On the prediction of mean monthly ocean temperatures

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ABSTRACT

The conservation of thermal energy equation is used as a basis for predicting month-to-month changes in surface ocean temperature. The equation includes horizontal advection of heat by mean ocean currents and by turbulent eddies, as well as heating by radiation, evaporation and turbulent vertical transport of sensible heat. The numerical experiments show that the predictions have skill. A comparative study is undertaken of the influence of the different factors that enter in the prediction, and a method is developed to test different options of the parameters.

1. Introduction

In previous numerical studies on temperature prediction (Adem 1963, 1964, 1965) an equation of the following type has been used for the upper layer of the oceans:

$$\bar{H}_s \frac{\partial \bar{T}_s}{\partial t} = \bar{E}_s - \bar{G}_2 - \bar{G}_3 \qquad (1)$$

where the variables with a bar are average values over a given time interval, T_s is the surface ocean temperature; $H_s = c\varrho_s c_s h$; ϱ_s, a constant density; c_s a constant specific heat and h, the depth of the layer. c is a fractional constant which was taken as one half, and in the most recent experiments as one; E_s, the radiation energy absorbed by the layer; G_2, the sensible heat lost by turbulent vertical transport at the surface and G_3, the heat lost by evaporation. In equation (1) the horizontal transport of thermal energy is neglected as well as the vertical transport at the bottom of the layer.

Equation (1) has been used, together with the conservation of thermal energy in the lower troposphere and with adequate parameterizations of the heating functions, to compute the surface temperature in the oceans and continents and the mean temperature in the troposphere.

Evaluations of 30-day predictions of temperature, using this thermodynamic model, have been reported recently (Adem & Jacob, 1968; Adem, 1969), and show encouraging skill. The best skill of the model seems to be in the prediction of month-to-month changes of ocean temperature (Adem, 1969).

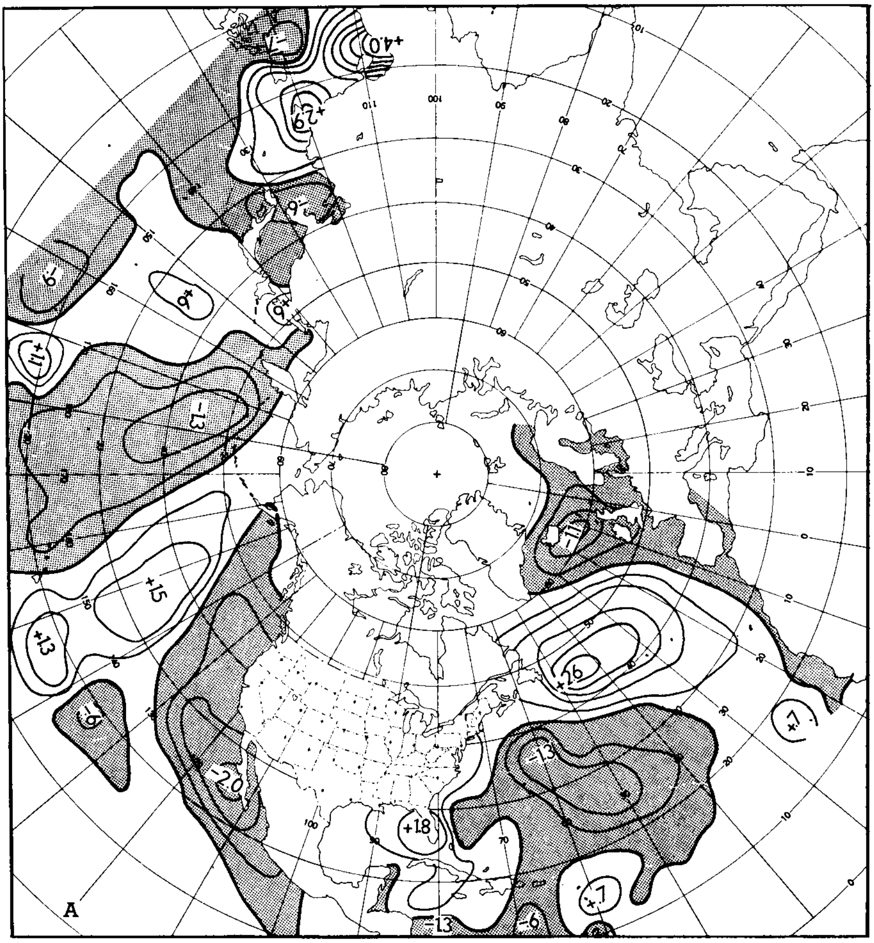
The predictions depend strongly on the sea-air interactions; and one of the most important factors is the storage of energy in the oceans, which give to the predictions a strong dependance on the initial surface ocean temperature.

The ocean temperature is a predicted variable which has emerged from the experience gained by the extensive numerical experiments as a primary variable which influences the surface temperature in the continents and the mid-tropospheric temperature in the whole region of integration. Furthermore, despite the crudness of the ocean model the skill in predicting the ocean temperature is very encouraging and open to improvement.

The main purpose of the present investigation is to develop an ocean temperature prediction model in which, besides the heating processes included in equation (1), the transport of heat by ocean currents is taken into account.

2. The thermal energy equation applied to the upper layer of the oceans

The basis for the prediction method presented in this paper is the first law of thermodynamics, which, when applied to a unit volume of water, in a way similar to that given by Miller (1950), can be written



$$\varrho_s c_s \left[\frac{\partial \overline{T}_s^*}{\partial t} + \nabla \cdot \overline{\mathbf{v}}_{s\,T}^* \overline{T}_s^* + \frac{\partial}{\partial z}(\overline{w}\,\overline{T}_s^*) + \nabla \cdot \overline{\mathbf{v}_{s\,T}^{*\prime} T_s^{*\prime}} + \frac{\partial}{\partial z}\overline{(w' T_s^{*\prime})} \right] = \overline{E}_s^* - \overline{G}_3^* + \overline{R} \qquad (2)$$

where the variables with prime are the departures from the time-average values; ∇, the two-dimensional gradient operator; z, the downward vertical coordinate; T_s^*, the temperature; $\mathbf{v}_{s\,T}^*$, the horizontal velocity vector; w, the vertical component of the velocity; E_s^*, the rate at which thermal energy is added by radiation; G_3^*, the rate at which heat is subtracted by evaporation; and R, the rate at which heat is added by compression, friction and molecular conduction.

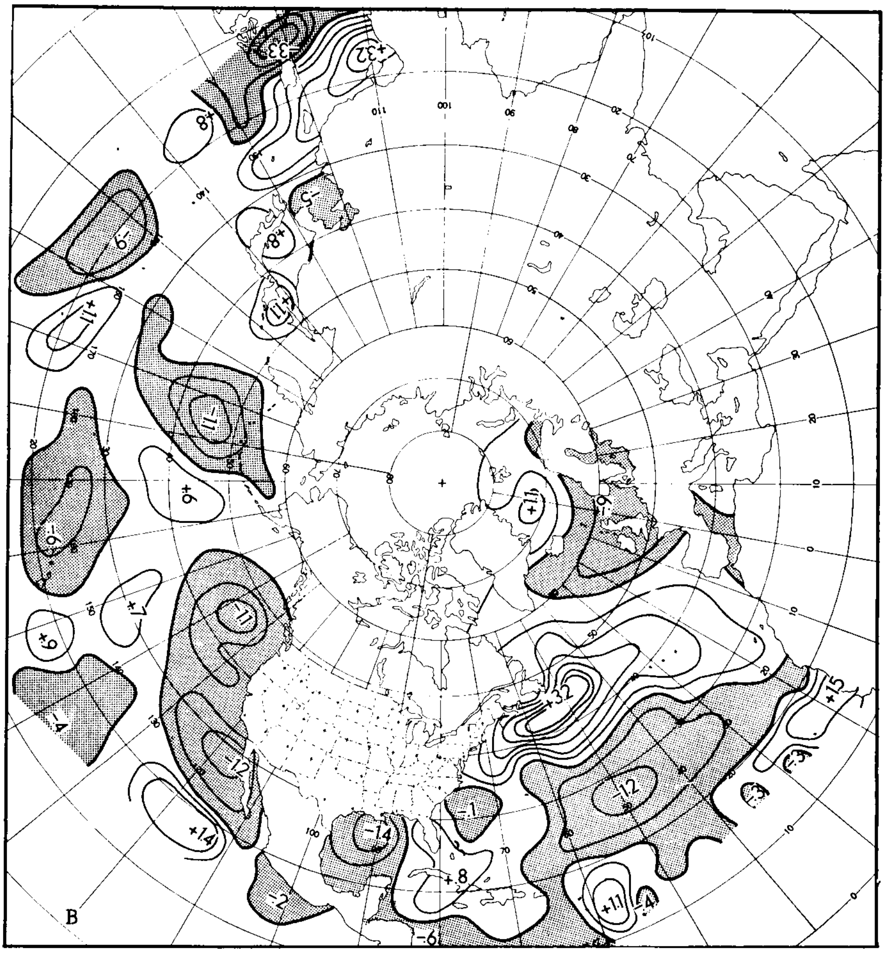
We shall integrate (2) vertically over the upper layer of the oceans to the depth $\overline{h}$. In this layer we will make the following assumptions;

(*a*) The layer is wind stirred, and therefore the temperature is assumed to be independent of depth. However, at the depth $\overline{h}$ a discontinuity in the gradient of temperature exists, which represents the beginning of the thermocline. Therefore

$$\frac{\partial \overline{T}_s^*}{\partial z} = 0 \quad \text{for} \quad z < \overline{h}$$

$$\frac{\partial \overline{T}_s^*}{\partial z} \neq 0 \quad \text{at} \quad z = \overline{h}$$

(*b*) The averaged variables satisfy the continuity equation, i.e.



$$\nabla \cdot \bar{\mathbf{v}}^*_{sT} + \frac{\partial \bar{w}}{\partial z} = 0 \tag{3}$$

Integrating (3) from the surface ($z=0$) to $\bar{h}$ we obtain

$$\nabla \cdot \bar{\mathbf{v}}_{sT} = -\bar{w}_h \tag{4}$$

where $\bar{w}_h$ is the vertical velocity at $z=\bar{h}$, and $\mathbf{v}_{sT}$ the mean horizontal velocity in the layer; and where we have used the condition that $(w)_{z=0}=0$.

(*c*) The horizontal turbulent transport term is parameterized using an "Austausch coefficient, K_s, i.e.

$$\overline{\mathbf{v}_s^{*\prime} T_s^{*\prime}} = -K_s \nabla \bar{T}_s^*$$

(*d*) Of the heating components on the right hand side of equation (2), the term $\bar{R}$ is believed to be negligibly small compared with the other terms, therefore we will assume that $\bar{R}=0$.

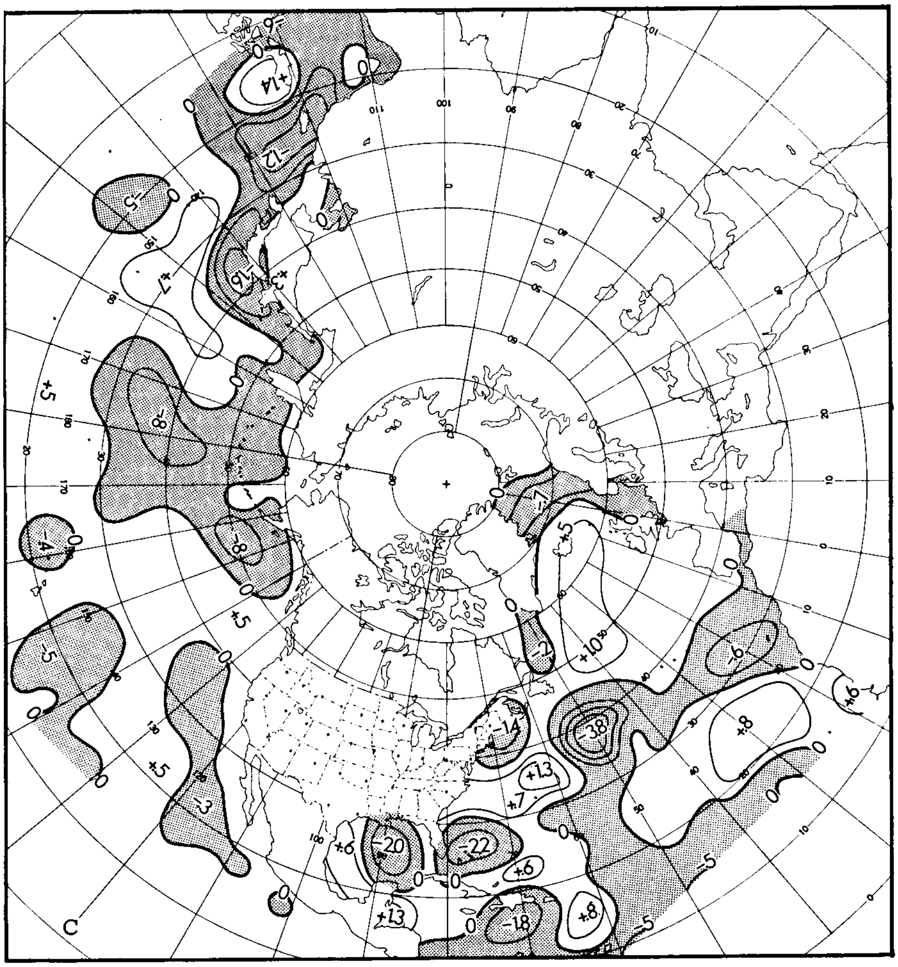
Integrating (2) from zero to $\bar{h}$ and using assumptions (*a*) (*c*), (*d*), and equation (4) of (*b*) we obtain:

$$\bar{h}\left[\frac{\partial \bar{T}_s}{\partial t} + \bar{\mathbf{v}}_{sT} \cdot \nabla \bar{T}_s - K_s \nabla^2 \bar{T}_s\right] + \bar{W} = \frac{1}{\varrho_s c_s}(\bar{E}_s - \bar{G}_3 - \bar{G}_2) \tag{5}$$

where $\bar{T}_s$ is the surface temperature,

$$\bar{\mathbf{v}}_{sT} = \frac{1}{\bar{h}}\int_0^{\bar{h}} \bar{\mathbf{v}}^*_{sT}\, dz, \quad \bar{E}_s = \int_0^{\bar{h}} \bar{E}^*_s\, dz,$$

$$\bar{G}_3 = \int_0^{\bar{h}} \bar{G}^*_3\, dz$$



and where $\bar{G}_2 = -(\overline{w' T_s^{*'}})_{z=0}$

is the sensible heat given off to the atmosphere by vertical turbulent transport.

The term $\overline{W}$, which is given by

$$\overline{W} = (\overline{w' T_s^{*'}})_{z=\bar{h}} \tag{6}$$

represents the heat given off to the thermocline.

The evaluation of $\overline{W}$ has many uncertainties. However, to estimate its order of magnitude we shall follow Wyrtki (1961), and assume that in the thermocline, equation (2) becomes

$$\frac{\partial}{\partial z}(\bar{w}\,\overline{T_s^*}) + \frac{\partial}{\partial z}(\overline{w' T_s^{*'}}) = 0 \tag{7}$$

therefore within the thermocline

$$\bar{w}\,\overline{T_s^*} + \overline{w' T_s^{*'}} = F(x, y) \tag{8}$$

where F is a function of the horizontal coordinates x and y.

Furthermore we assume that at the bottom of the thermocline $\overline{w' T_s^{*'}} = 0$. Therefore

$$F(x, y) = \bar{w}_B \overline{T}_B \tag{9}$$

where T_B is the temperature at the bottom of the thermocline; and w_B and T_B are functions of x and y only.

Applying (8) at $z = \bar{h}$, using (9) and (6) and assuming that $\bar{w}_B = \bar{w}_h$ we obtain

$$\overline{W} = -\bar{w}_h(\overline{T}_s - \overline{T}_B) \tag{10}$$

which is the expression obtained by Wyrtki (1961).

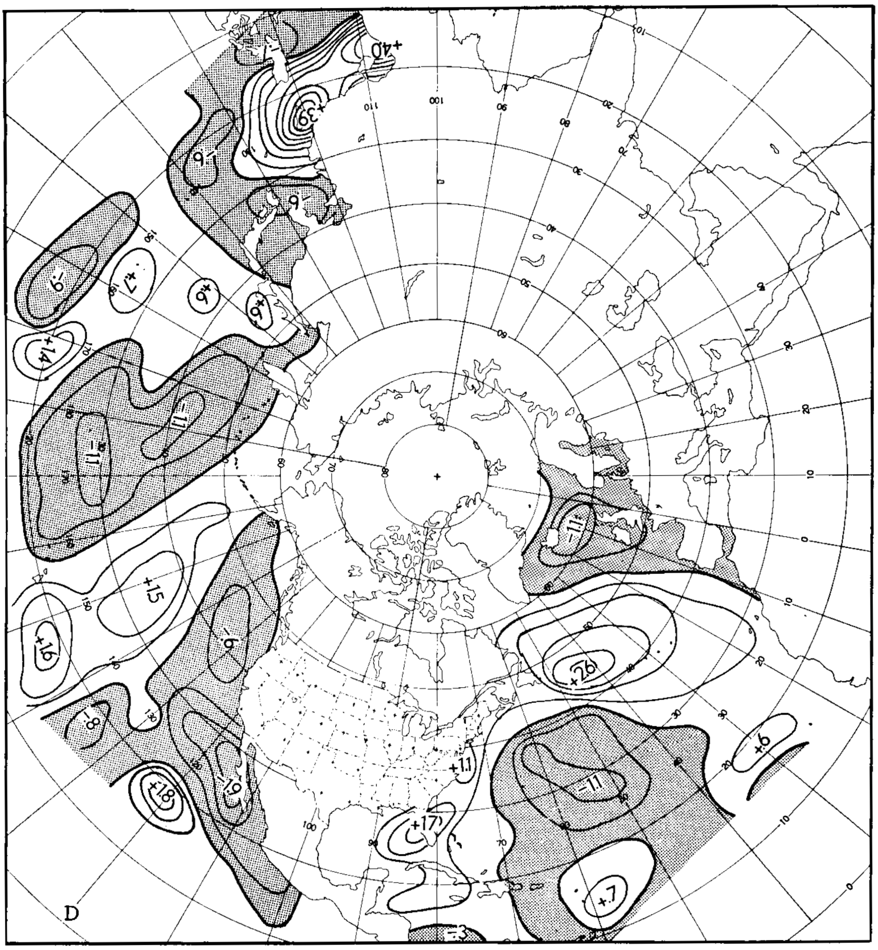


Fig. 1. Predicted changes of the anomalies of surface ocean temperatures, in Celsius degrees, from December 1967 to January 1968 based on horizontal advection by mean ocean currents: Parts *A* and *B* show the changes due to anomalies in pure drift currents (AD_1) which are computed using formula (20) with $\theta = 45^\circ$ and $\theta = 90^\circ$ respectively; *C* shows the changes computed using observed winter normal ocean currents (AD_2); and *D*, computed as *A*, but using normal ocean surface temperatures for December instead of the observed December 1967 values.

In the remaining part of the paper we will deal only with time-averaged variables, which for the sake of simplicity will be denoted without a bar.

3. Advection by mean ocean currents

For the total ocean current $\mathbf{v}_{s_T}^*$, we shall assume, following Arthur (1966), that

$$\mathbf{v}_{s_T}^* = \mathbf{v}_{s_w}^* + (\mathbf{v}_s^* - \mathbf{v}_{s_N}^*) \tag{11}$$

where $\mathbf{v}_{s_w}^*$ is the normal[1] seasonal ocean current, $\mathbf{v}_s^*$ is the pure drift current and $\mathbf{v}_{s_N}^*$ is the corresponding normal pure drift current.

To evaluate $\mathbf{v}_s^*$ we shall use Ekman's formulas for a pure drift current which are the following (Ekman, 1902; see also Neumann & Pierson, 1966, p. 210):

$$u_s^* = V_0\, e^{-(\pi/h_1)z} \cos\left(45^\circ - \frac{\pi}{h_1} z\right) \tag{12}$$

$$v_s^* = V_0\, e^{-(\pi h_1/\, z} \sin\left(45^\circ - \frac{\pi}{h_1} z\right) \tag{13}$$

[1] Normal values are defined as long-term monthly means at each geographical point.

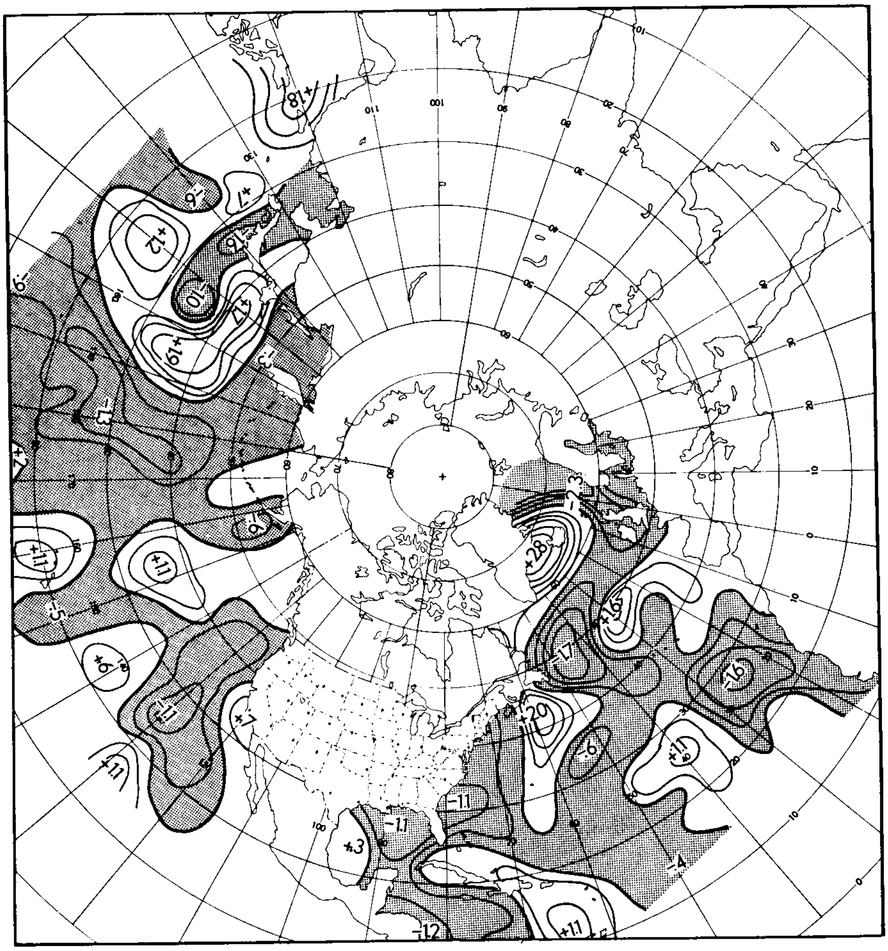


Fig. 2. Observed changes of surface ocean temperatures anomalies, in Celsius degrees, from December 1967 to January 1968.

where u_s^* and v_s^* are the x and y components of the ocean current, z is the depth; and the surface wind direction is along the positive y-axes.

Thorade (1914) has determined empirically the values of the depth of "frictional influence" h_1 and of the surface current speed, V_0. According to him (see also Neumann & Pierson, 1966, p. 210):

$$V_0 = \frac{0.0259\sqrt{|\mathbf{v}_a|}}{\sqrt{\sin\phi}} \quad \text{for} \quad |\mathbf{v}_a| \leqslant 6 \tag{14}$$

$$V_0 = \frac{0.0126|\mathbf{v}_a|}{\sqrt{\sin\phi}} \quad \text{for} \quad |\mathbf{v}_a| > 6 \tag{15}$$

$$h_1 = \frac{3.67\sqrt{|\mathbf{v}_a|^3}}{\sqrt{\sin\phi}} \quad \text{for} \quad |\mathbf{v}_a| \leqslant 6 \tag{16}$$

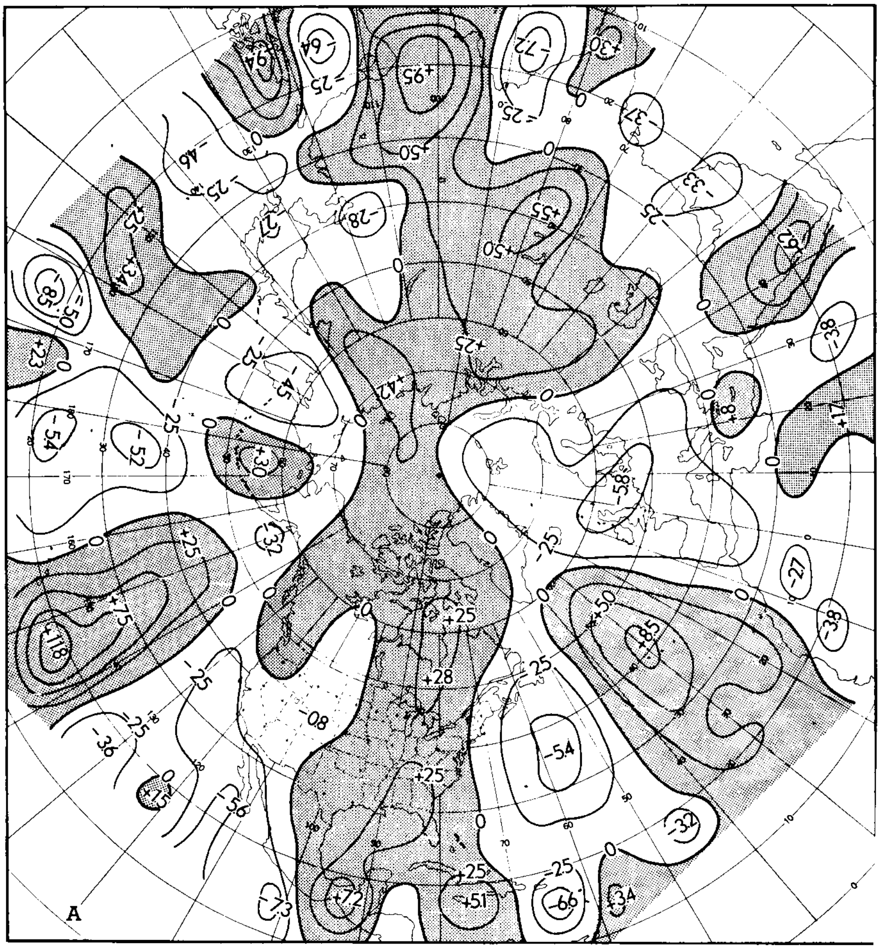
$$h_1 = \frac{7.6|\mathbf{v}_a|}{\sqrt{\sin\phi}} \quad \text{for} \quad |\mathbf{v}_a| > 6 \tag{17}$$

where V_0 is in m sec^{-1}, h_1 is in meters; $\mathbf{v}_a$ is the surface wind vector and $|\mathbf{v}_a|$ is the surface wind speed in m sec^{-1}.

The normal drift current $\mathbf{v}_{sN}^*$ is computed from the normal values of the surface wind, also using the above formulas.

From formulas (12) and (13) it follows that at the surface the vector pure drift current is directed 45° to the right of the wind direction and from (15) it follows that, for $|\mathbf{v}_a| > 6$ m/sec, its magnitude, V_0, is proportional to the surface wind speed and inversely proportional to the square root of the sine of the latitude.

The resultant current in the layer of fric-



tional influence (of depth h_1) is directed 90° to the right of the direction of the surface wind and its magnitude is equal to 0.225 V_0.

Due to the simplifying assumptions with which formulas (12) and (13) were derived, and the approximate nature of formulas (14), (15), (16) and (17), we should use these formulas only as a guide for constructing a numerical model. The best choice of some of the coefficients or parameters appearing in the formulas may then be obtained from the results of the numerical experiments.

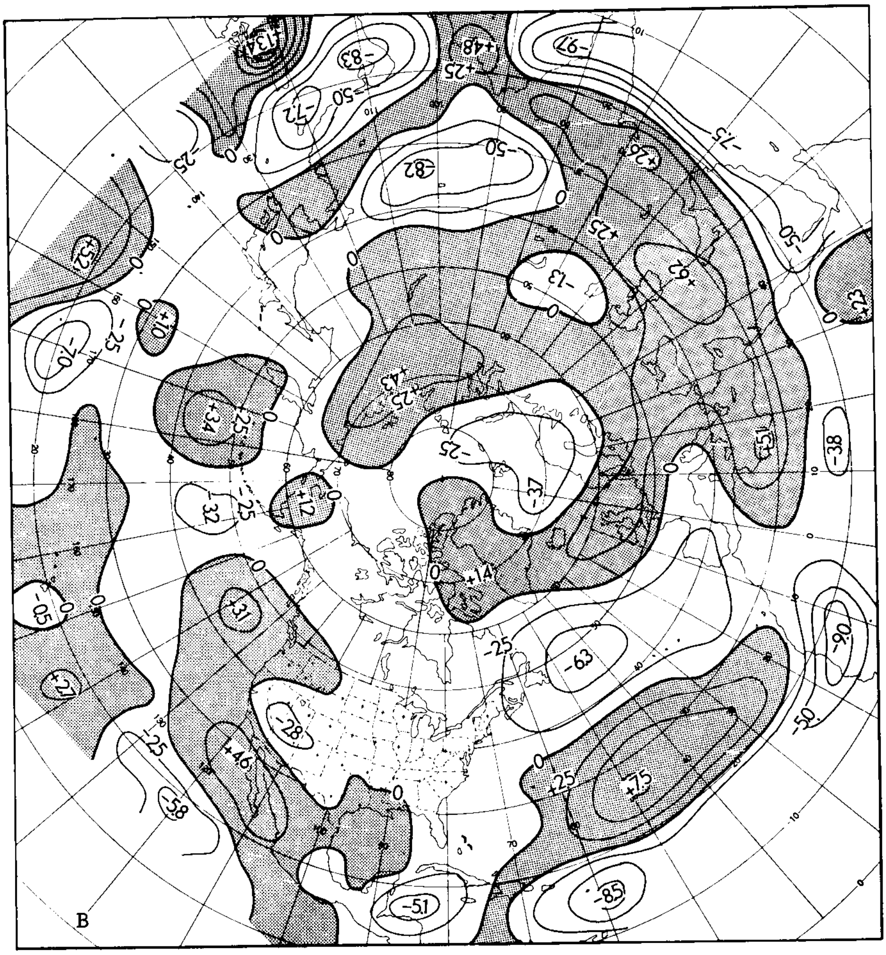
We shall consider, therefore, the following parameters:

The angle that measures the direction of the vector surface ocean current to the right of the surface wind direction, which will be denoted by θ; and the depth of the considered layer h, which is not necessarily equal to h_1. We shall also include as parameter a coefficient C_1 in the magnitude of the ocean current. The components of the ocean current will therefore be expressed by

$$u_s = C_1 \frac{0.0126}{\sqrt{\sin \varphi}} (u_a \cos \theta + v_a \sin \theta) \qquad (18)$$

$$v_s = C_1 \frac{0.0126}{\sqrt{\sin \varphi}} (v_a \cos \theta - u_a \sin \theta) \qquad (19)$$

where the directions of the coordinate axis are arbitrarily chosen; u_s and v_s are the x and y components respectively, of the resultant pure drift current in the layer of depth h; and u_a and v_a are the x and y components of the surface wind respectively.



The range of values of θ and C_1 are arbitrarily limited to

$$45^\circ \leqslant \theta \leqslant 90^\circ$$

$$1 \leqslant C_1 \leqslant 0.225$$

For $\theta = 45^\circ$ and $C_1 = 1$ we have the resultant pure drift surface current in a very shallow layer. For $\theta = 90^\circ$ and $C_1 = 0.225$ we have the resultant pure drift current in the whole frictional layer of depth $h = h_1$.

Namias (1959) and more recently Eber (1961) and Jacob (1967) have made estimates of changes in ocean temperature anomalies, due to advection by mean ocean currents. The relations that they use to evaluate the ocean currents correspond to the case $\theta = 45^\circ$ and $C_1 = 1$.

Using formulas (18) and (19) and assuming geostrophic wind, that part of the horizontal advection term in equation (5) due to the anomalies in the pure drift currents becomes:

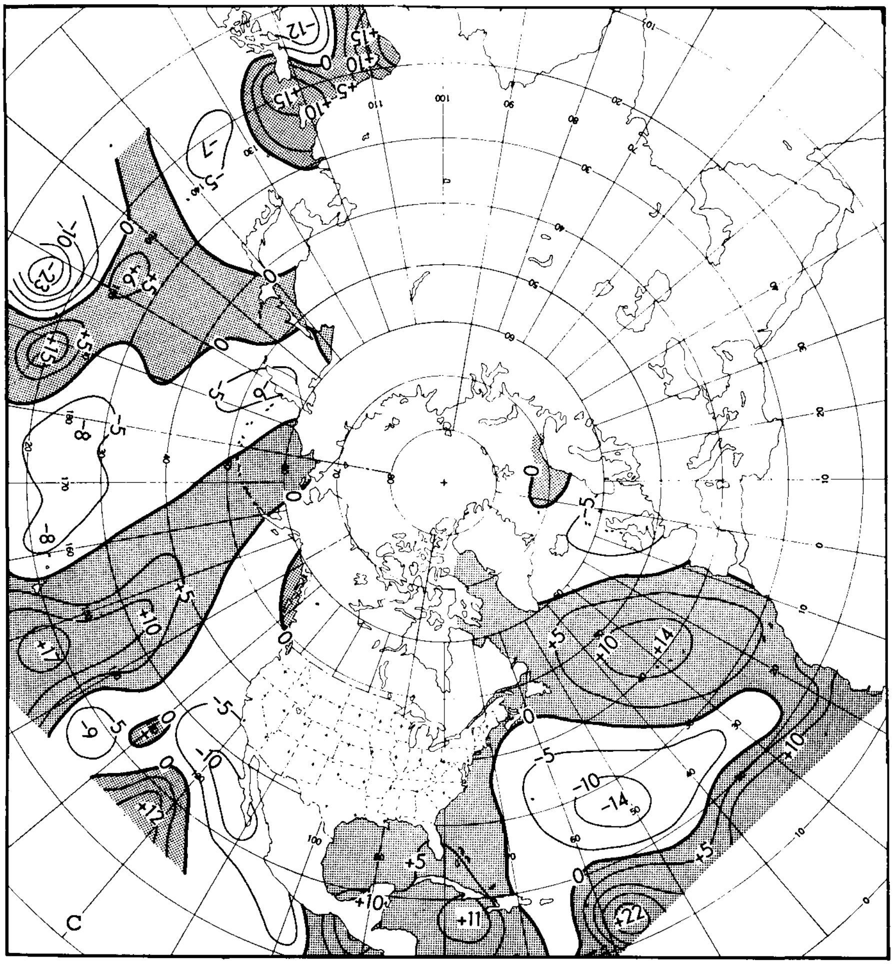
$$h(\mathbf{v}_s - \mathbf{v}_{s_N}) \cdot \nabla T_s = h\,\frac{0.0126}{\varrho_a f \sqrt{\sin\varphi}}$$

$$\times [\cos\theta\, J(p_a - p_{a_N}, T_s) + \sin\theta\, \nabla(p_a - p_{a_N}) \cdot \nabla T_s] \quad (20)$$

where J is the Jacobian

$$\mathbf{v}_s = \frac{1}{h}\int_0^h \mathbf{v}_s^*\, dz$$

$$\mathbf{v}_{s_N} = \frac{1}{h}\int_0^h \mathbf{v}_{s_N}^*\, dz$$



and where f is the Coriolis parameter, p_a is the surface air pressure, p_{a_N} is the corresponding normal surface air pressure, and ϱ_a is the surface air density which will be taken as a constant.

For $|\mathbf{v}_a| \leq 6$ m sec^{-1} we have to use the factor $0.0259/\sqrt{|\mathbf{v}_a|}$ instead of 0.0126 in formulas (18) and (19).

The total horizontal advection by mean ocean currents in equation (5) is given by

$$h\,\mathbf{v}_{s_T}\cdot\nabla T_s = h[\mathbf{v}_{s_w} + (\mathbf{v}_s - \mathbf{v}_{s_N})]\cdot\nabla T_s \qquad (21)$$

where

$$\mathbf{v}_{s_w} = \frac{1}{h}\int_0^h \mathbf{v}^*_{s_w}\,dz$$

4. The heating functions

For the heat lost by evaporation at the surface and the turbulent vertical transport of sensible heat at the surface we will use the so-called "bulk" formulas

$$G_3 = K_4|\mathbf{v}_a|[0.981\, e_s(T_s) - Ue_s(T_a)] \qquad (22)$$

$$G_2 = K_3|\mathbf{v}_a|(T_s - T_a) \qquad (23)$$

where K_4 and K_3 are constants, $|\mathbf{v}_a|$ is the ship-deck wind speed; T_a is the ship-deck air temperature; $e_s(T_s)$ and $e_s(T_a)$ are the saturation vapor pressure at the surface ocean temperature and at the ship-deck air temperature respectively; and U is the ship-deck air relative humidity.

For the saturation vapor pressure we shall

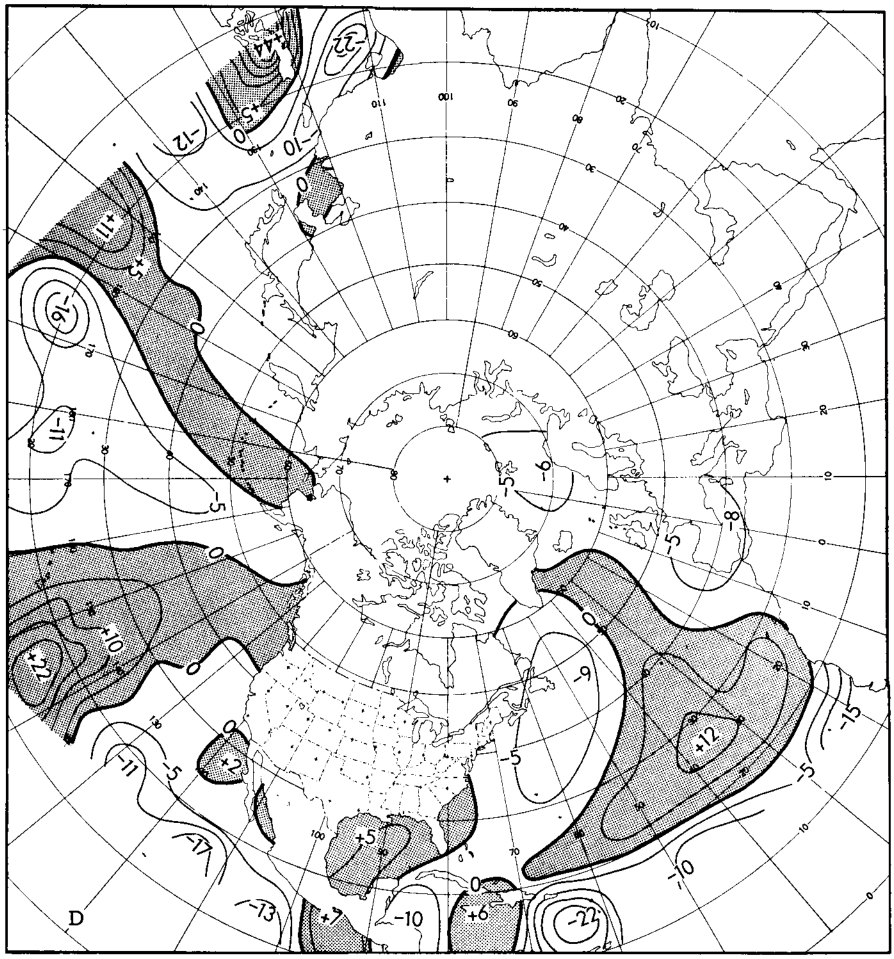


Fig. 3. Geostrophic wind anomalies, in meters per sec, and the corresponding wind drift ocean current anomalies, in cm per sec, for December 1967: Parts *A* and *B* are respectively the meridional and zonal components of the geostrophic wind anomalies; and parts *C* and *D* are respectively the meridional and zonal components of the anomalies of the ocean currents computed from formulas (18) and (19) for $\theta = 45°$ and $C_1 = 1$.

use the approximate formula:

$$e_s(t^*) = a_1 + b_1 t^* + c_1 t^{*2} + d_1 t^{*2} + l_1 t^{*4} \quad (24)$$

where e_s is in millibars and where $t^* = T^* - 273.16°C$; T^* is the absolute temperature; $a_1 = 6.115$, $b_1 = 0.42915$, $c_1 = 0.014206$, $d_1 = 3.046 \times 10^{-4}$ and $l_1 = 3.2 \times 10^{-6}$ (Adem, 1967).

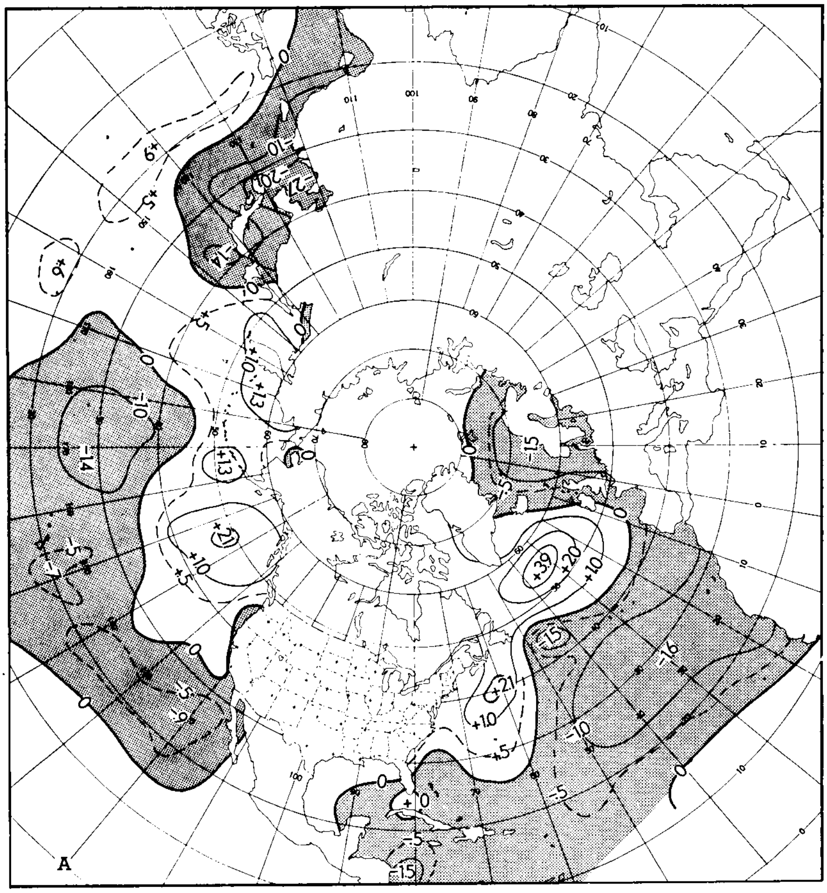
We shall also carry out experiments with the linear formulas used in the present thermodynamic model (Adem, 1964); which are the following:

$$G_3 = G_{3N} + K_4 B |\mathbf{v}_{aN}| [0.981(T_s - T_{sN}) - U_N(T_m - T_{mN})] \quad (25)$$

$$G_2 = G_{2N} + K_3 |\mathbf{v}_{aN}| [(T_s - T_{sN}) - (T_m - T_{mN})] \quad (26)$$

where T_m is the 700 mb temperature, G_{3N}, G_{2N}, T_{sN} and T_{mN} are the normal values of G_3, G_2, T_s and T_m respectively; K_3, K_4 and B are constants; $|v_{aN}|$ is the normal value of the surface wind speed and U_N is the normal value of the surface relative humidity.

Formulas (25) and (26) were developed by Clapp et al. (1965) for use in the thermodynamic model (Adem, 1965). They are obtained from (22) and (23) by assuming a normal lapse rate



and normal values of $|\mathbf{v}_a|$ and U; and approximating e_s by a linear function of temperature.

To compute E_s we shall use the radiation model developed previously (Adem, 1962), which yields the following formula:

$$E_s = F_{34} + \varepsilon F'_{34} + F_{35} T'_a + F_{36} T'_s + \alpha_1 I \qquad (27)$$

where ε is the fractional cloudiness, $T'_a = T_a - 288$, $T'_s = T_s - 288$, $\alpha_1 I$ is the short wave radiation absorbed by the ocean layer; F_{34}, F'_{34}, F_{35} and F_{36} are constants given by

$$F_{34} = -F(T_{s_0})$$

$$F'_{34} = F(T_{c_2})$$

$$F_{35} = 4\sigma T^3_{s_0} - \left(\frac{\partial F}{\partial T^*}\right)_{T^* = T_{s_0}}$$

$$F_{36} = -4\sigma T^3_{s_0}$$

where $\sigma = (8\,215)\ 10^{-14}$ cal cm^{-2} K^{-4} and $T_{s_0} = 288°$K; T_{c_2} is the temperature at the bottom of the layer of clouds considered in the radiation model. $F(T^*)$ is a function of the temperature T^*, which is given in a previous paper (Adem, 1962).

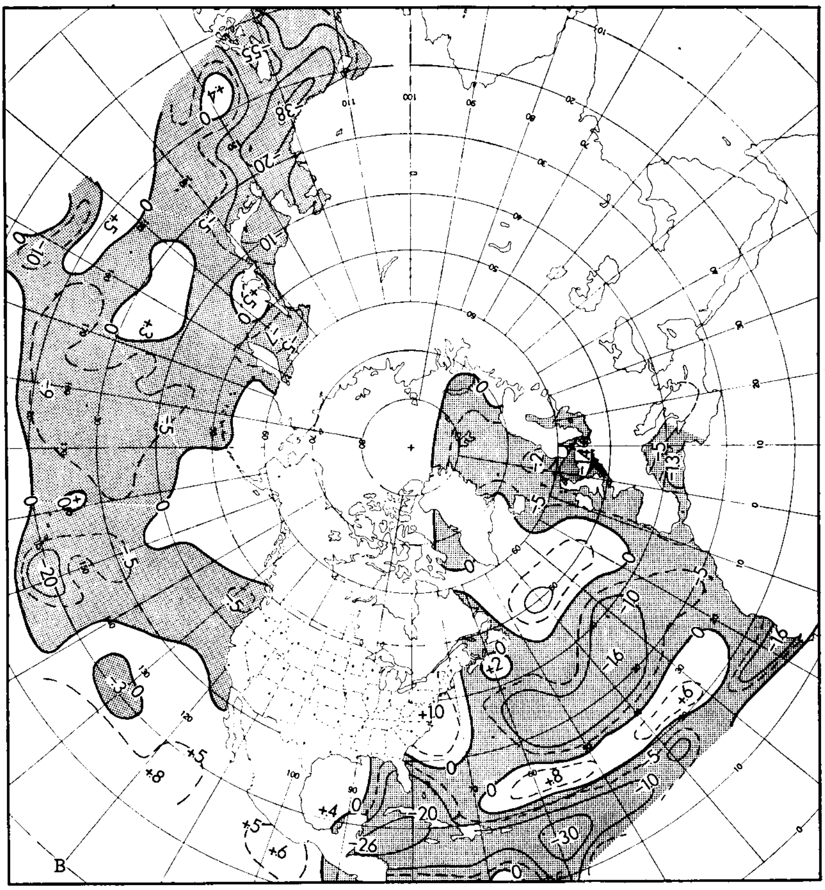
For $\alpha_1 I$ we will use, as previously (Adem, 1964), the formula

$$(Q + q)_0 [1 - (1 - k)\varepsilon](1 - \alpha)$$

where $(Q + q)_0$ is the total radiation received by the surface with clear sky, k is a function of latitude and α is the albedo of the surface of the oceans.

5. The numerical experiments

The local rate of change of the surface ocean temperature can be obtained from (5) and is given by



$$\frac{\partial T_s}{\partial t} = \mathrm{AD}_1 + \mathrm{AD}_2 + \mathrm{TU} + \mathrm{HE} - \frac{W}{h} \tag{28}$$

where

$$\mathrm{AD}_1 = -(\mathbf{v}_s - \mathbf{v}_{s_N}) \cdot \nabla T_s \tag{29}$$

$$\mathrm{AD}_2 = -\mathbf{v}_{s_W} \cdot \nabla T_s \tag{30}$$

$$\mathrm{TU} = K_s \nabla^2 T_s \tag{31}$$

$$\mathrm{HE} = (1/\varrho_s c_s h)(E_s - G_2 - G_3) \tag{32}$$

For the coefficients we shall use the values $\varrho_s = 1$ gm cm^{-3}, $c_s = 1$ cal gm^{-1}; and $K_s = 3 \times 10^8$ cm^2 sec^{-1}. The order of magnitude of this value of K_s agrees with the one suggested by Svedrup (1954) for ocean-wide areas.

Formula (28) will be applied to time-averaged periods of a month, and a sub-index will be used to specify which month is considered. Therefore, for the ith month will be written

$$\left(\frac{\partial T_s}{\partial t}\right)_i = (\mathrm{AD}_1)_i + (\mathrm{AD}_2)_i + (\mathrm{TU})_i + (\mathrm{HE})_i - \left(\frac{W}{h}\right)_i \tag{33}$$

Two options for the time derivative will be considered: the Euler formula

$$\left(\frac{\partial T_s}{\partial t}\right)_i = \frac{(T_s)_{i+1} - (T_s)_i}{\Delta t} \tag{34}$$

and the mid-point rule:

$$\left(\frac{\partial T_s}{\partial t}\right)_i = \frac{(T_s)_{i+1} - (T_s)_{i-1}}{2\Delta t} \tag{35}$$

In the present thermodynamic model, where the linear formulas (25), (26) and (27) have been used to evaluate $(\mathrm{HE})_i$ and the terms $(\mathrm{AD}_1)_i$, $(\mathrm{AD}_2)_i$, $(\mathrm{TU})_i$ and $-(W/h)_i$ have been

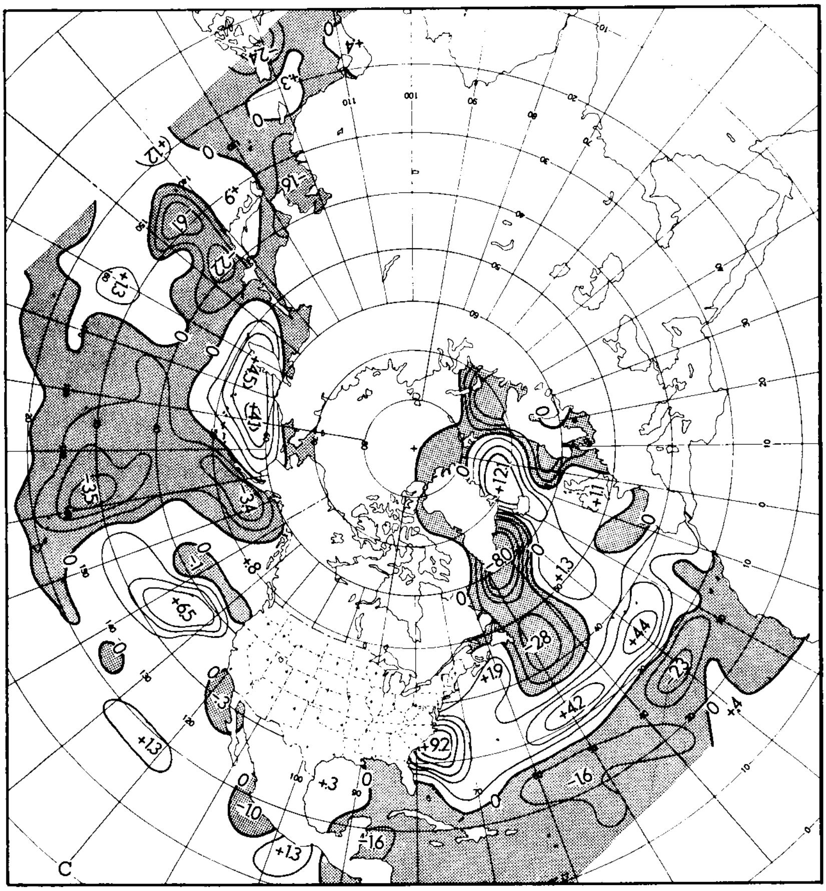


Fig. 4. Changes of surface ocean temperature anomalies, in Celsius degrees, from December 1967 to January 1968 due to heating by radiation, evaporation and vertical turbulent transport of sensible heat: Part A is the solution using the linear formulas (25) and (26), and $h = 100$ m; and B and C correspond to the case where the nonlinear formulas (22) and (23) are used. In part B, $h = 100$ m is used while in D the depth of the layer has been computed from formulas (16) and (17).

neglected, we have used, instead of (34) or (35), the backward finite difference formula

$$\left(\frac{\partial T_s}{\partial t}\right)_i = \frac{(T_s)_i - (T_s)_{i-1}}{\Delta t} \tag{36}$$

as described in a previous paper (Adem, 1965).

Substituting (34) in (33) we obtain

$$(T_s)_{i+1} - (T_s)_i = \Delta t\left[(\mathrm{AD}_1)_i + (\mathrm{AD}_2)_i + (\mathrm{TU})_i + (\mathrm{HE})_i - \left(\frac{W}{h}\right)_i\right] \tag{37}$$

When (35) is used, (33) becomes

$$(T_s)_{i+1} - (T_s)_i = 2\Delta t\left[(\mathrm{AD}_1)_i + (\mathrm{AD}_2)_i + (\mathrm{TU})_i + (\mathrm{HE})_i - \left(\frac{W}{h}\right)_i\right] - [(T_s)_i - (T_s)_{i-1}] \tag{38}$$

Formula (37) (or (38)) allows us to compute the change in ocean temperature from the ith month to the $i+1$st month using the observed values in the ith month. When (38) is used we need also $T_{s_{i-1}}$.

Therefore, one can attempt to develop a method for predicting monthly and seasonal temperature which includes advection by the ocean currents and by turbulent eddies as well as heating by radiation, evaporation and

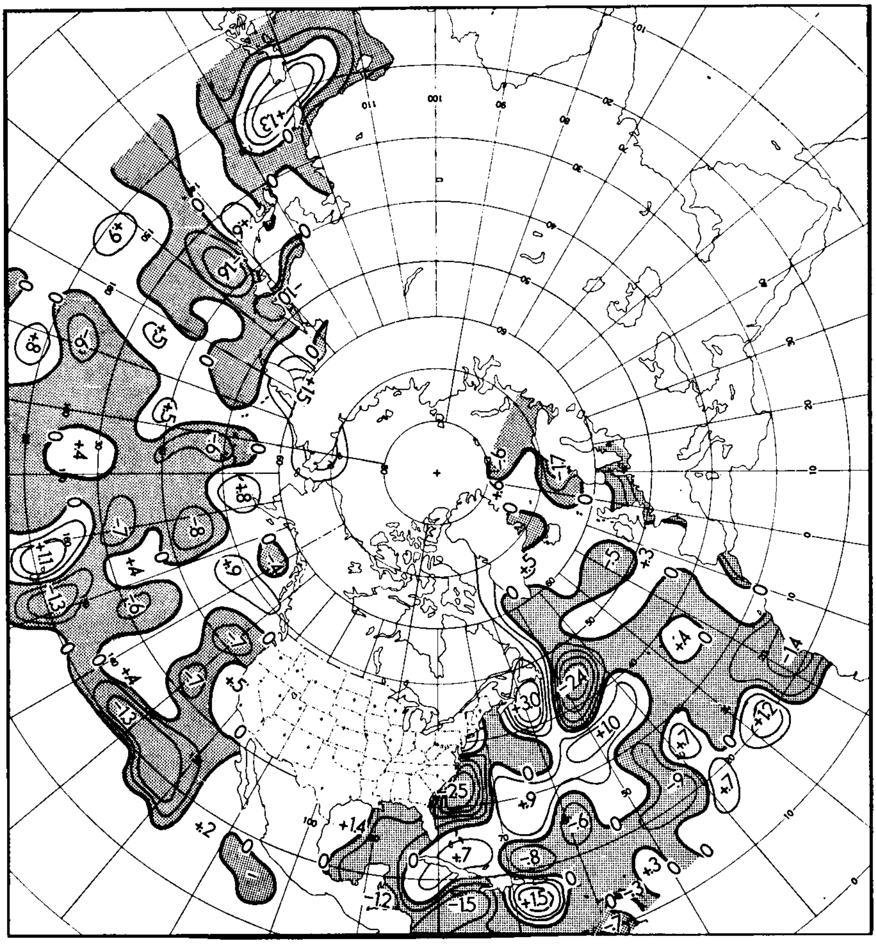


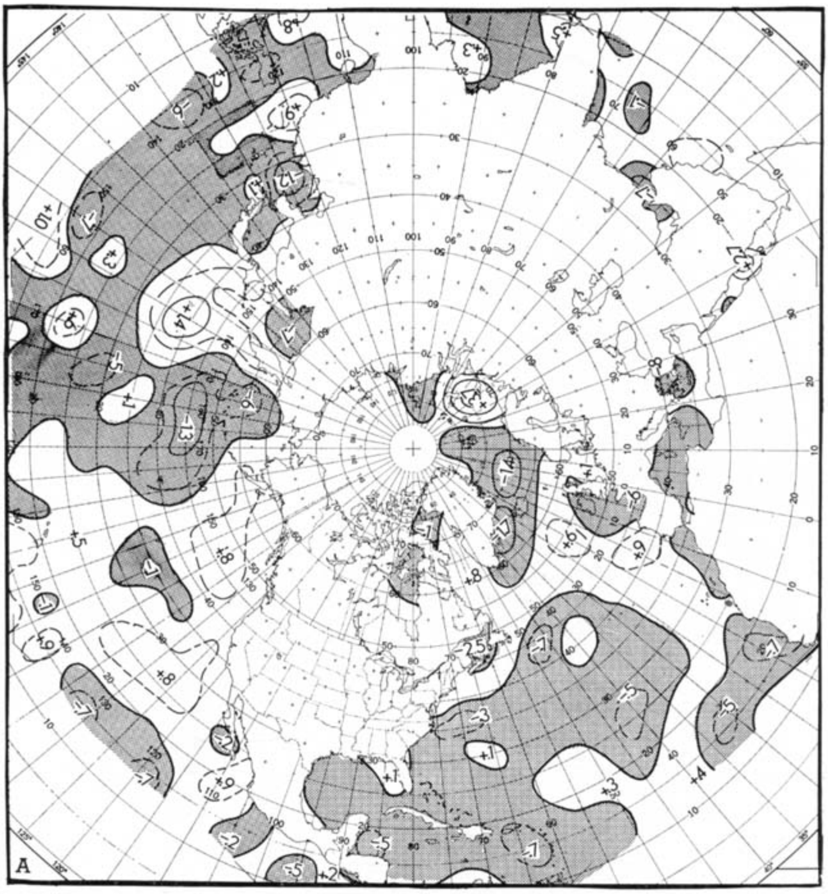
Fig. 5. Changes of surface ocean temperature anomalies, in Celsius degrees, from December 1967 to January 1968 due to horizontal turbulent transport of heat with $K_s = 3 \times 10^8$ cm^2 sec^{-1}.

vertical turbulent transport of sensible heat. Except for the use of forward or centered differences in equation (28) instead of backward, the method is essentially the same as the one used in the present thermodynamic model (Adem 1964, 1965). That is, one makes a prediction using normal initial conditions and another using the observed initial conditions corresponding to the month considered. The month-to-month change of the anomalies of surface ocean temperature is obtained by subtracting from the computed change for the given month, the corresponding computed normal change.

To evaluate the spatial derivatives we will use the mid-point rule and the same grid interval as in the previous work (Adem, 1964).

We shall carry out experiments using (37) and (38) with different options for the advection and the heating terms. The experiments reported here have been confined to the prediction of the change of anomalies from December 1967 to January 1968. In the experiments described below we use formula (37) and whenever (38) is used we will mention it.

Fig. 1 shows the results of the computations including only the mean currents horizontal advection terms. Fig. 1A and 1B are the predicted changes of the anomalies of temperature due to anomalies in pure drift currents (AD_1), from December 1967 to January 1968 using formula (20) with $C_1 = 1$ and with $\theta = 45°$ and $\theta = 90°$ respectively. Fig. 1C is the predicted change when the anomalies in pure drift currents

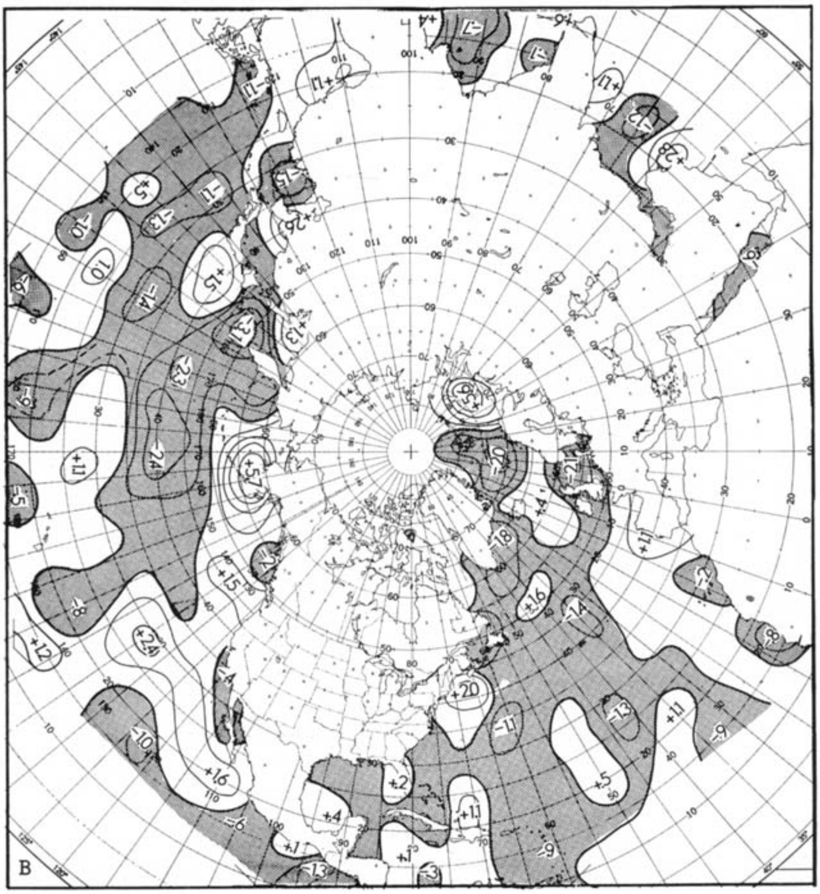


are neglected and using prescribed normal winter values of the ocean currents (AD_2) taken from published charts (U.S. Department of Commerce, 1959). Fig. 1 D is the predicted anomaly change due to the anomalies in pure wind drift current, using formula (20) with $C_1 = 1$ and $\theta = 45°$, but using normal December values of sea temperature instead of the observed values of December 1967 as in the case of 1 A.

Comparison of these four cases with the observed change (Fig. 2) shows that all of them have some skill, especially those shown in Figs. 1 A and 1 D in the Pacific ocean, where they predict some of the changes of the anomalies remarkably well. Furthermore, Fig. 1 A and 1 D are for practical purposes identical, showing that the term $(\mathbf{v}_s - \mathbf{v}_{sN}) \cdot \nabla(T_s - T_{sN})$ can safely be neglected, in agreement with Namias (1959), Eber (1961) and Jacob (1967).

To determine the main variables on which horizontal advection depends, we show in Fig. 3 the geostrophic wind and the corresponding computed drift currents used in the prediction shown in Fig. 1 A.

Figs. 3 A and 3 B are respectively the meridional and zonal components of the geostrophic wind anomalies, and 3 C and 3 D are respectively the meridional and zonal components of the anomalies of the ocean currents, computed from formulas (18) and (19) for $\theta = 45°$ and $C_1 = 1$. Comparison of Fig. 1 A with 3 C and 3 D shows the strong dependence of the anomalies of the advection term on the anomalies of ocean currents. In particular, the meridional component of the ocean current (Fig. 3 C) shows strong correlation with the prediction (Fig. 1 A). Furthermore, comparison of 3 A with 3 C shows strong dependence of the anomalies of the meridional component of ocean currents on the meridional component of surface wind (when $\theta = 45°$). However, when $\theta = 90°$ the meridional



component of the anomalies of the ocean currents is obviously related to the zonal wind anomalies. This is reflected (in the case when $\theta = 90°$) in a strong negative correlation of the change in anomalies (Fig. 1B) with the zonal component of the surface wind (Fig. 3B).

The remarkable agreement of the predictions shown in Figs. 1A and 1D shows that the change in anomalies of temperature associated with the anomalies of wind drift ocean currents is almost independent of the initial anomalies of temperature.

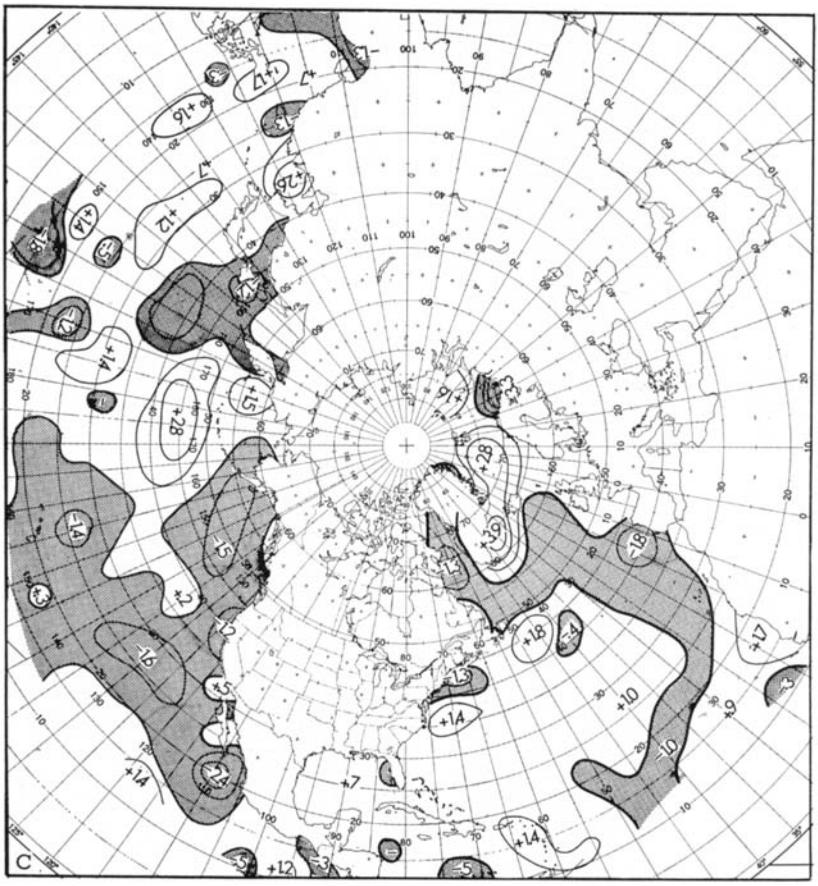
In contrast with these results the predictions based on the other terms in the right hand side of equation (33), have a strong dependance on the *initial ocean temperature anomalies*.

Fig. 4 shows the predictions when only the "heating terms" (HE) are included. Fig. 4A is the solution using formulas (25) and (26) with $h = 100$ m; and Figs. 4B and 4C correspond to the case when the non-linear formulas (22) and (23) are used. In 4B we have used $h = 100$ m, while in 4C the depth of the layer has been computed from formulas (16) and (17).

Comparison of the predictions of Fig. 4 with the observed values in Fig. 2 reveals, that they have some skill.

Fig. 5 shows the change of anomalies due to the turbulent transport term $K_s \nabla^2 T_s$, whose contribution to the total change appears to be important. Furthermore, comparison of Fig. 5 with Fig. 2, shows that the prediction using only this term has high skill.

The order of magnitude of the contribution due to the heat added at the bottom of the layer can be evaluated using (10). Assuming that the anomalies of T_B and w_h are negligibly small; and using formula (10), and the values $h = 100$ m and $\Delta t = 1$ month, the changes of the anomalies of the surface temperature due to



the term $-(W/h)_i$ in formula (37) is given by

$$(T_s - T_{s_N})_{i+1} - (T_s - T_{s_N})_i = 2.6 \times 10^2 w_h (T_s - T_{s_N})_i \tag{39}$$

According to Wyrtki (1961)

$$-w_h = 2 \times 10^{-5} \text{ cm sec}^{-1}$$

is a likely value of the upward velocity through the discontinuity layer. Substituting this value in (39) we obtain

$$(T_s - T_{s_N})_{i+1} - (T_s - T_{s_N})_i = -5.2 \times 10^{-3} (T_s - T_{s_N})_i$$

Since the observed change of anomalies are of the same order of magnitude as the anomalies themselves, this result shows that, on the basis of the above approach and using Wyrtki's values of vertical velocity, the changes of anomalies due to the term $-(W/h)_i$ are negligible.

To obtain non-negligible changes of anomalies we require upward velocity values of the order of $(1/2.6) \times 10^{-2}$ cm sec^{-1}.

Next we shall carry out a comparative study of the skill of the predictions, using a variety of options in the model. At this preliminary stage it is convenient to introduce an objective method to evaluate the skill of the predictions, which at the same time is simple and practical enough to test innumerable possibilities.

As in previous works (Adem & Jacob, 1968; Adem, 1969) we will evaluate only the skill of predicting the correct sign of the month-to-month change in the anomalies.

Table 1 shows the percentage of signs of the change in ocean temperature anomalies predicted correctly by some of the cases described above as well as for the cases in which combinations of the different terms are included.

The Fig. number corresponding to each case is given in the first column. In the cases when

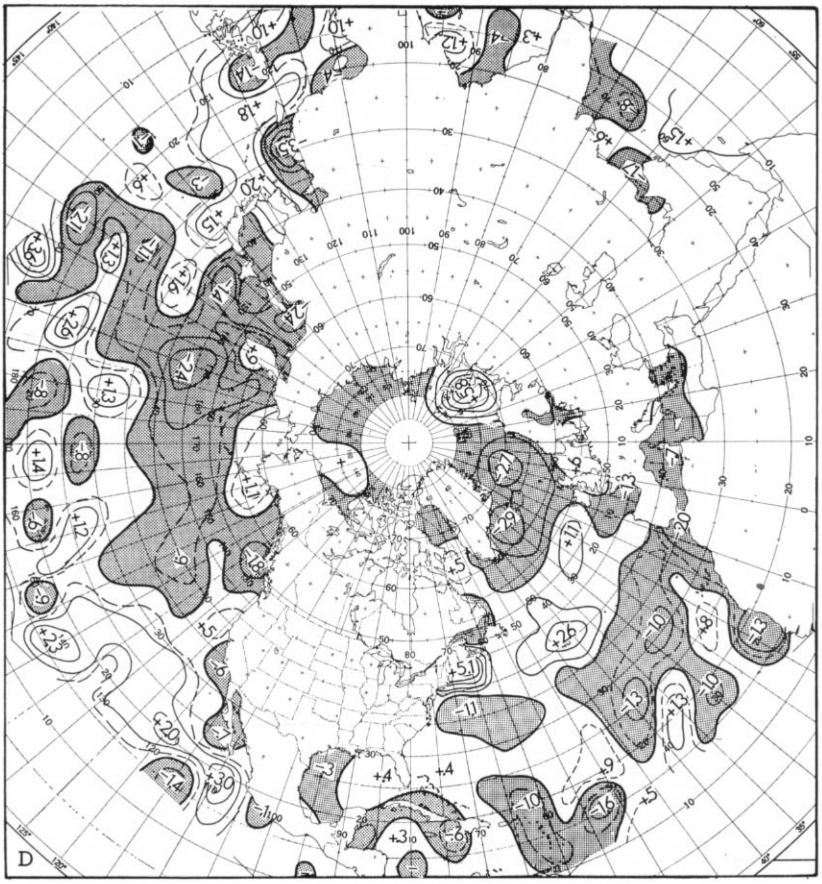


Fig. 6. Changes of surface ocean temperature anomalies, in Celsius degrees, from September 1969 to October 1969: Part *A* is the prediction of the changes based in a non-advective model (equation 1); part *D* the prediction of the changes including besides "heating" the advection of heat by ocean currents (equation 5); part B, observed changes. Part C shows the anomalies for September 1969, whose opposite sign gives the predicted changes of the sign of the anomalies by "return to normal".

terms are combined, the sum of the corresponding figures is given. The type of terms included is shown in the second column. The percentage values are given for the Pacific and for the Pacific and the Atlantic oceans combined. The scores using the Euler method (forward differences) correspond to the figures used as illustrations and whose numbers are given in the first column of the table. The scores using the mid-point rule (centered differences) are also listed. (Figures not shown.)

A first glance at the table reveals that the scores for the Pacific Ocean are better than those for both oceans, and therefore those of the Atlantic (not shown) are poorest.

Another result is that, in general, the scores using centered differences are better than the ones using forward finite differences. This was also found by Eber (1961) and Jacob (1967) for the cases that they studied, but they compared the results using centered differences with the corresponding cases using Euler-backward finite differences.

In the cases where advection by ocean currents is the only term included, the scores are disappointingly low, and the best scores correspond to $\theta = 45°$ (in formulas (18) and (19)). The highest scores correspond to the Pacific ocean for $\theta = 45°$ and when only the wind drift anomaly term (AD_1) is included.

The addition of advection by normal seasonal currents (AD_2) to the wind drift anomaly

Table 1. *Percentage of signs of the changes from December 1968 to January 1969 in ocean temperature anomalies predicted correctly by the model with different options*

Figure corresponding to case	Terms included	Pacific: Euler method	Pacific: Mid-point rule	Pacific and Atlantic: Euler method	Pacific and Atlantic: Mid-point rule
1A	AD_1	64.2	65.1	56.4	58.4
1B	AD_1	52.3	57.8	52.5	54.5
1C	AD_2	55.0	53.2	57.9	55.4
1D	AD_1	63.3	65.1	54.5	58.9
1A+1C	AD_1+AD_2	57.8	63.3	54.0	59.4
1B+1C	AD_1+AD_2	48.6	56.0	50.5	55.0
4A	HE	63.3	63.3	62.9	60.9
4B	HE	55.0	62.4	55.4	60.4
4C	HE	52.3	58.7	52.0	56.9
1A+4A+5	AD_1+HE+TU	66.1	69.7	64.4	65.3
1B+4A+5	AD_1+HE+TU	64.2	67.9	62.4	65.8
1A+4B+5	AD_1+HE+TU	66.1	67.9	61.9	64.4
1B+4B+5	AD_1+HE+TU	61.5	67.9	61.4	63.4
1B+4C+5	AD_1+HE+TU	63.3		57.9	
1A+1C+4A+5	AD_1+AD_2+HE+TU	65.1	67.0	63.4	63.9
1B+1C+4A+5	AD_1+AD_2+HE+TU	63.3	65.1	63.9	65.8
1A+1C+4B+5	AD_1+AD_2+HE+TU	66.1	67.9	62.9	64.4
1B+1C+4B+5	AD_1+AD_2+HE+TU	61.5	69.7	60.4	66.3
1B+1C+4C+5	AD_1+AD_2+HE+TU	59.6	61.5	56.9	60.4

advection (AD_1) decreases the scores, especially in the Pacific. However, when centered differences are used the scores for both oceans are the highest in this case.

When only the term AD_2 is used the scores for both oceans together are comparable to those in which only AD_1 is used. Although for the Pacific alone, the scores are much lower than those corresponding to the use of the term AD_1 alone. Therefore, it follows that for the Atlantic ocean the scores (not shown) are much higher for AD_2 than for AD_1 alone.

In the cases of Fig. 4, where only "heating" (HE) is included, the best scores correspond to the case when linear functions are used.

The worst score corresponds to the case when the depth of the layer is computed from formulas (16) and (17). In this case there are some regions with unrealistically large values in the predicted temperature changes which correspond to the very small values of the computed depth of the layer.

When all the terms are included (AD_1 + TU + HE or AD_1+AD_2 + TU + HE) there exists considerable improvement with respect to the cases when either advection by mean ocean currents alone (AD_1 or AD_1+AD_2) or heating alone (*HE*) are included.

The term *TU* has contributed in an important way to the prediction in which all the terms are included. It is interesting to point out that, in this particular example, the prediction using only this term has skill (not shown in Table 1) comparable to the case with all the terms included.

We have evaluated objectively only the skill in predicting the correct sign of the month-to-month change in the anomalies. Furthermore, from a comparison of Figs. 1, 4 and 5 with 2 it is evident that the position of some of the maxima and minima is well predicted. Also, the magnitude of the predicted changes of the anomalies is, in general, of the correct order of magnitude, and could be adjusted by an adequate selection of the parameters and coefficients, based on the numerical experiments.

For the particular case studied in this paper, the magnitude of the predicted changes of anomalies is in better agreement with the observations when non-centered differences are used. The predictions using centered differences (not shown) yield magnitudes about twice larger

Table 2. *Averages for seasons and for the whole period of the percentage of signs of month-to-month changes in ocean temperature anomalies predicted correctly by the model and by return to normal, for the period from December 1965 to September 1969*

	Pacific			Pacific and Atlantic		
	Model	Return to normal	Difference	Model	Return to normal	Difference
Winter	65.7	58.1	7.6	63.6	58.8	4.8
Spring	64.1	57.6	6.5	63.2	59.2	4.0
Summer	61.6	58.2	3.4	63.8	62.0	1.8
Fall	61.7	57.8	3.9	60.5	57.9	2.6
Average for whole period	63.4	57.9	5.5	62.8	59.5	3.3

than the corresponding non-centered predictions.

A computation similar to the one shown in Table I was carried out for the prediction of the change of the anomalies from September 1969 to October 1969, and the results confirm the above general conclusions.

6. Evaluation of 46 predictions based in a non-advective model

As mentioned in the introduction, predictions based on a thermodynamic model in which transport of heat by ocean currents is neglected have been carried out routinely, and a preliminary evaluation has been published (Adem, 1969). To illustrate this type of prediction we shall consider the case for October 1969.

Fig. 6A shows the predicted change of the monthly anomalies of surface ocean temperature from September to October 1969; and Fig. 6B the corresponding observed values. In Fig. 6C are shown the observed anomaly values in the previous month (September 1969), which are used, besides other fields, as input data for the prediction.

Comparison of Fig. 6C with Fig. 6B shows that there is a high negative correlation between them. Therefore, the opposite sign of the anomalies for September 1969 yields a good prediction of the sign of the change from September to October 1969. This is an observed fact that is true not only for the single case under consideration but that applies to any month of any year. This "return to normal" prediction is the simplest one that we can make which has significant skill. It therefore provides a good control for evaluating predictions by any other method.

The predicted sign of the change of anomalies by the thermodynamic model (Fig. 6A) compared with the observed values (Fig. 6B) also shows skill, and compared with Fig. 6C shows a striking similarity to the prediction by "return to normal".

The significant difference between the model prediction and return to normal is that the former is the result of applying the law of conservation of energy to the ocean-atmosphere system. In fact, the predicted anomalies in surface ocean temperature are due mainly to the predicted anomalies of the heat lost by evaporation and by turbulent vertical transport of sensible heat, which are negatively correlated with the predicted change of anomalies and positively correlated with the anomalies of temperature in the previous months.

Regarding the skill of predicting sign changes we have shown (Adem, 1969) that the model in which the transport of heat by ocean currents is neglected, applied to 26 cases yields skill higher than the return to normal prediction. This is confirmed, using the larger sample of 46 cases summarized in Table 2, which includes the 26 cases.

The prediction for October 1969 in which all the terms in equation 5 are included is shown in Fig. 6D. Its comparison with Fig. 6B shows better agreement than Fig. 6A in the position of some of the centers, and in the horizontal scale and magnitude of the change of the anomalies, especially in the Pacific Ocean.

7. Concluding remarks

We have shown that, using a model without transport of heat by ocean currents, there is skill in the prediction of the month-to-month changes of the anomalies of surface ocean temperature for a sample of 46 cases.

Furthermore, the results for two cases suggest that inclusion of horizontal advection by mean ocean currents and by turbulent eddies may yield an improved model.

A model that includes all of the above options is now available for numerical experimentation. Extensive application of it will throw more light on this subject, and will eventually yield the best choice for an improved operational prediction model.

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О ПРЕДСКАЗАНИИ СРЕДНЕЙ МЕСЯЧНОЙ ОКЕАНИЧЕСКОЙ ТЕМПЕРАТУРЫ

Уравнение сохранения термической энергии используется для предсказания изменений температуры поверхности океана от месяца к месяцу. Уравнение содержит горизонтальную адвекцию тепла средними океаническими течениями и турбулентными вихрями, а также радиационное нагревание, испарение и турбулентный вертикальный перенос тепла. Численные эксперименты показывают, что предсказание успешно. Сравнивается влияние различных факторов, входящих в схему предсказания, и приводится метод проверки различного выбора параметров.