The influence of mean wind shear on the propagation of Kelvin waves[1]

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ABSTRACT

The long Kelvin waves observed in the equatorial lower stratosphere transport sufficient westerly momentum upwards to account for the westerly acceleration of the quasi-biennial oscillation, provided that this momentum is absorbed in the westerly shear zone of the mean wind. The behavior of linearized Kelvin waves as they propagate through such a shear zone is deduced by scaling the wave equations with the ratio of the latitudinal scale to the zonal scale (the aspect ratio) which is a small parameter for the observed waves. The resulting system is reduced to a single elliptic equation which is solved numerically for various configurations of the mean zonal wind.

It is found that passage through a westerly shear zone with no critical level modifies the waves as follows:

(1) Both the latitudinal and vertical scales of the waves decrease. (2) The upward momentum transport becomes concentrated towards the equator. (3) The latitudinally averaged vertical momentum transport remains nearly constant.

If the zonal wind profile contains a critical level the waves are nearly totally absorbed at that level. Furthermore, the wave amplitude has a pronounced maximum at the equator just below the critical level. For cases in which the zonal wind has lateral as well as vertical shear it was found that the Kelvin waves are effectively absorbed at the height of the critical level at the equator even when the critical level height rises rapidly away from the equator.

1. Introduction

Holton & Lindzen (1968) have shown that the linearized wave equations for the equatorial beta-plane with a constant basic state zonal flow have a special solution with the following characteristics:

1. The perturbation meridional velocity is identically zero.
2. The perturbation pressure and zonal velocity distributions are symmetric about the equator and are in geostrophic equilibrium.
3. Amplitude decays exponentially away from the equator.
4. In the longitude-height plane the structure is that of an internal gravity wave in which phase propagates eastward and downward.

An analogous solution for a barotropic atmosphere was discussed by Matsuno (1966) who called this solution an atmospheric Kelvin wave.

The meteorological importance of equatorial, long period, vertically propagating gravity waves, such as the Kelvin wave, has been stressed by Lindzen & Holton (1968) who showed that such waves can provide the momentum source necessary to explain the downward propagating quasi-biennial oscillation in the zonal winds of the equatorial stratosphere. In Lindzen & Holton's model it was assumed that the momentum of the waves is absorbed by the zonal wind at a *critical level* where the zonal wind speed equals the horizontal phase speed of the waves. Such a critical level absorption was demonstrated theoretically by Booker & Bretherton (1967) for two dimensional vertically propagating gravity waves in a nonrotating system. The theory was extended by Jones (1968) to include inertia gravity waves on a rotating plane. However, attempts to study the critical layer problem analytically for waves on an equatorial beta-plane have been unsuccessful due to the nonseparability of the resulting partial differential equation.

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In the present paper a numerical model is used to examine the propagation of long Kelvin waves in the presence of both vertical and horizontal shear of the mean zonal wind. Lindzen (1970) has used a similar model to study Kelvin waves as well as the mixed Rossby-gravity wave mode. However, his model permits only vertical shear of the mean zonal wind and assumes that the Richardson number is very large.

It can be easily shown that a true Kelvin wave (with meridional velocity *identically* zero) can not exist if there is vertical shear of the basic zonal flow. However, as will be shown below, only a small meridional velocity component is required to maintain the characteristic Kelvin wave structure of the zonal wind, temperature, and pressure oscillations when a pure Kelvin wave encounters shear of the mean wind. Such waves will be referred to here as *Kelvin Waves*, even though they don't exactly meet the criterion of zero meridional velocity.

Observational studies by Wallace & Kousky (1968) indicate that oscillations resembling Kelvin waves do exist in the equatorial stratosphere, even at levels where the vertical shear of the mean zonal wind is quite strong. These oscillations are particularly strong in the lower equatorial stratosphere during the easterly phase of the quasi-biennial oscillation, and have their maximum amplitude in the shear zone below the descending westerlies. The observed oscillations have a period on the order of 15 days an amplitude of about 10 m $\sec^{-1}$ in the zonal wind component, but no detectable meridional wind component.

Most of the energy of the observed waves appears to be in zonal wave number one. Thus, for a period of 15 days these oscillations must have a westerly phase velocity of >30 m $\sec^{-1}$ relative to the ground. Wallace and Kousky point out that these Kelvin waves carry sufficient westerly momentum upward to account for the westerly accelerations which must occur as the westerly shear zone of the quasi-biennial oscillation descends. However, the zonally averaged westerlies in the quasi-biennial oscillation rarely exceed 15 m $\sec^{-1}$. Therefore, it would appear that no critical level exists for the Kelvin waves in the lower stratosphere. It is thus of some importance to determine theoretically whether Kelvin waves require a critical level in order to exchange momentum with the mean flow.

2. A theoretical model for linearized, long Kelvin waves

It is convenient to choose a coordinate system which moves at the zonal phase speed of the waves. The perturbation equations of motion, the hydrostatic approximation, the continuity equation, and the thermodynamic energy equation may then be written as follows (asterisks denote dimensional variables):

$$\bar{u}^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial \bar{u}^*}{\partial y^*}+w^*\frac{\partial \bar{u}^*}{\partial z^*} = -\frac{\partial \phi^*}{\partial x^*}+\beta y^* v^* - \varkappa u^* \quad (1)$$

$$\bar{u}^*\frac{\partial v^*}{\partial x^*} = -\frac{\partial \phi^*}{\partial y^*} - \beta y^* u^* - \varkappa v^* \quad (2)$$

$$\frac{\partial \phi^*}{\partial z^*} = \frac{RT^*}{H} \quad (3)$$

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}+e^{z^*/H}\frac{\partial}{\partial z^*}(e^{-z^*/H}w^*)=0 \quad (4)$$

$$\bar{u}^*\frac{\partial T^*}{\partial x^*}+w^*\left(\frac{R\bar{T}^*}{c_pH}+\frac{\partial \bar{T}^*}{\partial z^*}\right)+v^*\frac{\partial \bar{T}^*}{\partial y^*} = -\varkappa T^* \quad (5)$$

Here $\bar{u}^*$ is the basic state zonal velocity relative to the phase speed of the waves; u^*, v^*, w^* are the perturbation zonal, meridional and vertical velocities respectively; ϕ^* is the perturbation geopotential, T^* is the perturbation temperature, and $\bar{T}^*$ the zonally averaged temperature. The coordinates x^*, y^*, z^* are directed eastward, northward, and upward respectively, with $z^* = -H ln\ (p/p_s)$ where H is the scale height, p is the local pressure, and p_s is a standard reference pressure. Constants included in these equations are the gas constant for dry air, R; specific heat at constant pressure, c_p; and $\beta \equiv 2\Omega/a$ where Ω is the angular velocity of the earth and a the radius of the earth. The rate coefficients for Rayleigh friction and Newtonian cooling are set equal for simplicity, and designated by $\varkappa$. Thus, dissipation is included in the model without raising the order of the differential equations.

Equations (1)–(5) may be nondimensionalized by scaling the variables as follows:

$$x^* = xk^{-1}$$
$$y^* = y(c/\beta)^{1/2}$$
$$z^* = z\lambda^{-1}$$
$$u^* = uU \quad (6)$$

$$
\begin{aligned}
v^* &= vU\delta \\
w^* &= wUk\lambda^{-1} \\
T^* &= TcU\lambda HR^{-1} \\
\phi^* &= \phi cU \\
\bar{u}^* &= c\bar{u}
\end{aligned} \tag{6}
$$

Here k is the zonal wave number and $(c/\beta)^{1/2}$ is the latitudinal half width of the Kelvin wave (Matsuno, 1966), so that $\delta \equiv k(c/\beta)^{1/2}$ is the ratio of the latitudinal scale to the zonal scale; i.e., the horizontal *aspect ratio*. Also,

$$\lambda = \left[\frac{R}{H}\left(\frac{g}{c_p} + \frac{\partial T}{\partial z}\right)\right]^{1/2} / c$$

is the vertical wavelength of a gravity wave whose horizontal phase speed is c, and U is the amplitude of the zonal wind perturbation. In the following development λ is assumed to be constant, i.e., spatial variations of the static stability are neglected.

By applying the thermal wind equation for the basic state,

$$y^* \frac{\partial \bar{u}^*}{\partial z^*} = -\frac{R}{H}\frac{\partial T^*}{\partial y^*} \tag{7}$$

the horizontal derivative of the basic state temperature may be eliminated from (5).

Substituting from (6) into (1) through (5), eliminating T^* between (3) and (5), and assuming solutions of the form

$$\begin{Bmatrix} u \\ v \\ w \\ \phi \end{Bmatrix} = \begin{Bmatrix} u' \\ v' \\ w' \\ \phi' \end{Bmatrix} e^{ix + z/2\lambda H} \tag{8}$$

leads to the following nondimensional set:

$$i(\bar{u} - iD)\, u' + v' \frac{\partial \bar{u}}{\partial y} + w' \frac{\partial \bar{u}}{\partial z} = i\phi' + yv' \tag{9}$$

$$i\delta^2(\bar{u} - iD)\, v' = -\frac{\partial \phi'}{\partial y} - yu' \tag{10}$$

$$iu' + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} - \frac{w'}{2\lambda H} = 0 \tag{11}$$

$$i(\bar{u} - iD)\left(\frac{\partial \phi'}{\partial z} + \frac{\phi'}{2\lambda H}\right) - yv' \frac{\partial \bar{u}}{\partial z} + w' = 0 \tag{12}$$

where $D \equiv \varkappa\delta^{-1}(\beta c)^{-1/2}$ is nondimensional dissipation.

For zonal wave numbers one and two, <1. Therefore, from (10), it is apparent that for long waves the zonal wind oscillation is geostrophic correct to $0(\delta^2)$ even when $\bar{u} \equiv \bar{u}(y, z)$, provided that ϕ' is *symmetric* about the equator (i.e., $(\partial\phi'/\partial y) = 0$ for $y = 0$).

Formally, the variables in (9)–(12) can be expanded in a perturbation series in δ^2. The resulting zero order equations may then be combined by eliminating u', v', and w' to give a single equation in ϕ':

$$y(y - \bar{u}_y)\,\phi'_{zz} + \phi'_{yy} + 2y\bar{u}_z\phi'_{yz} - A\phi'_y - B\phi'_z + C\phi'/(\bar{u} - iD) = 0 \tag{13}$$

where

$$
\begin{aligned}
A(y, z) &\equiv y^{-1} + S_y/S - y\bar{u}_{zz} + y\bar{u}_z S_z/S \\
B(y, z) &\equiv -2y^2\bar{u}_z\bar{u}_{zz} - y\bar{u}_{yz} + y^2\bar{u}_z^2 S_z/S + y\bar{u}_z S_y/S \\
C(y, z) &= yS_y/S - y^2\bar{u}_{zz} + y^2\bar{u}_z S_z/S
\end{aligned}
$$

and

$$S(y, z) = y - \bar{u}_y - y\bar{u}_z^2$$

Terms of order $\frac{1}{2}\lambda H$ have been neglected in (13) since they are small for motions of interest here.

The zero order velocity components may then be expressed in terms of ϕ' as follows:

$$u' = -\phi'_y/y \tag{14}$$

$$v' = i(\phi' - \bar{u}\phi_y/y - \bar{u}\bar{u}_z\phi_z)/S \tag{15}$$

$$w' = yv'\bar{u}_z - i(\bar{u} - iD)\,\phi'_z \tag{16}$$

For realistic distributions of $\bar{u}(y, z)$ in the equatorial stratosphere the discriminant of (13)

$$y^2(1 - \bar{u}_z^2 - \bar{u}_y/y) \tag{17}$$

is greater than zero. Thus (13) is elliptic and either ϕ' or its normal derivative must be specified along a closed boundary region. If $D = 0$ there is a singularity in (13) for $\bar{u} = 0$ (i.e., at the critical level). The purpose of introducing dissipation in the form of Rayleigh friction and Newtonian cooling with equal rate coefficients is primarily to remove the singularity in a simple manner without raising the order of the equations. From a mathematical point of view this type of dissipation is equivalent to adding a small negative imaginary part of $\bar{u}$ which is often done purely as an artifice to remove the singularity. Eq. (13) is also singular for $y - (\partial \bar{u}/\partial y) = 0$.

However, this condition of vanishing absolute vorticity is dynamically unstable and would not occur for the mean zonal currents in the tropical stratosphere.

3. Numerical solutions and discussion

The primary goal of this study is to deduce the behavior of linearized Kelvin waves as they propagate through a region with mean wind shear. Thus, it is appropriate to choose as the lower boundary condition a perturbation geopotential field which corresponds to a pure Kelvin wave incident from below.

The pure Kelvin wave is obtained from (9)–(12) by letting $v' \equiv 0$ and $\bar{u}$ be a negative constant (which corresponds to a westerly phase velocity). It is then easily verified that the solution corresponding to upward energy propagation (downward phase propagation) is

$$\phi' = \exp\,[y^2/2\bar{u} + iz/\bar{u}] \qquad (18)$$

In the special case $\bar{u}$ constant, $D = 0$, (18) is the solution of (13) corresponding to upward energy propagation, provided that the following boundary conditions are applied:

$$\phi'_y = 0 \quad \text{at} \quad y = 0 \qquad (19)$$

$$\phi' \to 0, \; y \to \infty$$

$$\phi' = e^{+y^2/2\bar{u}}, \; z = 0$$

For the more general situation with $\bar{u} = \bar{u}(y, z)$, a finite difference analogue to (13) with boundary conditions (19) was solved for ϕ' using the direct inversion technique discussed by Lindzen & Kuo (1969).

For most of the results reported here a grid network of 32 points in the lateral direction with spacing of 1.5° latitude, and 100 points in the vertical with spacing of 125 m or 250 m was used. The smaller vertical spacing was used for critical level cases since it appeared to be necessary to properly represent the wave amplitude near the critical level. In order to prevent spurious reflection from the upper boundary (where the condition $\phi' = 0$ was imposed) the dissipation parameter D, was allowed to increase linearly with height in the region below the top boundary for cases with no critical level. This insured that the wave energy was absorbed before reaching the top boundary.

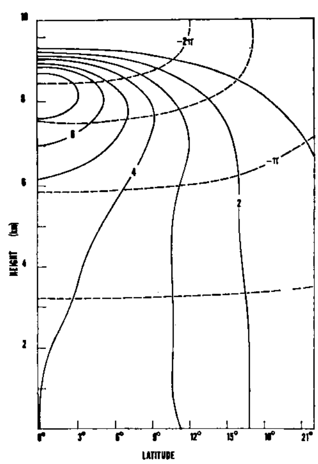

Fig. 1. Meridional cross section of the amplitude (solid lines) and phase (long dashes) of the perturbation zonal wind oscillation for Case I. Amplitude in m sec^{-1} and phase in radians. The critical level is at 9.25 km height.

For the purpose of discussion it is convenient to divide the numerical experiments into two classes:

(*a*) Cases where the mean wind profile contains a critical level; and

(*b*) Cases with no critical level.

In each class experiments were run with only vertical shear, and with both verical and horizontal shear of the mean wind.

Cases with a critical level

In Case I the zonal wind profile

$$\bar{u} = -0.4 + 0.6 \tanh (z - 3)$$

was specified, so that a critical level occurred at $z \approx 3.8$. D was set equal to 0.05, corresponding to a 40 day damping time. Fig. 1 shows a meridional cross section of the amplitude and phase of the density weighted perturbation zonal momentum oscillation for this case. Amplitude has been dimensionalized by assuming that the incident Kelvin wave at the lower boundary has a zonal wind amplitude of 4 m sec^{-1} at the equator. The vertical scale has been dimensional-

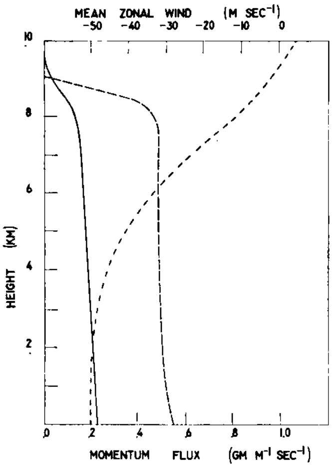

Fig. 2. Vertical profiles of the mean zonal wind (short dashes), latitudinally averaged vertical momentum flux (solid line), and vertical momentum flux at the equator (long dashes) for Case I.

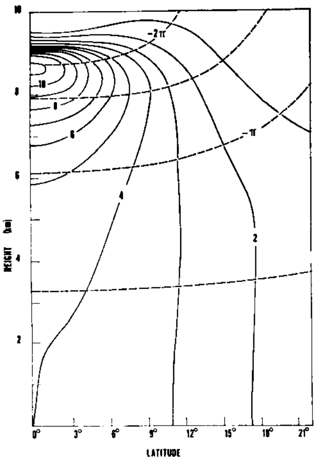

Fig. 4. Meridional cross section of the amplitude (solid lines) and phase (long dashes) of the perturbation zonal wind oscillation for Case II.

ized by letting $c = -50$ m sec^{-1}, $H = 7$ km, and assuming an isothermal atmosphere so that $\lambda^{-1} = 2.5$ km. The lower boundary $(z = 0)$ is assumed to be at the tropopause. Fig. 2 indicates the profile of $\bar{u}$ for this case. In addition the zonally averaged momentum flux, $\overline{u'w'}$, is plotted in Fig. 2. The solid line is momentum flux averaged in latitude from 0° to 30°, while the dashed line is the momentum flux profile at the equator.

It is evident from these figures that the Kelvin wave is nearly completely absorbed at the critical level. Moreover, the latitudinal half width and vertical wavelength decrease as the wave approaches the critical level. This decrease in the lateral and vertical scales is qualitatively consistent with the pure Kelvin wave solution (18) in which both scales are proportional to $\bar{u}$. Associated with the decrease in lateral scale, and consequent increase in the meridional gradient of ϕ', is a maximum in the pertubation zonal wind at the equator just below the critical level. The plotted amplitudes are scaled by the square root of density so that the maximum in perturbation wind *velocity* would be larger than the plotted values by a factor of $e^{z^*/2H}$.

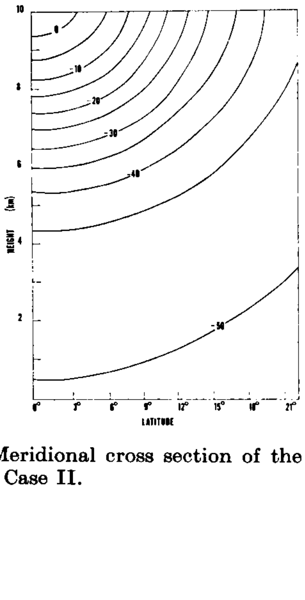

Fig. 3. Meridional cross section of the mean zonal wind for Case II.

In Case II the zonal wind profile was specified as

$$\bar{u}(y, z) = -1.0 + (0.6 + 0.6 \tanh (z - 3))e^{-y/2}$$

so that the vertical shear at the equator was the same as in Case I but decreased rapidly poleward as shown in Fig. 3. It should be recalled that $\bar{u}$ is the zonal velocity relative to the horizontal phase speed of the waves. Thus for the observed Kelvin waves with phase speeds of ~ 30 m sec^{-1}, $\bar{u} = -50$ m sec^{-1} corresponds to a zonal wind relative to the ground of -20 m sec^{-1}.

Figs. 3 and 4 contain the same information for Case II as was presented for Case I in Figs. 1 and 2 respectively. The results for these two cases are surprisingly similar despite the fact that in Case II the critical level curves upward rapidly away from the equator, and extends laterally little more than 4° from the equator.

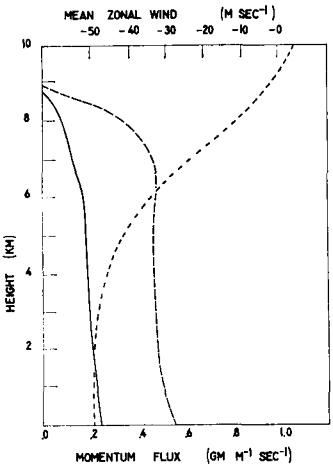

Fig. 5. Vertical profiles of the mean zonal wind at the equator (short dashes), latitudinally averaged vertical momentum flux (solid line), and vertical momentum flux at the equator (long dashes) for Case II.

Physically it appears that the tight lateral coupling imposed by geostrophy, and the consequent exponential decay of amplitude away from the equator, force the Kelvin waves to be absorbed at approximately the height of the critical level at the equator. Therefore, even a narrow westerly jet centered at the equator can nearly completely absorb the momentum of the Kelvin waves provided that a critical level exists in the jet.

Cases with no critical level

In Cases III and IV the zonal wind profile was $\bar{u} = -0.65 + 0.35 \tanh(z-3)$. For Case III the damping parameter, D, was set equal to zero below $z=6$ (i.e., 15 km). Above $z=6$ the damping increased linearly with height to a value $D=0.2$ at the top boundary. This damping layer effectively absorbed all the energy of the waves well below the upper boundary. For Case IV D was set equal to a constant value of 0.05, corresponding to a 40 day damping time. Fig. 6 shows a meridional cross section of the amplitude and phase of the density weighted perturbation zonal momentum for Case III.

The e-folding latitude for the perturbation amplitude ranges from $\sim 21°$ below the shear layer to $\sim 12°$ above the shear layer, which is consistent with the $\bar{u}^{1/2}$ dependence of the lateral scale for a pure Kelvin wave given by (18).

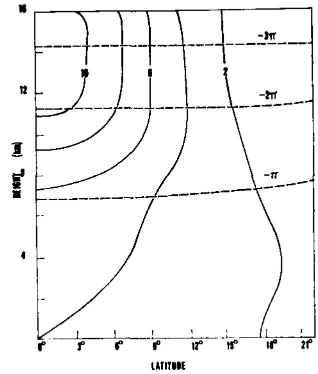

Fig. 6. Meridional cross section of the amplitude (solid lines) and phase (long dashes) of the perturbation zonal wind oscillation for Case III. The viscous damping layer above 16 km is omitted from the figure.

Fig. 7 contains vertical profiles of the mean zonal wind, the latitudinally averaged momentum flux, and the flux at the equator. The latitudinally averaged momentum flux is approximately constant with height in the dissipation free region below 15 km. The smalll oscillation in the flux profile in the shear zone may be only an artifice of the constraints in the model, rather than a real effect. However, the momentum flux at the equator almost doubles between the base and top of the shear zone. Thus, the shear zone is an adjustment region in which the Kelvin wave narrows in lateral extent,

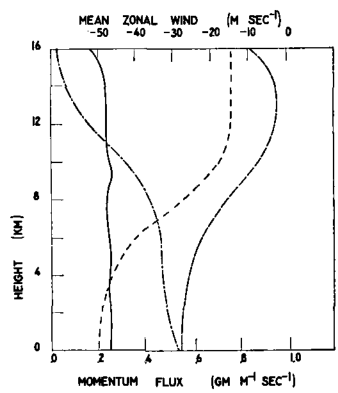

Fig. 7. Vertical profile of the mean zonal wind (short dashes), latitudinally averaged vertical momentum flux (solid line), and vertical momentum flux at the equator (long dashes) for Case III. Also shown is the vertical momentum flux at the equator for Case IV (short-long dashes).

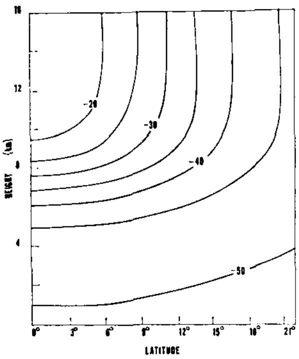

Fig. 8. Meridional cross section of the mean zonal wind for Case V.

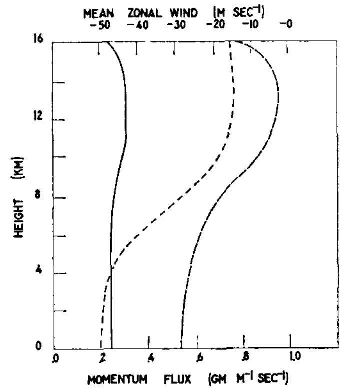

Fig. 10. Vertical profile of the mean zonal wind (short dashes), latitudinally averaged vertical momentum flux (solid line), and vertical momentum flux at the equator (long dashes) for Case V.

and decreases in vertical wavelength, while maintaining a nearly constant net vertical momentum flux. Using (14) and (15) the horizontal eddy momentum flux $\overline{v'u'}$ associated with the small meridional motion in the shear zone may be calculated. This flux turns out to nearly balance the divergence of $\overline{w'u'}$ near the equator, so that

$$\frac{\partial}{\partial y}\overline{(u'v')} + \frac{\partial}{\partial z}\overline{(u'w')} \approx 0$$

which is equivalent to the energy equation derived by Eliassen & Palm (1960) for the special case of negligible meridional eddy heat flux.

The momentum flux profile at the equator for Case IV which is also plotted in Fig. 7 indicates that even a moderate damping rate is quite effective in dissipating the waves above the shear layer where the doppler shifted period is quite long. Thus, even in the absence of a critical level, it is unlikely that Kelvin waves would be observed to propagate very far into the stratospheric westerlies since Newtonian cooling should damp out such short vertical wavelength waves very effectively.

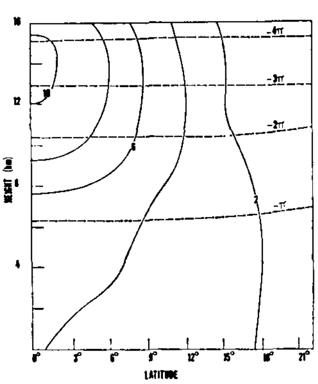

Fig. 9. Meridional cross section of the amplitude (solid lines) and phase (long dashes) of the perturbation zonal wind oscillation for Case V.

Finally, the profile

$$\bar{u}(y, z) = -0.1 + [0.35 + 0.35 \tanh(z-3)]e^{-y^2}$$

shown in Fig. 8 together with the profile of D for Case III was specified as Case V. This case bears the same relationship to Case III as Case II bears to Case I. Again the results obtained with the inclusion of lateral shear suggest that the value of $\bar{u}$ at the equator is the crucial parameter. Only two significant differences between Cases III and V are readily apparent: (*a*) the vertical wavelength is longer for Case V, and (*b*) in Case V the average momentum flux actually increases as the wave passes through the shear zone. However, in the real atmosphere this effect would probably be completely masked by dissipation due to radiative damping.

In conclusion, the results of this study indicate that linearized Kelvin waves interact with the mean flow almost like ordinary two dimensional gravity waves. In the absence of dissipation, the waves are absorbed only at a critical level. Thus, it appears that a linear wave theory can not completely explain the observations of Wallace and Kousky discussed in Section 1. However, order of magnitude scaling indicates that nonlinear advection can not be neglected for the observed waves. Thus the momentum exchange between the Kelvin waves and mean zonal wind is very likely due to nonlinear distortion of the waves as they propagate into the westerly shear zone.

Acknowledgments

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ВЛИЯНИЕ СДВИГА СКОРОСТИ СРЕДНЕГО ВЕТРА НА РАСПРОСТРАНЕНИЕ ВОЛН КЕЛЬВИНА

Длинные волны Кельвина, наблюденные в нижней экваториальной стратосфере, переносят вверх достаточно направленного на восток количества движения, чтобы привести к ускорению к востоку квазидвухгодичных колебаний, при условии, что это количество движения поглащается в зоне широтного градиента западного ветра. Поведение линеаризованных волн Кельвина по мере их распространения через такую зону сдвига скорости ветра выводится путем оценки порядков величин членов волнового уравнения с учетом того, что отношение масштаба по долготе к масштабу в зональном направлении является малым параметром для наблюдавшихся волн. В результате система уравнений сводится к одному эллиптическому уравнению, которое решается численно для различных конфигураций среднего зонального ветра. Найдено, что прохождение через зону сдвига западного ветра без критического уровня модифицирует волны следующим образом:

1) уменьшаются как широтный, так и вертикальный масштабы волн;

2) перенос количества движения вверх концентрируется к экватору;

3) осредненный по широтам перенос вертикального количества движения остается почти постоянным. Если профиль зонального ветра имеет критический уровень, то волны почти полностью поглощаются на этом уровне. Далее, амплитуда волны имеет значительный максимум на экваторе сразу ниже критического уровня. Для случаев, когда зональный ветер имеет сдвиг как по широте, так и по высоте было найдено, что волны Кельвина эффективно поглощаются на высоте критического уровня на экваторе, даже если высота критического уровня быстро растет по мере удаления от экватора.