Solar radiative transfer in clouds using Eddington's approximation

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ABSTRACT

The present investigation deals with solar radiative transfer in horizontally stratified clouds. The approach taken is based on the Eddington approximation of radiative intensities applied to Chandrasekhar's (1960) exact formulation of the radiative transfer equation. All important physical processes such as absorption by water vapor and cloud droplets as well as multiple scattering by the cloud droplets are taken into consideration. Comparisons are made with results derived from the exact solution of the radiative transfer equation. The weakness of Eddington's approximation is discussed and an extension of the method is recommended.

1. Introduction

There are essentially two approaches to the solution of the radiative transfer equation (RTE). The first solution is based on an exact formulation of the RTE introduced by Chandrasekhar (1960), which is extremely complicated but achieves great accuracy. This approach was initially used by Coulson et al. (1960) to solve the Rayleigh scattering problem. Herman & Browning (1965) developed a numerical scheme to solve the exact form of the RTE which is applicable to particulate as well as molecular scattering. Yamamoto et al. (1966) and Korb & Zdunkowski (1970), among others, have used entirely different approaches to solve the exact form of the RTE.

The second approach represents a simplified method originally introduced by Schuster (1905) which is not based upon the exact form of the RTE. Over the years, this method was subsequently improved by, among others, Mecke (1921), Albrecht (1933) and again Mecke (1944). These simplified methods enjoy great simplicity at the expense of accuracy.

If radiative transfer processes are to be considered on a routine basis in conjunction with the dynamical equations of the atmosphere, then one cannot use the exact solution of the RTE due to present computer limitations. Since the simplified equations are not based on

the true physical concept of the multiple scattering mechanism, it is necessary to look for still another computational scheme of radiative transfer. A system is required that is based on the exact formulation of the RTE, yet it must be simple and accurate enough to be employed, for example, in prediction models of fog and cloud dynamics.

A successful step in this direction is made by Shettle & Weinman (1970) (later, simply referred to as SW) who employ Eddington's approximation to calculate radiative transfer through turbid atmospheres of moderate optical thickness. It is the purpose of this paper to extend their analysis and to apply Eddington's approximation to radiative transfer of solar energy in clouds of varying optical thickness. Results are then compared with exact solutions for the identical model cases. The cloud model is due to Best (1951) for a liquid water content of 0.1 g/m³. Different droplet distribution functions do not affect the conclusions.

Although SW give solutions of the RTE for conservative as well as non-conservative atmospheres, their set of solution equations is not complete. For particular positions of the sun and some cloud models, their formulas yield infinitely large fluxes. This resonance case is treated in the present paper and special solutions are given.

In addition, the Eddington approximation at

Tellus XXVI (1974), 3

times yields total downward fluxes near the cloud top which increase with increasing optical depth. This effect may represent real physical conditions, as demonstrated by SW, if the lower cloud section has a highly reflective property or if the ground albedo is high. The same increase is also produced by the Eddington approximation in case of zero ground reflection and optically thin cloud layers for some solar positions. It is shown that this particular phenomenon is an undesirable property of the Eddington approximation and does not represent real physical conditions.

2. Mathematical development

2.1. Solution of the radiative transfer equation

The analysis is based on the formulation of the RTE as discussed by Korb & Zdunkowski (1970) (hereafter, called KZ). For details, refer to that paper. The basic terminology of KZ is extracted, for convenience, and given below:

 $I_{\lambda}(\tau_{\lambda}, \mu, \phi)$ Radiative intensity at wave length λ , at optical depth τ_{λ} and directions μ, ϕ .

 $au_{\lambda} = -\int_{H}^{z} (eta_{\mathrm{ext}_{\lambda}} + K_{\lambda}) dz$. Optical depth in terms of the droplet extinction coefficient $eta_{\mathrm{ext}_{\lambda}}$, and the absorption coefficient K_{λ} of atmospheric gases. z stands for the height and H represents the altitude of the cloud top.

 θ , ϕ Zenith and azimuthal angles of the scattered radiative intensity, with $\mu = \cos \theta$.

 θ' , ϕ' Zenith and azimuthal angles of the incident radiative intensity with $\mu' = \cos \theta'$.

 $-\mu_0, \, \phi_0$ Direction of the parallell solar radiation

 $J_{\lambda}(\tau_{\lambda}, \mu, \phi)$ Source function.

 $R_{1,\lambda}^{(m)}(\mu,\mu')$, $F_{1,\lambda}^{(m)}(\mu,\mu_0)$. Series expression representing the phase function.

 p_1 Expansion coefficient of the phase function.

 E_0 , λ Intensity of solar radiation at the upper boundary of the cloud.

 k_{λ} Total absorption coefficient divided by the total extinction coefficient: absorption quantity.

If the phase function and the radiative intensity are introduced into the RTE as series ex-

pressions (eqs. 2, 3, 4 and 5, KZ), the RTE splits up into m components, i.e.

$$\mu \frac{dI^{(m)}(\tau,\mu)}{d\tau} = I^{(m)}(\tau,\mu)$$

$$-\frac{(1-k)}{2} \int_{-1}^{+1} I^{(m)}(\tau,\mu') R_1^{(m)}(\mu,\mu') d\mu'$$

$$-(2-\delta_{0,m}) \frac{(1-k)}{4\pi} E_0 e^{-\tau/\mu_0} F_1^{(m)}(\mu,\mu_0)$$
(1)

The subscript λ , indicating monochromatic radiation, is omitted for simplicity. Integration over the spectral interval is carried out as explained by KZ. For the computation of fluxes, only the component m=0 is required. Next introduce the Eddington approximation

$$I^{(m=0)}(\tau, \mu) = I_n(\tau) + \mu I_1(\tau) \tag{2}$$

into (1). To the resulting equation, apply the operators $\int_{-1}^{+1} d\mu$ and $\int_{-1}^{+1} \mu d\mu$. As demonstrated by SW, one obtains two simultaneous differential equations for I_0 and I_1 . In the notation of KZ, these assume the forms

$$\begin{split} \frac{dI_1(\tau)}{d\tau} &= 3I_0(\tau) \\ &- \frac{3}{4} (1-k) \, I_0(\tau) \int_{-1}^{+1} d\mu \int_{-1}^{+1} R_1(\mu,\mu') \, d\mu' \\ &- \frac{3}{4} (1-k) \, I_1(\tau) \int_{-1}^{+1} d\mu \int_{-1}^{+1} \mu' \, R_1(\mu,\mu') \, d\mu' \\ &- \frac{3}{8\pi} (1-k) \, E_0 \, e^{-\tau/\mu_0} \int_{-1}^{+1} F_1(\mu,\mu_0) \, d\mu \end{split} \tag{3}$$

$$\begin{split} \frac{dI_0(\tau)}{d\tau} &= I_1(\tau) \\ &- \frac{3}{4} \left(1 - k \right) I_0(\tau) \int_{-1}^{+1} \mu d\mu \int_{-1}^{+1} R_1(\mu, \mu') \, d\mu' \\ &- \frac{3}{4} \left(1 - k \right) I_1(\tau) \int_{-1}^{+1} \mu d\mu \int_{-1}^{+1} \mu' \, R_1(\mu, \mu') \, d\mu' \\ &- \frac{3}{8\pi} \left(1 - k \right) E_0 \, e^{-\tau/\mu_0} \int_{-1}^{+1} \mu F_1(\mu, \mu_0) \, d\mu \end{split} \tag{4}$$

Tellus XXVI (1974), 3

Eqs. (3, 4) simplify by observing certain orthogonality relations and by introducing some abbreviations.

$$\begin{split} a_1 &= \int_{-1}^{+1} d\mu \int_{-1}^{+1} R_1(\mu,\mu') \, d\mu' = 4 \\ c_1 &= 3 \left[1 - \frac{(1-k)}{4} \, a_1 \right] = 3k \end{split} \tag{5.1}$$

$$\begin{split} \alpha_2 &= \int_{-1}^{+1} d\mu \int_{-1}^{+1} R_1(\mu, \mu') \, \mu' \, d\mu' = 0 \\ c_2 &= -\frac{3}{4} \, (1-k) \, \alpha_2 = 0 \end{split} \tag{5.2}$$

$$a_{3} = \int_{-1}^{+1} F_{1}(\mu, \mu_{0}) d\mu = 2$$

$$c_{3} = -\frac{3}{8\pi} (1 - k) E_{0} a_{3} = -\frac{3}{4\pi} (1 - k) E_{0}$$
(5.3)

$$\begin{aligned} a_4 &= \int_{-1}^{+1} \mu d\mu \int_{-1}^{+1} R_1(\mu, \mu') \, d\mu' = 0 \\ c_4 &= -\frac{3}{4} \, (1-k) \, a_4 = 0 \end{aligned} \tag{5.4}$$

$$a_{5} = \int_{-1}^{+1} \mu d\mu \int_{-1}^{+1} \mu' R_{1}(\mu, \mu') d\mu' = \frac{4}{9} p_{1}$$

$$c_{5} = 1 - \frac{3}{4} (1 - k) a_{5} = 1 - \frac{p_{1}}{3} (1 - k)$$
(5.5)

$$\begin{aligned} \alpha_{8} &= \int_{-1}^{+1} \mu F_{1}(\mu, \mu_{0}) \, d\mu = -\frac{2}{3} \, p_{1} \, \mu_{0} \\ c_{6} &= -\frac{3}{8\pi} \, (1 - k) \, E_{0} \, \alpha_{8} = (1 - k) \frac{E_{0}}{4\pi} \mu_{0} \, p_{1} \end{aligned}$$
 (5.6)

Eqs. (3, 4) now assume the following forms

$$\frac{dI_{1}(\tau)}{d\tau} - c_{1}I_{0}(\tau) = c_{3}e^{-\tau/\mu_{0}}$$
 (6)

$$\frac{dI_0(\tau)}{d\tau} - c_5 I_1(\tau) = c_6 e^{-\tau/\mu_0}$$
 (7)

As pointed out by SW, the solutions to eqs. (6, 7) cannot in general be given due to variation of meteorological and radiative attenua-

tion parameters. However, by subdividing the atmosphere into homogeneous layers, it is possible to write down analytical solutions for each layer. For $k \neq 0$ one obtains on the requirement that $\mu_0^2 \neq (c_1c_5)^{-1}$

$$I_{0}(\tau) = B_{1} e^{\sqrt{c_{1} c_{5} \tau}} + B_{2} e^{-\sqrt{c_{1} c_{5} \tau}} + \frac{(c_{3} c_{5} \mu_{0} - c_{6})}{(1 - \mu_{0}^{2} c_{1} c_{5})} \mu_{0} e^{-\tau/\mu_{0}}$$

$$(8)$$

$$I_{1}(\tau) = \sqrt{\frac{c_{1}}{c_{5}}} B_{1} e^{\sqrt{c_{1} c_{5}} \tau} - \sqrt{\frac{c_{1}}{c_{5}}} B_{2} e^{-\sqrt{c_{1} c_{5}} \tau}$$

$$+ \frac{(c_{1} c_{6} \mu_{0} - c_{3})}{(1 - \mu_{0}^{2} c_{1} c_{5})} \mu_{0} e^{-\tau/\mu_{0}}$$
(9)

The arbitrary constants B_1 and B_2 for each layer are obtained from continuity of intensities I_0 , I_1 at the interface of the homogeneous layers and from the boundary conditions.

For some cloud models and a particular choice of μ_0 , the third R.H. terms however may become infinite. This resonance effect requires separate solutions which are not given by SW. Employing the method of variation of parameters with $\mu_0^2 = 1/(c_1c_5)$ one obtains

$$\begin{split} I_{0}(\tau) &= B_{1} e^{\tau/\mu_{0}} + B_{2} e^{-\tau/\mu_{0}} - \left(\frac{c_{6} \mu_{0} + c_{3} c_{5} \mu_{0}^{2}}{4} \right) e^{-\tau/\mu_{0}} \\ &+ \left(\frac{c_{6} - c_{3} c_{5} \mu_{0}}{2} \right) \tau e^{-\tau/\mu_{0}} \end{split} \tag{10}$$

$$\begin{split} I_{1}(\tau) = & \frac{B_{1}}{\mu_{0}} c_{5} e^{\tau/\mu_{0}} - \frac{B_{2}}{\mu_{0}} c_{5} e^{-\tau/\mu_{0}} - \left(\frac{c_{6} + c_{3} c_{5} \mu_{0}}{4 c_{5}}\right) e^{-\tau/\mu_{0}} \\ & - \left(\frac{c_{6} - c_{3} c_{5} \mu_{0}}{2 \mu_{0}}\right) \tau e^{-\tau/\mu_{0}} \end{split} \tag{11}$$

For k = 0, the conservative case, the solution is more simple and is given by

$$I_0(\tau) = A_0 + c_5 A_1 \tau + \alpha_0 e^{-\tau/\mu_0}$$
 (12)

$$I_1(\tau) = A_1 + \alpha_1 e^{-\tau/\mu_0} \tag{13}$$

The quantities A_0 , A_1 are arbitrary constants to be found from boundary conditions and α_0 , α_1 are given by

$$\alpha_0 = -\mu_0(c_5 \alpha_1 + c_6), \quad \alpha_1 = -c_3 \mu_0$$

The net flux of diffuse radiation is defined by

Tellus XXVI (1974), 3

24 - 742892

$$F_{\text{net}}(\tau) = F_1(\tau) + F_2(\tau) \tag{14}$$

where

$$F_1(\tau) = \pi I_0(\tau) + \frac{2}{3}\pi I_1(\tau) \tag{15}$$

$$F_2(\tau) = -\pi I_0(\tau) + \frac{2}{3}\pi I_1(\tau) \tag{16}$$

The direct still parallel solar radiation is not included in the diffuse downward flux F_2 but is accounted for by an extra term. Also note carefully that numerical values of F_1 and F_2 normally differ in sign.

Next, proper boundary conditions must be supplied. For simplicity, it is assumed that the diffuse illumination at the cloud top is zero and that the cloud base is illuminated by the uniformly reflecting ground of albedo A_g . With T referring to the total optical depth of the cloud, the boundary conditions are:

$$F_2(0) = 0$$

$$F_1(T) = A_0(-F_2(T) + \mu_0 E_0 e^{-T/\mu_0})$$
(17)

These simple boundary conditions are sufficient for the purpose of comparison. Absorption above and below the cloud can be taken into account by obvious modification.

2.2. Some remarks on Eddington's approximation

- (1) Eddington's approximation strictly conserves energy for k = 0, i.e. the total net flux, which includes the parallel solar radiation, is constant.
- (2) For a small solar zenith angle, in some cases the total downward flux near the cloud top increases with τ . This is a realistic feature for optically thick clouds or strongly reflecting ground surface as pointed out by SW. When the total optical thickness is small and ground albedo is zero, this increase near the upper cloud boundary should not occur, but nevertheless, is predicted by Eddington's approximation. This is easily demonstrated for k=0, the case of no absorption.

Differentiate the equation for the total downward flux (diffuse plus parallel) with respect to τ and set the result equal to zero. This gives

$$\tau_{\text{max}} = -\mu_0 \ln \left(\frac{\pi c_5 A_1}{(E_0/2) (1 - \frac{3}{2} \mu_0)} \right)$$
 (18)

which is the optical depth to which the total downward flux increases. From boundary conditions and $A_g = 0$, one obtains

Table 1. Total optical thickness T of clouds as function of solar zenith angle for which the total downward flux increases throughout the cloud, $\lambda = 1.01$ microns

μ_0	1.00	0.95	0.90	0.85	0.809
$_0^{\mu_0}$	0°	18°	25°	32°	36°
T'	0.609	0.483	0.341	0.171	Approx. 0

$$A_{1} = \frac{\alpha_{0}(1 - e^{-T/\mu_{0}}) - \frac{2}{3}\alpha_{1}(1 + e^{-T/\mu_{0}})}{\frac{4}{3} + c_{5}T}$$

$$\alpha_{0} = -\frac{3}{4\pi}\mu_{0}^{2}E_{0}$$

$$\alpha_{1} = \frac{3\mu_{0}}{4\pi}E_{0}$$
(19)

In certain cases, the total downward flux increases throughout the cloud. To find the total optical thickness T of such clouds, set T equal to $\tau_{\rm max}$ in eq. (18) and solve for T. The result is shown in Table I. When $\mu_0 < 2/3$, the argument of the logarithm in eq. (18) becomes negative and the total downward flux cannot increase with optical thickness.

3. Discussion of results

The Eddington approximation is used to evaluate the distribution fo solar radiation fluxes within clouds of various thicknesses and for different solar zenith angles. In one selected case, the fluxes are used to obtain the transmissivity within the cloud. In all cases considered, the cloud base is taken at 500 meters above the ground and the cloud depth is counted from the cloud top. To facilitate comparison with exact computations, no attenuation is assumed above the cloud so that the incident solar flux at the cloud top is parallel radiation. This simplification does not affect the conclusions at all.

Temperature and pressure distribution is taken from the US Standard Atmosphere Tables. The relative humidity is taken as 70% below the cloud, 100% within. The liquid water content of the model clouds is assumed to be 0.1 g/m³, a typical value for stratus clouds and fogs. The cloud droplet distribution formula for all computations is due to Best (1951). The cloud attenuation parameters and the phase

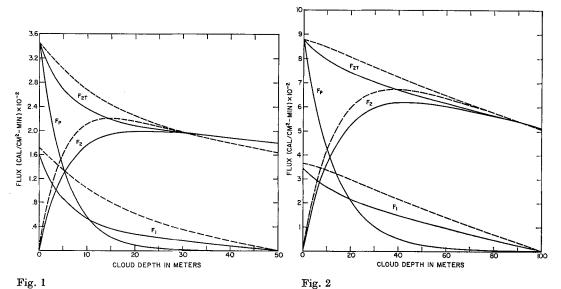


Fig. 1. Comparison of radiation flux components with exact computations (dashed lines) for a 50 meter cloud and solar zenith angle of 78.5°. F_1 , F_2 , F_p and F_{2T} denote the upward diffuse, downward diffuse, parallel, and total downward fluxes, respectively. $\lambda = 0.92$ microns.

Fig. 2. Comparison of radiation flux components with exact computations (dashed lines) for a 100 meter cloud, solar zenith angle of 60° and $\lambda = 0.92$ microns.

functions used in this study are based on this droplet distribution function as tabulated by Zdunkowski & Strand (1969). The ground albedo is assumed to be zero, except where noted otherwise.

The investigation pertains to two sub-intervals within the solar spectral region: (1) the non-absorbing interval centered at $\lambda = 1.01$ and extending over the wave-lengths interval 0.99—

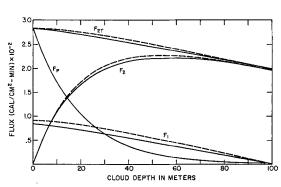


Fig. 3. Comparison of radiation flux components with exact computations (dashed lines) for a 100 meter cloud, solar zenith angle of 45° and $\lambda = 1.01$ microns.

1.03 microns, and (2) the interval from 0.85-0.99 microns ($\lambda=0.92$) containing the complete $\varrho\sigma\tau$ water vapor band. In order to test the accuracy of the method, comparisons are made with

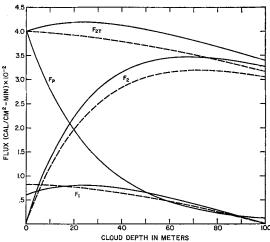


Fig. 4. Comparison of radiation flux components with exact computations (dashed lines) for a 100 meter cloud, solar zenith angle of 0° and $\lambda = 1.01$ microns.

Tellus XXVI (1974), 3

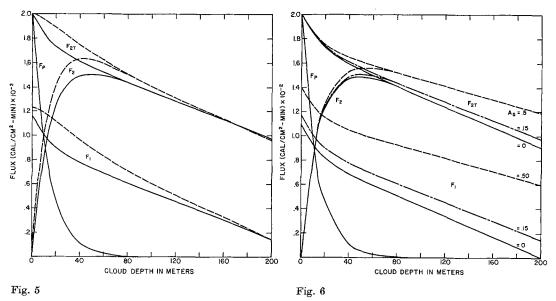


Fig. 5. Comparison of radiation flux components with exact computations (dashed lines) for a 200 meter cloud, solar zenith angle of 60° and $\lambda=1.01$ microns. The ground albedo $A_G=0.15$.

Fig. 6. Comparison of the components of the radiation field determined by the Eddington approximation for a 200 meter cloud, solar zenith angle of 60° and $\lambda = 1.01$ microns. The ground albedo is variously 0 (solid line), 0.15 (broken line) and 0.5 (dashed line).

available exact calculations using the method of KZ. Only the magnitudes of the fluxes are depicted without regard to sign. To facilitate discussion, similar figures are grouped together: Figs. 1 and 2 compare the distribution of solar fluxes in Region 2 for the Eddington approximation and the exact case for a 50 meter cloud ($\mu_0 = 0.2$) and a 100 meter cloud ($\mu_0 = 0.5$). Note that in the upper layers of the cloud, where the parallel radiation is rapidly diminished, there is a large disparity with the Eddington approximation which is consistently low for both the upward and downward diffuse fluxes. The difference is markedly decreased for the 100 meter cloud where μ_0 is larger. Fig. 3 gives the same comparisons for a 100 meter cloud with $\lambda = 1.01$ and $\mu_0 = 0.707$. For this particular zenith angle, the error is much lower and is nearly minimal for this type of approximation. These findings are qualitatively the same for different spectral intervals and cloud thicknesses, therefore, results are not shown. A superficial inspection of Fig. 4 reveals an apparently anomalous situation: for $\mu_0 = 1$, the total downward flux increases within the upper layers of the cloud implying a solar transmission greater than one. These values do not violate the principle of conservation of energy, since the net flux is

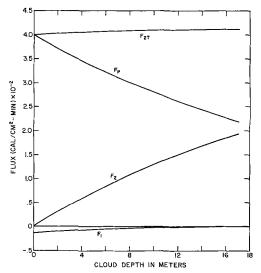


Fig. 7. Components of the radiation field determined by the Eddington approximation for a 17.26 meter cloud, solar zenith angle of 0° and $\lambda = 1.01$ microns.

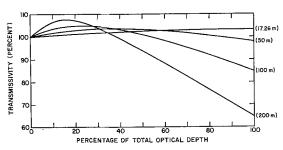


Fig. 8. Transmissivity for 17.26, 50, 100 and 200 meter clouds as a percentage of the total optical depth of the cloud. The solar zenith angle is 0° and $\lambda = 1.01$ microns.

precisely constant in the non-absorbing band. In fact, for all bands, the sum of the transmissivity, reflectivity and absorptivity are precisely equal to one at any level within the cloud. SW suggest that this increase is due to multiple reflections within very deep clouds, or moderately thick clouds with a high ground albedo. For this case, however, the ground albedo is zero and the cloud is of moderate thickness. A comparison with the exact case verifies that this unrealistic increase is due to the Eddington approximation which confirms the discussion of Section 2.2.

Fig. 5 compares the exact with the Eddington case for a 200 meter cloud and a uniform ground albedo of 15%; the same type of behavior is found as for the previous situations. Next, consider Fig. 6 which compares the Eddington results for the same cloud and solar zenith angle of $\mu_0 = 0.5$ for ground albedos of zero, 15 and 50%. Fig. 7 depicts a 17.26 meter cloud whose total optical depth T = 0.609 coincides with the predicted maximum of the total downward flux as discussed in Section 2.2. The region is non-absorbing so that the net flux remains necessarily constant throughout the cloud. Morever, the ground albedo is zero so that the net flux at the cloud base equals the total downward flux. Recall that normally F_1 and F_2 differ in sign, with F_2 less than zero. However, in this case, the total downward flux increases (negatively) throughout the cloud which requires, to keep the net flux constant, that F_1 and F_2 have the same sign. To point

out this physically unrealistic result more clearly, the upward flux is shown as a negative quantity.

Fig. 8 depicts the transmissivity within a 17.26, 50, 100, and 200 meter cloud for $\mu_0 = 1$. The abscissa is scaled as a percentage of total cloud depth. This type of graph clearly outlines the limitations of Eddington's approximation.

4. Conclusion

The Eddington approximation makes it possible to determine very economically radiative solar fluxes not only at the cloud boundaries, but also at any level within the cloud. Uniform ground albedo can be easily accounted for. For the case of zero absorption (purely conservative atmosphere) a special solution is obtained. For non-zero absorption the basic equations may yield infinitely large fluxes for certain atmospheric models. Therefore, additional equations are given to handle the case of resonance.

The great simplicity of the Eddington approximation results in a loss of accuracy which is particularly severe near the upper cloud boundary. Nevertheless, Eddington's approximation is still a useful tool since, with the exception of optically thin clouds, the emerging fluxes at the cloud base are in reasonable agreement with the exact results. The downward flux arriving at the earth's surface is of particular importance since it is required in the boundary statement of the heat conduction equations of the soil and the air layer near the ground. Furthermore, if the solar zenith angle is near 45 degrees, the Eddington approximation gives good agreement with more accurate solutions even within optically thin clouds.

It is recognized that the Eddington's approximation represents the leading terms of the intensity expansion into a series of Legendre polynomials. Taking only two additional terms into account improves greatly the accuracy of the approximation. The extension of the method results in a rather involved analytical solution which is still capable of rapid evaluation. Results are in preparation for publication.

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ИССЛЕДОВАНИЕ ПЕРЕНОСА СОЛНЕЧНОГО ИЗЛУЧЕНИЯ В ОБЛАКАХ С ИСПОЛЬЗОВАНИЕМ ПРИБЛИЖЕНИЯ ЭДДИНГТОНА

В настоящем исследовании рассматривается перенос солнечного излучения в горизонтально стратифицированных облаках. Выбранный подход основан на использовании приближения Эддингтона для интенсивностей в строгой формулировке Чандрасекара (1960) уравнения переноса излучения. Приняты во внимание все важные физические

процессы, такие как поглощение водяным паром и облачными частицами, а также многократное рассеяние на облачных каплях. Проводится сравнение с результатами, полученными путем точного решения уравнения переноса излучения. Обсуждаются недостатки приближения Эддингтона и даются рекомендации по развитию этого метода.