model, we thus have $\Delta x = \Delta y = d = 150$ km. $L_x = L_y = 4d$ is the wave that will give the maximum frequency. For $c_0 = 300\ \mathrm{m \cdot s^{-1}}$ we have $\omega \sim 2.8 \cdot 10^{-3}$, which corresponds to a period of 37 minutes. The observed period for the external waves is of the order 50 minutes (Fig. 2). Obviously, the dominating wavelength is longer than $4d$. (Shorter waves can not be represented due to aliasing.) The observed period 50 minutes corresponds to a wave having a wavelength of 1200 km ($8d$).

An interesting question is how the presence of artificial gravity waves affects the synoptic waves. From Fig. 2 we stated that the divergence, owing to the presence of gravity waves oscillates around a time-dependent mean value, which is connected with the Rossby mode. From the continuity equation we then realize that the gravity waves must have a similar influence on the vertical velocity. Thus, forecasts of atmospheric processes governed by the vertical velocity, such as precipitation, must by necessity be affected by these waves. The influence on other processes, less dependent of the vertical velocity, is harder to establish, but experiments indicate (i.e. Økland, 1970), that the influence is of minor importance, at least for practical purposes.

2.2. *The balance equation applied to even as well as odd fields*

In a *third experiment* the *balance equation* was applied not only to the even fields but also to the odd ones. In this case the balanced even fields should not be disturbed by the interplay with unbalanced odd fields during the forecast. Both sets would be equally good and the gravity waves might possibly be reduced. Curve *e* in Fig. 1 shows P_t for even timesteps as a function of time in this forecast. The P_t-curves obtained from the odd timesteps utilizing the two initialization techniques in the second and third experiments are shown in Fig. 3. The magnitude of the tendencies is lower in the last experiment for the odd as well as for the even timesteps but the differences are small indicating that the initial unbalance in the odd fields, if evaluated by interpolation, is not much different than if the balance equation were used to get these fields. Thus the solution of the balance equation for the odd fields seems unnecessary.

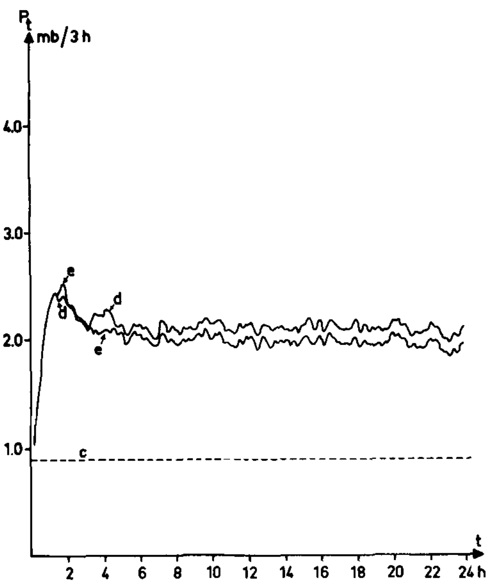

Fig. 3. Surface pressure tendencies as a function of time obtained during a 24-hour forecast from 1 November 1969 00Z without orography. Curve *d*: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from odd timesteps when the balance equation is solved only for even grid. (The same as curve *d* in Fig. 1.) Curve *e*: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_t$ from odd timesteps when the balance equation is solved for both even and odd timestep fields.

It is of interest to see if the surface pressure at a specific point is different due to the different initialization techniques and therefore the difference between the surface pressure fields obtained from the last two forecasts was evaluated. The maximum difference after a 12-hour forecast at any of the 2000 grid-points was 0.5 mb. A corresponding comparison of the vertical velocities was also performed. Roughly the patterns were the same for the two forecasts with a difference not exceeding 10%. The conclusion is that the two initialization procedures may be considered to be equally good for practical purposes.

Forecasts were also made with orography included both in the initial fields and during the subsequent forecasts. Fig. 4 shows P_t for even timesteps when only even fields were balanced initially,

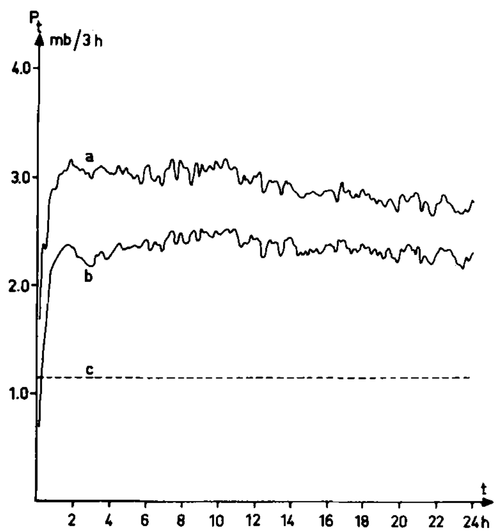

Fig. 4. Surface pressure tendencies as a function of time obtained during a 24-hour forecast from 1 November 1969 00Z. Orography is included both in the initialization and in the forecast. Curve *a*: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_i$ from even timesteps with the balance equation solved for the even timestep fields. Curve *b*: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_i$ from even timesteps when the balance equation is solved for both even and odd timestep fields. Curve *c*: $(1/N)\Sigma_{i=1}^{N}|\partial p_s/\partial t|_i$ as obtained from a 12-hour forecast minus a 9-hour forecast. Orography is included in the forecast.

and also the resulting P_t-curve when both even and odd fields were balanced initially. The difference between the two curves is here considerably larger. The gradients of temperature and pressure fields in sigma-surfaces are large in mountain areas. If only the even timestep fields are balanced initially and the odd fields are obtained by interpolation the effects of orography on the temperature and pressure fields will be smoothed in the odd grid. This seems to cause a more serious unbalance between the fields when orography is included and an improvement is obtained if both sets of fields are balanced initially.

The magnitude of the tendencies is larger when orography is included than without orography. This is also the case for the 3-hour tendencies which have increased with about 0.4 mb/3 hours. Thus the larger P_t-values in the cases with orography is not only due to more high-frequency oscillations but also to larger pressure-tendencies for the synoptic waves.

3. Conclusions

The purpose of the present investigation has been to study the effect of different initialization schemes for models using alternating grid. Normally the initial fields for odd timesteps are obtained by interpolation from the initial even fields. It has been shown that in this case the surface pressure tendency (P_t) for odd timesteps is larger than for the even timesteps during the first hours of the forecast, but later they level off at the same value. The simplest way to reduce this greater unbalance for the odd timesteps is to solve the balance equation initially also for the odd timestep fields. The experiments show, however, that neither the synoptic pattern nor the remaining gravity waves are noticeably affected by this method, but when orography is included an improvement is found.

The gravity waves introduced by the incomplete balance between the initial mass and wind fields remain throughout the 24-hour forecast. The pressure tendencies are considerably larger than those obtained by evaluating 3-hour surface pressure tendencies in the model.

Attention has also been paid to the influence of gravity waves on model forecasts of different atmospheric processes. Processes governed by the vertical velocity were found to be affected, while the influence on other processes for practical purposes seems to be of minor importance.

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ИНИЦИАЛИЗАЦИЯ ДЛЯ МОДЕЛИ ПРИМИТИВНЫХ УРАВНЕНИЙ, СФОРМУЛИРОВАННОЙ ДЛЯ АЛЬТЕРНАТИВНОЙ СЕТКИ

для численного предсказания по модели примитивных уравнений с использованием для интегрирования альтернативной сетки необходимо определить начальные данные для двух наборов точек сетки. Показано, что резулвтаты получаются лучше, если решать уравнение боланса для обоих наборов, а не для одного, с получением другого набора путем интерполяции, хотя улучмение является умеренным. Налагаемое уравнением баланса требование нулевой начальной дивергенции приводит к возникновению гравитационных волн. Изучается поведение этих волн, и найдено, что они имеют некоторое значение для атмосферных процессов, связанных с вертикальной скоростью.