An axisymmetric boundary layer solution for an unsteady vortex above a plane

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ABSTRACT

A given axisymmetric potential swirling flow is bounded by a plane perpendicular to the axis of symmetry. By the no slip condition, viscous effects will be important over the plane and since the circumferential velocity must be zero on the axis of symmetry, viscous effects will also be important in the core of the vortex. These two viscous regions will overlap near the intersection of the axis of symmetry with the plane. Thus the flow field can be divided into four regimes, the viscous core, the boundary layer on the plane, a 'stagnation point' regime and the given potential flow which provides the outer boundary conditions for each of the first two regimes. Such a model could be useful for studying meteorological flow systems such as tornadoes. The viscous core regime has already been studied (Rott, 1958, 1959; Bellamy-Knights, 1970, 1971).

1. Introduction

In this paper the existence of a boundary layer regime is verified and analysed. By introducing a suitable similarity variable, the boundary layer equations for the unsteady axisymmetric flow of an incompressible fluid are reduced to two ordinary differential equations which are solved numerically. When the radial velocity in the outer potential flow satisfies certain conditions, boundary layer solutions are possible. These conditions are compatible with those required for the existence of multicellular core structures.

Certain meteorological flow systems, such as tornadoes, have been modelled by a viscous vortex core embedded in a potential flow with constant circulation above a plane boundary; such work has been reviewed by Morton (1966). For example, Burgers (1940, 1948), Rott (1958, 1959) and Sullivan (1959) considered such a model for a steady flow vortex core. For steady flow to be possible, the reduction in circumferential velocity at any radius due to the diffusion of the vorticity must be balanced by the circumferential velocity increase resulting from the inwards convection of fluid with higher angular momentum. Hence the solutions of Burgers, Rott and Sullivan have radial inflow in the outer potential flow.

When the radial inflow ceases to be maintained the vortex core will diffuse radially outwards with increasing time and the flow will necessarily be unsteady. Such unsteady core flows have been considered by Oseen (1911), Rott (1958, 1959) and Bellamy-Knights (1970, 1971) in the hope that they may help to explain the eventual dissolution of such phenomena as tornadoes.

All the aforementioned core solutions belong to the class

$$u = u(r, t), \quad v = v(r, t), \quad w = zW(r, t) \qquad (1.1)$$

where r, θ, z are cylindrical polar co-ordinates and u, v, w are the corresponding radial, tangential and axial components of velocity and t is the time. Thus it is seen that one weakness of these solutions is their failure to satisfy the no-slip condition on the ground, $z = 0$. The purpose of the present work is to obtain a boundary layer solution for the unsteady flow vortex model. In particular, a boundary layer solution on the plane $z = 0$ is sought for radii sufficiently large for the flow external to the boundary layer to be the assumed potential flow solution to which the viscous core solution asymptotes for large

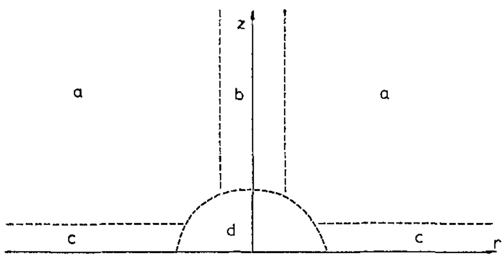

Fig. 1. A meridian section through the vortex showing the flow regimes of the mathematical model: (*a*) is the outer potential flow, (*b*) is the viscous core, (*c*) is the boundary layer regime, and (*d*) is the stagnation point regime.

radius. As in earlier work on unsteady vortex cores (Bellamy-Knights 1971), this outer potential flow is specified by the velocity components

$$u = -\gamma r/2t, \quad v = K_c/r, \quad w = \gamma z/t \qquad (1.2), (1.3), (1.4)$$

where $2\pi K_c$ is the circulation and the parameter γ characterizes the magnitude and direction of the radial flow.

The mathematical model for the entire flow thus consists of four regimes as illustrated in Fig. 1. First, there is the potential flow regime described above. Secondly there is the viscous core regime in which Bellamy-Knights (1971) found that various core structures were possible provided that $-1 < \gamma < \frac{1}{2}$. When $\gamma < -1$, no core solutions could be obtained and when $\gamma > \frac{1}{2}$ only two isolated solutions existed corresponding to one- and two-cell analytical solutions. Thirdly, there is a boundary layer which is the subject of this paper. Lastly, there is a region around the stagnation point which has yet to be studied. It may be noted that these last three regimes will expand with increasing time. For example, the core radius will increase and the boundary layer thicken with time.

The mathematical procedure for considering the boundary layer region is as follows. The usual boundary layer assumption, that variations along the boundary are much smaller than variations normal to the boundary, is made. Then certain viscous terms can be neglected. An order of magnitude analysis shows that the axial momentum equation reduces to $\partial p/\partial z = 0$, where p is the pressure. Hence, in the radial momentum equation for the boundary layer, $\partial p/\partial r$ can be replaced by its value in the outer potential flow. The aforementioned solutions for the core region show that only for large values of $r^2/\nu t$ does the flow become approximately potential; ν is the kinematic viscosity. Thus only for these large values of $r^2/\nu t$ is the present boundary layer solution valid as only then is the flow external to the boundary layer given approximately by (1.2)–(1.4). Moreover, in the present work it is assumed that $r^2/\nu t$ is sufficiently large for the radial momentum equation to be essentially independent of swirl velocity. (The required magnitude of $r^2/\nu t$ will depend on the size of the tangential Reynolds number K_c/ν.) After making the above simplifications, the radial momentum equation can be solved for the stream function. Knowing the stream function, the circumferential momentum equation can be solved for the circulation function, $K = rv$.

It is found that for each value of γ within the range $-1 < \gamma < \frac{1}{2}$, numerical solutions for the boundary layer regime can be obtained. This is precisely the same range of γ in which various core solutions were possible.

2. Mathematical formulation of the problem

In cylindrical polar coordinates, the Navier-Stokes equations for unsteady axisymmetric flow of a viscous incompressible fluid are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\varrho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \frac{\partial^2 u}{\partial z^2}\right] \qquad (2.1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \nu\left[\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{\partial^2 v}{\partial z^2}\right] \qquad (2.2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] \qquad (2.3)$$

and the continuity equation is

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{2.4}$$

where ϱ is the density.

For a boundary layer flow along the plane $z=0$, gradients along the boundary are much smaller than the corresponding gradients normal to the boundary. Consequently, in the viscous terms in the above equations, derivatives with respect to r can be neglected in comparison with derivatives with respect to z. In the present investigation, we consider the boundary layer flow at radii larger than the radius of the diffusing viscous vortex core, i.e. at large values of $r^2/\nu t$ for which the flow outside the boundary layer is approximately given by (1.2) to (1.4). Hence, if δ is the thickness of the boundary layer, then u changes from zero at $z=0$ to $-\gamma r/2t$ at $z=\delta$ and v increases from zero at $z=0$ to K_c/r at $z=\delta$. Thus v^2 is of order K_c^2/r^2. Hence in (2.1), the ratio of v^2/r to the remaining acceleration terms is of order $(K_c/\nu)^2(\nu t/r^2)^2$ which is small provided that $r^2/\nu t$ is sufficiently large. In this case the swirl term in the radial momentum equation can be neglected. In each of the equations (2.1) to (2.3), the unsteady acceleration term is of the same order as the convective acceleration terms and the viscous terms are assumed to be of the same order of magnitude as the acceleration terms. It follows that the boundary layer thickness, δ, increases as the square root of the time. Moreover, since $r^2/\nu t$ is large, it follows that δ^2/r^2 is small. This result is in accordance with the initial boundary layer hypothesis.

Continuing the order of magnitude analysis, it is found from (2.3) that $\partial p/\partial z$ is of order $(\partial p/\partial r)\,\delta$. Hence the change in pressure across the boundary layer may be neglected and the term $\partial p/\partial r$ in (2.1) may be replaced by its value just outside the boundary layer in the outer potential flow. Then

$$-\frac{1}{\varrho}\frac{\partial p}{\partial r}=\frac{\gamma r}{4t^2}(\gamma+2) \tag{2.5}$$

Using the above results, (2.1) reduces to

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\frac{\gamma r}{4t^2}(\gamma+2)+\nu\frac{\partial^2 u}{\partial z^2} \tag{2.6}$$

and (2.2) reduces to

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\frac{\partial^2 v}{\partial z^2} \tag{2.7}$$

To satisfy the continuity equation (2.4), Stokes' stream function, ψ, is introduced where

$$u=-\frac{1}{r}\frac{\partial \psi}{\partial z}, \tag{2.8}$$

$$w=\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{2.9}$$

Dimensional analysis of (2.6) and (2.7) suggests a similarity variable, η say, where

$$\eta=\tfrac{1}{2}z/\sqrt{\nu t} \tag{2.10}$$

the factor $\frac{1}{2}$ being introduced for future convenience. Moreover, dimensional analysis suggests that ψ can be expressed in terms of η by the equation

$$f(\eta)=\frac{1}{r^2\gamma}\sqrt{\frac{t}{\nu}}\,\psi \tag{2.11}$$

Then (2.8), (2.9) and (2.11) give

$$u=-\frac{\gamma r}{2t}f'(\eta), \tag{2.12}$$

$$w=2\gamma\sqrt{\frac{\nu}{t}}\,f(\eta) \tag{2.13}$$

where primes denote differentiation with respect to η. Equation (2.6) then reduces to

$$\tfrac{1}{2}f'''+\eta f''+2f'+\gamma(f'^2-2ff'')=\gamma+2 \tag{2.14}$$

It is now assumed that the circulation is also a function of η only, i.e.

$$g(\eta)=rv/K_c \tag{2.15}$$

Hence (2.7) reduces to

$$g''+2\eta g'-4\gamma f g'=0 \tag{2.16}$$

Thus the boundary layer equations (2.6) and (2.7) have been reduced to two ordinary differential equations. These are subject to the following boundary conditions. On the ground, $z=0$, the velocity components u, v and w are zero by the no-slip condition. Hence

$$f(0)=0,\quad f'(0)=0,\quad g(0)=0 \quad (2.17), (2.18), (2.19)$$

As $z \to \infty$, the velocity components in the boundary layer regime tend asymptotically to the values in the outer potential flow and therefore, as $\eta \to \infty$,

$$f \to \eta - C \tag{2.20}$$

and

$$g \to 1 \tag{2.21}$$

where C is a constant.

Equation (2.14), being independent of g, can be solved for f subject to the boundary conditions (2.17), (2.18) and (2.20) and then g can be obtained by solving (2.16) subject to boundary conditions (2.19) and (2.21).

The range of values of γ for which solutions may be expected can be found by considering an asymptotic formula for f, valid for large values of η. This may be obtained by substituting

$$f = \eta - C + \varepsilon F(\eta) \tag{2.22}$$

into (2.14). Here ε is small and so quadratic terms in ε can be neglected. Hence (2.14) reduces to the following linear equation for F,

$$F''' + [(2-4\gamma)\eta + 4\gamma C]F'' + 4(1+\gamma)F' = 0 \tag{2.23}$$

On putting

$$x = -(1-2\gamma)[\eta + 2\gamma C/(1-2\gamma)]^2 \tag{2.24}$$

and

$$\phi(x) = F'(\eta),$$

(2.23) reduces to the confluent hypergeometric equation

$$x\frac{d^2\phi}{dx^2} + (\tfrac{1}{2} - x)\frac{d\phi}{dx} - \left(\frac{1+\gamma}{1-2\gamma}\right)\phi = 0$$

It follows that

$$\phi \sim D_1F_1\left(\frac{1+\gamma}{1-2\gamma}; \tfrac{1}{2}; x\right)$$

where D is a constant and ${}_1F_1$ is the confluent hypergeometric function. Now from the theory of such functions (Morse & Feshbach, 1953), ϕ will tend to zero for large values of η only if

$$-1 < \gamma < \tfrac{1}{2} \tag{2.25}$$

This range of γ is confirmed by numerical results and is identical to that required for the existence of families of core structures in regime (*b*) of Fig. 1; see Bellamy-Knights (1971).

Similarly, an asymptotic formula for g can be obtained by substituting (2.22) and

$$g = 1 + \varepsilon G(\eta)$$

into (2.16) and neglecting quadratic terms in ε. This leads to the equation

$$G'' + [2\eta(1-2\gamma) + 4\gamma C]G' = 0$$

Hence integration yields

$$G'(\eta) = E \exp(x)$$

where E is a constant. In order that $G' \to 0$ as $\eta \to \infty$, it is required that $x < 0$ and so from (2.24), $\gamma < \frac{1}{2}$. This is in accordance with inequality (2.25).

Finally, the shearing stress, "volume flow thickness" and "mass flow thickness" associated with a specified direction (Lighthill, 1958) will be introduced since these quantities are related to certain constants in the boundary conditions. The shearing stress in the direction of r increasing, τ_r say, is given by $\tau_r = \mu(\partial u/\partial z)_{z=0}$ and so (2.10) and (2.12) give

$$\tau_r = -\tfrac{1}{4}\gamma r(\varrho\mu/t^3)^{\frac{1}{2}} f''(0) \tag{2.26}$$

The radial volume flow thickness, δ_r say, and the radial mass flow thickness, θ_r say, are defined by

$$\delta_r = \int_0^\infty (1 - u/U)\,dz \tag{2.27}$$

and

$$\theta_r = \int_0^\infty (1 - u/U)(u/U)\,dz \tag{2.28}$$

respectively, where U is the radial velocity just outside the boundary layer. Then C in (2.20) is related to δ_r by the equation

$$\delta_r = 2\sqrt{\nu t}\,C \tag{2.29}$$

The radial momentum integral equation reduces to

$$\theta_r + \left(\frac{1+\gamma}{3\gamma}\right)\delta_r = \left(\frac{4t^2}{3\gamma^2 r}\right)\frac{\tau_r}{\varrho} \tag{2.30}$$

Hence, (2.26), (2.29) and (2.30) give

$$\theta_r = -\sqrt{\nu t}\,\{2(1+\gamma)C + f''(0)\}/3\gamma \qquad (2.31)$$

In a similar manner, the shearing stress and volume and mass flow thicknesses associated with the circumferential direction can be obtained. These two thicknesses are also proportional to the square root of the time.

3. Numerical solution of equations

The two-point boundary value problem of solving (2.14) and (2.16) subject to boundary conditions (2.17) to (2.21) is expressed as an initial value problem. This method of solution has been described in an earlier paper (Bellamy-Knights, 1971) and so will receive only a brief description here.

Two tentative boundary conditions are introduced at $\eta = 0$ in place of the two boundary conditions at infinity. Shooting techniques are used to obtain values of the tentative boundary conditions for which the numerical solution is found to satisfy the boundary conditions at infinity. For example, (2.14) is solved subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad f''(0) = A$$

where the constant A is related to τ_r by (2.26). Numerical values of A, for which boundary condition (2.20) is satisfied, are obtained iteratively. Having obtained f, a solution of (2.16) subject to boundary conditions (2.19) and (2.21) can be obtained in closed form, i.e.

$$g = N\int_0^\eta \exp\left[-s^2 + 4\gamma\int_0^s f(q)\,dq\right] ds,$$

where

$$\frac{1}{N} = \int_0^\infty \exp\left[-s^2 + 4\gamma\int_0^s f(q)\,dq\right] ds.$$

For numerical convenience, however, g is solved simultaneously with f subject to the boundary conditions

$$g(0) = 0, \quad g'(0) = 1.$$

Then, since (2.16) is linear in g, the numerical results are normalised in order to satisfy boundary condition (2.21).

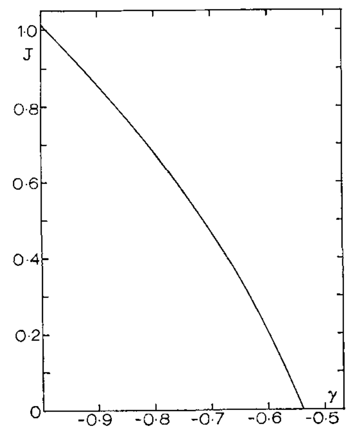

Fig. 2. J is plotted as a function of γ.

4. Discussion of results

It has been found (Bellamy-Knights, 1971) that values of γ leading to one, two and three-cell core solutions lie in the range $-1 < \gamma < \frac{1}{2}$. This is in accordance with the range of γ obtained in section 2 by considering the asymptotic form of f. Hence solutions of the present boundary layer equations are sought in this same range. The numerical results show that for each value of γ in the range $-1 < \gamma < \frac{1}{2}$, various values of A give permissible solutions of (2.14) and (2.16), that is solutions which satisfy the outer boundary conditions (2.20) and (2.21). It is not clear what additional criterion should be adopted to render the solution unique. This may be determined by matching this boundary layer region of the flow to the other regions (see Fig. 1). However, a solution for the stagnation point regime, satisfactory for this purpose, has still to be obtained. In the absence of a better criterion, we adopt the condition used by Hartree (1937) when solving the Falkner-Skan boundary layer equations, that is the condition that as $\eta \to \infty$, $f'(\eta)$ shall tend to 1 as rapidly as possible. This condition excludes solutions in which the radial velocity profile overshoots its asymptotic value. The value of A which gives this unique solution will be called J. Referring to (2.26) shows that A, and hence J, is proportional to the shearing stress in the radial direction.

Fig. 2 shows the variation of J with γ. The curve cuts the γ axis when $\gamma = -0.538 = \gamma_s$, say. Negative values of γ correspond to external

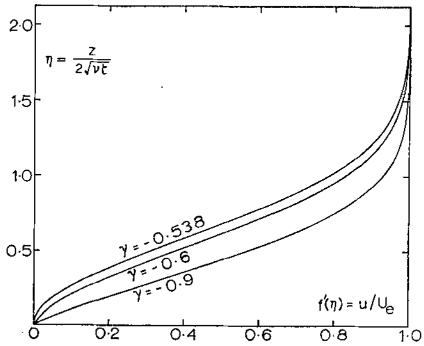

Fig. 3. Radial velocity profiles.

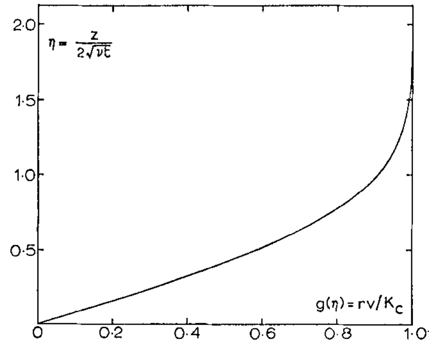

Fig. 5. The circulation profile for $\gamma = -0.6$.

flows which are radially outwards; see (1.2). Normal unseparated boundary layer profiles occur when $J > 0$, i.e. when $\gamma < \gamma_s$. The separation profile, $J = 0$, is obtained when $\gamma = \gamma_s$. When $\gamma > \gamma_s$, J is negative and so there would be reversed flow in the boundary layer. Solutions of this type involving separated flows will not be considered further. The following discussion will therefore be limited to flows for which $-1 < \gamma \leqslant \gamma_s$.

In Fig. 3, radial velocity profiles are shown for several values of γ in the above range. When $\gamma = -0.6$, there is a point of inflexion near $\eta = 0$; see Fig. 3. As $\gamma \to \gamma_s$, this inflexion becomes more pronounced as the separation profile is approached. On the other hand, as γ decreases from -0.6 to -1, the inflexion will become less pronounced; see Fig. 3. The values of J corresponding to $\gamma = -0.538$, -0.6 and -0.9 are 0, 0.206 and 0.856, respectively.

The behaviour of the functions $f(\eta)$ and $g(\eta)$ are qualitatively similar throughout the range $-1 < \gamma \leqslant \gamma_s$. In an earlier paper, Bellamy-Knights (1971), detailed results were presented for the outer flow at one particular value of γ, namely $\gamma = -0.6$, yielding a typical three-cell structure in the core region of the vortex (i.e. regime (*b*) in Fig. 1). The present results when $\gamma = -0.6$ are also fairly typical and will be described further below. This will facilitate the correlation of the results presented here for the boundary layer flow with those presented earlier for the core flow.

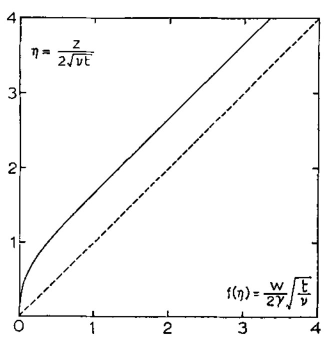

Fig. 4. The axial velocity profile for $\gamma = -0.6$ (solid line). The profile $f = \eta$ is also plotted (broken line).

When $\gamma = -0.6$, Fig. 4 shows $f(\eta)$, which is proportional to the axial velocity component. The curve $f = \eta$ is also shown in broken lines. It is seen that these curves become parallel for large values of η where the horizontal displacement between the two curves is C; see (2.20). Fig. 5 shows the circulation function $g(\eta)$.

When $\gamma = -0.6$, the corresponding value of J is 0.206. Inserting these values into (2.26), the shearing stress in the radial direcion is

$$\tau_r = 0.031\, r(\varrho\mu/t^3)^{\frac{1}{2}}$$

The numerical results give $C = 0.652$ and so, from (2.29), the radial volume flow thickness is

$$\delta_r = 1.303 \sqrt{\nu t}$$

Hence from (2.31), the radial mass flow thickness is

$$\theta_r = 0.404 \sqrt{\nu t}$$

5. Conclusions

In the present work it has been shown that boundary layer solutions on the plane $z = 0$ are possible when the outer flow consists of a potential vortex and an unsteady radial flow of the type given by (1.2), provided that $r^2/\nu t$ is sufficiently large; the required magnitude of $r^2/\nu t$ depends on the size of the tangential Reynolds number, K_c/ν. Thus a weakness of certain earlier work on viscous vortex cores is partially removed in so far as there exists a boundary layer solution for the external potential flow to which the core solutions asymptote. This leaves only the region (*d*), around the stagnation point, needing further attention.

In particular, for each value of γ in the range $-1 < \gamma \leqslant -0.538$ a unique solution may be determined by stipulating that $f' \to 1$ as rapidly as possible as $\eta \to \infty$. In this range the flow outside the boundary layer is radially outwards and the separation profile is obtained at that extremity of the range for which the radial velocity is lowest, namely $\gamma = -0.538$. The above range includes values of γ for which three-cell core structures can be obtained in the core region (*b*) of Fig. 1; see Bellamy-Knights (1971). Other mathematical solutions were found throughout the range $-1 < \gamma < \frac{1}{2}$ but these were not considered because they corresponded to separated flows or because f' did not tend to unity sufficiently rapidly.

For a steady outer potential flow as in Burgers's (1940, 1948) or Sullivan's (1959) core solutions, it was found that a boundary layer solution of the present type could not be obtained. Such a solution would correspond to a value of γ greater than $\frac{1}{2}$ and in this case the outer boundary conditions cannot be satisfied.

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ОСЕСИММЕТРИЧНОЕ РЕШЕНИЕ ТИПА ПОГРАНИЧНОГО СЛОЯ ДЛЯ НЕСТАЦИОНАРНОГО ВИХРЯ НАД ПЛОСКОСТЬЮ

Осесимметричное потенциальное вихреобразное течение ограничено плоскостью, перпендикулярной оси симметрии. Вследствие условия прилипания эффекты вязкости играют существенную роль вблизи плоскости, и так как трансверсальная скорость должна обращаться в нуль на оси симметрии, эффекты вязкости также существенны в центральной части вихря. Эти две вязкие области перекрываются вблизи пересечения оси симметрии с плоскостью. Таким образом, поле течения можно разделить на четыре режима: вязкую центральную часть, пограничный слой над плоскостью, режим «точки застоя» и потенциальное течение, которое обеспечивают внешние граничные условия для каждого из двух первых режимов. Такая модель может быть полезна при изучении метеорологических течений типа торнадо. Режим вязкой центральной части уже изучался (Ротт, 1958, 1959, Беллами–Найтс, 1970, 1971). В данной статье подтверждается и анализируется существование режима пограничного слоя. Путем введения из соображений подобия соответствующей переменной уравнения пограничного слоя для нестационарного осесимметричного течения несжимаемой жидкости сводятся к двум обыкновенным дифференциальным уравнениям, которые решаются численно. Когда радиальная скорость во внешнем потенциальном течении удовлетворяет определенным условиям, возможны решения типа пограничного слоя. Эти условия сходны с теми, которые требуются для существования многоячеистой структуры центральной части.