SHORTER CONTRIBUTION

The effect of coastline and continental shelf variations on a coastal current

By AILEEN CREEGAN, *School of Mathematics, Kingston Polytechnic, Kingston upon Thames, Surrey, England*

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ABSTRACT

A homogeneous model for the boundary layer over a continental shelf is considered in order to investigate the effect of the shape of the coastline and shelf features on the structure of the coastal current.

This note considers the response of the model of Hill & Johnson (1975) (hereafter called HJ) in the presence of variations in the coastline and continental shelf. Their model describes an $E^{1/4}$-shelf layer matching with a homogeneous interior, and takes into account upwelling flow in the Ekman layers over the shelf and the associated longshore current at an eastern boundary.

A description of the model and subsequent analysis is given in detail in HJ and only the salient features are included here. The theory is adapted for the case of a generally southward-facing coast. The model configuration and (dimensionless) co-ordinates used here are illustrated in Fig. 1, the boundary being defined by $y = q(x)$. The shelf slope is $\alpha(x)$ so that the equation of the shelf is $z = \alpha(x)(y - q(x))$. Scaled co-ordinates appropriate to the $E^{1/4}$-shelf layer are

$$Y = \frac{y - q(x)}{E^{1/4}}, \quad \zeta = \frac{z}{\alpha(y-q)}$$

and the lowest order horizontal velocity components in the shelf region are (U_0, V_0). The β-plane is used, with $f = 1 + \beta y$. The analysis is then analogous to HJ, yielding the differential equation

$$\frac{\partial^2}{\partial \eta^2}\left(\frac{\bar{f}^2 U_0}{\alpha}\right) - \frac{\partial}{\partial X}\left(\frac{\bar{f}^2 U_0}{\alpha}\right) = 0 \tag{1}$$

where

$$\bar{f} = 1 + \beta q(x), \ \eta = \alpha Y/\bar{f}, \ X = -\int_0^x \frac{\alpha(1+q'^2)}{2F_N \bar{f}^2}\, dx'$$

and $F_N^2 = \frac{1}{2}\bar{f}/(1 + \alpha^2 + \alpha^2 q'^2)$. Over the shelf the upwelling transport is carried in the lower Ekman layer, and at the boundary matches with the surface Ekman flux to yield the condition

$$U_0(x, 0) = 2F_N(\tau^x + q'\tau^y)/\bar{f}(1 + q'^2)\big|_{y=q(x)} \tag{2}$$

where $\tau = (\tau^x, \tau^y, 0)$ is the surface wind stress. Positive U_0 at the coast corresponds to upwelling, negative U_0 to downwelling.

The solution of (1) subject to $\bar{f}^2 U_0(X, 0)/\alpha = 1$, $\bar{f}^2(0)\, U_0(0, \eta)/\alpha(0) = 0$ is given in Carslaw & Jaeger as

$$\frac{\bar{f}^2 U_0}{\alpha} = \operatorname{erfc}(-\eta/2\sqrt{X}).$$

Use of Duhamel's theorem leads to the solution subject to the boundary condition (2) as

$$\frac{\bar{f}^2 U_0}{\alpha} = \int_{-\infty}^{X} \frac{dH}{ds} \operatorname{erfc}(-\eta/2\sqrt{X - s})\, ds \tag{3}$$

where

$$H(X) = \frac{2F_N \bar{f}}{\alpha}\left(\frac{\tau^x + q'\tau^y}{1 + q'^2}\right)\Bigg|_{y=\eta=0}$$

and $\left.\dfrac{\bar{f}^2 U_0}{\alpha}\right|_{X\to-\infty} = 0.$

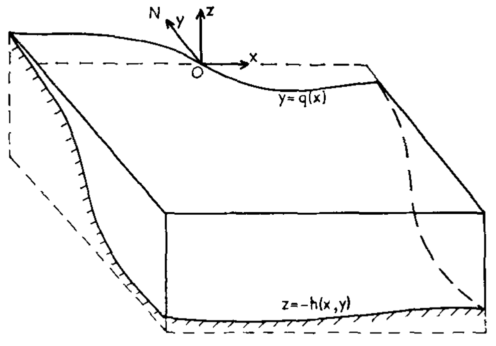

Fig. 1. Model configuration.

This latter condition was used in HJ and makes the assumption of no coastal current crossing the equator.

Integrating the longshore flow across the section of the shelf layer and rewriting in the original variables gives for the longshore transport T_L,

$$T_L = E^{1/2} \int_x^{q^{-1}(-1/\beta)} \frac{\tau^x + q' \tau^y dx'}{\bar{f}}. \tag{4}$$

Thus it may be seen that the local wind stress governs the magnitude of upwelling at any point on the coast, but that the coastal current is influenced by the net effect of the wind stress along the coast. This feature is discussed in HJ and results because the upwelling flux in the lower Ekman layer originates in the shelf layer, and must therefore be drawn along the coast as a coastal current in order to satisy the upwelling requirements at each latitude.

Johnson (1975) discusses the effect of coastal

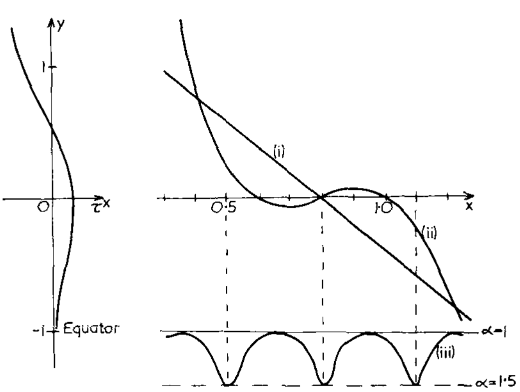

Fig. 2. Diagrams of wind stress, coastal and shelf configurations.

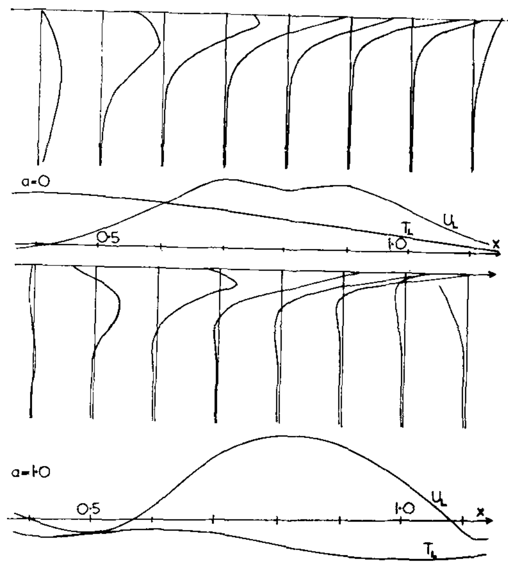

Fig. 3. U_0 profiles across the shelf layer for $a = 0$, 1.0 and $\alpha(x) = 1$, case *(a)*. U_L is longshore component of current at the coast.

variation on the position of centres of upwelling. The purpose of this note is to investigate the structure of the coastal current in the presence of coastal irregularities and variations in the shelf slope. The wind stress used throughout the results here is

$$\tau^y = a\tau^x, \ \tau^x = \bar{f} \cos \frac{\pi}{6}(1 + 4\beta y) \quad (a \text{ constant})$$

as used by Johnson (1975). This is a convenient choice as both τ and its curl are zero at the equator. It represents westerly winds at the equator and easterly winds further north. Solutions have been obtained for a variety of cases, two of which are illustrated in Fig. 2:

(*a*) curved boundary (ii) with $\alpha(x) = 1$. The configuration was suggested by the Guinea coast of West Africa.

(*b*) oblique coast, curve (i), with $\alpha(x)$ given in curve (*iii*).

Values of the constant a used are $a = 0$, 1.0. Fig. 3 shows the results for case (*a*) and Fig. 4 for case (*b*).

Case (a)

An intense, narrow coastal current is produced along the portion of coast where the wind stress acts most nearly in the longshore

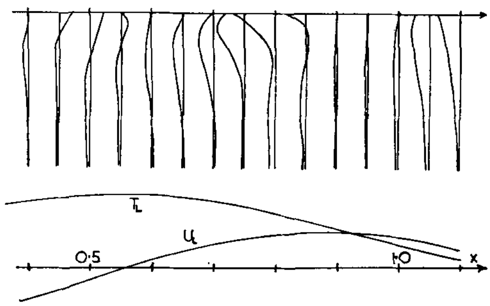

Fig. 4. Case (*b*). Profiles show difference in U_0 when results obtained in presence and absence of canyons, $a = 0$.

direction. In particular, where $a = \tau^y = 0$, the longshore velocity at the coast has a double maximum corresponding to the two turning points of the cubic curve. The sign of U_0 at the coast depends on the relative orientation of the wind stress and the coast. Given that the transport in the layer must vanish at the equator, the transport into the layer in the north compensates for any imbalance between the upwelling and downwelling flux. At the changeover from one regime to the other, countercurrents develop offshore. As a increases, a more complex current pattern emerges due to an increasingly frequent transition from an upwelling to a downwelling situation. For $a = 1.0$ there are positions at which there is a triple current structure. A new current caused by the change in the type of wind stress pushes the existing currents offshore and is itself confined to a narrow coastal region. As the currents move further offshore they become exponentially small.

Thus the shelf flow is modified by the presence of coastal irregularities and the variations superposed on the basic flow give rise to a current structure exhibiting complex currents and countercurrents.

Case (*b*)

This corresponds to canyons across the shelf at regular intervals along the coast. The effect of such features on the flow is not dramatic, but a definite pattern emerges. Firstly it should be noted from (4) that T_L is independent of α, and that the upwelling depends only on the wind stress and orientation of the coast. Therefore only a redistribution of flow across the shelf may occur. Irrespective of the sign of U_0 at the coast, i.e. whether upwelling or downwelling is occurring, there is a decrease in magnitude of the current near the coast over a canyon. This is to be expected since the deepening layer will give rise to a less intense current. However, further offshore, the magnitude of the coastal current may become *greater* than it would be in the absence of a canyon, then decreases once more before the difference becomes exponentially small. As with case (*a*) the response depends not only on the local configuration, but also on the surrounding features. A single canyon produces a general slackening of the current above it, while a group of canyons as illustrated will produce the "wave" in the solution described above.

Since the model takes no account of stratification, its limitations for describing the detailed processes of upwelling are obvious. The result which emerges clearly here is how such a coastal current may carry information about what is happening elsewhere along the coast. The basic transport calculation (4) shows how the wind stress and coastal orientation, which together govern the upwelling, influence the solution along the coast, and the integrated effect of the shelf slope is included in the solution (3). Increasing variability in coastal features causes an increase in complexity of the current structure. Comparison with observations may give insight into the relative importance of such effects in the flow regime. Together with the equivalent model on a predominantly westward facing coast, it could therefore be of particular interest in areas of coastal upwelling.

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