Spectral analysis of traveling planetary scale waves: Vertical structure in middle latitudes of Northern Hemisphere

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ABSTRACT

Results of analysis of about three years of Northern Hemisphere geopotential data, from 850 to 10 mb, using a technique of discrete spectral analysis of traveling waves are presented. The vertical structure of the traveling planetary scale waves (zonal wave-numbers 1 and 2) in middle latitudes is described. The implications of the results in regard to vertical energy propagation are discussed. The waves are approximately barotropic, with relatively small variation of phase with height, for frequencies from $\frac{1}{2}$ to about 8 cycles per month.

Introduction

The ultra-long, planetary-scale, waves of the atmosphere are of considerable amplitude in the Northern Hemisphere, and are quasistationary, which causes them to appear conspicuously on monthly and seasonal average charts. The Northern Hemisphere winter average 500 mb chart, for instance, shows definite displacement of the contours toward the northern Pacific sector, which is associated with wave-number 1 in a zonal Fourier analysis. Wave-numbers 2 and 3 are also well-defined in the mean height fields. The waves appear to be caused mainly by a combination of ground topography and large-scale patterns of heating and cooling (Charney & Eliasen, 1949, and many other workers).

The fluctuations of the planetary-scale waves have been studied by a number of authors. Kubota & Iida (1954) described westward traveling planetary-scale waves superimposed on the quasi-stationary waves. Deland (1964, 1965) and Eliasen & Machenhauer (1965, 1969) analyzed hemispheric isobaric fields for the Northern Hemisphere, showing that the planetary-scale fluctuations in the troposphere are largely due to the presence of apparent westward-traveling Rossby-type waves, with amplitudes on the average somewhat less than the amplitudes of the mean waves. Hirota (1967) and Deland & Johnson (1968) analyzed the apparent traveling waves in the troposphere and stratosphere of the Northern Hemisphere. Theoretical studies of the fluctuations include that of Hirota (1971) who solved the quasi-geostrophic equations numerically, and concluded that the apparent westward propagation of the planetary-scale waves in the stratosphere is due to the response of the atmosphere to propagation upward of energy from below, apparently arising from variable interaction of the zonal wind with the earth's topography. Wiin-Nielsen (1971) obtained analytic solutions to the baroclinic equations on the beta-plane. He found theoretical modes corresponding roughly to the apparent barotropic behavior of the fast-moving (of the order of 60 degrees of longitude per day for wave-number 1) westward-traveling waves, and additional slow-moving baroclinic modes, with marked variations of amplitude and phase with height, which have not yet been observed.

Deland & Johnson (1968) made a detailed statistical study of the structure of the apparent traveling planetary-scale waves, but their analysis of the vertical structure was based on correlation procedures that did not separate

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the traveling component from the other fluctuations. There are presumably random fluctuations due to observational and analysis errors, and there may be fluctuations of amplitude of the stationary waves, perhaps forced from below in the manner investigated by Hirota (1971). These variations are likely to have different vertical and latitudinal structures than the freely propagating waves, presuming that these exist. The problem of separating the traveling and other types of variations has been recently discussed by Deland (1972*a*, 1972*b*). It is evident from inspection of the time-variations of the Fourier harmonics (e.g. Deland, 1972*a*) that waves of more than one speed are present, or alternatively, the wave speed is not constant with time. The phase velocity of the traveling component is of theoretical interest: several authors have attempted to estimate it from simplified models of the atmosphere (Diiki & Golytsyn, 1969; Eliasen & Machenhauer, 1969; Wiin-Nielsen, 1971; Deland, 1970). Various estimates of the average speed of different harmonics have been made, ranging from 20 to 70 degrees per day (westward) for the largest scale waves at 500 mb in summer, for instance. The differences in estimates of the average speed are partly due to the different methods of analysis. Differentiating with respect to time, as in the autoregression analysis method used by Deland & Johnson (1968), "blues" the spectrum, increasing the contributions of high frequencies compared to low frequencies.

Deland (1972*a*) has presented a method of spectral analysis of the traveling component of the fluctuations, which has been used to describe the structure of the traveling planetary-scale waves in the SIRS radiance data (Deland, 1973). In this paper, some results of the analysis of two to three years of Northern Hemisphere pressure levels from 850 to 10 mb, using the spectral analysis procedure of Deland (1972*a*), are presented and compared with other observational results and theoretical studies. This paper is restricted to the vertical structure of the traveling planetary-scale waves in middle latitude (35° to 55° N).

Data and analysis procedure

The geopotential data for standard isobaric levels from 850 mb to 10 mb, for two to three years (not all levels for all months) were provided to us on magnetic tape by the National Center for Atmospheric Research. Daily geopotentials heights, at 00Z, were averaged over 10-degree latitude bands centered at 20°, 30°, 40°, 50°, 60°, 70°, 80° N, the data for the lowest latitudes being linearly extrapolated to a constant value at the equator. The results were subjected to zonal Fourier analysis, up to wavenumber 6. Fourier components at each level and latitude, as functions of time, were then Fourier analyzed with respect to time over periods of one month and two months, and the results analyzed for the traveling wave component according to the method of Deland (1972*a*). This method is the discrete Fourier equivalent of the quadrature spectral analysis method used by Deland (1964), and is related to the method of Hayashi (1971), but with important differences that are discussed by Deland (1972*a*). The discrete spectral analysis procedure is advantageous in that the phase information is preserved; it is lost in the continuous spectral analysis methods based on the Fourier transform of the autocorrelation function. We are thus able to analyze the variation of the phase, as well as amplitude, of the traveling waves with latitude and height, from which we can make some deductions about the dynamics of these fluctuations, based on the theoretical work of Eliassen & Palm (1961). The ratio of the variance accounted for by the computed traveling component to the total variance associated with that particular frequency (referred to hereafter as R) is calculated. The statistical distribution of R has not been derived. If the four coefficients which determine R [see Deland (1972*a*)] are assumed to be uncorrelated and normally distributed about zero, the mean value of R is easily estimated using a table of random normal numbers to be approximately 0.50 ($\pm$0.01).

In the following section we present the results of the computations, comparing them with theoretical models only where doing so is helpful in justifying the method of presentation. Interpretation of these observations in terms of the dynamics of the waves is postponed to a later section.

Results

As an example, the results of the spectral analysis of six winter and six summer months, of the geopotential data at 500 mb and 30 mb, zonal wave-number 1 at 40° N, are listed in Table 1. We will discuss these results in some detail and then describe the vertical structure of the waves as deduced from the larger collection of statistical results.

In the table, the frequency in cycles per month corresponds to phase velocity

$$c = \frac{N_T}{N_z}\left(\frac{360}{n_m}\right)$$

where N_T is frequency in cycles per month, N_z is zonal wave-number, and n_m is the number of days in the month, which can be approximated as 30, so that the speed for wave-number 1 is approximately equal to 12 N_T degrees of longitude per day. The phase angle θ is defined by the following expression (see Deland, 1972*a*)

$$W_k(x, t) = \sum_w R_{k,w} \cos\left[kx + w\left(t - \frac{\theta}{w}\right)\right]$$

so that it corresponds to the time at which the wave maximum crosses a particular meridian. In the case of westward traveling waves (positive w) the longitudinal position of a wave maximum at a given time shifts negatively (westward) for decreasing θ, that is in the same sense. For eastward traveling waves (negative w) the longitudinal position shifts positively (eastward) for decreasing θ, that is in the opposite sense. The amplitude of the traveling component is given in geopotential meters, and is shown as a positive number for eastward traveling components, negative for westward traveling components.

If we examine the summer spectra in Table 1, we see that the westward traveling components predominate from about 3 to 7 cycles per month, i.e. speeds of about 36 to 84 degrees of longitude per day, averaging about 60 degrees of longitude per day. On the average, the westward motion extends to the lowest frequencies, down to one cycle per month in these spectra. The spectra seem fairly consistent from month to month in the middle range of frequencies, but vary greatly at 1 or 2 cycles per month. The amplitude of the traveling waves is approximately constant with height. The coefficient R is greater than 0.5 for most of the westward traveling components in the middle frequency range, as can be seen by inspection of Table 1; R is greater than or equal to 0.50 (rounded), with westward traveling components, for 20 out of the 35 harmonics from 3 to 7 cycles at 500 mb, and for 9 of these harmonics R is greater than or equal to 0.9.

It is not possible, from this method of analysis, to tell whether the slow and fast moving components are simultaneously present during the whole of the period analyzed, or, in June 1965 at 500 mb for example, the waves move slowly eastward for part of the month and more rapidly westward at other times. Since this question involves the relations between different spectral components we may not have enough data to answer this question, in any case.

Although the amplitudes of the traveling waves are widely scattered, they appear to increase from 500 to 30 mb, on the average. The spectra are considerably more erratic in winter than in summer but westward motions are still predominant in winter from about 1 to 6 cycles per month. The winter spectra are shifted to low frequencies compared to the summer spectra: the greatest amplitudes in winter are for 1 to 3 cycles per month.

The difference in phase between two levels, for the same frequency, is meaningful only if the harmonic is traveling in the same direction at the two levels. Inspection of the phases in the table indicate that phases of corresponding components at the two levels are related for the summer months given in Table 1. Of the 35 components from 3 to 7 cycles per month, 24 move westward at both levels and have phase differences in the range $-90°$ to $+90°$. The average phase shift for these cases is approximately zero ($<5°$). In winter the phases are more erratic, but for those harmonics for which the speed is of the same sign at the two levels the phase differences again appear to average close to zero. Although the number of cases is small, phase shifts in summer from 500 to 30 mb appear to be positive and relatively large on the average for those harmonics for which R is close to 1.0 at both levels.

The traveling wave amplitude spectra, as analyzed for separate months, for 40° N and 50° N, are plotted for zonal wave-numbers 1 and 2 in summer (June, July, August) and winter (December, January, February) in Figs. 1–4. In plotting and analyzing these spectra the computed spectral amplitudes are taken as

Table 1. *Traveling-wave spectral results for 500 and 30 mb, 40° N, six winter and six summer months*

Phase (θ) in degrees, amplitude (A) in meters

Cycles/ month	1	2	3	4	5	6	7	8	9	10
A. *Summer*										
64/7					*30 mb*					
θ	140	185	330	122	279	152	202	296	286	131
A	−7.4	−4.7	−4.6	−3.9	−7.8	−1.0	−6.9	−5.1	3.2	.4
R	.47	.91	.52	.19	.91	.11	.94	.65	.28	.01
64/7					*500 mb*					
θ	40	103	349	350	102	138	147	252	215	146
A	−15.8	−19.7	−10.7	−10.2	5.1	−10.5	1.6	−4.7	3.7	3.0
R	.85	.86	.44	.64	.29	.74	.94	.85	.88	.60
64/8					*30 mb*					
θ	101	16	218	351	233	69	175	341	219	298
A	2.1	4.4	−3.8	−2.8	−7.3	−4.8	−6.8	1.9	−1.9	2.7
R	.08	.14	.12	.41	.56	.63	.96	.16	.38	.90
64/8					*500 mb*					
θ	280	84	304	339	217	60	111	241	111	99
A	−14.2	−9.6	−2.0	−11.5	−8.1	−5.1	−6.9	1.7	−1.3	−.4
R	.61	.83	.80	1.00	.92	.53	.98	.19	.14	.02
65/6					*30 mb*					
θ	360	27	101	195	170	130	299	77	210	27
A	−10.8	−8.4	−17.3	−7.3	−6.7	−4.9	−1.3	−4.5	−1.8	1.8
R	.95	.99	.90	.59	.77	.90	.19	.94	.40	.94
65/6					*500 mb*					
θ	12	224	154	169	174	325	254	78	206	226
A	13.2	11.8	−12.2	−4.0	−12.8	2.0	2.4	−5.3	−1.2	−2.1
R	.57	.87	.62	.43	.76	.40	.29	.06	.27	.50
65/7					*30 mb*					
θ	87	13	206	122	102	65	199	217	229	146
A	3.1	−9.6	−4.0	−5.1	.5	−8.9	2.3	−1.3	−1.7	1.6
R	.14	.98	.18	.92	.00	.93	.53	.80	.57	.43
65/7					*500 mb*					
θ	21	49	263	278	202	342	237	173	293	29
A	−9.6	−8.8	−7.8	1.1	−6.0	−4.6	−4.2	−2.8	−.4	.6
R	.73	.92	.45	.32	.89	.94	.64	.85	.83	.45
65/8					*30 mb*					
θ	7	244	134	257	145	336	305	74	123	184
A	−4.7	−.8	−12.0	−12.9	−10.0	−5.3	2.4	1.5	1.4	1.4
R	.71	.97	.89	1.00	.98	.70	.68	.28	.29	.38
65/8					*500 mb*					
θ	119	134	74	221	89	350	159	122	114	181
A	7.2	−7.5	−2.8	−9.0	−7.9	−6.5	1.3	2.1	1.6	2.4
R	.35	.26	.70	.98	.88	.89	.10	.36	.38	.89
66/6					*30 mb*					
θ	136	218	317	124	54	234	349	278	7	151
A	−6.3	−8.6	−8.4	−6.2	−9.2	−3.3	−6.9	−1.9	−3.6	2.1
R	.90	.88	.46	.76	.70	.70	.93	.80	.32	.77
66/6					*500 mb*					
θ	62	203	252	148	64	223	308	254	269	230
A	7.1	−13.6	−4.0	−11.7	−5.5	−8.2	−6.2	−2.5	−1.7	−1.9
R	.24	.56	.74	.67	.78	.72	.97	.70	.81	.81
64/12					*30 mb*					
θ	66	205	147	135	33	212	32	57	229	314
A	−55.0	−19.7	−11.3	−20.2	.9	−7.9	5.0	2.9	−3.3	−1.1
R	.75	.73	.33	.94	.02	.94	.82	.71	.38	.06

Table 1 cont.

Cycles/ month	1	2	3	4	5	6	7	8	9	10
64/12					*500 mb*					
θ	80	116	31	228	63	222	60	257	330	333
A	−15.8	−6.4	4.5	3.8	7.1	−5.0	3.9	−.6	−3.8	1.5
R	.23	.13	.56	.12	.66	.89	.60	.02	.66	.26
65/1					*30 mb*					
θ	234	93	130	39	45	103	43	234	84	230
A	−35.0	11.7	−35.3	−18.3	−8.0	−14.2	4.3	2.7	4.4	2.1
R	.77	.82	.95	.96	.94	.75	.40	.52	.57	.63
65/1					*500 mb*					
θ	190	253	176	4	332	55	127	167	185	13
A	36.2	6.6	−1.6	−17.2	−8.5	−7.3	.4	2.6	1.4	−.9
R	.37	.22	.04	.81	.77	.67	.01	.57	.41	.07
65/2					*30 mb*					
θ	35	40	311	80	40	249	256	272	208	75
A	−22.0	−18.2	−8.9	−8.1	−8.4	10.3	5.0	2.3	3.0	−2.7
R	.37	.55	.37	.85	.62	.51	.87	.68	.12	.14
65/2					*500 mb*					
θ	56	61	162	197	9	160	123	91	286	189
A	−28.9	−21.9	−12.3	2.6	−2.8	−7.1	1.1	−1.0	3.4	−1.5
R	.83	.57	.61	.35	.71	.40	.15	.08	.53	.21
65/12					*30 mb*					
θ	284	96	341	65	273	121	99	120	85	144
A	10.7	−18.2	−12.3	−5.5	5.7	4.8	−7.3	2.4	2.6	3.3
R	.07	.22	.92	.45	.83	.55	1.00	.39	.47	.90
65/12					*500 mb*					
θ	86	302	347	331	226	306	50	20	250	314
A	17.7	7.3	−14.9	−18.1	−11.1	2.8	−3.9	2.2	1.3	1.2
R	.69	.34	.94	.64	.72	.18	.58	.42	.16	.10
66/1					*30 mb*					
θ	332	224	103	41	167	258	157	200	71	326
A	−4.5	−55.9	−29.6	−9.8	−7.6	−5.6	−4.1	−1.9	1.8	−1.9
R	.10	.89	.85	.47	.65	.77	.30	.08	.16	.66
66/1					*500 mb*					
θ	20	276	78	26	50	13	177	236	99	187
A	−33.9	−4.8	−3.3	−10.5	−8.0	−2.8	2.9	3.1	−1.7	3.1
R	.82	.10	.25	.71	.39	.13	.19	.78	.12	.66
66/2					*30 mb*					
θ	6	27	248	281	178	343	267	244	355	301
A	−12.4	−13.7	−17.2	−15.0	−5.1	−16.0	−8.4	1.6	3.2	1.9
R	.28	.31	.99	.64	.95	.95	.96	.46	.40	.33
66/2					*500 mb*					
θ	59	105	84	313	179	356	232	102	41	54
A	−41.0	5.1	−16.1	7.2	−4.3	−8.3	−8.5	2.4	2.7	3.5
R	.90	.42	.62	.73	.26	.63	.92	.10	.24	.39

estimates of the (presumed or hypothetical) population spectral values: all the values for the season are plotted together without distinguishing the results for the different months and the two latitudes, except that the results for 1969, plotted as crosses, are separated because there are fewer pressure levels for this year. The other months, represented by circles, are equally represented at all the levels from 850 to 10 mb except that one summer month is missing for 100 mb due to a computer error. The amplitudes rather than the more conventional squared amplitudes are shown in the figures for convenience in plotting on a

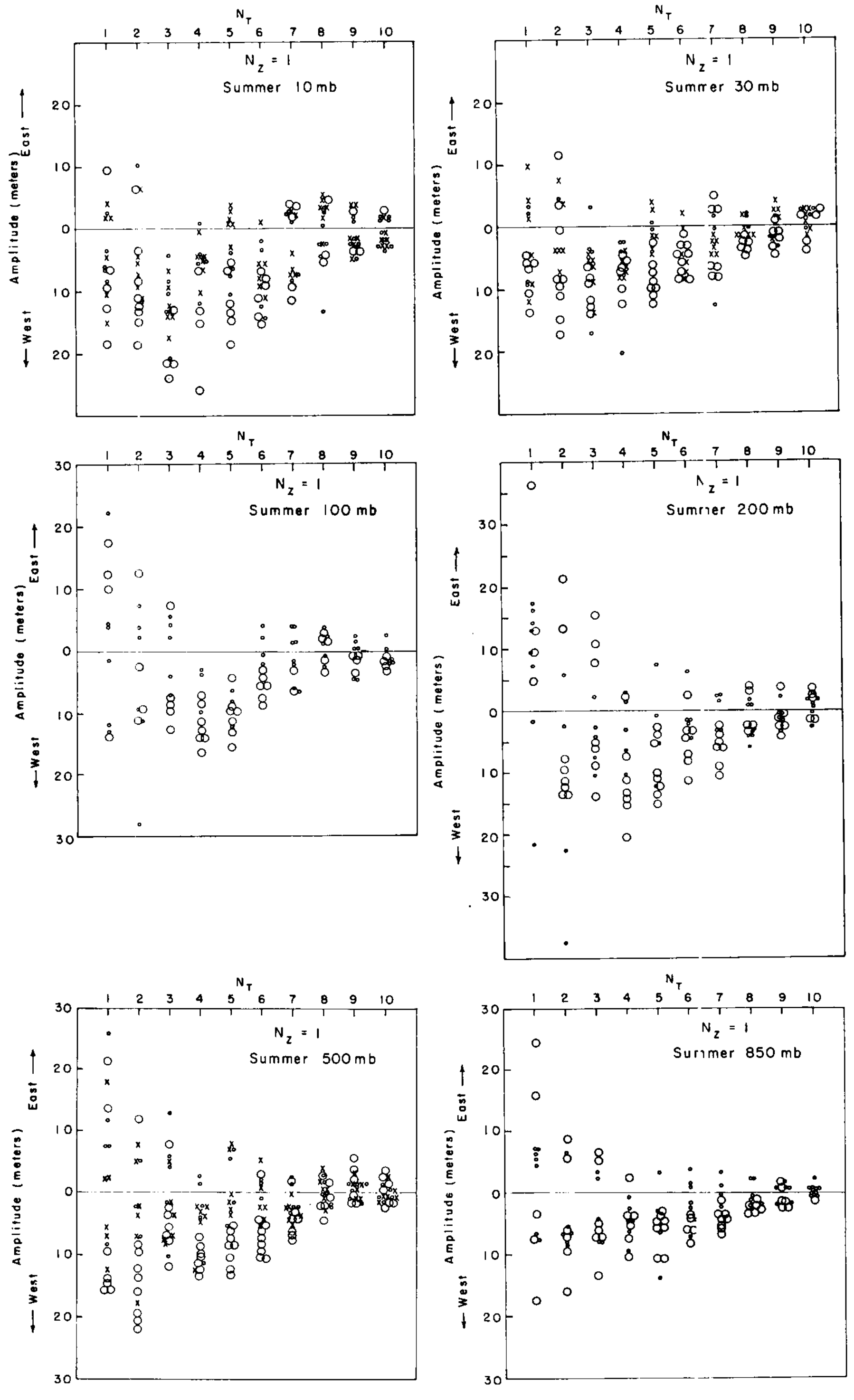

Fig. 1. Traveling-wave spectra, zonal wave-number 1, in summer (June, July, August) 40° and 50° N. Circles for 1964–1966 spectra, crosses for 1969 spectra. Positive amplitudes represent eastward-traveling components, negative westward. Spectra computed for one-month periods, frequencies in cycles per month.

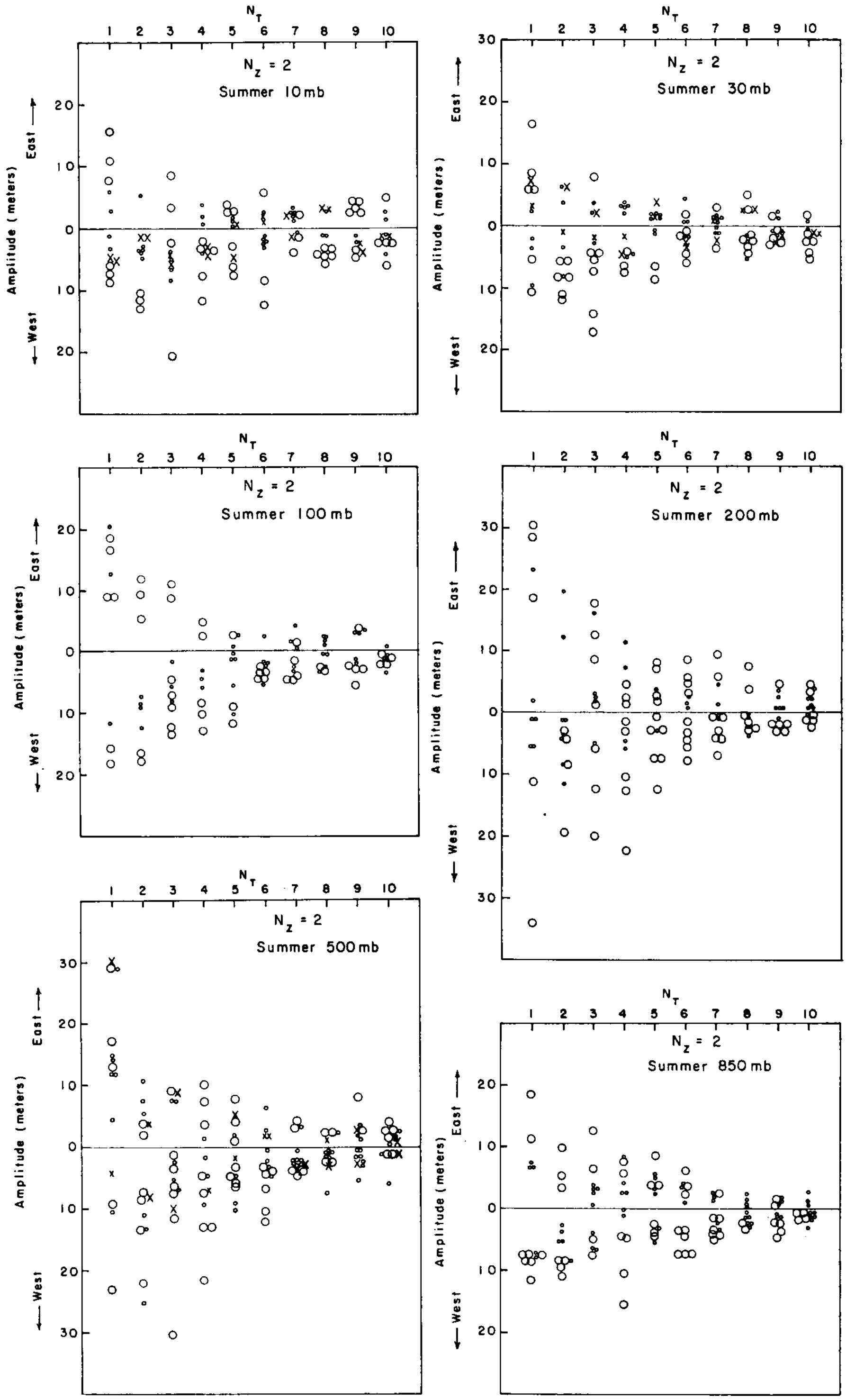

Fig. 2. Traveling-wave spectra, zonal wave-number 2, in summer, otherwise as for Fig. 1.

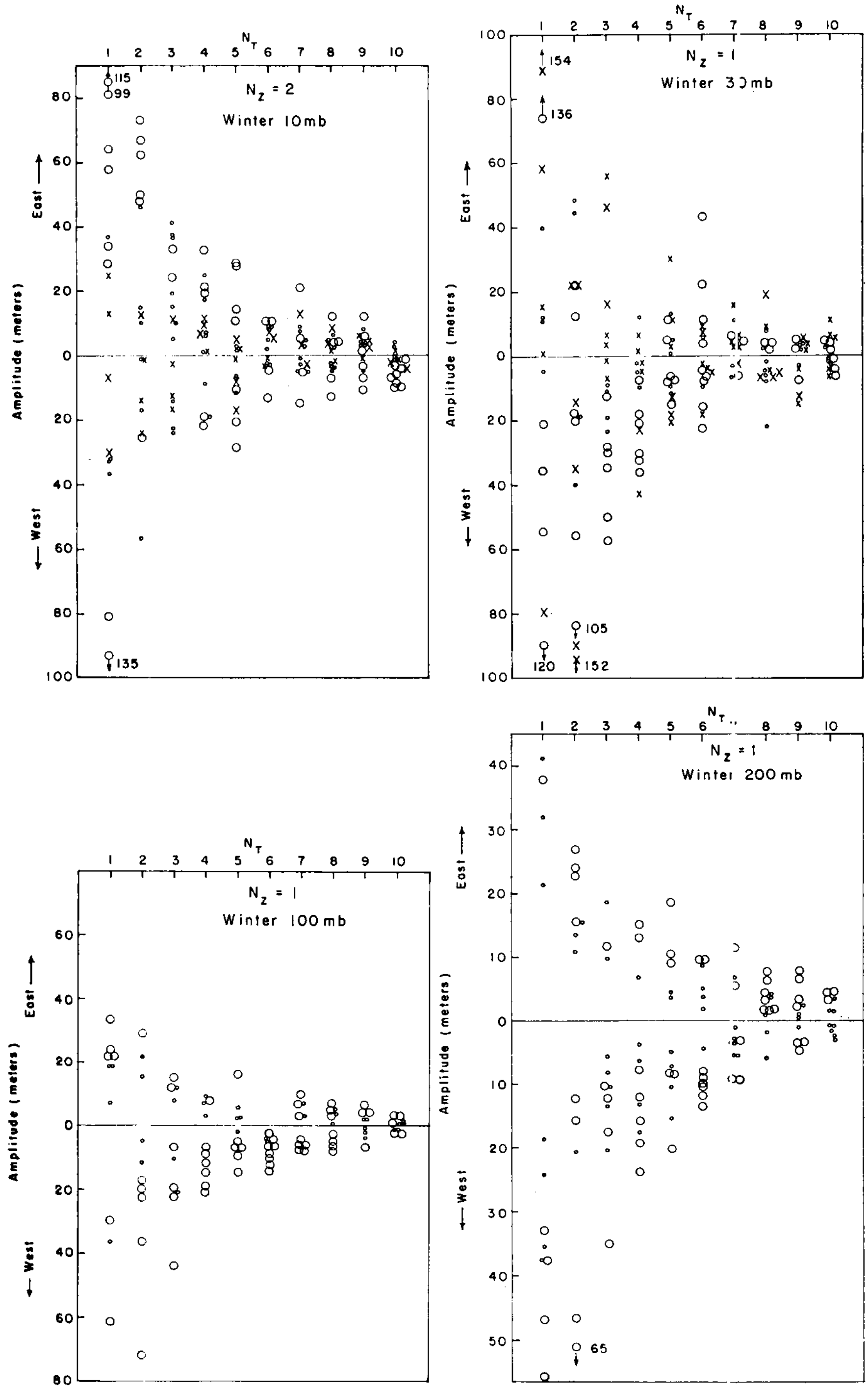

Fig. 3 a–d.

Fig. 3. Traveling-wave spectra, zonal wave-number 1, in winter, otherwise as for Fig. 1.

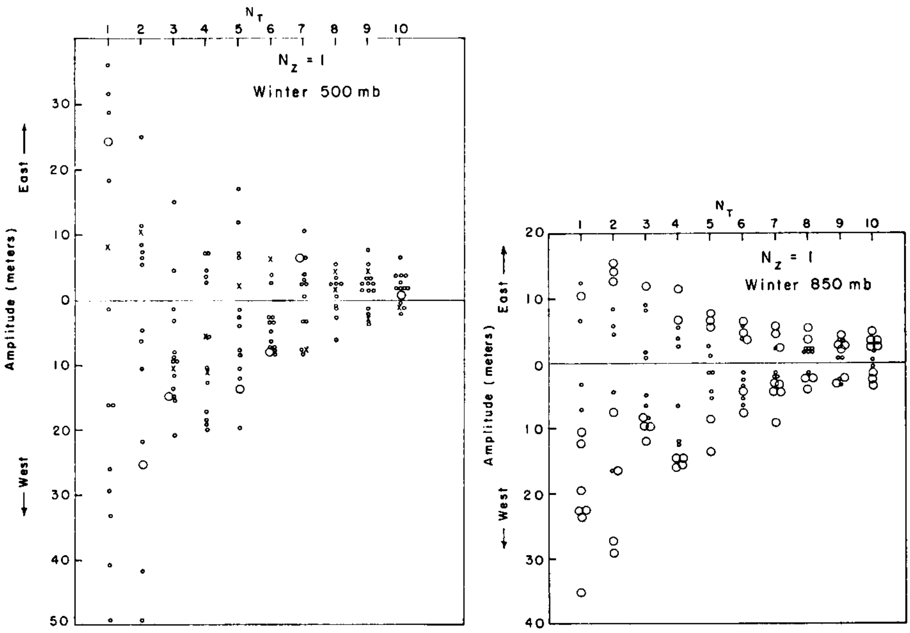

Fig. 3 e–f.

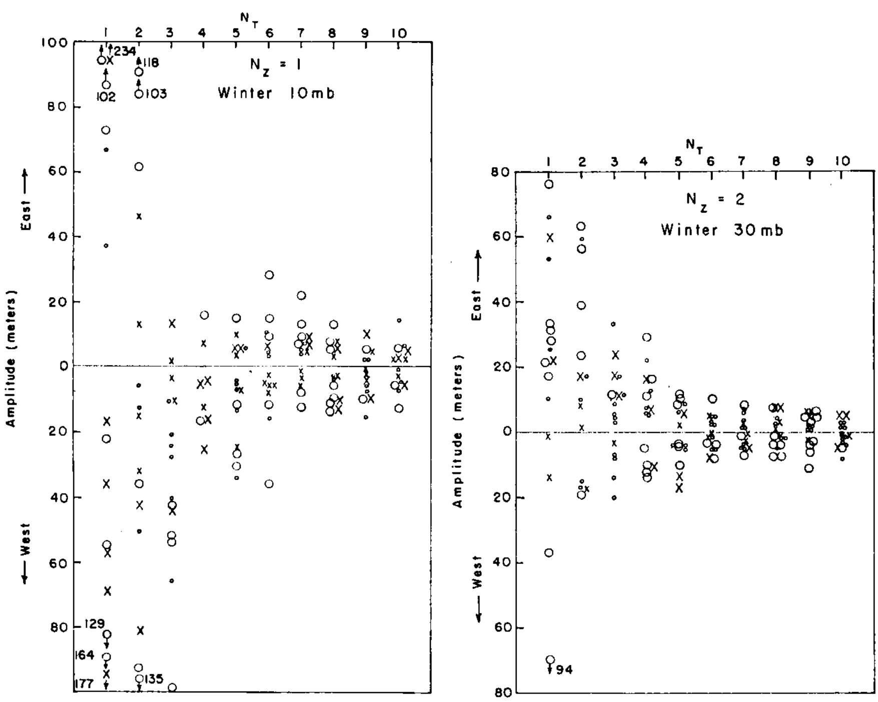

Fig. 4. a–b.

Fig. 4. Traveling-wave spectra, zonal wave-number 2, in winter, otherwise as for Fig. 1.

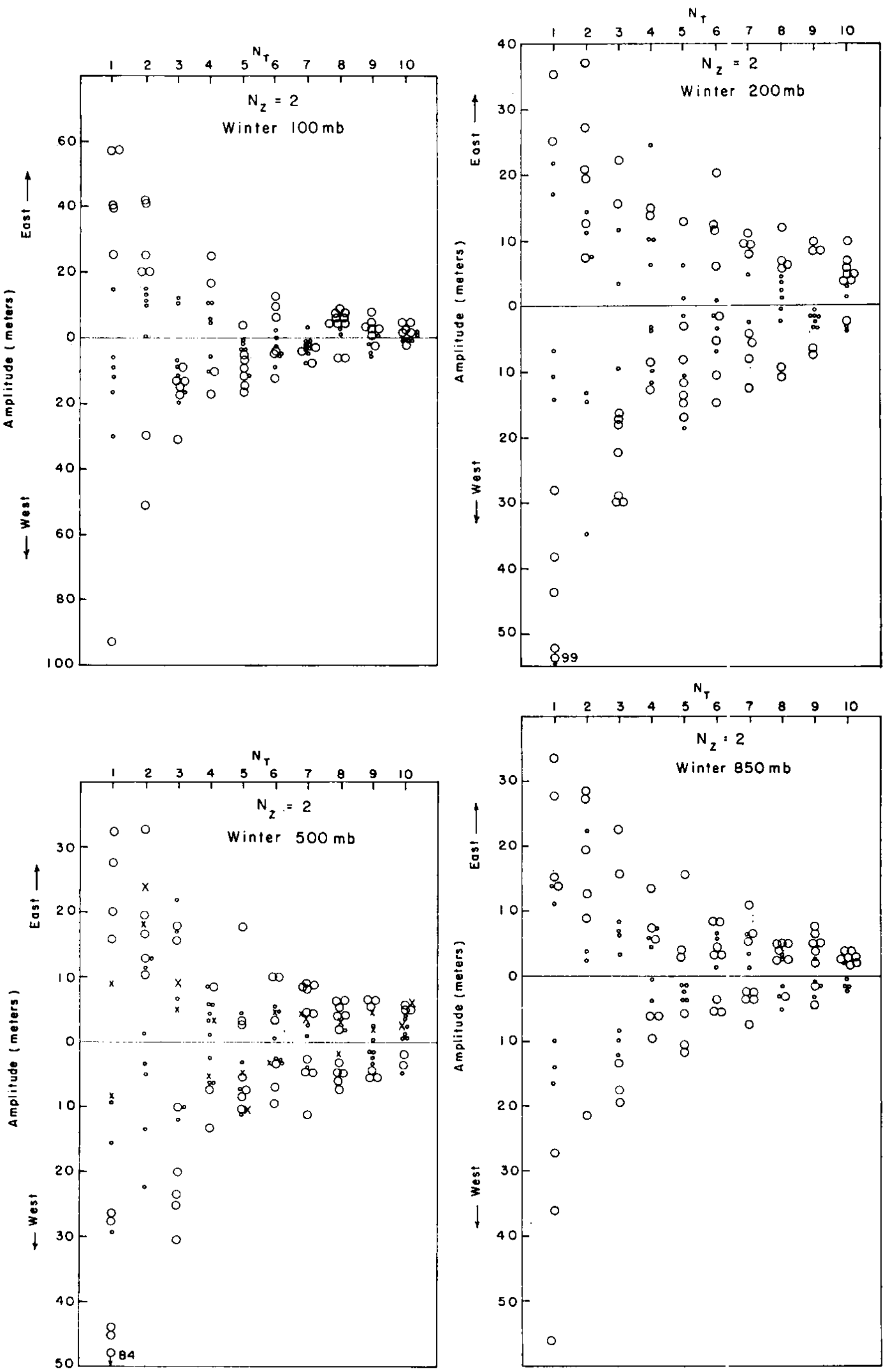
N_T
1 2 3 4 5 6 7 8 9 10
N_z = 2
Winter 100 mb
East
West
Amplitude (meters)
N_z = 2
Winter 200 mb
99
N_z = 2
Winter 500 mb
84
N_z = 2
Winter 850 mb

Fig. 4 c–f.

linear scale, on which the eastward traveling and westward traveling components can be plotted as positive and negative amplitudes respectively. It is to be expected that the harmonics for which the coefficient R, the ratio of the variance due to the traveling wave to the total variance, is greater than 0.5 should be more representative of the traveling waves in the atmosphere than those for which R is less than 0.5, so the harmonics for which R is greater than or equal to 0.50 (rounded off) are represented by larger symbols than those for which R is less than 0.50. Most of the discussion of these spectra will be in terms of their average properties as far as they can be estimated from this ample, which is relatively small by usual statistical standards.

The variation of the wave-vector with height, which corresponds to a wave in the temperature field, is due to variations with height of both amplitude and phase of the zonal harmonic. As will be seen later, the variation with height of the phase of the traveling waves is considerably less than that of the quasi-stationary waves, as estimated from the means for the period analyzed. Because of the relatively small variations of phase of the traveling waves, it is practicable to describe the variations of amplitude and phase separately.

The continuity of the spectral estimates between different levels is important in lending credibility to the calculated values as representing physical traveling waves in the atmosphere, rather than an artifact resulting from the application of the particular method of analysis to random data. Since we can only compare phases for a given harmonic at different levels if the direction of motion is the same at the two levels, the comparison of phrases later in the paper will effectively filter out the traveling waves for which there is vertical continuity, and which will be presumed to represent physical traveling waves. The results of the phase analysis supports this interpretation, as will be seen below.

Amplitude spectra in summer

The zonal wave-number 1 spectra in summer (Fig. 1) are rather similar at all levels analyzed. The spectra appear to be about the same shape at all levels, with an apparent small increase in amplitude of the order of 50 %, from 850 to 10 mb, most of the increase being from 850 to 500 mb and from 100 to 10 mb. Westward traveling waves predominate at all levels from about two cycles to about 7 cycles per month, as we have already seen for 500 mb at 40° N. The traveling waves appear to be most clearly defined at about four or five cycles per month. In the two-month spectra, not shown, there are westward traveling waves as slow as 1 cycle per month at 850 and 500 mb, but at higher levels up to 30 mb the slowest motions are mainly eastward, in the summers of 1964 and 1965 (but not in 1969). Since the structure of the lowest frequency harmonics is of particular interest in view of the results of Wiin-Nielsen (1971), they will be analyzed in more detail later.

The zonal wave-number 2 results in summer (Fig. 2) are much more random than those for wave-number 1. Negative amplitudes representing westward traveling waves are in the majority but not conspicuously so at all levels. The spectra are shifted toward lower frequencies compared to wave-number 1, amplitudes for frequencies greater than 6 per month (5 at some levels) appearing to be random. The amplitudes of the traveling waves appear to be almost constant with height, with no evidence of increase with height as is observed for wave-number 1.

At the lowest frequency, 1 cycle per month, there is the same shift from westward motion at lower levels (in this case 850 mb only) to eastward motion at higher levels (100, 50 and 30 mb) as was observed for zonal wave-number 1.

Winter spectra

The spectral amplitudes for zonal wave-number 1 in winter (Fig. 3) increase upward apparently monotonically by a factor of about 2 or 3 from 850 mb up to 10 mb. The shape of the spectra appears to change with increasing height, with the relative contribution of the lower frequencies increasing with height, that is the average speed appears to be less at the higher levels.

The average westward wave speed for wave-number 1 appears considerably less in winter than in summer. Westward moving components predominate down to one cycle per month, with maximum amplitudes appearing to be at about 1 or 2 cycles per month in winter.

For wave-number 2 in winter (Fig. 4) eastward and westward motions are in general about equally represented. In the stratosphere, however, at low frequencies, 1 or 2 cycles per month, eastward motions predominate, consistent with the pattern for the other cases as described above. The amplitude increases with height, for wave-number 2 in winter but it is only for the slowest components in the stratosphere, one or two cycles per month, that the direction of motion (eastward) is sufficiently consistent that we can consider the spectra to represent real traveling waves. These components increase with height in the stratosphere by a factor of two or three from 100 to 10 mb.

Variation of phase with height

The difference of phase between corresponding traveling components at different levels represents both a time-shift and a longitudinal displacement. For instance, for a westward-traveling component, if it leads at a higher level it is tilted westward with increasing height. For eastward traveling components, westward tilt with increasing height corresponds to higher levels lagging behind lower levels. In the following, the time-wise phase shifts for eastward traveling components are changed in sign so that they represent longitudinal displacement. In the case of wave-number 2, the displacement in degrees of longitude is one-half the phase-shifts given. The phase-shifts from the one-month spectral computations, for 1 to 10 cycles per month, will be discussed first followed by the low frequency results from the two-month computations. The phase-shifts, for the one-month computations, from 850 to 500 mb, from 500 to 200 mb, from 200 to 100 mb, and from 100 to 10 mb, are plotted with respect to frequency in Figs. 5 and 6 separately for all the winter (December, January and February) months of the two years July 1964 to June 1966, for zonal wave-numbers 1 and 2, for 40° and 50° N. Westward traveling components are plotted as circles, eastward traveling components as crosses.

For both wave-numbers, in summer and winter in middle latitudes, the variation of phase for the westward-traveling components is generally small from 850 to 200 mb, averaging approximately zero from 1 to about 9 cycles per month. The westward-traveling components are thus approximately vertical in the troposphere. The eastward traveling components, on the other hand, are tilted westward with increasing height on the average from 850 to 200 mb.

From 200 to 100 mb, the westward-traveling components in winter, for both wave-numbers 1 and 2, tilt eastward with increasing height, on the average. The eastward-traveling components tilt westward on the average. In summer (not shown) for wave-number 1 there is considerable scatter and the phase shift is approximately zero, but for wave-number 2 there is a definite eastward tilt of the westward traveling components. Wave-number 3, which is not plotted, behaves similarly to wave-number

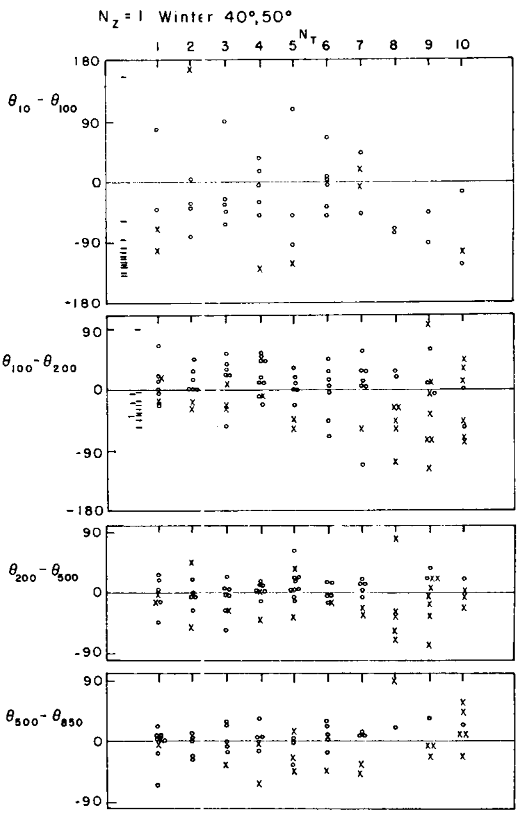

Fig. 5. Longitudinal phase-shifts. One month computations, for 850–500 mb, 500–200 mb, 200–100 mb, and 100–10 mb, at 40° and 50° N. Winter months, 1964–1966, zonal wave-number 1. Circles represent westward, crosses eastward traveling components.

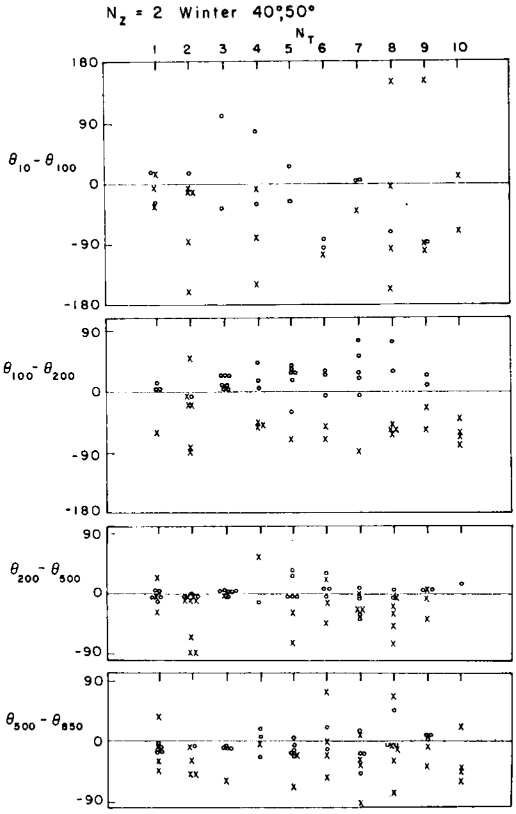

Fig. 6. Longitudinal phase-shifts. Zonal wave-number 2, otherwise as for Fig. 5.

2, with a well-defined eastward tilt of the westward components from 200 to 100 mb.

Both westward and eastward-traveling components tilt westward on the average from 100 up to 10 mb for both wave-numbers 1 and 2. The computed phase shifts scatter widely but the westward tilt averages about 50 degrees from 100 to 10 mb for the westward traveling components, the eastward traveling components being tilted more, of the order of 100 degrees on the average. Also shown in Fig. 5 for wave-number 1 are the phase-shifts of the mean wave for the six winter months. It is apparent that the westward tilt from 100 to 10 mb of the traveling waves in winter is considerably less than the tilt of the mean waves. In summer (not shown), on the other hand, when traveling waves tilt more than they do in winter, the mean waves are almost vertical.

The phase-shifts between levels, when the average is different from zero, as in the 200–100 mb layer, appears to decrease approximately linearly with values at the lowest frequency, so that at low frequencies they correspond to roughly constant (with respect to frequency) time differences. Deland (1973) has pointed out a similar feature of the phase shift between the traveling temperature waves in different layers of the stratosphere, analyzed from NIMBUS III SIRS radiances.

The estimate of the phase of the traveling wave is affected by the presence of a stationary wave whose amplitude varies periodically with the wave frequency (Deland, 1972*a*), in fact if a traveling wave and an oscillating quasi-stationary wave are both present, the computed phase for the traveling wave will be roughly the weighted average of the phases of the traveling and stationary components. Since it is likely that there will be fluctuations of amplitude of the mean wave, associated especially with upward propagation of energy, it is to be expected that the apparent tilt of the traveling waves shown in Figs. 5 and 6 will reflect to some extent the tilt of the mean waves, or if their vertical structure is similar, the average phase error will be zero. If the traveling waves are excited to a significant extent by the upward propagation of energy associated with the qusi-stationary waves, in the manner illustrated by Hirota's (1971) model, the mean phase shifts of the traveling waves might be biased by the longitudinal displacement of the stationary fluctuations with height.

Since the coefficient R represents the fraction of variance associated with the traveling wave, we can expect that the higher its value, the less would be the effect of the quasi-stationary waves. The maximum phase error due to the presence of a stationary fluctuation decreases with increasing R. It is easy to show that the maximum phase error is approximately 20° for R equal to 0.70, and less than 10° for R equal to 0.90. The phase-shifts were plotted against the coefficient R, averaged for the two levels, and also against R for the two levels as separate variables in an attempt to determine whether the structure of the waves varies with respect to these ratios. The phase-shifts appear to be independent of the coefficient R for all layers, for both harmonics, and for both seasons. In all cases, restriction to larger values of R resulted in reducting the scatter of the phase-shifts, as plotted in Figs. 5 and 6 for example, without apparent change of the average.

Very slow traveling waves

Wiin-Nielsen (1971) has suggested that slow-moving baroclinic ultra-long waves may be present in the atmosphere. According to his theoretical model, there may be waves moving slowly westward with respect to a stationary atmosphere, with almost opposite phases near the bottom and top, and also higher modes with more complicated vertical structure. Corresponding waves in the real atmosphere should be moving slowly westward in summer, at perhaps 5 degrees of longitude per day, and eastward at about the same speed in winter, if we may presume that they are translated by the average, over the Northern Hemisphere atmosphere, of the zonal wind. It therefore appears worthwhile to examine the vertical structure of the slowest components in more detail.

For the purpose of better definition of the slowest components, traveling wave-spectra were computed for two-month periods. Harmonics were calculated for 1, 2, 3 cycles per two months, i.e. ½, 1, 1½ cycle per month. A frequency of ½ cycle per month corresponds to longitudinal speeds of 6 and 3 degrees of longitude per day at 45° N for wavenumbers 1 and 2 respectively. In Figs. 7 and 8 the longitudinal phases of the first three harmonics, 1, 2, and 3 cycles per two months, are plotted for zonal wave-numbers 1 and 2 at 40° N and 50° N, for summer and winter, with all levels from 850 to 10 mb with the exception of 50 mb plotted on the same diagram. As plotted the phases represent the longitudinal position of the maximum at the beginning of the two-month period analyzed. For comparison with the phases of the traveling components, the envelope of the longitudinal phases of the two-month mean harmonic (the corresponding harmonic of the two-month mean field) is represented on the figures by dashed lines. Points representing harmonics (for the same period) moving in the same direction are connected by lines, continuous for adjacent levels, and dotted for separated levels up to 100 mb. Above 100 mb, the variations of phase between harmonics moving the same direction at separated levels with contrary motion in between are large and appear to be random.

These slow components appear to be remarkably uniform in vertical structure for the two seasons and for the two wave-numbers, in the two latitude bands. They do not appear to differ much from the faster components, being almost vertical on the average in the troposphere, and with a moderate westward slope, less than that of the mean waves, in the stratosphere. For 1 and 1½ cycles per month the westward-traveling components tilt westward on the average in the 200–100 mb layer similarly to the faster-moving waves, but at ½ cycle per month they tilt west in this layer as do the eastward traveling components on the average.

The slow components show considerable vertical coherence, extending in many cases from 850 to 10 mb with relatively small phase shifts. The slope of an individual harmonic is commonly consistent over several levels, especially from 850 to up 100 mb. The slow eastward motions at the top levels described previously (Figs. 1–4) are apparent in these figures as eastward-traveling components that are not coherent with similar components at lower levels.

Since the majority of the harmonics are coherent in the vertical, the variation of amplitude with height at one cycle per month in the one-month computations, as plotted in Figs. 1–4, is representative of the variation of amplitude with height of the coherent components. The amplitude is approximately constant with height in summer, and increases upwards in winter by about a factor of 3 for zonal wave-number 1, and 1½ or two times for wave-number 2. There is no evidence in these results of the presence of the slow-moving baroclinic modes that were theoretically deduced by Wiin-Nielsen (1971).

Energetics of traveling planetary-scale waves

It is well-known that for waves with westward phase speed relative to the atmosphere, as for stationary waves in the zonal westerlies, westward tilt of the waves corresponds to upward propagation of energy (Eliassen & Palm, 1961). In the troposphere, the approximately zero phase shift with height of the westward-travel-

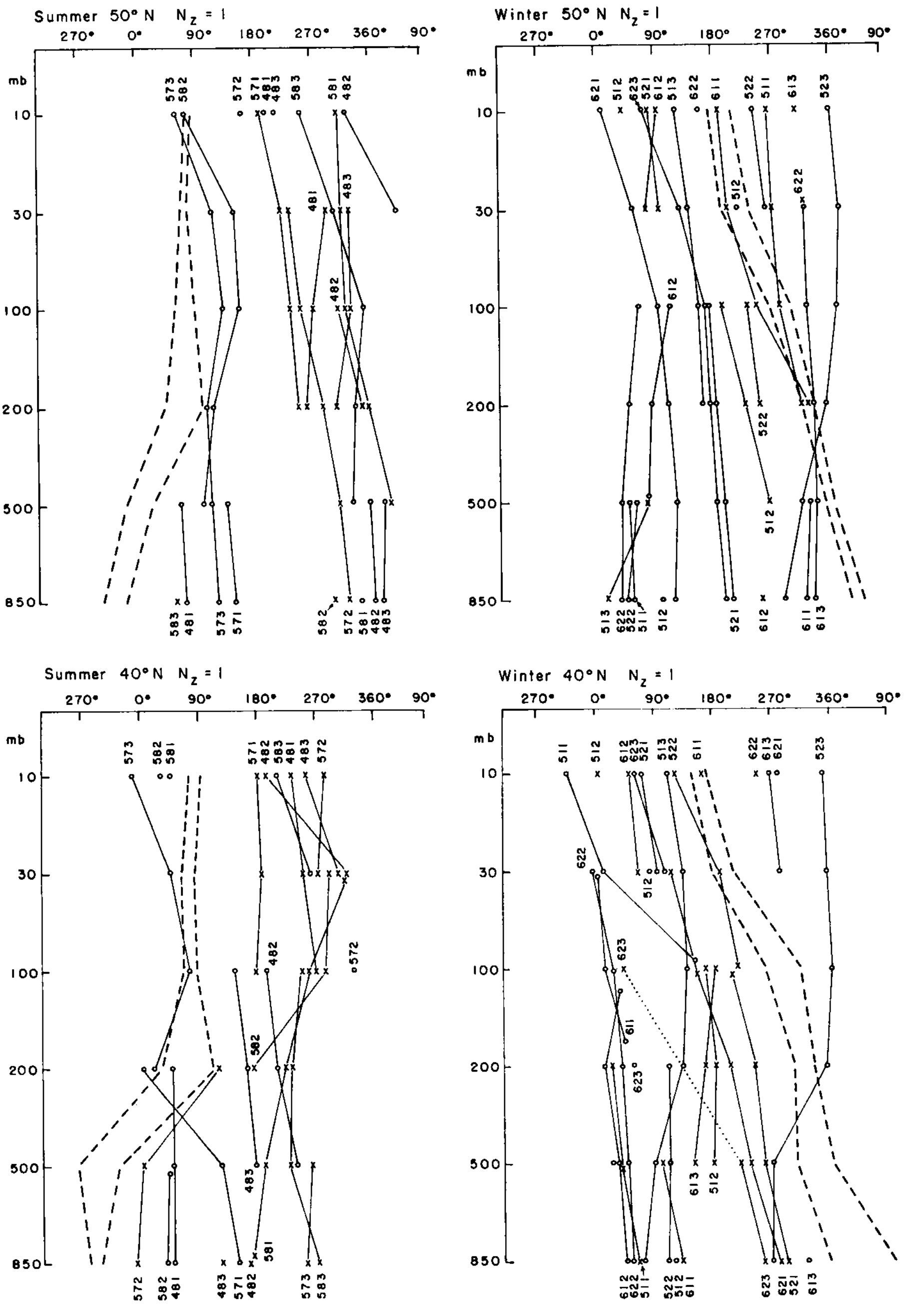

Fig. 7. Longitudinal phases of slowest harmonics, 1, 2, and 3 cycles in two months, for zonal wave-number 1, summer and winter periods at 40° and 50° N, from 850 to 10 mb. Each harmonic is identified by year, two-month period, and frequency, thus: 512 represents 1965/January (December 1964–January 1965)/2 cycles in two months; 481 represents 1964/July–August/1 cycle in two months. Circles represent westward-traveling harmonics crosses eastward.

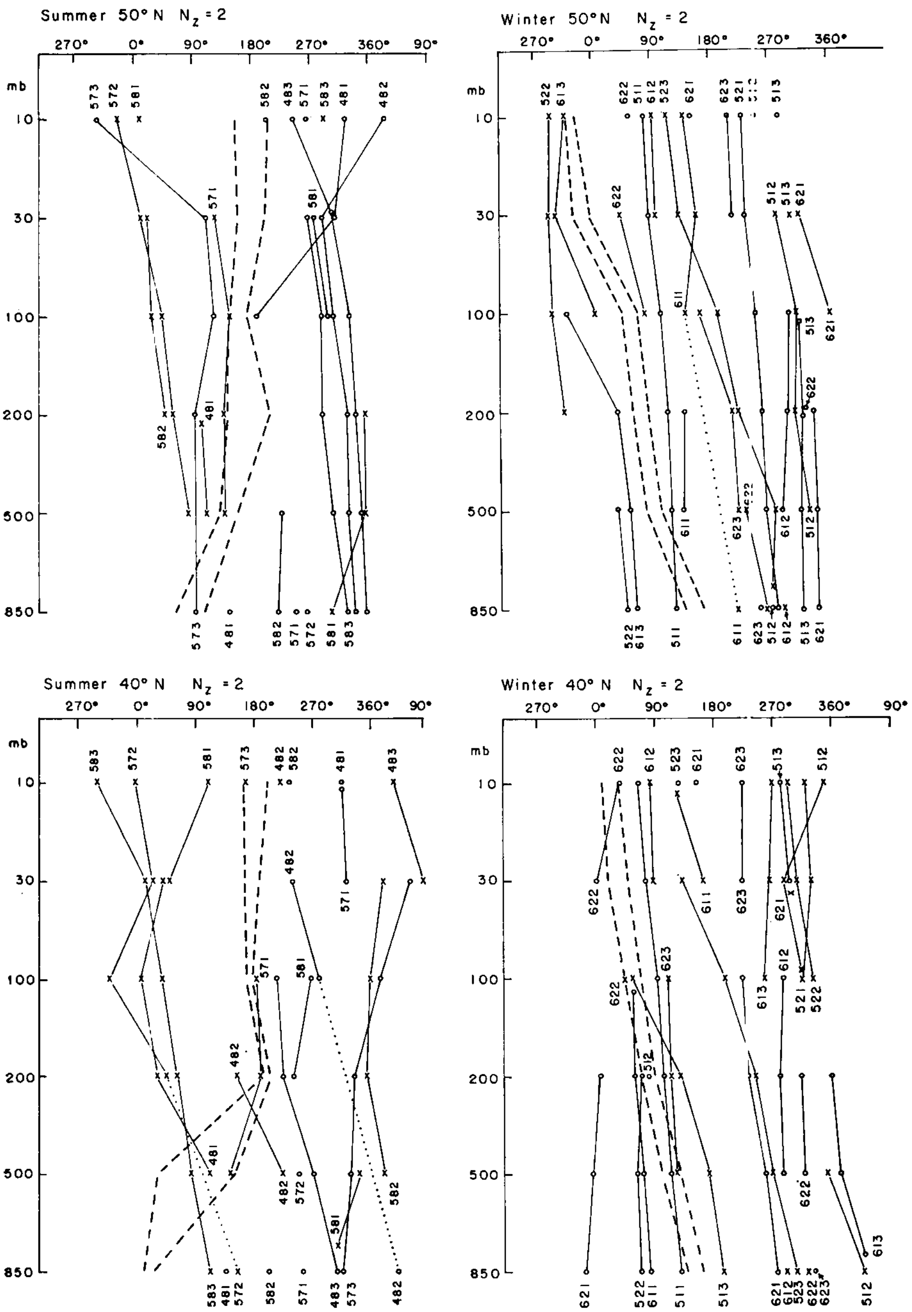

Fig. 8. Longitudinal phases of slowest harmonics, for zonal wave-number 2, otherwise as for Fig. 7.

ing components indicates that the average vertical flux of energy associated with these waves is relatively small in the troposphere. Downward energy flow to compensate for frictional or other losses at the ground may be balanced by upward energy flow due to forcing at the boundary, theoretially studied by Hirota (1971).

The westward tilt of the eastward traveling components might be associated with upward propagation of energy for the slowest components, which are traveling west relative to the atmosphere, but at the higher frequencies, greater than one or two cycles per month in the case of wave-number 1, these waves are propagating east relative to the mean flow in the troposphere, corresponding to downward energy propagation. The uniformity of the average vertical structure of the traveling components, with respect to frequency and season, suggests that the apparent eastward traveling components are not bound by energy requirements. They may result from observational deficiencies and analysis errors, in which case they might be expected to bear some resemblance to both the stationary and westward traveling components, as indeed they do. Alternatively, the eastward traveling components may be the results of applying the method of traveling wave spectral analysis to stationary waves of varying amplitude, and the dynamics of these waves probably cannot be usefully analyzed in terms of a (linear) superposition of traveling waves. In the stratosphere, the general westward tilt of the traveling components is presumably associated with upward energy propagation, as discussed by Hirota (1971) in the case of the traveling waves.

We can apply the Eliassen & Palm (1961) stationary wave theory to the traveling waves in more detail if we consider the coordinate system to move with the phase-speed of the waves, so they are regarded as stationary waves in a modified (faster westerlies for westward wave-speed) zonal wind field. Eliassen & Palm (1961), derived the following approximate (linearized "beta-plane") equation for the zonally averaged wave-energy [their equation (10.3)], in the same (standard) notation with the same meanings for all symbols.

$$(\overline{\phi v})_y + (\overline{\phi \omega})_p = -U_y \overline{uv} - U_p \overline{u\omega} + \frac{fU_p}{\sigma} \overline{v\phi_p}$$

The left hand side represents horizontal and vertical divergence of energy flux, the energy generation terms are on the right.

Let us apply the above equation in middle (or somewhat higher) latitudes in winter, where both zonal wind and wave-amplitude have maximum values. Then U_y can be approximated as zero. The eastward (wave) wind component u is also approximately zero at this latitude. We therefore conclude that the generation of wave energy can be approximated by the last term

$$C = \frac{fU_p}{\sigma} \overline{v\phi_p}$$

For reference:

f is Coriolis parameter
σ represents the static stability, defined by $\sigma = -(\theta_p/\varrho_0 \theta)$
θ is mean (equilibrium) potential temperature
v is northward component of (wave) wind speed
ϕ is geopotential
ϱ_0 is equilibrium density

The generation of wave-energy is thus proportional to the product of the zonal wind shear and the meridional eddy heat flux.

The above expression for the generation of wave-energy is independent of zonal wind, and thus of the moving coordinate system assumption.

In the 200–100 mb layer the westerlies decrease with height, so that U_p is positive. The eastward tilt of the westward traveling components, in which case $\overline{v\phi_p}$ is positive, thus corresponds to positive generation of wave-energy in this layer. Above 100 mb, up to the stratospause, the zonal wind in winter increases with height, and the generation of wave-energy is positive as pointed out by Eliassen & Palm (1961) and discussed by Dickinson (1969) in the case of the stationary waves.

The westward tilt of the eastward traveling components and the very slowest westward-traveling components ($\frac{1}{2}$ cycle per month) in the 200–100 mb layer, corresponding to energy absorption, suggests that these components are due to variable forcing from below, similarly to the stationary waves. It is in this layer that the differences are most marked between the apparently freely (for the most part, with relatively little interaction with other components) traveling planetary-scale waves, whose energy appears to be maintained directly by conversion from zonal energy, and the quasi-stationary waves whose energy derives, at least to a considerable extent, from the zonal flow through interaction with the boundary.

The time variation of wave and zonal energy associated with the traveling waves has been discussed by various authors, especially Hirota (1971). When the traveling component of a planetary wave is in phase with the quasi-stationary forced wave the total amplitude is large, and when the waves are out of phase the total amplitude is small. The resulting variation of wave energy may be supplied from the zonal flow through the interaction with the earth's topography, resulting in the observed (Hirota and Sato, 1969) negative correlation between wave-energy, for wave-number 1, and the energy of the zonal flow. Theoretical analysis of the interaction of the traveling and stationary waves with the zonal wind is difficult, for stationary conditions cannot be assumed.

Concluding remarks

The eastward slope of the westward traveling planetary-scale waves in the lower stratosphere, with its associated conversion of zonal to eddy energy (assuming the applicability of the Eliassen & Palm approximations) should require compensating generation of zonal energy by mean meridional motions (Dickinson, 1969). The traveling planetary-scale waves thus appear to make a positive contribution to, and are consistent with, the well-known (deduced) indirect circulation of the lower stratosphere in the Northern Hemisphere (Newell, 1963).

It is of interest to compare these results with the theoretical model of Hirota (1971). The observations and Hirota's theoretical results agree very closely in the stratosphere. In the troposphere, the small phase-shift with height of the observed waves indicates that they are maintained mainly by energy generation in situ, rather than by forcing from below. The observation of similar traveling waves in the Southern Hemisphere stratosphere (Deland 1973), where the smaller amplitude of the stationary waves indicates that upward forcing is much weaker than in the Northern Hemisphere, is additional evidence that the waves are, for the most part, internally generated.

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REFERENCES

Charney, J. G. & Eliassen, A. 1949. A numerical method for predicting the perturbations in the middle latitude westerlies. *Tellus 1*, 38–54.

Deland, R. J. 1964. Traveling planetary waves. *Tellus 16*, 271–273.

Deland, R. J. 1965. Some observations of the behavior of spherical harmonic waves. *Monthly Weather Review 93*, 307–312.

Deland, R. J. 1970. On the vertical structure of planetary-scale Rossby waves. *Quart. J. Royal Meteor. Soc. 96*, 756–757.

Deland, R. J. 1972*a*. On the spectral analysis of traveling waves. *J. Meteor. Soc. Japan 50*, 104–109.

Deland, R. J. 1972*b*. A note on the analysis of quasi-stationary and traveling waves. Manuscript submitted to *J. Meteor. Soc. Japan*. January 1972.

Deland, R. J. 1973. Analysis of NIMBUS III SIRS Radiance data: Traveling planetary-scale waves in the stratospheric temperature field. Manuscript submitted to *Monthly Weather Review*, April 1972.

Deland, R. J. & Johnson, K. W. 1968. A statistical study of the vertical structure of traveling planetary-scale waves. *Monthly Weather Review 97*, 12–22.

Dickinson, R. E. 1969. Vertical propagation of planetary Rossby waves through an atmosphere with Newtonian cooling. *J. Geophys. Res. 74*, 929–939.

Diiky, L. A. & Golytsyn, G. S. 1969. Calculation of the Rossby wave velocities in the Earth's atmosphere. *Tellus 20*, 314–317.

Eliasen, E. & Machenhauer, B. 1965. A study of the fluctuations of the atmospheric flow patterns. *Tellus 17*, 220–238.

Eliasen, E. & Machenhauer, B. 1969. On the observed large-scale atmospheric wave motions. *Tellus 21*, 149–166.

Eliassen, A. & Palm, E. 1961. On the transfer of energy in stationary mountain waves. *Geofyz. Publikajnoner 12*, 1–23.

Hayashi, Y. 1971. A generalized method of resolving disturbances into progressive and retrogressive waves by space Fourier and time cross-spectral analyses *J. Meteor. Soc. Japan 49*, 125–128.

Hirota, I. 1967. The vertical structure of the stratospheric sudden warming. *J. Meteor. Soc. Japan 45*, 409–421.

Hirota, I. 1971. Excitation of planetary Rossby waves in the winter stratosphere by periodic forcing. *J. Meteor. Soc. Japan 49*, 439–449.

Hirota, I. & Sato, Y. 1969. Periodic variation of the winter stratospheric circulation and intermittent vertical propagation of planetary waves. *J. Meteor. Soc. Japan 47*, 390–402.

Kubota, S. & Iida, S. 1954. Statistical characteristics of the atmospheric disturbances. *Papers in Meteorology and Geophysics* (*Tokyo University*) *5*, 22–34.

Newell, R. E. 1963. Transfer through the tropopause and within the stratosphere. *Q. J. Royal Meteor. Soc. 89*, 167–204.

Wiin-Nielsen, A. 1971. On the motion of various vertical modes of transient, very long waves. Part 1. Beta-Plane Approximation. *Tellus 23*, 87–98.

СПЕКТРАЛЬНЫЙ АНАЛИЗ ДВИЖУЩИХСЯ ПЛАНЕТАРНЫХ ВОЛН: ВЕРТИКАЛЬНАЯ СТРУКТУРА В СРЕДНИХ ШИРОТАХ СЕВЕРНОГО ПОЛУШАРИЯ

Представлены результаты анализа данных о геопотенциале за 3 года для поверхностей от 850 *мб* до 10 *мб*, использующего методы дискретного спектрального анализа движущихся волн. Описана вертикальная структура движущихся волн планетарного масштаба (зональные волновые числа 1 и 2) в средних широтах. Обсуждаются полученные из этих результатов выводы относительно вертикального распространения энергии. Волны приблизительно баротропны с относительно малыми вариациями фазы по высоте для частот от 1/2 до, приблизительно, 8 циклов в месяц.