A theoretical framework for the description of estuaries

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ABSTRACT

A theoretical framework for the description of an estuarine system is outlined, with particular reference to the Baltic. This description, in terms of continuous functions of salinity and time, may be considered as an improvement of existing "box models", which are inherently incapable of making efficient use of hydrographic data. Relations between deep water supply and internal mixing properties, as well as the equations controlling diffusion of substances in general, are derived and their applicability discussed.

1. Introduction

Despite the great amount of work (see, e.g. Fonselius, 1969) that has been devoted to the understanding of the Baltic, it seems that we are still missing an appropriate quantitative description of the general circulation in the system. Obviously such a description is vital for a correct evaluation (or even estimation) of the fluxes involved in the overall ecological balance.

There is a gap between our relatively detailed knowledge of the hydrographic state and the very crude estimates of the fluxes involved; the latter are based on box models in one form or another. Indeed our understanding of the deep layer exchange is founded on the so-called Knudsen relations involving only three quantities related to observation, i.e. the net fresh water supply M_0, a deep and a surface layer salinity (s_1 and s_2) representing in some vague sense the conditions in the Kattegatt and the Baltic. M_0, s_1 and s_2 are related in the following way to satisfy continuity of volume and salt.

$$M_0 + M_1 = M_2$$
$$M_1 s_1 = M_2 s_2$$

or

$$M_1 = M_0 \frac{s_2}{s_1 - s_2}$$

where M_1 represents the flux into the Baltic of water having salinity s_1, while M_2 represents the flux leaving the Baltic with salinity s_2. Although

some refinements on this description are possible (see Fonselius, 1969), it should be recognized that our present ideas about the mean circulation in the Baltic rely almost completely on the above simple calculation.

The basic assumption needed in using the above estimate of M_1 is that the more saline, deeper parts of the Baltic can be considered as an essentially homogeneous entity. We may then estimate fluxes of various nutrients by multiplying volume flux from the lower to the upper layer (given by or closely related to M_1) with the mean concentrations in the lower layer.

Although such calculations are of primary importance for our present knowledge, it should be recognized that the result may be wrong even with respect to order of magnitude. The reason is simply that the concentrations involved may well vary by an order of magnitude within the deep layer, a situation which is in fact typical for any strongly stratified fluid. Consequently we may make an order of magnitude error when representing the lower layer as a single homogeneous entity (i.e. when we attribute a single concentration value to the lower layer).

In this paper I will try to develop the first step in an effort to cope with the above-mentioned difficulty. This will amount to describing the system (possibly divided into a number of subsystems) with continuous functions of one space-like variable and time. The choice of the space-like variable is not obvious. I have chosen salinity
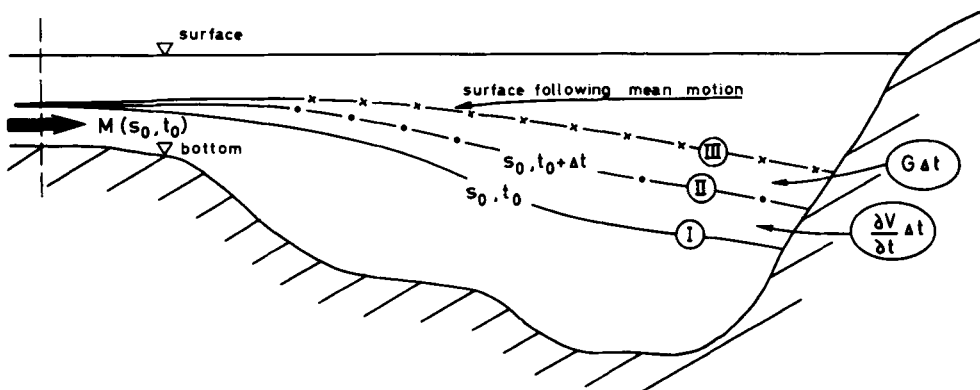
Fig. 1. Section through basin with illustration of control surfaces used in derivation of continuity relations. Surface I: Isohaline surface s_0 at time t_0. Surface II: Isohaline surface s_0 at time $t_0 + \Delta t$. Surface III: Position at time $t_0 + \Delta t$ of surface which at time t_0 was identical with I but has followed the mean (non-turbulent) motion thereafter.

mainly because the density variation is dominated by the salt concentration, which most likely implies that the variation of any other concentration involved will tend to be small along surfaces of constant salinity.

2. Definitions

We consider a semi-enclosed basin like the Baltic, i.e. a region enclosed by the free surface facing the atmosphere, the bottom and fixed vertical control surfaces across the connections with adjacent ocean basins (Fig. 1).

In the following the salinity s and time t will be used as independent variables.

The *hydrographic state* in the basin is represented by the function $V(s,t)$ defined as the volume of all the water in the basin with salinity larger than s. We will also make use of the related functions $v(s,t)$ and $S(s,t)$ defined by

$$v(s,t) = -\frac{\partial V}{\partial s} \tag{1a}$$

$$S(s,t) = \int_s^\infty v \cdot s \, ds \tag{1b}$$

It is easily recognized that $v\,ds$ equals the volume of water with salinity in the interval $(s, s + ds)$ while $S(s,t)$ equals the salt content in the volume $V(s,t)$.

The *circulation* in the basin is described by the function $G(s,t)$ representing the volume flux through the isohaline surface with salinity s.

The *interaction* with adjacent basins is described by the function $M(s,t)$ defined as the volume flux of water with salinity larger than s into the basin.

We may also define a function m related to M by

$$m(s,t) = -\frac{\partial M}{\partial s} \tag{2}$$

meaning that $m\,ds$ will represent the influx of water in the salinity interval $(s, s + ds)$.

To represent the rate of *mixing* in the basin we introduce the function $F(s,t)$, giving the diffusive flux of salt through the isohaline surface with salinity s. With diffusive flux we mean essentially the non-advective part of the flux, i.e. the flux through a surface following the "mean" motion. Since some confusion may arise as to the precise meaning of F we refer the reader to the discussion in the next section where F is introduced into the salt balance.

The *chemical* and/or *ecological* state of the system is described by functions $C_\nu(s,t)$ giving the total content of some particular dissolved substance (referred to by the index ν) in the volume $V(s,t)$.

We will also make use of the "concentration" $c_\nu(s,t)$ defined in such a way that

$$C_\nu(s,t) = \int_s^\infty c_\nu v \, ds \tag{3}$$

This definition of c_ν implies that $c_\nu v \, ds$ is the content of "ν" in the salinity interval $(s, s + ds)$, i.e. c_ν must be the mean concentration in the incremental volume $v\,ds$.

Since the volume in ordinary space may be thought of as a sheet of fluid with small but variable thickness we may interpret c_v as a weighted area mean value of the local concentration in which the weighting function should be proportional to the thickness of the sheet. Since the sheet is composed of the space between two "neighbouring" isohalines having usually very small slope, it follows that the weighting function is closely approximated by $\partial z/\partial s$ where z is the depth of the isohaline.

Finally, we define functions $F_v(s,t)$ so that F_v represents *the diffusive flux of the substance "v"* through the isohaline surface with salinity s (see next section also).

3. Continuity of mass (volume) and salt

Let us now formulate the conservation principles for mass and salt in terms of the concepts introduced in the previous section. First we note that with exceedingly good accuracy conservation of mass implies conservation of volume (Boussinesq approximation) for the type of phenomena under consideration.

In the following we will thus use volume conservation as the basic principle rather than mass conservation. In order to make use of conservation of volume, we consider the rate of change of the volume $V(s,t)$. In accordance with the definition of M and G (see Fig. 1) we have

$$\frac{\partial V}{\partial t} = M - G \qquad (4)$$

(Note that the volume between I and III in Fig. 1 equals $M(s_0, t_0) \cdot \Delta t$.)

Let us now consider continuity of salt and begin by a quantitative (hopefully unambiguous and appropriate) definition of the function $F(s,t)$ introduced in the previous section. Let us first recognize that a diffusive flux (whether turbulent or molecular), contrary to an advective flux, is a flux within the fluid itself, i.e. a flux through a surface following the motion of the fluid.

Regarding turbulent diffusion we should consider the flux through a surface following the mean (non-turbulent) motion. As usual we may run into difficulties if the mean and turbulent motions cannot be clearly distinguished, i.e. if the turbulent motion ranges over scales all the way up to the time and length scales of the "mean" motion. These

difficulties are always present in the description of turbulent motion and will not be discussed any further in this context.

Let us now consider a surface $\mathscr{S}$ which at time t_0 coincides with the isohaline s_0 and which is displaced by the mean motion field. In the time interval $(t_0, t_0 + \Delta t)$. $\mathscr{S}$ is displaced to some new position (illustrated by III in Fig. 1). We then define $F(s,t)$ in order that the salt flux through $\mathscr{S}$ in the time interval $(t_0, t_0 + \Delta t)$ approaches $F(s_0, t_0)\Delta t$ when $\Delta t \to 0$.

The "ecological" and chemical fluxes $F_v(s,t)$ are defined in a precisely analogous manner in order that $F_v(s_0, t_0)\Delta t$ equals the flux of "v" through $\mathscr{S}$ in the time interval $(t_0, t_0 + \Delta t)$.

As will be discussed and utilized in the next section the fluxes F_v and F are not independent since they are caused by the same turbulent activity.

Let us now consider the rate of change of the salt content, $S(s_0, t)$, below the isohaline s_0. We note that we have three contributions to this quantity associated with (i) the inflow $M(s,t)$, (ii) the advection G and (iii) the diffusion F. In accordance with the definition of these quantities we obtain

$$\frac{\partial S}{\partial t} = -G \cdot s + \int_s^\infty m \cdot s \, ds - F \qquad (5)$$

Equations (4) and (5) form the basis for our theoretical framework. It should be noted that (4) and (5) are exact within the Boussinesq approximation. We may also consider (4) and (5) as definitions for the functions G and F which makes it unnecessary to introduce explicitly the surface $\mathscr{S}$ used in the above discussion.

Differentiating (4) and (5) with respect to s, multiplying the first with s and subtracting from the latter we obtain

$$\frac{\partial F}{\partial s} = -G \qquad (6a)$$

or when combined with (4)

$$\frac{\partial V}{\partial t} = M + \frac{\partial F}{\partial s} \qquad (6b)$$

Taking an average over a sufficiently long time (denoted by an overbar) we obtain

$$\frac{\partial \bar{F}}{\partial s} = -\bar{M} \quad \text{or} \quad \bar{F} = \int_s^\infty \bar{M} \, ds + \text{const.} \qquad (7)$$

Equation (7) represents in the most simple way how the deep water supply is related to the overall "vertical" (i.e. cross-isohaline) mixing properties in the basin.

Making appropriate measurements of the supply function $M(s,t)$ we may thus gather information about the mean rate of mixing through any isohaline surface without doing a single measurement inside the basin.

According to the definition of F and G the total saltflux through an isohaline surface $H(s,t)$ towards lower salinity is given by

$$H(s,t) = F(s,t) + G(s,t) \cdot s \qquad (8a)$$

or making use of the volume balance (4)

$$H = F + Ms - s\frac{\partial V}{\partial t} \qquad (8b)$$

Making again an average over a sufficiently long time we have

$$\bar{H} = \bar{F} + \bar{M}s \qquad (9)$$

Combining with (7) and integrating by parts we obtain the relation

$$\bar{H} = \int_s^\infty \bar{m}s\,ds$$

between H and the mean salt supply from adjacent regions.

4. Continuity of some other substance

Let us now consider continuity of some other quantity "ν". We do not assume that "ν" is conservative, i.e. there may be sources and sinks inside the basin. We thus define a source function $Q_\nu(s,t)$ so that Q_ν represents the production of "ν" in all the water with salinity exceeding s.

The continuity relation for the substance "ν" may then be written (compare derivation of (5)):

$$\frac{\partial C_\nu}{\partial t} = -G \cdot c'_\nu + \int_s^\infty m\gamma_\nu\,ds - F_\nu + Q_\nu \qquad (10)$$

In eq. (10) we have been forced to introduce two new symbols γ_ν and c'_ν; γ_ν representing the concentration in the inflowing water, and c'_ν the mean concentration in the incremental volume $G\Delta t$. Neither of these concentrations is in general equal to c_ν defined in section 2. c_ν and c'_ν are mean values over isohaline surfaces formed with

different weighting functions, while γ_ν represents a local concentration of "ν". Only when the local concentration is constant over the isohaline surfaces are c'_ν and γ_ν identical with c_ν. In what follows we will keep γ_ν as a variable distinct from c_ν but approximate c'_ν by c_ν. The latter approximation is absolutely vital for our description. In fact the accuracy which can be obtained is usually limited by this and the corresponding approximation underlying eq. (11). The reason to keep γ_ν as a distinct quantity, which might seem inconsistent, is that γ_ν is likely to represent an extreme value of the local concentration in the system. Replacing c'_ν by c_ν is, on the other hand, justified if the correlations between the local concentration and the weighting function for c_ν and c'_ν are weak. This is however not necessarily the case. If, e.g. the flux G is concentrated to regions where the local concentration is small and the stratification strong (i.e. $\partial z/\partial s$ small), we are in difficulties. In such cases we are probably forced to divide the system into parts within which the deviations of the local concentration from c_ν (or some other mean value) are sufficiently small.

In order to use eq. (10) we have to introduce some relation for F_ν, i.e. make explicit use of the fact that F_ν and F are not independent. In what follows we will use the relation

$$F_\nu = F \cdot \frac{\partial c_\nu}{\partial s} \qquad (11)$$

Equation (11) essentially involves an approximation of the same type and order as replacing c'_ν by c_ν. This may be seen by considering local fluxes f and f_ν so that $F_\nu = \int f_\nu\,dA$ and $F = \int f\,dA$, where $\int dA$ represents integration over an isohaline surface. We now assume that f and f_ν satisfies

$$\frac{f}{\partial s/\partial n} = \frac{f_\nu}{\partial c'_\nu/\partial n} \quad \text{or} \quad f_\nu = f\frac{\partial c_\nu}{\partial s} \qquad (12)$$

where n is a coordinate perpendicular to the isohaline s and c'_ν is the local concentration of "ν". The validity of eq. (12) is not undisputable. It may be violated when molecular processes are of importance, e.g. in so-called double diffusion. Otherwise no restriction on the variation in space and time of the mixing intensity is involved. Physically (12) implies that the substance "ν" and salt are mixed by the same field of turbulence.

Applying the integral operator $\int dA$ on (12) we obtain

$$F_\nu = \int f \frac{\partial c_\nu^l}{\partial s} \, dA = F \frac{\partial c_\nu''}{\partial s} \qquad (13)$$

The last part of eq. (13) defines another "mean value", c_ν'', such that $c_\nu''/\partial s$ equals a weighted mean value of $\partial c^l/\partial s$. Equation (11) is thus obtained by replacing c_ν'' by c_ν.

Replacing c_ν^l with c_ν and making use of (11) we have from (10)

$$\frac{\partial}{\partial t} C_\nu = \int_s^\infty m\gamma_\nu \, ds - F \frac{\partial c_\nu}{\partial s} - Gc_\nu + Q_\nu \qquad (14)$$

Equation (14) is the required continuity relation for "ν", which together with salt and volume continuity form the basis for our description of the chemical state and development.

Differentiating (14) with respect to S and making use of (6a) we obtain

$$c_\nu \frac{\partial v}{\partial t} + v \frac{\partial c_\nu}{\partial t} = m\gamma_\nu + c_\nu \frac{\partial G}{\partial s} + F \frac{\partial^2 c_\nu}{\partial s^2} - \frac{\partial Q_\nu}{\partial s}$$

Making use of volume continuity (i.e. operate with $c_\nu(\partial/\partial s)$ on (4) and subtract) we obtain

$$v \frac{\partial c_\nu}{\partial t} = m(\gamma_\nu - c_\nu) + q_\nu + F \frac{\partial^2 c_\nu}{\partial s^2} \cdot \qquad (15)$$

where $q_\nu = -\partial Q_\nu/\partial s$.

Equation (15) may be considered as a diffusion equation for "ν" in s-space, in which the function v, m and F describing the salt balance enter as coefficients. Equation (15) however represents just one possible way to manipulate eq. (14) (with the aid of the salt and mass balances). Which form of the balance equations to use depends of course on the type of data available.

The flux of "ν", $H_\nu(s,t)$, through the isohaline s is given by

$$H_\nu(s,t) = F_\nu + Gc_\nu^l$$

or within the approximations involved in deriving (14)

$$H_\nu = F \frac{\partial c_\nu}{\partial s} + Gc_\nu \qquad (16)$$

Taking the average we obtain

$$\overline{H_\nu} = \overline{F \cdot \frac{\partial c_\nu}{\partial s}} + \overline{G \cdot c_\nu} \qquad (17)$$

Equation (17) becomes much more useful if it may be approximated by

$$\overline{H_\nu} = \overline{F} \cdot \frac{\partial \overline{c_\nu}}{\partial s} + \overline{G} \cdot \overline{c_\nu} \qquad (18)$$

or one of the alternative forms

$$\overline{H_\nu} = \overline{F} \frac{\partial \overline{c_\nu}}{\partial s} + \overline{M} \cdot \overline{c_\nu} = \overline{F} \frac{\partial \overline{c_\nu}}{\partial s} - \frac{\partial \overline{F}}{\partial s} \cdot \overline{c_\nu}, \text{ etc.}$$

It has to be carefully recognized that the step from (17) to (18) is far from trivially justified. It is thus strongly recommended to reconsider the situation any time this type of approximation is needed.

5. Indirect determination of $F(s,t)$

We have found that the flux $H_\nu(s,t)$ in general may be determined if the state of the system [$V(s,t)$ and $C_\nu(s,t)$] and the function $F(s,t)$ is accurately known (having assumed of course that the system satisfies our basic assumptions). The purpose of the analysis is to arrive at more efficient ways of making use of our knowledge of the state of the system. It is thus natural to consider for the moment $V(s,t)$, $C_\nu(s,t)$ (and thus the derived functions $v(s,t)$ and $c_\nu(s,t)$) as known with sufficient accuracy. The crucial question is then how to obtain information about the function $F(s,t)$.

Obviously the most straight-forward approach is to measure $M(s,t)$ and calculate $F(s,t)$ with the aid of (6b). We then need a boundary condition on $F(s,t)$. As we will find in (5.1) it is most natural and sometimes even necessary to assume the existence of a finite amount of water having the maximum salinity present in the system (s_m). This means that the functions $V(s,t)$, $G(s,t)$, $M(s,t)$ not necessarily tend to zero when $s \to s_m$. To obtain a boundary condition on $F(s,t)$ we consider eqs. (5) in the limit $s \to s_m$ obtaining

$$s_m \frac{\partial}{\partial t} \int_s^{s_m} v \, ds = -G \cdot s_m + s_m \int_s^{s_m} m \, ds - F$$

when $s \to s_m$ or in view of the definitions of v and m

$$s_m \frac{\partial}{\partial t} V(s,t) = G(s,t) \cdot s_m + s_m M(s,t) - F(s,t)$$

where $s - s_m \rightarrow 0$.

Making further use of volume continuity as expressed by eq. 4 we obtain

$$F(s,t) = 0 \qquad s \rightarrow s_m \qquad (18)$$

which is the required boundary condition on $F(s,t)$. We thus have

$$F(s,t) = \int_s^{s_m} M(s,t)\, ds \qquad (19)$$

irrespective of whether M, V, G vanish at $s = s_m$ or not.

Although the direct observation of M and subsequent determination of F is a most attractive approach, it is far from always possible because of observational difficulties. It is thus of the utmost importance if we can determine F in some indirect way from available knowledge about the state functions $C_v(s,t)$ and $V(s,t)$.

In sections (5.1)–(5.3) we will discuss briefly some such possibilities. The discussion is by no means complete; the purpose is only to point out some of the more obvious possibilities and difficulties. Most important, we have left out completely any discussion of computations based on time-dependent tracer distributions, i.e. when the time dependence is used as a source of information rather than eliminated through the formation of mean values.

5.1. *Computations based on a conservative tracer*

If we can ignore the heat supply from radiation, which is usually the case in the stratified part of the system under consideration, the temperature $T(s,t)$ becomes a conservative tracer, i.e. a tracer with zero source function. Equation (15) then simplifies to

$$v \frac{\partial T}{\partial t} = m(\gamma_T - T) + F \frac{\partial^2 T}{\partial s^2} \qquad (20a)$$

where $\gamma_T(s,t)$ is the temperature distribution in the incoming (or outgoing) water.

Differentiating (6b) we obtain

$$\frac{\partial v}{\partial t} = m - \frac{\partial^2 F}{\partial s^2} \cdot \qquad (20b)$$

Assuming now that the state functions v, T and γ_T are known, eqs. (20) provide an ordinary

differential equation for the determination of m or F. The equation is of second order and we thus need two boundary conditions, e.g. on F.

Let us illustrate the situation with the "steady state" case. We thus assume either that m, F and v are truly stationary, or that variations in these functions are only weakly correlated with variations in T and γ_T (i.e. that $\overline{v \cdot \partial T / \partial t}$ may be approximated by $\bar{v} \cdot \partial \bar{T} / \partial t$ etc.).

We then obtain

$$\bar{m}(\bar{\gamma}_T - \bar{T}) + \bar{F} \frac{d^2 \bar{T}}{ds^2} = 0 \qquad (21a)$$

$$\bar{m} - \frac{d^2 \bar{F}}{ds^2} = 0 \qquad (21b)$$

As discussed above the only boundary condition available is that F should vanish at $s = s_m$. In fact eqs. (21) clearly demonstrate that we *cannot* require

$$\bar{M} = - \frac{d\bar{F}}{ds} = 0$$

If so the homogeneous system (20) has in general only the trivial solution $F = 0$. (An exception is provided by the case when (21) is singular at $s = s_m$, i.e. when $(\bar{\gamma}_T - \bar{T}) = 0$ at $s = s_m$).

We thus have to accept that we need some additional information from which we may calculate a second boundary condition on F.

In some cases, e.g. when we consider the Baltic as a whole, this information may be provided by the overall salt balance. We will not go into such details here but only emphasize this basic difficulty encountered when the computations are based on a conservative tracer.

5.2. *Radioactively decaying tracers*

For a radioactively decaying tracer we have

$$q_v = - \lambda v c_v \qquad (22a)$$

where λ is the decay-constant for the substance having concentration c_v. Considering again a "steady state" under the same condition as in section 5.1 we have from (15) and (6b)

$$\bar{m}(\bar{\gamma}_v - \bar{c}_v) - \lambda \bar{v}\, \bar{c}_v + \bar{F} \frac{d^2 \bar{c}_v}{ds^2} = 0 \qquad (22b)$$

$$\bar{m} - \frac{d^2 \bar{F}}{ds^2} = 0 \qquad (22c)$$

As in (5.1) we have only one boundary condition

$$F = 0 \quad \text{at } s = s_{max} \tag{22d}$$

unavailable and (22) is thus not a complete system.

Unlike eqs. (21), eqs. (22) are, however, not homogeneous in $\bar{m}$ and $\bar{F}$. This probably means that solutions to (22) are not as sensitive to the missing boundary condition as are solutions to (21).

Let us now assume that we have at our disposal observations of a radioactive tracer c_ν described by (22) and a conservative tracer, e.g. temperature described by (21).

We now have the possibility of combining eqs. (21) and (22), e.g. by eliminating $\bar{m}$ between (21a) and (22b), obtaining

$$\bar{F} = \cfrac{\lambda \bar{v} \bar{c}_\nu}{\cfrac{d^2 \bar{c}_\nu}{ds^2} - \cfrac{\bar{\gamma}_\nu - \bar{c}_\nu}{\bar{\gamma}_T - \bar{\bar{T}}} \cfrac{d^2 \bar{T}}{ds^2}} \tag{23}$$

This very elegant formula provides information *locally* (i.e. without solving any differential equation) on $F(s,t)$ from the fields c_ν, T, γ_ν and γ_T. Whether (23) will become anything but an academic curiosity depends, however, on whether a suitable radioactive tracer is available on which dense enough data can be obtained.

For the Baltic, tritium has a suitable time constant and is in fact supplied from atomic power stations. It is highly questionable, however, whether enough useful data will ever become available because of the relatively expensive procedures when analyzing sea water for tritium.

5.3. *Non-conservative tracers with unknown source functions*

Generally the description of this type of tracer represents the goal of the analysis, i.e. we want to compute the unknown source function and/or the flux $H_\lambda(s,t)$ for the tracer in question. It may be worth pointing out however that we often encounter two or more tracers with related source functions, e.g. we might have

$$q_\nu = \alpha q_\mu \tag{24a}$$

where α is a constant.

We then have from (15)

$$v \frac{\partial c_\nu}{\partial t} = m(\gamma_\nu - c_\nu) + \alpha q_\mu + F \frac{\partial^2 c_\nu}{\partial s^2}$$

$$v \frac{\partial c_\mu}{\partial t} = m(\gamma_\mu - c_\mu) + q_\mu + F \frac{\partial^2 c_\mu}{\partial s^2}$$

Multiplying the second equation with α and subtracting we obtain

$$v \frac{\partial}{\partial t} c' = m(\gamma' - c') + F \frac{\partial^2 c'}{\partial s^2} \tag{24b}$$

where

$$c' = c_\nu - \alpha c_\mu \tag{24c}$$

$$\gamma' = \gamma_\nu - \alpha \gamma_\mu \tag{24d}$$

From (24b) we can see that the linear combination $c' = c_\nu - \alpha c_\mu$ behaves as a conservative tracer, which may be used in a similar way as temperature.

6. Application to the Baltic

It is indubitably an urgent task to obtain reliable estimates of various cross-isohaline fluxes in the Baltic, e.g. the fluxes of nutrients or biologically important trace substances. The theoretical framework discussed in this paper has been developed for this purpose in particular.

For background information regarding the hydrographic and chemical state of the Baltic the reader is referred to the work by Fonselius (1969).

The most attractive approach would be to consider the Baltic as one single basin, i.e. described by a single set of functions V, F and c_ν. It would then be sufficient for many purposes to observe the inflow function M at some suitably located section. In fact such observations are presently being made (Petrén & Walin, 1976). It is, however, most likely that the local concentrations of the substances "ν" have a substantial variation along the isohalines because of the considerable length of the basin, thereby violating our basic assumptions. The deeper parts of the Baltic of interest here may however be described as a number of basins separated by sills and/or relatively narrow trenches and it seems probable that the description developed in this paper would be applicable to each of these basins separately rather than to the entire Baltic.
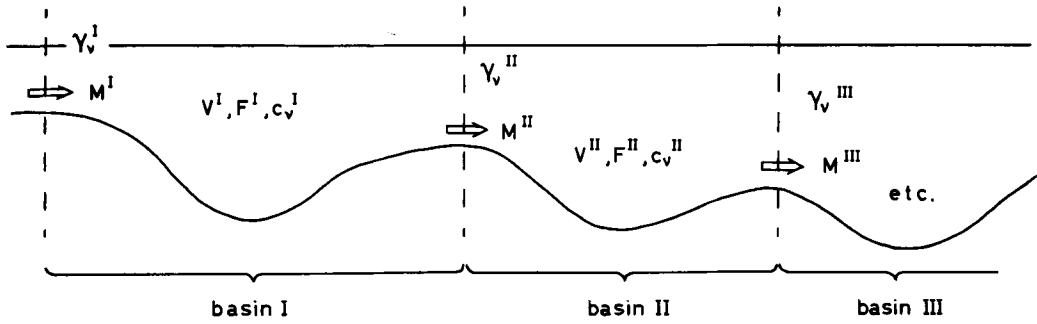
Presumably the simplest practically useful

Fig. 2. Illustration of multibasin model. Each basin is described by an individual set of functions, e.g. $(V^{II}, F^{II}, c_v^{II})$. M^N describes the flow from basin N-1 into basin N while γ_v^N is the concentration distribution in this flow.

description may be obtained by dividing the system into just two basins, essentially the Bornholm and the Gotland basins connected by the so-called Stolpe trench.

Whatever division of the Baltic we choose we are forced into a slight generalization of our theoretical framework. We thus have to introduce an individual set of functions V^N, F^N, C_v^N for the Nth basin.

We also need functions M^N and γ_v^N, representing volume flux from basin N-1 into basin N and the concentration of "v" in that flux respectively (Fig. 2).

Obviously M^I will correspond to our previous M, i.e. the supply from the surroundings. The modifications required to make the continuity relations derived in previous sections applicable to the Nth basin are fairly obvious. Equation (15b) for example will take on the form

$$v^N \frac{\partial c_v^N}{\partial t} = m^N(\gamma_v^N - c_v^N) - m^{N+1}(\gamma_v^{N+1} - c_v^{N+1})$$

$$+ q_v^N + F^N \frac{\partial^2 c_v^N}{\partial s^2} \qquad (25)$$

Of course we greatly increase the number of unknowns passing over to a multiple basin description. It should be pointed out however that a reasonable approximation may be to identify in (25) the concentration γ_v^N with c_v^{N-1}. This amounts to assuming that the concentration of c_v in the "incoming" water is determined by the concentration in the "up-stream" basin. Such an assump-

tion requires of course that the concepts up- and down-stream make sense, i.e. that the motion is mainly unidirectional between the basins (e.g. from the basin N-1 to basin N).

It is very unlikely that all of the flux functions M^N (or m^N) may be observed directly with sufficient accuracy. In the case of a two-basin description of the Baltic we may probably not obtain anything better than an order of magnitude estimate for M^{II}, i.e. the function which would describe the flow through the Stolpe trench. This means that indirect computations of F^N from observations of c_v^N, as outlined in the previous section, become vital.

At the oceanographic department in Gothenburg we are presently conducting flow measurements in the Bornholm strait (as mentioned above). The distribution thus observed is intended to be used as the function M^I in a model of the circulation and fluxes in the deep, stratified parts of the Baltic.

We will next try to organize existing data in order to determine as closely as possible the functions $c_v^N(s,t)$ and $V^N(s,t)$ for suitably chosen parts of the Baltic. We also plan to arrange automatic observations of the salt s and temperature T structure in the centre of the main basins. This would give us estimates of the functions $V^N(s,t)$ and $T^N(s,t)$ to be used in continuity calculations as outlined in section (5). Note that the temperature $T^N(s,t)$ should be considered as one of the tracer concentrations c_v; indeed a most important one for the purpose of indirect determination of the unknown functions M^N and F^N.

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ТЕОРЕТИЧЕСКИЕ ОСНОВЫ ДЛЯ ОПИСАНИЯ УСТЬЕВ

Намечены теоретические основы для описания устьеобразных систем, имея ввиду, прежде всего Балтийское море. Описание дается в терминах непрерывных функций солености и времени и может рассматриваться как улучшение существующих «боксовых моделей», которые принципиально не могут эффективно использовать гидрографические данные. Выводятся соотношения между притоком глубинны вод и величинами внутреннего перемешивания, а также общие уравнения, описывающие диффузию примесей. Обсуждается применимость этих соотношений и уравнений.