Some features of instantaneous point source diffusion within a turbulent boundary layer

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ABSTRACT

Diffusion of helium gas from an instantaneous point source within a neutral boundary layer has been studied. Concentrations from a simulated point source, located at a fixed height of 8 inches above a smooth surface, were measured for several downstream cross-sections of the diffusing cloud. Separation of the total dispersion into two components, spreading and meandering, is obtained on the basis of Gifford's fluctuating model. The results of this analysis are used to determine the Hay-Pasquill scale parameter which relates the Lagrangian and Eulerian scales of turbulence. Meandering is shown to have a significant effect on the scale parameter.

Introduction

Atmospheric diffusion is a subject of considerable activity and research in the area of fluid dynamics and meteorology. At the present time, our knowledge of transport mechanisms is far from being complete and consequently, a great deal of faith cannot be placed in the quantitative predictions based on it. Since a great deal of practical interest centers at the diffusion in the lower atmosphere, it is significant that the lower atmospheric conditions can be simulated in a wind tunnel within the boundary layer generated near the wall.

Gifford (1959) suggested a mathematical model for diffusion from a continuous source which provides for the fluctuations of the plume to occur by separating the total plume dispersion into two components, spreading and meandering. Based on this model, he was able to deduce the various properties of the resulting material concentration distribution. Under the assumption of Gaussian distribution for the mean plume concentration, Gifford derived the following relation:

$$\frac{\text{Peak concentration}}{\text{Average concentration}} = \frac{\overline{Y^2}+\overline{D^2}}{\overline{Y^2}} \tag{1}$$

where $\overline{Y^2}$ is interpreted as the variance due to dispersion by spreading, and $\overline{D^2}$ is the variance due to meandering. Gifford further showed from theoretical reasoning that

$$\frac{\text{Peak concentration}}{\text{Average concentration}} - \begin{cases} \propto (t-t_0)^{-1}, & (t-t_0) \text{ small} \\ \simeq \text{constant}, & (t-t_0) \text{ large} \end{cases}$$

Hay & Pasquill (1959) treated the problem of relating the spread of particles released serially from a fixed point to features of turbulent flow which can be measured or estimated. Starting with Taylor's equation for steady, homogeneous turbulence, they obtained the following relation between the spread $\overline{Y^2}$ and spectrum function $F_E(n)$:

$$\overline{Y^2} = \overline{v'^2}\, T^2 \int_0^\infty F_E(n) \left[\frac{\sin\,(\pi n T/\beta)}{(\pi n T/\beta)}\right]^2 dn \tag{2}$$

where β is the ratio of the Lagrangian to the Eulerian time scales. This form of Taylor's equation displays a basic property of diffusion from a continuous point source, namely, as travel time from the source increases and the plume grows in size, the smaller turbulent eddies become increasingly less effective in further diffusion of the plume. The filter function $\sin^2 (\pi n T)/(\pi n T)^2$ is equivalent to smoothing the velocity record by an averaging period T.

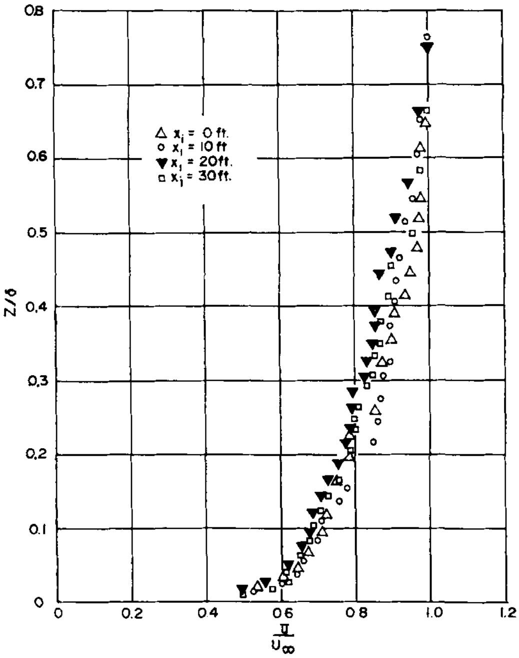

Fig. 1. Velocity profiles.

Thus, the dispersion of particles after a time of travel T from a continuous point source is determined completely by β, $\overline{v'^2}$, and the form of $F_E(n)$, and the observations of dispersion and turbulence may be used to evaluate $\bar{\beta}$.

Hay & Pasquill (1959) tested their analysis scheme on a series of 8 diffusion experiments where σ_t^2, the variance of the arc-wise tracer distribution, was measured at a travel distance of 100 m by computing the appropriate value of β for each experiment. They obtained β ranging, for most part, from 1 to 10 with an average value of 4 and further showed that the use of this average value was of significant practical interest in predicting σ_t^2.

In the present work, diffusion data were obtained by allowing helium puffs to diffuse into the air stream within a neutral boundary layer. The total dispersion was divided into the components of spreading and meandering on the basis of Gifford's fluctuating model. The results were then used to calculate the Hay–Pasquill scale parameter which relates the Lagrangian and Eulerian scales of turbulence.

It must be noted that the plume width and the general plume meandering are usually dependent upon two widely different eddy sizes. Thus, the contribution to dispersion by plume spreading, $\overline{Y^2}$, is statistically independent of that due to meandering, $\overline{D^2}$, and hence, Gifford's technique of obtaining the separation of the spreading and meandering effects does not depend either on the presence of a spectrum gap or on limitation to any special meteorological conditions. Thus, the independence of spreading and meandering is a general property of dispersion and is not restricted to the validity of the assumptions of the fluctuating plume model of Gifford.

The experimental data

The experimental data reported in the study of this paper were taken in Fluid Dynamics and Diffusion Laboratory at Colorado State University. The air entered the 6 ft × 6 ft test-section of the wind tunnel, and the free-stream velocity was maintained at 20 ft/sec for all experiments. The performance characteristics of the wind tunnel and the details of the

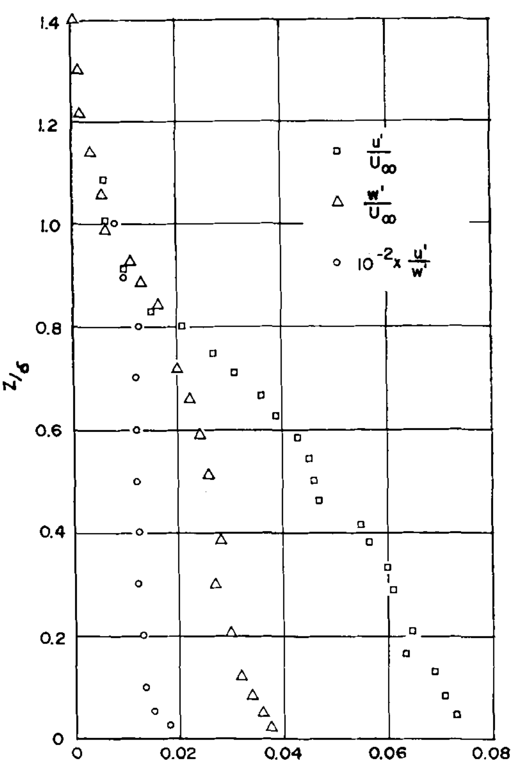

Fig. 2. Variation of turbulence parameters as a function of Z/δ.

instantaneous source and the sampler system are described in Chandra (1967). Figs. 1, 2, and 3 summarize the performance characteristics of the wind tunnel in the form of dimensionless velocity profiles, turbulence parameters, and the spectral energy distribution. Fig. 4 gives the schematic layout of the sampling method. A known volume of the helium gas was released from the source as a puff, and a leak-detector type mass spectrometer was used to measure the concentrations of the helium-air mixtures. The height (8 in) and the location of the source were same for all experiments. At the location of the source in the wind tunnel the boundary layer thickness was approximately 16 in.

The concentration data at various locations along the lateral (y) and vertical (z) directions for fixed horizontal distances of $X = 1, 2, 3, 4$ ft are presented in dimensionless form in Figs. 5 and 6. Assuming that the source behaves like a continuous source of short duration, $\bar{C}$ is the time-averaged concentration at a point (x, y, z). In Figs. 5 and 6 the concentration ratio $\bar{C}/\bar{C}_{max}$ is plotted against y/σ_y or z/σ_z, where σ_y^2 and σ_z^2 are respectively the variances of the $\bar{C}$-distribution in the y-direction for $z = 8$ in and in the z-direction for $y = 0$.

By considering the helium puff from the instantaneous source as being squeezed into a disk, Gifford's (1959) ideas can be usefully applied to diffusion for this disk and, on this

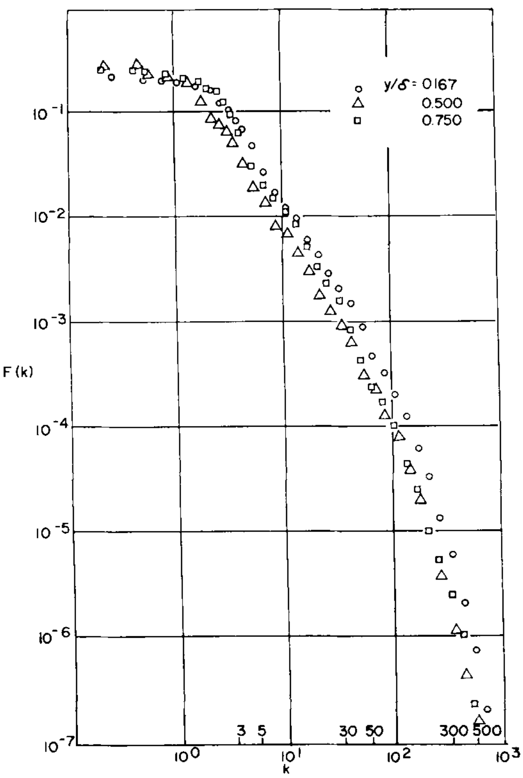

Fig. 3. Spectral energy distribution of longitudinal component.

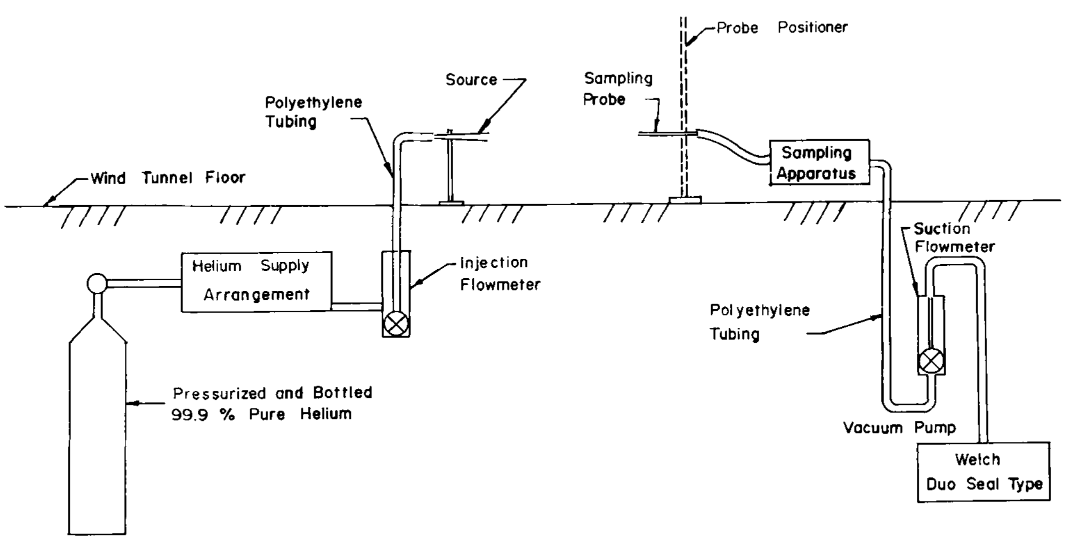

Fig. 4. Schematic layout of the sampling method.

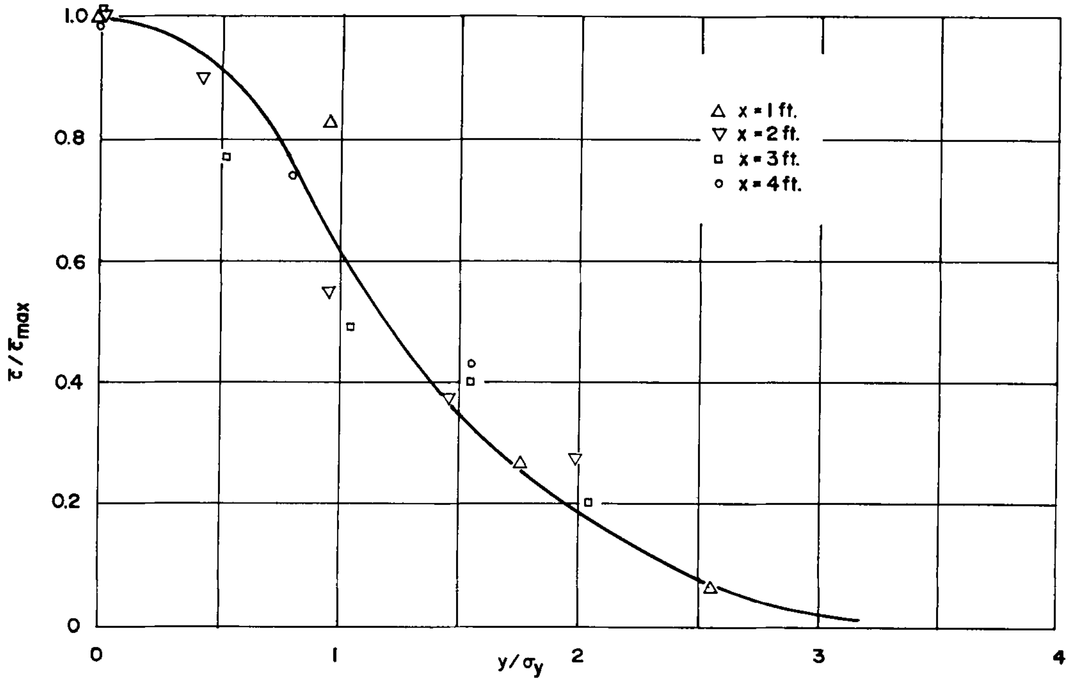

Fig. 5. Dimensionless concentration distribution in lateral direction.

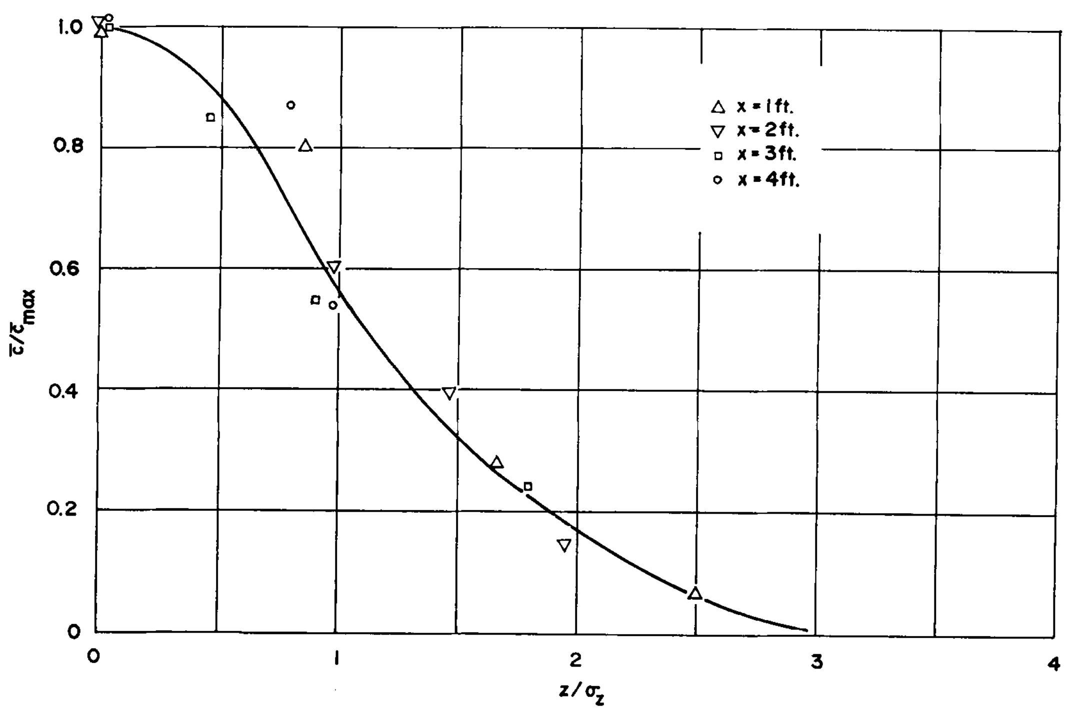

Fig. 6. Dimensionless concentration distribution in vertical direction.

basis, the components of spreading and meandering can be separated. The mean values of the spreading variance, $\overline{Y_y^2}$, and the meandering variance, $\overline{D_y^2}$, are plotted as a function of the travel distance X in Fig. 7.

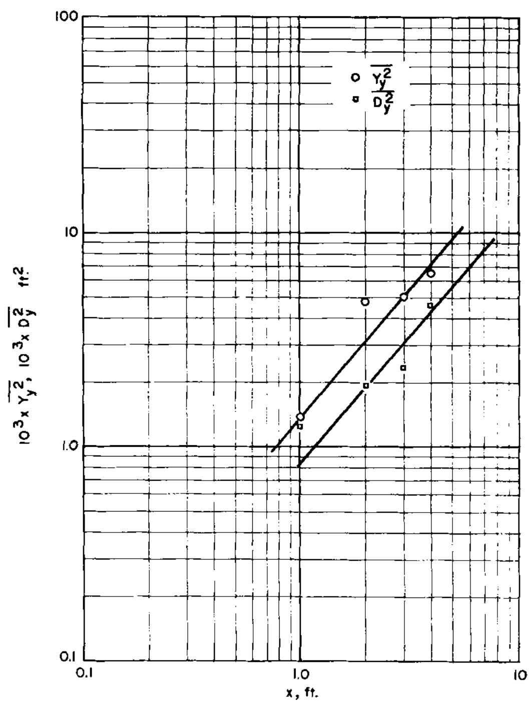

Fig. 7. Spreading and meandering variance versus travel distance.

The Hay–Pasquill scale parameter

Hay & Pasquill (1959) analyzed diffusion data from a continuous ground level point source by assuming that the Lagrangian and Eulerian correlograms have similar shapes but different scales (ratio β:1). With the help of this scale parameter β, they showed that the turbulent spread of particles can be derived from wind fluctuation records directly. Their method, based on this scale parameter β, has been applied to various wind tunnel and field data of different investigators, and predictions of particle spread have been obtained as a function of the travel distance on the basis of an average value of β. The diffusion data of the author's study were used to estimate the scale parameter β at the four stations, $X = 1, 2, 3, 4$ ft, using Eq. (2) and the values of $\overline{Y_y^2}$, $\overline{v'^2}$, dispersion time T, and the form of the spectrum function. The following results were obtained:

Station	*Scale parameter β*
(1′, 0,8″)	18.3
(2′, 0,8″)	21.1
(3′, 0,8″)	5.5
(4′, 0,8″)	3.7

If the meandering effect is also included in calculating β, the latter should be based on the combined variance for spreading and meandering ($\overline{Y_y^2} + \overline{D_y^2}$). The results in this case were:

Station	*Scale parameter β*
(1′, 0,8″)	44
(2′, 0,8″)	39
(3′, 0,8″)	12.4
(4′, 0,8″)	11.5

The above results indicate that the meandering of the diffusion cloud produces a significant increase in the values of the Hay-Pasquill scale parameter, It must be noted, however, that the assumption of similarity in shap of Eulerian add Lagrangian correlograms is a good assumption only in the case of fine scale motion which disregards any large scale meandering. By examining the results of other investigators (e.g., Frenkiel's (1949) computed values of $\overline{Y^2}$ for assumed forms of R_ξ, the Lagrangian correlation coefficient), it can be shown that a substantial change in the *shape* of the correlogram is much less important than a severalfold change in *scale*. Hence, as long as the condition of similarity in shape is valid roughly, the assumption of precise similarity is not likely to cause any significant error. Since the derivation of Eq. (2) is based on the above assumption, it would be more realistic to determ!ne the scale parameter β by using $\overline{Y_y^2}$ in Eq. (2) rather than she combined variance ($\overline{Y_y^2} + \overline{D_y^2}$). It should also be noted that, although there is variation in the β-values with travel distance, conclusions regarding any systematic pattern of this variation cannot be firmly established on the basis of the above results.

Hay & Pasquill (1959] measured the variance of the particle spread at a distance of 100 m downwind from a continuous point source, by

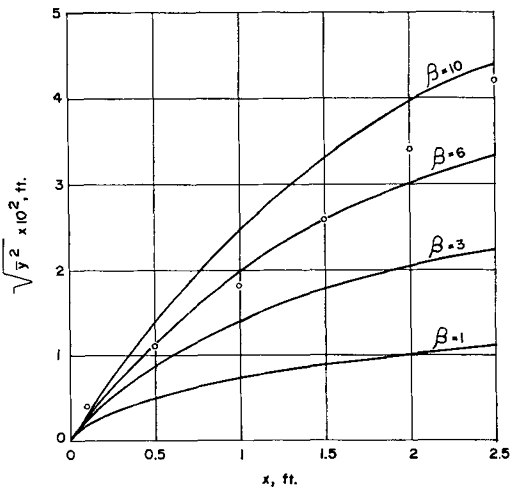

Fig. 8. Values of the Hay–Pasquill scale parameter for Mickelsen's data.

carrying out experiments over downland grass of length 1 to 2 inches, in a variety of stability conditions. Simultaneous measurements were made of the fluctuations in wind speed and direction at the source. They tested their analysis scheme, represented by Eq. (2), on a series of 8 diffusion experiments by computing the appropriate value of β for each experiment. They obtained β ranging from 1 to 10 with an average of 4, and concluded that this average value was independent of wind speed and stability and therefore was of great significance in predicting the variance of particle spread at different stations.

Baldwin and Mickelsen (1962) considered their data on diffusion of helium from a continuous point source in the center-line region of a fully turbulent pipe flow and obtained rough values of β from 4 to 18, depending on the mean flow rate. Their diffusion results were obtained within a 2-inches radius core of ambient air flowing through a commercial 8-inches diameter pipe.

Some data on a small scale of turbulence are reported by Mickelsen (1955), who measured the spread of helium injected continuously at a fixed point in the air stream. The turbulent field was generated in an 8-in diameter duct having an inlet length-to-diameter ratio of 36. The scale of turbulence was approximately 0.02 m. Using Mickelsen's spectrum data and the assumed values of β, Hay & Pasquill (1959) calculated $(\overline{Y^2})^{\frac{1}{2}}$ from Eq. (2) for stream velocity of 50 ft/sec and 100 ft/sec, and found that no single value of β gave a fit over the whole range of travel up to 2.5 ft. However, for distances up to 1 ft, i.e. up to 15 times the scale of turbulence, a value of 6 provided a close approximation at 100 ft/sec and a value of 4 was more suitable at 50 ft/sec. The β-values were roughly between 4 and 9 for the travel distances up to 2.5 ft. Fig. 8 represents the above conclusion for Mickelsen's data at 50 ft/sec.

Haugen (1966) analyzed selected Prairie Grass diffusion data to determine the scale parameter β. Thirty-five experiments were analyzed. Of these only thirteen experiments were found to give β-values between 1 and 10. In nine cases β-values were greater than 10; the maximum value of β being 160. In the remaining cases, the scale parameter was found to have values of less than 1. Haugen concluded that non-stationary conditions inherently produce nonsystematic variations of β with travel distance as well as the unexpected result of $\beta < 1$.

The results of the various field and laboratory studies of diffusion described above yield a fairly wide scatter for the values of the Hay-Pasquill scale parameter. The results for β-values, based on the present study, are no exception. The β obtained on the basis of the spreading variance $\overline{Y_y^2}$ has values which are more in agreement with the β-values of Mickelsen, Baldwin & Mickelsen, Hay & Pasquill, and Haugen. When β is calculated by taking the total plume dispersion into consideration, the β-values of the present study are higher than those of others, with the exception of Haugen whose results include β-values of 160, 32, 28, 26, 24, 21, 20 on the high side. It shows that meandering affects the β-values significantly. Both Baldwin & Mickelsen (1962), and Mickelsen (1955) conducted their experiments in a narrow duct or a circular pipe in which only a restricted degree of meandering could be produced at small distances downstream from the source. On the other hand, Haugen's high β-values might be attributed to the significant meandering effect in the atmosphere. This apparently explains why the β-values are higher when the meandering is not disregarded. The results for β from the diffusion data of the above investigators also show that β is essentially insensitive to the scale of turbulence.

In spite of the scatter in the β-values, the possibility remains that practical and useful results can be obtained since the analysis of Hay and Pasquill, based on the scale parameter β, leads to a simple method of deriving the turbulent spread of particles directly from wind fluctuation records.

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REFERENCES

Baldwin, L. V. & Mickelsen, W. R. 1962. Turbulent diffusion and anemometer measurements. *Journal of Engineering Mechanics 88*, 37–69.

Chandra, S., August 1967. *Diffusion from an instantaneous point source into a turbulent boundary layer*. Ph. D. dissertation, Colorado State University, Fort Collins, Colorado. 141 pp.

Frenkiel, F. N. 1949. *Proc. U.S. Naval Ordnance Laboratory Research Symposium*, White Oak, Maryland.

Gifford, F. 1959. Statistical properties of a fluctuating plume dispersion model. *Advances in Geophysics 6*, 117–137. Academic Press.

Haugen, D. A. 1966. Some Lagrangian properties of turbulence deduced from atmospheric diffusion experiments. *Journal of Applied Meteorology 5*, 646–652.

Hay, J. S. & Pasquill, F. 1959. Diffusion from a continuous source in relation to the spectrum and scale of turbulence. *Advances in Geophysics 6*, 345–365.

Mickelsen, W. R. 1955. *An experimental comparison of the Langrangian and Eulerian correlation coefficients in homogeneous isotropic turbulence*. National Advisory Committee for Aeronautics, TN 3570, Washington.

НЕКОТОРЫЕ ОСОБЕННОСТИ ДИФФУЗИИ ОТ МГНОВЕННОГО ТОЧЕЧНОГО ИСТОЧНИКА ВНУТРИ ТУРБУЛЕНТНОГО ПОГРАНИЧНОГО СЛОЯ

Изучается диффузия гелия от мгновенного точечного источника внутри нейтрально стратифицированного пограничного слоя. Концентрация газа, выделяемого модельным точечным источником на фиксированной высоте 8 дюймов над гладкой поверхностью, измерялась для нескольких сечений диффундирующего облака в направлении по ветру. Разделение полной дисперсии на две компоненты, расширение и меандрирование, получено на основе флуктуационной модели Гиффорда [1]. Результаты этого анализа используются для определения параметра Хэя и Паскуилла, который связывает лагранжев и эйлеров масштабы турбулентности. Показано, что меандрирование оказывает существенное влияние на величину этого параметра.