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Global mass and energy requirements for glacial oscillations and their implications for mean ocean temperature oscillations

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ABSTRACT

Conservation equations for water in all forms and thermodynamic energy are developed for the complete atmosphere–hydrosphere–lithosphere–cryosphere system, allowing the possibility for temporal variations of all significant components of the system that may be involved in long term climate changes. Using these equations we find an integral constraint on the fluctuations of the mean, mass-averaged, ocean temperature that accompany glacial fluctuations. Some *possible* inferences are discussed based on speculations concerning the variations of the net radiative flux at the top of the atmosphere. For one plausible qualitative estimate of the processes involved it is determined that the mean global ocean temperature should be increasing during the period of growth of ice coverage, reaching a maximum sometime after the time of maximum ice extent and a minimum sometime after the interglacial, a result that is in substantial accord with the ice-age mechanism discussed by Newell.

1. Introduction

The principles of conservation of mass and energy, when applied to the complete atmosphere–hydrosphere–lithosphere–cryosphere system, place certain integral constraints on the behavior of the system as it undergoes the major state variations associated, for example, with the late Cenozoic ice ages. We begin this discussion with a statement of these integral constraints (a) for water in all its forms and (b) for thermal energy. When these mass and energy constraints are combined, we obtain a formula relating the global rates of change of water vapor and ice, and of the internal and potential energy of the atmosphere, hydrosphere, and lithosphere, to the net solar and terrestrial radiative flux at the outer limit of the atmosphere.

The implications of this formula, given plausible estimates of the terms involved from both qualitative physical reasoning and from the results of recent numerical calculations, can be surprising. For example, the theoretical results of Saltzman & Vernekar (1975) suggest that at the time of maximum ice there is a net *positive* balance of incoming radiation at the top of the atmosphere, with the reverse, negative, balance prevailing during an ice minimum of which the present global state may be an example. If these radiation variations are valid the mean temperature of the oceans must increase as ice builds up, reaching a maximum after the maximum ice growth is achieved, much in accord with the scenario of Newell (1974). It is also implied that at present the oceans are cooling as a whole and we are likely to be in a phase of climatic variation precursive of more extensive ice formation and lower global air temperatures. The energy fluxes involved are too small to be measured, but their cumulative effects over thousands of years can be significant, as is implied by the existence of the ice ages. In any event, if reliable estimates of the palaeovariations of the deep ocean become available the integral constraint derived here can be used to determine the required net radiative variations at the top of the atmosphere.

2. The water mass balance

The forms of water on the earth are (1) atmospheric water vapor, (2) liquid and ice particles in the atmosphere, i.e., clouds and hydrometeors,

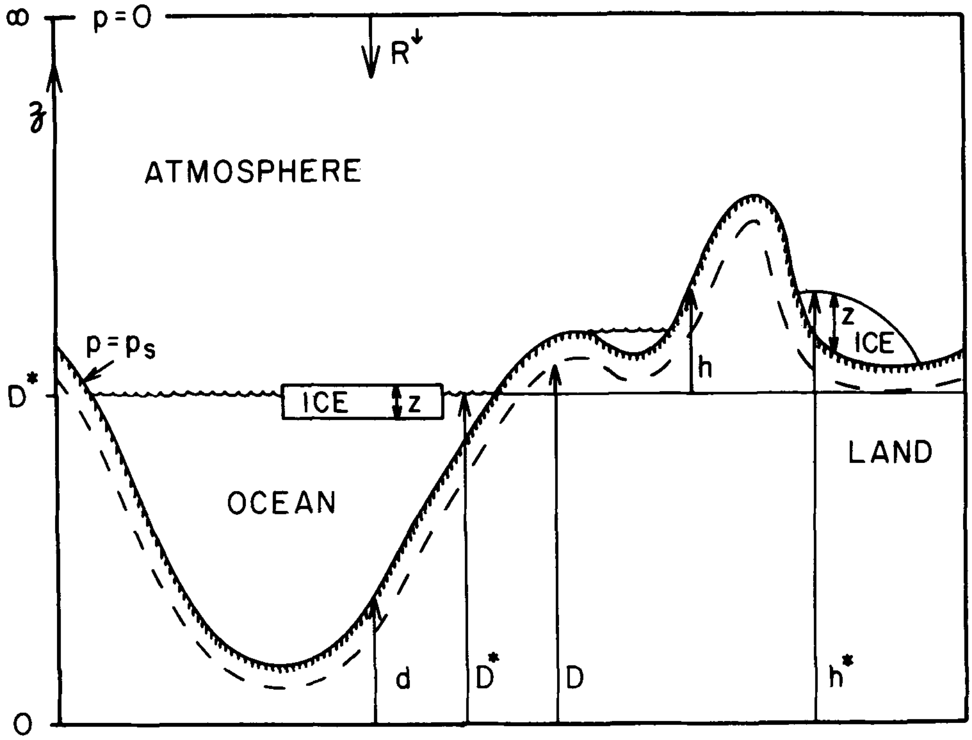

Fig. 1. Schematic representation of the land–ocean–ice atmosphere climatic system, showing symbols used to denote height above a subsurface reference level $z = 0$.

(3) liquid water in the oceans, and in lakes, rivers, and subsurface reservoirs (i.e., continental waters), and (4) solid water comprising ice sheets, ground ice, and sea, lake and river ice.

For the complete earth system portrayed schematically in Fig. 1, we assume that the total mass of water in all these forms is conserved, i.e.,

$$M_v + M_c + M_m + M_i = \text{constant} \tag{1}$$

where

M_v = mass of water vapor in atmosphere
M_c = mass of water in clouds
M_m = mass of liquid water in oceans and continents
M_i = mass of surface ice

A tabulation of the amounts of mass presently contained in each of these reservoirs is given by Flint (1971) showing that $M_m > M_i \gg M_v, M_c$.

Thus we have

$$\frac{\partial}{\partial t}(M_v + M_c + M_m + M_i) = 0 \tag{2}$$

or, assuming the long-term changes in the atmospheric water content can be neglected compared to the surface water content,

$$\frac{\partial}{\partial t}(M_m + M_i) \approx 0 \tag{3}$$

3. The global energy balance

For the complete atmosphere–hydrosphere–lithosphere–cryosphere system we can write a generalized statement of the first law of thermodynamics in the following form (c.f. Starr, 1951), excluding consideration of composition (e.g., salinity) changes in the oceans,

$$\frac{\partial}{\partial t}[\rho(U + k + \Phi)] = -\nabla_3\cdot\left[\rho\left(U + k + \Phi + \frac{p}{\rho}\right)\mathbf{V}_3\right] + \rho q \tag{4}$$

where t is time, ρ is density, U is internal energy function, $k = V_3^2/2$ is the kinetic energy per unit mass, ($\mathbf{V}_3$ = the three-dimensional velocity of the fluid portions of the earth), $\Phi = gz$ is the geopotential (g = acceleration of gravity, z = height above an arbitrary subsurface level depicted in Fig. 1), ∇_3 ($= \nabla + \mathbf{k}\partial/\partial z$) is the three-dimensional del-operator, p is pressure, and q is the rate of heat addition per unit mass excluding frictional heating. This heating function can be expanded in the form

$$q = q_r - \frac{1}{\rho}\left(\nabla\cdot\mathbf{H} + \frac{\partial H^{\uparrow}}{\partial z}\right) \tag{5}$$

where q_r represents internal heat sources due, for example, to radioactivity, $\mathbf{H}$ represents the horizontal flux of heat, and $H^{\uparrow}$ the upward vertical flux of heat due to radiation, conduction, and convection. We shall let

$$H^{\uparrow} = H^{(1)\uparrow} + H^{(2)\uparrow} + H^{(3)\uparrow}$$

where $H^{(1)\uparrow}$ denotes the net upward short-wave (solar) radiative flux, $H^{(2)\uparrow}$ the net upward long-wave (terrestrial) radiative flux, and $H^{(3)\uparrow}$ the combined heat flux due to conduction and convection.

If we integrate (4) over the complete system shown in Fig. 1, from a subsurface level $z = D$ (below which there is no active water content and negligible temperature change due to conduction from the lithosphere surface at $z = d$) to the "top" of the atmosphere denoted by the subscript T where z is a very large height z^* and $p \to 0$, we obtain,

$$\frac{\partial}{\partial t}\int \rho(U + k + \Phi)\,dV = \int \rho q\,dV$$

or

$$\frac{\partial}{\partial t}\int_M (U + k + \Phi)\,dm = \int_M q\,dm$$

$$= G + \sigma(\tilde{H}_T^{(1)\downarrow} - \tilde{H}_T^{(2)\uparrow}) + \int q_r\,dm \tag{6}$$

where dV ($= r^2 \cos\phi\, d\lambda\, d\phi\, dz$) is an element of volume (r = radial distance from the center of the earth, λ = longitude, ϕ = latitude), dm ($= \rho\,dV$) is an element of mass, G is the net "upward" geothermal flux normal to the lower boundary at $z = d$, and σ ($= 4\pi a^2$) is the surface area of a spherical earth of mean radius a. Thus we have defined an area average,

$$\tilde{f} = \frac{1}{4\pi a^2}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi} f a^2 \cos\phi\, d\lambda\, d\phi$$

Let us now decompose the total mass M as follows,

$$M = M_a + M_w + M_i + M_l$$

where M_a ($= M_d + M_v$) is the mass of atmosphere composed of "dry air" (M_d) and water vapor (M_v); M_w ($= M_{cw} + M_{m1} + M_{m2}$) is the mass of liquid water in the form of liquid cloud drops (M_{cw}), ocean (M_{m1}) and continental waters (M_{m2}); M_i ($= M_{ci} + M_{i1} + M_{i2}$) is the mass of ice in the form of cloud (M_{ci}), sea ice (M_{i1}) and continental ice (M_{i2}); M_l is the mass of lithosphere above $z = D$.

We can next specify the internal energy function for each of these basic component substances, in the forms,

$$U_a = (1 - \varepsilon)c_v T + \varepsilon[L_v + c_{vv}(T - 273) - 273\,R_v]$$
$$\approx c_v T + \varepsilon L_v$$

$$U_w = c_w(T - 273) \tag{8b}$$

$$U_i = -L_f + c_i(T - 273) \tag{8c}$$

$$U_l = c_l T \tag{8d}$$

where T is temperature (K), ε is the specific humidity, c_v the specific heat of air at constant volume, c_{vv} the specific heat of water vapor at constant volume, R_v the gas constant for water vapor, c_w the specific heat of liquid water, c_i the specific heat of ice, c_l the specific heat of land substances, L_v the latent heat of vaporization at 273 K, L_f the latent heat of fusion at 273 K. Here we have arbitrarily taken liquid water at 273 K to represent a state of zero enthalpy.

Using (7) and (8a–d) we obtain from (6)

$$\frac{\partial}{\partial t}[M_a(c_v\hat{T}^{(a)} + \hat{\Phi}^{(a)} + \hat{k}^{(a)}) + L_v M_v - L_f M_i + M_w(c_w\hat{\tau}^{(w)} + \hat{\Phi}^{(w)} + \hat{k}^{(w)}) + M_i(c_i\hat{\tau}^{(i)} + \hat{\Phi}^{(i)} + \hat{k}^{(i)}) + M_l(\widehat{c_l T^{(l)}} + \hat{\Phi}^{(l)})] =$$
$$G + \sigma(\tilde{H}_T^{(1)\downarrow} - \tilde{H}_T^{(2)\uparrow}) + \int q_r\,dm, \tag{9}$$

where

$$\hat{f}^{(\xi)} = \frac{1}{M_\xi}\int_{M_\xi} f\,dm$$

is the average value of f over the mass of a particular component denoted by ξ, and $\tau = T - 273$.

4. Application to paleoclimatic ocean temperature variations

Let us now consider the implications of (9) for long-term variations of mean ocean temperature accompanying the glaciations of the late Cenozoic, i.e. for variations on time scales ranging from 10^2–10^5 years. For this purpose, let us,

(i) make a further resolution of the total liquid water mass, M_w, into an ocean part (M_{m1}) and a continental part (M_{m2}), neglecting the very small cloud and hydrometeor component,

(ii) assume that the major build-ups of ice on the earth are at the expense of ocean water, with variations of atmospheric and continental waters negligible in comparison, so that we can rewrite (3) as

$$\frac{\partial M_{m1}}{\partial t} \approx - \frac{\partial M_i}{\partial t}, \tag{10}$$

(iii) assume that the mass of atmosphere and land remains constant, i.e.,

$$\frac{\partial M_a}{\partial t} = \frac{\partial M_l}{\partial t} = 0 \tag{11}$$

and (iv) note that from the hydrostatic principle we have

$$M_a(c_v \hat{T}^{(a)} + \hat{\Phi}^{(a)}) = M_a c_p \hat{T}^{(a)} + \sigma \widetilde{p_s h^*},$$

since

$$c_v = c_p - R, \quad p = \rho R T, \quad \text{and}$$

$$\int_{h^*}^{\infty} (-p + \rho\Phi)\, dz = - \int_{h^*}^{\infty} d(pz) = p_s h^*,$$

where c_p is the specific heat of air at constant pressure, p_s is the surface air pressure, and h^* is the height of the surface of the earth (i.e. the lower boundary of the atmosphere whether it be land or ocean) above the reference level $z = 0$.

We can thus rewrite (9) for the rate of change of mean ocean temperature in the form:

$$\frac{\partial}{\partial t} \hat{T}^{(m1)} = \frac{1}{M_{m1} c_w} \left[A - B - L_v \frac{\partial M_v}{\partial t} - \frac{\partial P}{\partial t} - \frac{\partial}{\partial t} \int k\, dm + G + \int q_r\, dm - c_w \hat{\tau}^{(m2)} \frac{\partial M_{m2}}{\partial t} + \sigma \tilde{R}^{\downarrow} \right] \tag{13}$$

where

$$A = [L_f + (c_w \hat{\tau}^{(m1)} - c_i \hat{\tau}^{(i)})] \frac{\partial M_i}{\partial t}$$

essentially represents the latent heat of fusion released during ice formation [note that $(c_w \hat{\tau}^{(m1)} - c_i \hat{\tau}^{(i)}) > 0$];

$$B = \left(M_a c_p \frac{\partial \hat{T}^{(a)}}{\partial t} + M_{m2} c_w \frac{\partial \hat{T}^{(m2)}}{\partial t} + M_i c_i \frac{\partial \hat{T}^{(i)}}{\partial t} + M_l c_l \frac{\widehat{\partial T^{(l)}}}{\partial t} \right),$$

represents the enthalpy increase of the atmosphere, continental waters, ice and land, respectively;

$$P = (\sigma \widetilde{p_s h^*} + M_w \hat{\Phi}^{(w)} + M_i \hat{\Phi}^{(i)} + M_l \hat{\Phi}^{(l)});$$

and

$$\tilde{R}^{\downarrow} = (\tilde{H}_T^{(1)\downarrow} - \tilde{H}_T^{(2)\uparrow})$$

represents the net downward radiative flux at the top of the atmosphere per unit horizontal area.

To evaluate the signs of the terms on the right hand side of this equation we shall make the following approximations, based to some extent on calculations reported in the literature, and to some extent on qualitative plausibility arguments:

(a) the amplitude of the variations of ice content in clouds M_{ci} is insignificant compared to that of the ice content of the surface of the earth, but, in any event, M_{ci} varies in phase with surface ice, i.e.,

$$\delta M_{ci} \ll \delta(M_{i1} + M_{i2}) \approx \delta M_i$$

where δf denotes a long term variation of f.

(b) the atmosphere is a fast response part of the climatic system so that on the time scales we are considering we may assume that air temperature and lower boundary ice coverage equilibrate rapidly, i.e. these two variables tend to be nearly in phase,

$$\delta \hat{T}^{(a)} \sim -\delta M_i$$

(c) it seems plausible also that on these long time scales the variations of the mean temperature of the surface layers of the continents, the ice sheets, and the surface waters of the continents, should be in phase with the air temperature, owing to the relatively low conductive capacities of these media. Thus,

$$\delta \hat{T}^{(m2)} \sim \delta \hat{T}^{(i)} \sim \delta \hat{T}^{(l)} \sim \delta \hat{T}^{(a)} \sim -\delta M_i,$$

or $B \sim -\delta M_i \sim -A$

(d) the water vapor content of the atmosphere, being highly temperature dependent, also varies inversely with M_i, i.e.,

$$\delta M_v \sim \delta \hat{T}^{(a)} \sim -\delta M_i \sim -A$$

The above items (b) and (d) are borne out by the ice age equilibrium calculations of Williams, et al. (1974), Saltzman & Vernekar (1975), and Gates (1976), and are consistent with presently observed seasonal variations.

(e) We shall neglect possible variations in the geothermal flux of heat measured by G. This heat flux has been estimated to be of the order of $6.3 \times 10^{-2}\,\mathrm{Wm}^{-2}$ at present for the world ocean floor (Lee & Uyeda, 1965) but its variations over the late Cenozoic are unknown.

(f) We shall also neglect heat sources due to radioactivity ($q_r \approx 0$), mass changes of continental waters ($\partial M_{m2}/\partial t \approx 0$), kinetic energy changes of the fluid portions of the earth ($\partial \int kdm/\partial t \approx 0$), changes in atmospheric potential energy due to changing sea-level and mass shifts relative to topography measured by $\sigma \overline{p_s h^*}$, and changes in the potential energy of the lithosphere associated with compression and rebound under varying ice loads.

(g) since an ice age is characterized by a build-up of continental ice sheets and a simultaneous lowering of sea-level there is a net potential energy increase of the system, measured by $\partial(M_w \hat{\Phi}^{(w)} + M_i \hat{\Phi}^{(i)})/\partial t$. A numerical estimate of this effect for the Wisconsin ice age shows that this term is at least one order of magnitude smaller than the latent heat term, A, in (12).

It follows from (a)–(g) applied to (13), that if there is always radiative balance (i.e., $\tilde{R}^{\downarrow} = 0$) the mean ocean temperature must be in phase with the ice coverage, with maxima of ocean temperature coinciding with maxima in ice. That is, the latent heat of fusion must be absorbed by the oceans, given the likelihood that all other components of the system are cooling during the development of an ice age. A plausible estimate of the implied mean ocean warming between an interglacial and the Wisconsin ice age of the order of 4 K.

There is a strong probability, however, that $\tilde{R}^{\downarrow}$ is not identically zero over all time, as discussed by Rossby (1959), and the variations of mean ocean temperature can depend critically on these variations. In this connection we note the following:

Neglecting possible variations of the solar constant, the net incoming solar radiation, $\tilde{H}_T^{(1)\downarrow}$, is highly dependent on the planetary albedo, which in turn is a strong function of ice and cloud coverage. There are some theoretical indications that a high surface ice coverage is accompanied by a somewhat reduced global cloud coverage (Paltridge, 1974; Gates, 1976), consistent with the fact that at present winter cloudiness is lower than summer cloudiness (Schutz & Gates, 1971, 1972). Thus, the relation implied by ice-albedo alone,

$$\delta \tilde{H}_T^{(1)\downarrow} \sim -\delta M_i,$$

may in fact be considerably reduced in magnitude or even reversed in sign if the cloud effect predominates.

The net outgoing long-wave radiation, $\tilde{H}_T^{(2)\uparrow}$, is proportional to atmospheric and surface temperature, and tends to be inversely proportional to water vapor and cloud amount. Thus, in this case too, the effects of cloud (and also of water vapor) can reduce or reverse the direct effect of temperature and ice cover which otherwise would be of the form,

$$\delta \tilde{H}_T^{(2)\uparrow} \sim \delta \hat{T}^{(a)} \sim -\delta M_i$$

Because of the dependence of both short- and long-wave radiation on ice cover and the associated surface and atmospheric temperature, it follows that, maxima or minima of the combined net radiative flux at the top of the atmosphere, $\tilde{R}^{\downarrow} = (\tilde{H}_T^{(1)\downarrow} - \tilde{H}_T^{(2)\uparrow})$, are likely to coincide with the maxima or minima of the ice coverage, but we cannot say *a priori* whether the variations are in-phase or

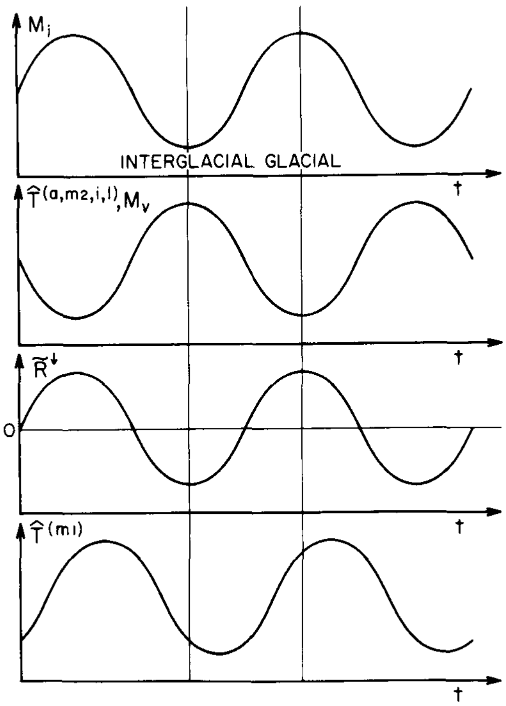

Fig. 2. Schematic representation of possible relative variations of quantities in the conservation equation (13), based on assumptions of Section 4.

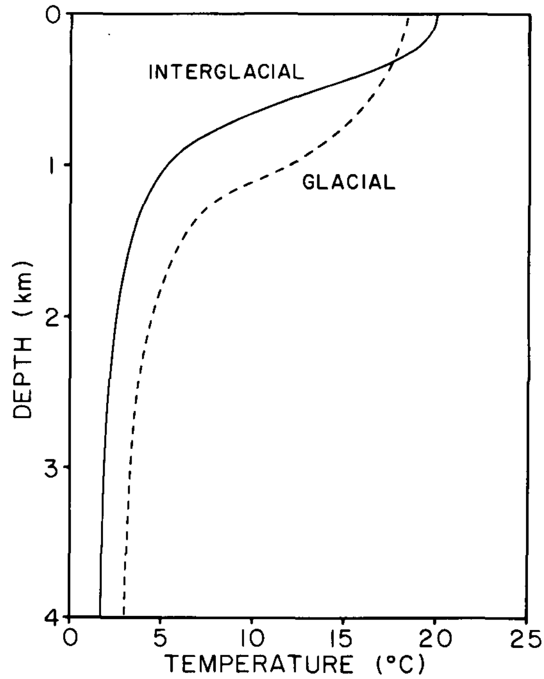

Fig. 3. Hypothetical variations of mean ocean temperature with depth during interglacial and glacial periods.

180 degrees out of phase. However, from the Northern Hemisphere equilibrium calculations made by Saltzman & Vernekar (1975), keeping cloud and water vapor effects in $\tilde{H}^{(1)\downarrow}$ and $\tilde{H}_T^{(2)\uparrow}$ fixed at present values, we find that $\tilde{R}^{\downarrow}$ is positive during maximum ice coverage at 18,000 BP and negative for present, nearly "interglacial", conditions—i.e. the negative feedback of longwave radiation with temperature dominates the positive feedback of short-wave radiation with the ice albedo. If we invoke this theoretical result we are led to postulate that $\delta\tilde{R}^{\downarrow} \sim -\delta\hat{T}^{(a)} \sim \delta M_i$ with $\tilde{R}^{\downarrow} = 0$ (i.e. radiative "balance") occurring sometime between the glacial maximum and minimum.

Assuming this radiative scenario to be correct, the variations of mean ocean temperature should lag the ice coverage with maxima in ocean temperature occurring after the ice age maxima, as depicted in Fig. 2. In this figure the upper curve represents a given ice-age oscillation portrayed in a schematic sinusoidal form. The second curve gives the probable relative variations of the temperature of all components of the system excluding the ocean temperature, and the variations of atmospheric water vapor mass. The third curve represents the hypothetical net radiation variations suggested by the equilibrium climate solutions. The bottom curve gives the implied mean ocean temperature variations.

It is now fairly certain that ocean *surface* temperatures during the last ice maximum (18,000 BP) were in fact *colder* on the average than at present, at least for summer (CLIMAP, 1976)—though not by as much as might have been expected (2.3 K). Thus, if the above results are valid, the relative warmth of the ocean during and just after the ice maximum must reside in the deeper ocean—either in the abyssal bottom waters, or in a deepened mixed layer above a lower main thermocline perhaps as depicted in Fig. 3. If paleo-observational studies do not reveal this relative warmth of the glacial-period ocean, it is implied that there exist considerable departures from pure radiative balance and from the particular radiative imbalance of the Saltzman and Vernekar solutions; i.e., there must be large net losses of radiant energy during glacial growth and gains during glacial decline, with the glacial maxima and minima being nodes for $\tilde{R}^{\downarrow}$. Such departures would probably be related to cloud variations in the atmosphere, though the possibility for solar constant variations cannot be ruled out.

The foregoing energy budget consistency arguments imply no "mechanisms" for the oscillations portrayed in Fig. 2. However, a plausible mechanistic model has been proposed by Newell (1974) that can, in principle, account for the behavior shown. In essence, Newell argues that a negative feedback exists between the mean temperature of the ocean and the extent of sea-ice cover, in which the sea-ice acts as an insulator preventing the loss of oceanic heat in high latitudes. As noted by Newell, the ocean warming phase should be slower than the cooling phase since the former will be more a *diffusive* process under more stable stratification, whereas the latter will involve a more *convective* process. Thus the sinusoidal variations portrayed in Fig. 2 could become sawtoothed in nature, as has been inferred from geological evidence (e.g. Broeker & van Donk, 1970). We note also that this "free" oscillatory process can, in principle, be supplemented by other forcing mechanisms such as variations in the solar constant or in the distribution and magnitude of solar heating due to earth orbital changes.

Before concluding, let us return briefly to the results of Saltzman & Vernekar (1975), that were quoted above. As noted, on further examining this solution for ice age climate we found that for a zonally-symmetric equilibrium model subject to 18,000 BP ice boundary conditions there was a net positive radiation balance, $\tilde{R}^{\downarrow} > 0$, that implies a net *downward* flux of heat at the earth's surface since $\partial(\hat{T}^{(a)}, M_i, M_v)/\partial t = 0$ in this equilibrium model. Conversely, for present ice conditions the solution shows a net negative radiation balance, $\tilde{R}^{\downarrow} < 0$, that implies a net upward flux of heat through the earth's surface. The values obtained for these net surface heat fluxes are $+4.27\ \mathrm{Wm}^{-2}$ (18,000 BP) and $-6.25\ \mathrm{Wm}^{-2}$ (present).

Though undetectable by observations, these values seem unreasonably large since they imply mean temperature changes for the entire world ocean of the order of 10×10^{-3} K/yr. But, even if the signs only are correct they are corroborative of the Newell hypothesis, and at the same time portend a progressive cooling of the oceans over the near future that may be precursive of more "glacial" conditions. It would be of great interest to learn the values of these quantities obtained in other equilibrium climate experiments, especially those based on the long-term integration of general circulation models containing explicit large-scale dynamics (e.g., Gates, 1976).

5. Summary

We have developed a set of global water mass and energy balance equations relevant to a discussion of long-term changes in the complete atmosphere–hydrosphere–lithosphere–cryosphere system. By applying physically plausible arguments and some recent theoretical results we infer some general evolutionary properties of the system.

If we assume that "radiative equilibrium" always prevails we are led to conclude that the mean ocean temperature must reach its maximum value at very nearly the same time that the ice volume reaches a maximum, mainly due to the release of latent heat of fusion during the ice formation. However, if we assume that the atmospheric climate tends to achieve a quasi-static equilibrium with the lower boundary ice coverage and ocean surface temperature, at least when the ice volume is near its maximum or minimum, it follows that there can be a net radiative flux at the top of the atmosphere equal in both magnitude and direction to the net flux of heat across the ocean surface. From the results of a statistical-dynamical model of climate based on prescribing this atmospheric equilibrium (Saltzman & Vernekar, 1971, 1975) we find that there should be a net downward flux of radiation (and, hence, of heat into the oceans) during a glacial maximum and a net upward flux during present, quasi-interglacial, conditions. If this finding is valid it follows that the mean ocean temperature should increase during the period of maximum ice formation and ice extent and reach its maximum after the maximum ice volume is achieved, a result that is in substantial accord with the ice-age scenario recently proposed by Newell (1975). Since it is generally agreed that the surface water temperatures were in fact *colder* during the ice age than at present (CLIMAP, 1976), we infer that these mass-averaged ocean temperature changes must reflect conditions of the deeper water masses with attendant changes in the strength and form of the main thermocline. A mechanistic, dynamical, theory that could account for the implied oscillation between ocean temperature and ice formation remains to be developed.

If, in the course of time, reliable estimates of the evolution of mean ocean temperatures become available, whether supportive of these conclusions or not, the equations derived can be used to determine the required net imbalance of radiation at the top of the atmosphere.

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ТРЕБОВАНИЯ ГЛОБАЛЬНОГО БАЛАНСА МАССЫ И ЭНЕРГИИ ДЛЯ ЛЕДНИКОВЫХ КОЛЕБАНИЙ И ИХ СЛЕДСТВИЯ ДЛЯ КОЛЕБАНИЙ СРЕДНЕЙ ТЕМПЕРАТУРЫ ОКЕАНА.

Уравнения сохранения для воды во всех формах и для термодинамической энергии выведены для полной системы атмосфера-гидросфера-литосфера-криосфера с учетом возможных временных вариаций всех существенных компонент системы, которые могут участвовать в долгопериодных изменениях климата. Используя эти уравнения, мы находим интегральное ограничение на флуктуации средней осредненной по массе температуры океана, сопровождающие ледниковые флуктуации. На основе спекуляций о вариациях полного потока радиации на верхней границе атмосферы обсуждаются некоторые возможные следствия. Для одной из правдоподобных качественных оценок рассматриваемых процессов найдено, что средняя глобальная температура океана должна увеличиваться в течение периода роста ледяного покрова, достигая максимума спустя некоторое время после времени достижения наибольшего ледяного покрытия, и минимума— спустя некоторое время после межледникового периода. Этот результат находится в существенном согласии с механизмом "ледниковых эпох", обсуждавшимся Ньюэллом.