Meridional transport of mean zonal kinetic energy from five years of hemispheric data

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(Manuscript received May 5, 1970)

ABSTRACT

The mean zonal kinetic energy transports across arbitrary latitude walls in the northern hemisphere are evaluated from five years of data. It is found that the stress-work processes at the boundaries are much more significant than the actual advection of zonal kinetic energy. The stress component associated with the earth's rotation accounts for a net northward transport of zonal kinetic energy across the equator.

Introduction

Recently, Starr & Sims (1970) presented in this journal an equation for the balance of the kinetic energy of the mean zonal flow. This equation contains boundary integrals representing the physical transport of kinetic energy across the free boundaries of a polar cap region. The vertical-boundary terms have been evaluated from five years of observations at 799 upper-air sounding stations, collected and processed by the Planetary Circulations Project at the Massachusetts Institute of Technology. (These data are described in some detail by Starr, Peixoto & Gaut, 1970.) Frequent reference will also be made to a subset of stations selected by Walker (1970). This subset was selected so as to reduce the contrast in station densities over land as compared with ocean areas of the northern hemisphere for the 5 year period.

For convenience, the notation and numbering of integrals is the same as that used by Starr & Sims. To save space the complete explanations are not repeated here. The integrals differ only in that they are written in the space and time domain. The overbar represents an average with respect to time while the prime represents a departure from this average. Brackets indicate averages with respect to longitude and asterisks departures from these averages. The integrals were, as usual, evaluated in pressure coordinates. For further details see also Sims (1969) and Starr, Peixoto & Sims (1970).

The boundary terms have not previously been investigated in much detail. This is presumably because most observational studies of the general circulation deal with one fixed volume and are therefore concerned with effects only on the fixed boundaries of that volume. In the present study, however, the results are presented in such a manner that the boundary effects can be examined at any latitude in the northern hemisphere. The integrals measured below are $\{7\}$, $\{8\}$, $\{9'\}$, $\{9''\}$ and $\{A\}$ as enumerated by Starr & Sims.

Discussion of individual terms

1. *Evaluation at latitude* ϕ_1, *of*

$$\int 2\pi\varrho\, R^2 \cos^2\phi_1 (\Omega\, R \cos\phi_1)\, [\bar{v}]\, \frac{[\bar{u}]}{R\cos\phi_1}\, dR \quad \{7\}$$

where u, v are the eastward and northward wind components, and ϱ is density. This integral represents the transport of kinetic energy of the mean zonal flow through work done by the stress component associated with the rotation of the earth, Ω, acting across the vertical surface at ϕ_1. Term $\{7\}$ is unique to the formulation of the energy equation presented by Starr & Gaut (1969), there being no boundary terms involving Ω in the traditional scheme. Due to the comparatively large magnitude of $\Omega R \cos\phi$, where R is essentially the earth's radius, the

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Table 1. *The 60 month and seasonal average values of the transport due to integral {7} across various latitude walls*

Units are 10^{20} erg sec^{-1}. Positive values indicate northward transports.

Lat. (N°)	60 month	Spring	Summer	Fall	Winter
80	+ 0.05	+ 0.03	− 0.09	+ 0.14	+ 0.03
70	+ 0.64	+ 0.39	+ 0.16	+ 1.49	+ 0.36
60	+ 0.50	− 1.40	− 1.52	+ 2.28	+ 1.98
50	− 6.84	− 9.91	−11.50	− 6.07	− 2.62
40	−12.20	−17.00	− 8.12	− 8.46	−27.50
30	− 0.59	− 8.87	+ 3.16	+12.60	+12.50
20	+ 6.82	− 3.50	+ 4.56	+21.50	+38.80
10	+ 3.87	+ 1.76	+12.70	+11.60	+32.10
0	+ 1.54	+ 1.40	+10.60	+ 3.29	+ 4.99

term might be expected to be large and sensitive to any errors in $[\bar{v}]$.

Table 1 contains the values of the vertical integral {7} for every ten degrees of latitude, and the meridional profiles of this integral are included in Figs. 1 and 2. For the long-term average there is a net northward transport in low latitudes, a net southward transport in middle latitudes and negligible transports at high latitudes. This general pattern holds also for the seasons. However, in spring the northward transport at low latitudes is, for the vertical average, quite small. Each seasonal value is a 15 month average. Thus for winter the months January, February and March were included for all 5 years, etc. for the other seasons.

To aid in studying the vertical distribution of the transport by term {7} (as well as the other transport terms) there are depicted in Fig. 3 meridional cross-sections through the

Table 2. *The 60 month and seasonal average values of the transport due to integral {8} across various latitude walls*

Units are 10^{20} erg sec^{-1}. Positive values indicate northward transports.

Lat. (N°)	60 month	Spring	Summer	Fall	Winter
80	0.00	0.00	−0.01	+0.01	−0.01
70	+0.03	+0.02	+0.01	+0.07	0.00
60	+0.04	−0.04	−0.05	+0.12	+0.09
50	−0.36	−0.48	−0.67	−0.37	−0.10
40	−0.75	−1.02	−0.48	−0.59	−2.19
30	−0.10	−0.48	−0.05	+0.55	−1.23
20	+0.09	−0.08	−0.17	+0.38	+1.76
10	−0.04	−0.02	−0.35	−0.05	+0.11
0	−0.02	−0.02	−0.19	−0.02	−0.03

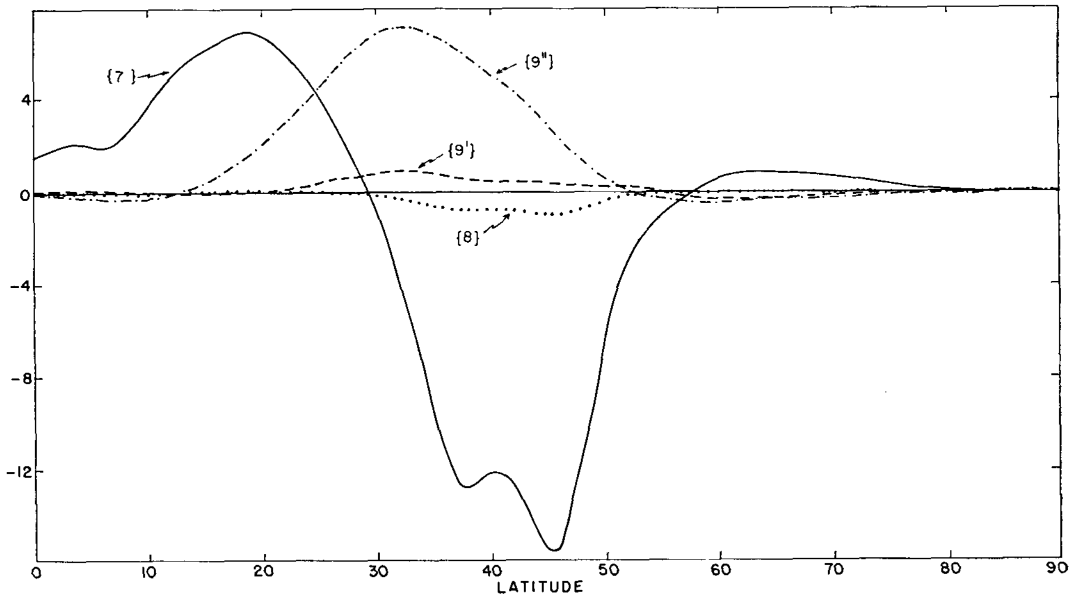

Fig. 1. Meridional profiles of the boundary integrals, shown in units of 10^{20} erg sec^{-1}. The curves are for the 60 month average conditions.

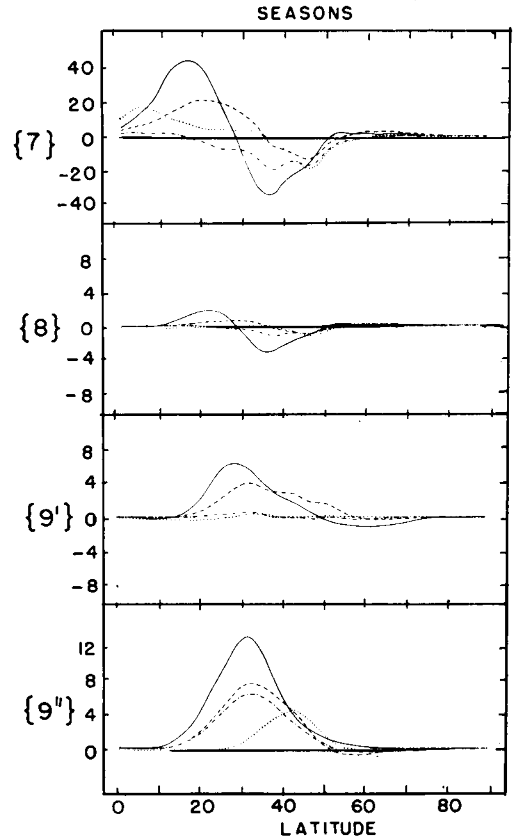

Fig. 2. Meridional profiles of the boundary integrals, shown in units of 10^{20} erg/sec^{-1}. —·—·—·, spring season; ...., summer; ---, fall; ——, winter.

atmosphere showing the meridional transport of zonal kinetic energy across any given latitude wall for the 60 month mean conditions. The corresponding seasonal conditions are shown in Fig. 4. As should perhaps be expected, the process is generally most effective near the ground and near the tropopause. It is at these levels, of course, that the mean meridional circulations are strongest. It might also be noted that the weak net northward transport at low latitudes in spring results from a compensation between a northward transport in the tropical stratosphere and a low level southward transport. Improved observations in the tropics might alter this balance significantly. The large transport in the tropical stratosphere in summer may also be affected by sparse data.

Another region in which high level transports are consistently opposed by transports nearer the surface is between 40° N and 50° N, especially in fall. As an indication of the sensitivity of this type of action to the data sample, use of Walker's stations (approximately 200) results in a value for {7} across 44° N, in fall, of -6.8×10^{20} erg sec^{-1} as compared to a value obtained from the full 799 stations of -13.1×10^{20} erg sec^{-1}.

Because of the importance of this transport term at low levels, it might be anticipated that mountains and other high terrain would have a considerable influence on the nature of the results. There is a need for more study of this point.

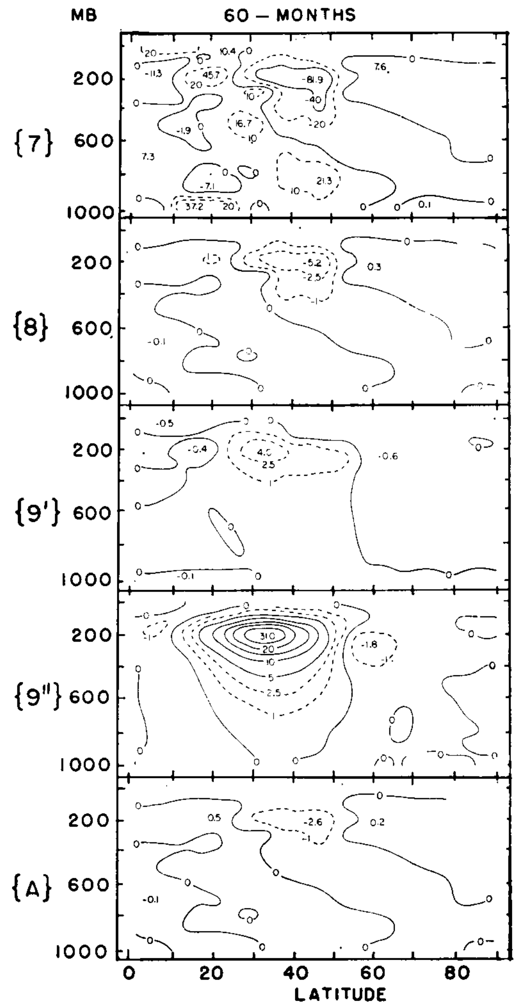

Fig. 3. Meridional cross-sections through the atmosphere showing the 60 month average distribution of the transports due to the integrands of {7}, {8}, {9'}, {9"} and {A}. Units are 10^{14} cm^{3} sec^{-1}.

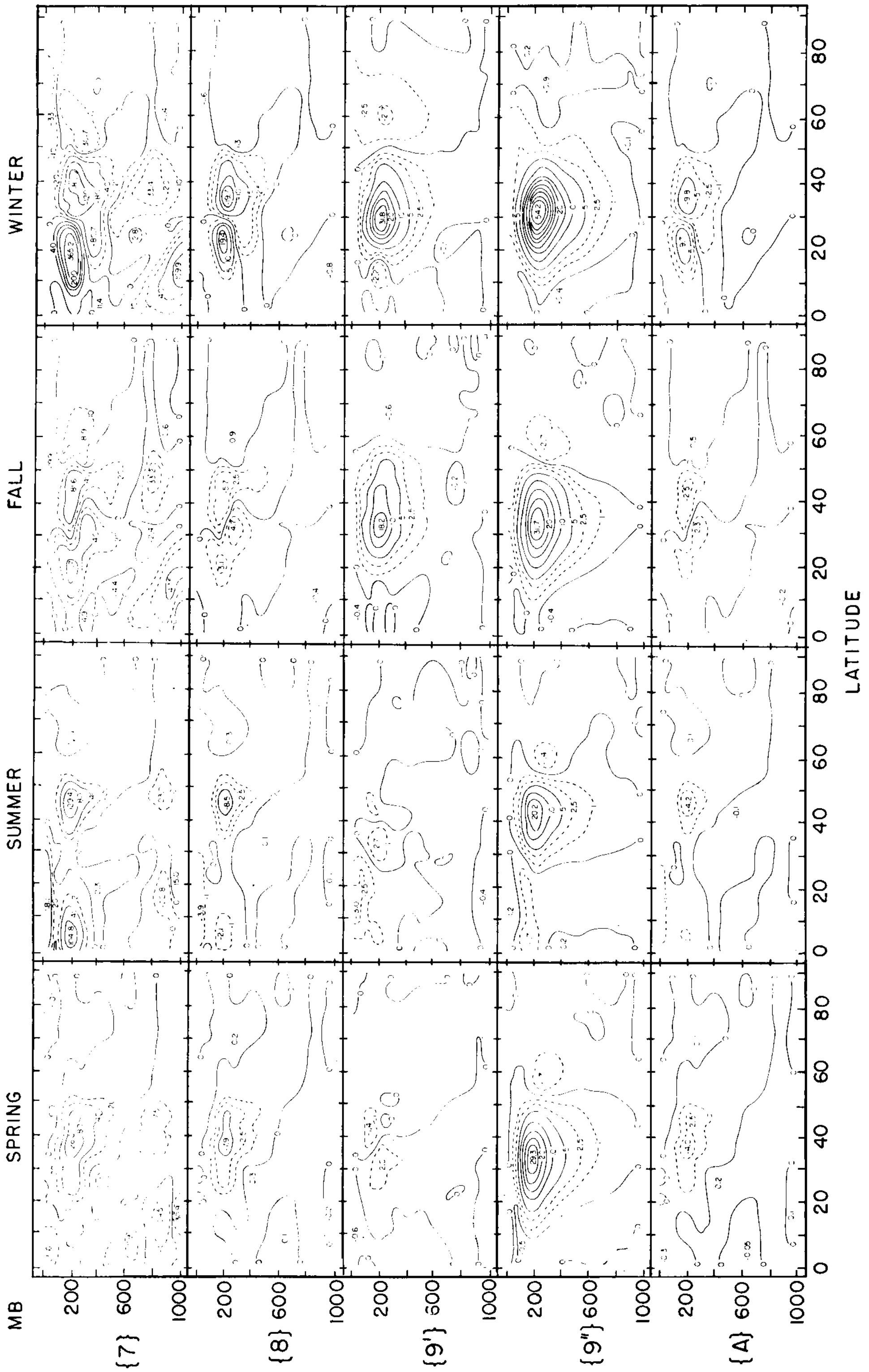

Fig. 4. Meridional cross-sections through the atmosphere representing the seasonal transport of zonal kinetic energy across latitude walls due to the integrands of {7}, {8}, {9′}, {9″} and {A}. Units are 10^{14} cm^3 sec^{-1}.

Table 3. *The 60 month and seasonal average values of the transport due to integral {9′} across various latitude walls*

Units are 10^{20} erg sec^{-1}. Positive values indicate northward transports.

Lat. (N°)	60 month	Spring	Summer	Fall	Winter
80	−0.01	0.00	0.00	−0.01	−0.03
70	−0.13	−0.08	−0.03	−0.19	−0.68
60	−0.28	−0.30	−0.03	−0.20	−1.24
50	+0.32	−0.12	+0.13	+1.66	−0.19
40	+0.49	−0.10	−0.02	+2.86	+2.39
30	+0.83	+0.45	+0.15	+3.68	+6.41
20	+0.04	+0.06	−0.38	+0.57	+1.79
10	+0.01	−0.03	−0.38	+0.01	−0.17
0	+0.03	−0.01	−0.08	+0.03	+0.03

2. *Evaluation at latitude* ϕ_1 *of*

$$\int 2\pi\varrho R^2 \cos^2\phi_1 [\bar{u}][\bar{v}] \frac{[\bar{u}]}{R\cos\phi_1} dR \qquad \{8\}$$

This integral represents the transport of kinetic energy of the mean zonal flow through work done by the stress component associated with the relative rotation, $[\bar{u}]$, acting across the vertical surface at ϕ_1. The integral occurs in the traditional form of the energy equation, but is usually neglected in hemispheric energy studies. That this assumption is valid is readily apparent upon examination of the profiles of the vertical integral of {8} in Figs. 1 and 2 and the tabulated values in Table 2. (Note that the vertical scale for term {7} in Fig. 2 is much larger than that for the other terms pictured therein.)

Because {8} is very similar in form to term {7}, much (but not all) of what was said concerning the distribution of the latter applies here also. Other than being much smaller in magnitude, {8} differs from {7} in the following important respects. The presence of $[\bar{u}]^2$ in {8} increases the relative importance of the process in the higher levels of the atmosphere, and the fact that the cosine function appears to only the first power decreases the relative importance of {8} in low latitudes. The only appreciable transports in the 60 month mean thus occur near the tropopause between 30° N and 50° N, as depicted in both the profile and in the cross section, Fig. 4. The net transports for this region and time period are southward. Only in summer is there a significant transport by term {8} across the equator, reflecting no doubt the intrusion of the southern-hemisphere Hadley cell into the northern hemisphere.

Significant northward transports occur only in fall and winter, between 13° and 35° N and between 9° and 27° N respectively. As pointed out earlier, the process is most significant near the jet-stream levels so that there is little tendency for cancelation between high and low level values at any given latitude. The net transport across a latitude wall being nearly free from this effect, it is measured fairly accurately. Computations based on Walker's data do not differ markedly from the values given in Table 2.

3. *Evaluation at latitude* ϕ_1 *of*

$$\int 2\pi\varrho R^2 \cos^2\phi_1 [\bar{u}^* \bar{v}^*] \frac{[\bar{u}]}{R\cos\phi_1} dR \qquad \{9'\}$$

This integral represents the transport of kinetic energy of the mean zonal flow through work done by the stress component associated with the standing eddy motions, $\bar{u}^*$ and $\bar{v}^*$, acting across the vertical surface at ϕ_1. Term {9′} arises in all more complete formulations of the zonal kinetic energy equation, but has not previously been studied in much detail.

The vertical integral of the term is presented pictorially in Figs. 1 and 2 and in tabular form in Table 3. The process can be examined in the vertical by reference to the seasonal cross-sections in Fig. 3 and the 60 month mean cross section in Fig. 4. In general, the standing eddy boundary transport term is characterized by negligible transports in the tropics, net northward transports in middle latitudes and small southward transports at high latitudes. It thus acts roughly opposite to terms {7} and {8}. As should be expected, the action is dominated by the level and latitude of the jet.

One of the most significant features of the standing eddy term is the extent to which it nearly vanishes in spring and summer. This is not too surprising when one considers that probably the most important single cause for the persistence of standing waves in the atmosphere is the temperature contrast between continents and oceans. This contrast is, of course, smallest in the summer half-year. Since, in the present study, spring and summer comprise the months April through September, these seasons correspond ideally to the warmest months of the year. The marked annual cycle of the strength of the standing eddies is not nearly as evident in the internal process, (see Starr, Peixoto & Sims, 1970), as it is in the boundary term, $\{9'\}$. This is presumably because the former involves the gradient of the angular velocity which does not weaken as much in the warm months as does the magnitude of the angular velocity. For some further comments on this question, see Macdonald & Frazier (1969).

Some evidence of a land–ocean bias exists in the station network when term $\{9'\}$ is computed using Walker's stations, although the effect is not generally large. Only in winter, as might be expected, is the difference significant, the maximum near 30° N being reduced from $+6.41$ to $+3.98 \times 10^{20}$ erg sec^{-1}.

4. *Evaluation at latitude* ϕ_1 *of*

$$\int 2\pi\varrho\, R^2 \cos^2\phi_1 [\overline{u'\,v'}] \frac{[\bar{u}]}{R\cos\phi_1}\, dR \qquad \{9''\}$$

This integral represents the transport of kinetic energy of the mean zonal flow through work done by the stress component associated with the transient eddy motions, u' and v', acting across the vertical surface at ϕ_1. Like terms $\{8\}$ and $\{9'\}$, term $\{9''\}$ is usually included as one of the boundary effects in the more traditional formulations of the zonal kinetic energy equation. Its variation in time and space has not been adequately investigated, however. This variation is depicted in the figures and in Table 4.

The transport across the equator is quite small as might be anticipated. In the 60 month mean, the transports are small but southward in the tropics and high latitudes and large and northward in middle latitudes. These reults are in agreement with the balance requirements of the angular momentum flux, which specify that there must be a net northward transport of angular momentum across 30° N. As will be discussed below, it is obvious from Fig. 1 that this necessary northward transport is, in the mean, accomplished almost entirely by the transient eddies.

Table 4. *The 60 month and seasonal average values of the transport due to integral* $\{9''\}$ *across various latitude walls*

Units are 10^{20} erg sec^{-1}. Positive values indicate northward transports.

Lat. (N°)	60 month	Spring	Summer	Fall	Winter
80	−0.04	−0.04	−0.02	−0.09	+0.03
70	−0.28	−0.18	−0.11	−0.23	−0.22
60	−0.45	−0.45	−0.27	−0.71	+0.21
50	+0.77	+0.56	+1.66	+0.36	+1.20
40	+5.20	+4.24	+4.13	+5.65	+6.41
30	+6.84	+6.23	+0.69	+7.33	+13.20
20	+2.01	+1.92	−0.25	+2.09	+4.52
10	−0.22	+0.02	−0.18	−0.03	+0.14
0	−0.07	+0.03	−0.13	+0.03	+0.08

The seasonal profiles of term $\{9''\}$ present (in Fig. 2) a reasonable seasonal progression of the latitude of maximum northward transport, although one might have expected the winter extreme to occur somewhat further south. The relative magnitudes of the maximum kinetic energy transports are also as would be expected, with greatest values in winter and smallest in summer. Perhaps the most striking feature of the profiles is their generally smooth, symmetric appearance. That such curves can be produced from a mass of scattered data must mean something about the methods used and the stability of the process itself.

From Figs. 3 and 4, it is evident that the term is most important near the tropopause. At the latitude of the maximum in all time periods, the transports are northward throughout essentially the depth of the atmosphere. It is interesting to note that at 60° N there is a center of southward transport at about 300 mb. This feature persists at the same latitude and height in all seasons, with only its magnitude varying. If the phenomenon, as seems likely, is real and not somehow a result of the numerical methods used, it is most interesting and deserves further investigation.

Walker's data give generally the same results as the full set of stations,with the values of the maxima less than 10% smaller. The maximum transport in winter occurs at 30° N as opposed to 32° N in the present study, giving an even more logical seasonal progression of this feature.

5. *Evaluation at latitude* ϕ_1 *of*

$$\int 2\pi\varrho\, R\cos\phi_1[\bar{v}]\frac{[\bar{u}]^2}{2}\,dR \qquad \{A\}$$

This integral represents the advection of kinetic energy of the mean zonal flow by $[\bar{v}]$ across the vertical surface at ϕ_1. Term $\{A\}$ is exactly equal to one-half the value of $\{8\}$, although it represents an entirely different process. Since term $\{8\}$ has already been discussed, only a few additional points need be made about $\{A\}$. The advection of zonal kinetic energy is generally quite small. The largest value of the integral for the 60 month mean occurs at 46° N and with a value of -0.48×10^{20} erg sec^{-1}, is hardly significant compared with the stress-work terms. In winter, however, the advection reaches a numerical maximum of -1.55×10^{20} erg sec^{-1}. While this is still comparatively small, it is by no means negligible when considering the overall balance of the zonal kinetic energy for the polar cap north of, say, 35° N.

It should be pointed out that there is, for any size polar cap, an internal process which exactly compensates the boundary integral $\{A\}$. The reader is referred to Starr & Sims (1970) for further remarks on this point.

General remarks

(1). Since 30° N marks the approximate mean boundary between the Hadley cell and the Ferrel cell, there should be little if any transports across this latitude by these mean cells. This is verified by observations as shown by the curves labeled $\{7\}$ and $\{8\}$ in Fig. 1. However, it was found by Sims (1969) that for the cap north of 30° N the net generation resulting from internal horizontal processes is less than the net destruction of kinetic energy resulting from internal vertical processes. The kinetic energy of the mean zonal flow is then, on the average, being reduced by actions in the volume. To maintain this kinetic energy at a nearly constant level requires the transport of additional energy, by some process, northward into the region. As already stated the mean cells cannot account for this transport through stress-work processes or by physical advection of kinetic energy. The additional energy must then be supplied by the eddies. This is shown graphically by the curves labeled $\{9'\}$ and $\{9''\}$ in Fig. 1. The same general reasoning can be used regarding the boundary between the Ferrel cell and the polar cell, although of course the magnitudes of the transports involved are much smaller.

(2). In fall and winter, every boundary term transports energy northward across 20° N, contributing to the general acceleration of the circulations at higher latitudes in these seasons (see Fig. 2). With the exception of term $\{7\}$, every boundary term transports energy southward across 20° N in summer, although the transports are considerably weaker. Since the contribution of $\{7\}$ still dominates, and the warm season slow-down north of 20° N is not aided by the boundary transports, the internal processes must on the average destroy kinetic energy during this period.

(3). From the profiles of the vertical-boundary transport terms in Fig. 1, one can obtain, by graphical addition, a single (60 month) curve representing a meridional profile of the vertical integral of all physical transport terms combined. (One can do this as well from Tables 1–4, keeping in mind the additional contribution from the advection term, $\{A\}$, which as stated is one-half as large as term $\{8\}$ at all latitudes.) Upon constructing the total transport curve one finds net northward transports at all latitudes south of about 34° N and north of about 61° N, with net southward transports between. Based on these results, the kinetic energy balance of the atmosphere north of 34° N could be considered without regard to vertical-boundary effects. In other words, this volume constitutes a closed system in the long term average, from the standpoint of zonal kinetic energy. Latitude walls, across which there are no net energy transports, are found in summer at 37° N, in fall at 39° N and in winter at 31° N. In spring there is no corresponding mid-latitude wall, the closest one being at about 15° N where all transports are small. There is a relative minimum of total transport, however, at about 30° N in spring. It is interesting to note that the locations of these latitudes, which are based on the vertical integrals of the transport terms, correspond within one degree to the latitudes at which the mean surface northerlies change to southerlies, i.e., at which $[\bar{v}]$ vanishes. (See, e.g., the mass streamlines of Starr, Peixoto & Gaut, 1970).

The reason for this correspondence is not known, although it might be more than coincidence.

(4). The transports across the equator are negligible except for term $\{7\}$. Even though in general, $\{7\}$ is the least accurately measured of the boundary terms, due to the uncertainties with $[\bar{v}]$, the fact remains that it indicates a net northward transport across the equator in all seasons. The cross sections show, further, that only in fall are there any appreciable compensating effects in the vertical at this latitude. Therefore, even though the absolute value may not be precise, the sign is probably positive. The conclusion then follows that the southern hemisphere, in the long term average and in fact in each season, is supplying zonal kinetic energy to the northern hemisphere.

(5). The traditional formulation of the zonal kinetic energy equation does not normally contain the complete set of physical transport terms evaluated above. It does, however contain $\{8\}$, $\{9'\}$ and $\{9''\}$ as boundary integrals, in the way this formulation is usually written. More comment upon this and other matters concerning the material of this paper will be included in future discussions.

Acknowledgements

The writer wishes to express his gratitude to Professor V. Starr for reading the manuscript and to Mr J. Welsh of the Travelers Research Corporation for performing the computations. This research was supported by the National Science Foundation under NSF Grant GA-1310X.

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МЕРИДИОНАЛЬНЫЙ ПЕРЕНОС СРЕДНЕЙ ЗОНАЛЬНОЙ КИНЕТИЧЕСКОЙ ЭНЕРГИИ ПО ДАННЫМ ЗА ПЯТЬ ЛЕТ В СЕВЕРНОМ ПОЛУШАРИИ

По данным за пять лет оцениваются переносы средней зональной кинетической энергии через произвольные широтные стенки в северном полушарии. Найдено, что вклад напряжений на границах значительно более существенен, чем фактическая адвекция зональной кинетической энергии. Компонента напряжения, связанная с вращением земли, ответственна за чистый перенос зональной кинетической энергии к северу через экватор.