Generation of mean ocean circulation by fluctuating winds

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ABSTRACT

The study of barotropic wind-driven, ocean circulation by means of a Fourier series expansion is extended in this paper to include systems with up to six components in each of the horizontal directions. The results agree with the qualitative results deduced from the 2×2 study reported earlier. The principal new result which is derived here is that a fluctuating wind (whose time-average vanishes) superimposed on a mean wind can significantly increase the mass transport of the mean circulation. The maximum increase derived here is 30 % of the mean circulation in the absence of a fluctuating wind. It is also shown that mean circulations arise from purely fluctuating winds.

1. Introduction

The present study has two purposes. One is to test a general physical hypothesis, the other is to extend the results of an earlier study (VERONIS, 1963) and to apply the hypothesis to a specific oceanographic problem. In this section the general physical hypothesis will be discussed first and the oceanographic application follows.

Three years ago in an unpublished study of the equatorial acceleration of the sun the author tried to answer the following question: Given a rotating spherical shell of fluid which is uniformly heated at the radius $r = R_0$ and uniformly cooled at a radius $r > R_0$ so that convection ensues. Because the sphere is rotating and because the component of rotation in the direction of gravity varies with latitude, being a maximum at the poles and vanishing at the equator, the inhibiting effect of this component on convection varies with latitude. Will such a variation in the inhibition of convection order the small-scale convective processes so that a net circulation results?

The investigation showed that a net circulation (with an accelerated flow at the equator) did, in fact, exist. However, because of our inadequate knowledge of turbulent convection, in general, and of convection on a rotating sphere, in particular, it was necessary to treat a thin shell of fluid and to apply second-order perturbation results of convection to the problem. The approach was inadequate for obtaining quantitative results. It was decided to try the same general idea on wind-driven ocean circulation since our (or at least the author's) knowledge of the physical processes involved is more advanced. The subsequent part of this paper is a discussion of the attempt to apply the idea to ocean circulation.

The process that is proposed to provide a mean circulation from a fluctuating force is based on the effects of an externally imposed ordering mechanism, viz., the rotation of the earth. It is also possible to generate mean, or ordered, circulations by resonant interactions of different scales of motion so that a flux of energy, momentum or vorticity to certain scales is selected by internal interaction. Such is the mechanism proposed by HASSELMANN (1961) in a study of waves in deep water. The latter is a weakly interacting non-linear system and the approach is made through perturbation theory.

Although the present problem can also be approached through perturbation theory, there is less reason to do so than in Hasselmann's study. In the first place the mechanism which provides the ordering is clear in our problem. It is a simple matter to show (by perturbation theory) that an ocean basin on a uniformly rotating plane system has no ordered response of the type which is characteristic of a basin on a β-plane or on a sphere. Secondly, at least some of the fluctuating motions in the real ocean are driven by winds with amplitudes considerably larger than that of the mean wind. Hence, amplitude considerations alone indicate a preference for a fully non-linear study. Finally, in paper A it was found that the response of a β-plane ocean is qualitatively different for different ranges of the Rossby number. Here, the Rossby number measures the degree of non-linearity. Hence, a perturbation approach must necessarily exclude systems whose response is essentially non-linear. The drawback to the fully non-linear study is that it must be carried out by numerical methods of some sort and the general parametric dependence must be explored through special cases.

The results of A are reviewed here briefly in order to set the stage for the present study. In A the non-linear, two-dimensional vorticity equation which described the flow of a barotropic fluid on a β-plane ocean of constant depth and driven by a constant (in time) windstress curl of the form, $-\sin x \sin y$, was analysed by a truncated Fourier representation. Two Fourier components in the directions x (for west–east) and y (for south–north) were retained and the truncated system was studied for different sets of values of the Rossby number, R, and the frictional parameter, ε. It was found that qualitatively different solutions occurred for different ranges of the parameters.

For sufficiently large values of ε the ocean responded to the driving wind by developing a single anti-cyclonic gyre which was nearly symmetric about the mid-point of the basin. Essentially this means that the frictional control was so strong that non-linearities and the β-effect were masked by the effect of friction.

For smaller values of ε three types of circulation resulted, depending on the value of R. When R was small (weak non-linear response), the steady circulation which resulted showed a strong westward intensification. The solution was described as being of the Sverdrup type because the essential balance was between the term due to the variation of Coriolis parameter with latitude (the so-called β-term) and the curl of the wind-stress. Such a balance was first described by SVERDRUP (1947). When R was sufficiently large, the non-linear response dominated the behavior of the system and the β-effect was very slight. For an intermediate value of R the ocean responded to the steady wind by fluctuating between states which were described by larger and smaller values of R. The cyclic response was very well-ordered and uniform but it was a function of the initial conditions. When the system was started off with values close to those of the Sverdrup solution, a steady-state Sverdrup solution was achieved.

The range of values of R for which a limit cycle appeared is an interesting one to explore further. In the real ocean the Rossby number based on the smallest-scale motions (i.e., Gulf Stream scales) is close to unity. The range of R for which the limit cycle behavior appeared in the 2×2 study and defined on the basis of the smallest scale of motions also borders on values close to unity. Hence it appears that a possible transient response of the ocean may be explored through a study using a more detailed representation than that of A. It is also apparent that, if the value of the Rossby number is close to the values required for limit-cycle behavior, then a fluctuating wind-stress superimposed on the mean stress could give rise to a response which is at least partially controlled by the dynamics of the limit cycle. Results of such an investigation are described below.

2. Statement of the problem

The non-dimensional form for the vorticity equation in terms of the stream function, ψ, of a barotropic, two-dimensional flow in a square oceanic basin on a β-plane is (cf. A for the derivation)

$$\nabla^2\psi_t + RT(\psi, \nabla^2\psi) + \psi_x = -\varepsilon\nabla^2\psi + \mathrm{curl}_z\boldsymbol{\tau}, \quad (1)$$

where ψ is the dimensionless stream function, subscripts correspond to partial derivatives, t is the dimensionless time, x and y are horizontal coordinates which are positive to the east and north respectively, and $\mathrm{curl}_z\boldsymbol{\tau}$ is the vertical component of the curl of the wind-stress. The Rossby number, R, and the (bottom) frictional parameter, ε, are defined by

$$R = \frac{W}{D\beta^2 L^3}, \quad \varepsilon = \frac{K}{\beta L}, \quad (2)$$

where W is the amplitude of the wind-stress, D is some appropriate depth, β is the measure of the north–south variation of the vertical component of the Coriolis parameter, L is the horizontal length scale of the basin and K is the bottom frictional coefficient. The boundary conditions are

$$\psi = 0 \quad \text{on} \quad x = 0, \pi \quad \text{and} \quad y = 0, \pi. \tag{3}$$

The values of R and ε which would prevail if the observed Gulf Stream were respectively controlled by non-linearities and by friction are

$$R \approx 10^{-4}, \quad \varepsilon \approx 10^{-2}.$$

In the present problem the values of R and ε will be chosen so that

$$R \leqslant \left(\frac{l}{L}\right)^2, \quad \varepsilon \leqslant \frac{l}{L}, \tag{4}$$

where l is the minimum scale describable by the representation used. Thus for the system with four Fourier components in x, $l/L = 1/4$. If this system is to be dominated by friction, then it is necessary that $\varepsilon \approx 1/4$. For dominance of non-linear terms, $R \approx 1/16$ (see Veronis (1964)[1] for a detailed discussion of these points).

The analysis is straightforward. A form for $\operatorname{curl}_z \boldsymbol{\tau}$ is chosen and the stream function is expanded in a double Fourier series

$$\psi = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \sin mx \sin ny, \tag{5}$$

where the a_{mn} are generally functions of time. This form satisfies the boundary conditions (3). The integers, M and N, are some finite integers which determine the degree of representation. It is a straightforward though lengthy procedure to verify that the following equations govern the Fourier coefficients:

$$\begin{aligned}
\dot{a}_{rs} = \frac{R}{4(r^2+s^2)} \Bigg\{ & \sum_{m=1}^{r} \sum_{n=1}^{s} a_{mn} a_{r-m,\, s-n} (n^2+m^2) \\
& \times [ms - nr] \\
& + \sum_{m=1}^{M} \sum_{n=1}^{N} a_{m+r,\, n} a_{m,\, n+s} [2mr + r^2 - 2sn - n^2] \\
& \times [ms + r(s+n)] \\
& - \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} a_{r+m,\, s+n} [nr - ms] \\
& \times [2mr + ns) + r^2 + s^2] \\
& + \sum_{m=1}^{r} \sum_{n=1}^{N} a_{m,\, s+n} a_{r-m,\, n} [m^2 + (s+n)^2] \\
& \times [nr + s(r-m)] \\
& - \sum_{m=1}^{r} \sum_{n=1}^{N} a_{mn} a_{r-m,\, s+n} [m^2+n^2][ms+nr] \\
& + \sum_{m=1}^{M} \sum_{n=1}^{s} a_{mn} a_{m+r,\, s-n} [n^2+m^2][ms+nr] \\
& - \sum_{m=1}^{M} \sum_{n=1}^{s} a_{m+r,\, n} a_{m,\, s-n} [(m+r)^2 + n^2] \\
& \times [ms + r(s-n)] \\
& + \frac{4r}{\pi(r^2+s^2)} \sum{}' \frac{m a_{ms}}{r^2-m^2} - \varepsilon a_{rs} + \text{Driving term.}
\end{aligned} \tag{6}$$

where there are $M \times N$ such equations and Σ' means summing over odd m for even r and even m for odd r.

The method of solution is to express equations (6) in finite difference form and to integrate them numerically using an implicit integration scheme (as described in A). The system is integrated in time from some initial state to a final steady state or, if the response is periodic, through several periods, or, for the case of a fluctuating wind, through a preassigned number of oscillations of the latter.

The principal obstacle in integrating this system is that the number of non-linear terms (those in curly brackets) increase somewhat faster than $M \times N$ so that the system quickly becomes unmanageable because of the large number of terms. Hence, cases with a maximum of 6×6 components were integrated. Details of only the 4×4 results are reported here since the computing time required for 6×6 was much longer and fewer runs with 6×6 components were made. Since only qualitative results are sought the 4×4 case suffices.

3. Specific cases

(A) Steady solutions

Figure 1 shows the steady solution for the stream function for the case $\varepsilon = 0.1$, $R = 0.01$ with the representations of 4×4 and 6×6 and an anti-cyclonic wind-stress of the form $-\sin x \sin y$. It is seen that in each case an anti-cyclonic

[1] Hereafter referred to as B.

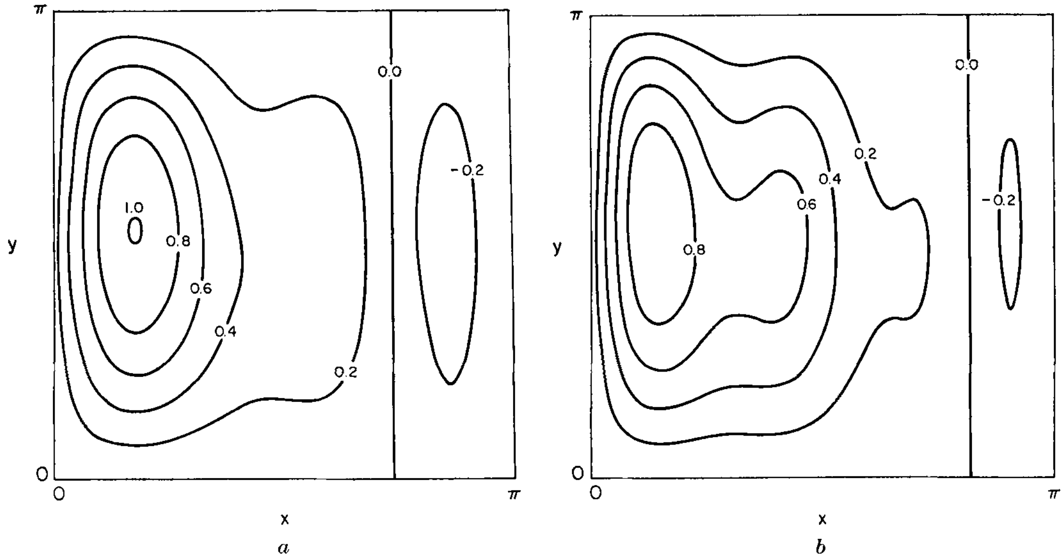


FIG. 1a. Streamline pattern for the 4×4 case with $\varepsilon = 0.1$, $R = 0.01$ and with a mean wind-stress of the form $-\sin x \sin y$. The pattern is normalized and $\psi_{max} = 1.73$.

FIG. 1b. Same as Fig. 1a but with 6×6 components. Here $\psi_{max} = 1.77$.

gyre exists in the western part of the basin and a cyclonic cell exists in the eastern part. The latter is a result of the inadequacy of the representation because the actual solution which should change smoothly from a peak value near the western boundary to zero at the eastern boundary cannot be represented smoothly by as little as six components in the x-direction.[1] This case represents an essentially linear solution with a rather large value for the frictional parameter, ε. (Cf. B for a more detailed discussion of the linear problem.) A noticeable feature in Fig. 1 is that the spurious cyclonic cell in the eastern part of the basin becomes weaker as the representation is improved. It was shown in B that this cyclonic cell vanishes when the representation is sufficiently detailed to represent the linear solution well.

The steady solution for the case $\varepsilon = 0.03$, $R = 0.01$ exhibits another feature peculiar to the Fourier representation. It was pointed out in B for the linear problem that, if the value of the frictional parameter, ε, determines a scale which is smaller than that which can be represented by the number of components available, a spatial oscillation of the stream function results. An equivalent phenomenon occurs here, viz., if *both* ε and R determine scales which are smaller than can be represented, then a spatial oscillation will occur. It is seen that Fig. 2 exhibits a north-south asymmetry with a stronger spatial oscillation in the northern half than in the southern half of the basin. This is the manifestation of non-linear behavior which results from the representation. The point can be made clearer by noting (cf. B) that ε can be interpreted as the ratio of the smallest scale controlled by friction to the scale of the basin. If the system were to be frictionally controlled, then it would be necessary that $\varepsilon \approx 1/N$ where N is the number of Fourier components. The Rossby number, R, is interpreted as the square of the ratio of the smallest scale to the scale of the basin. Thus, if $R \approx 1/N^2$ the system would have strong non-linear behavior. Hence, for the 4×4 case, if $\varepsilon \approx 0.25$ ($R \approx 0.06$) the system would exhibit strong frictional (non-linear) effects. Since, in fact, $\varepsilon = 0.03$ and $R = 0.01$ for the case shown in Fig. 2, neither of the above conditions

[1] This point is not pertinent in what follows. Our concern here is to recognize the different types of solutions as frictional, Sverdrup or non-linear, whether or not the representation admits spurious oscillatory behaviour. We confine our attention to qualitative integral features in the flow patterns and in the time fluctuating case in part (c) of this section.

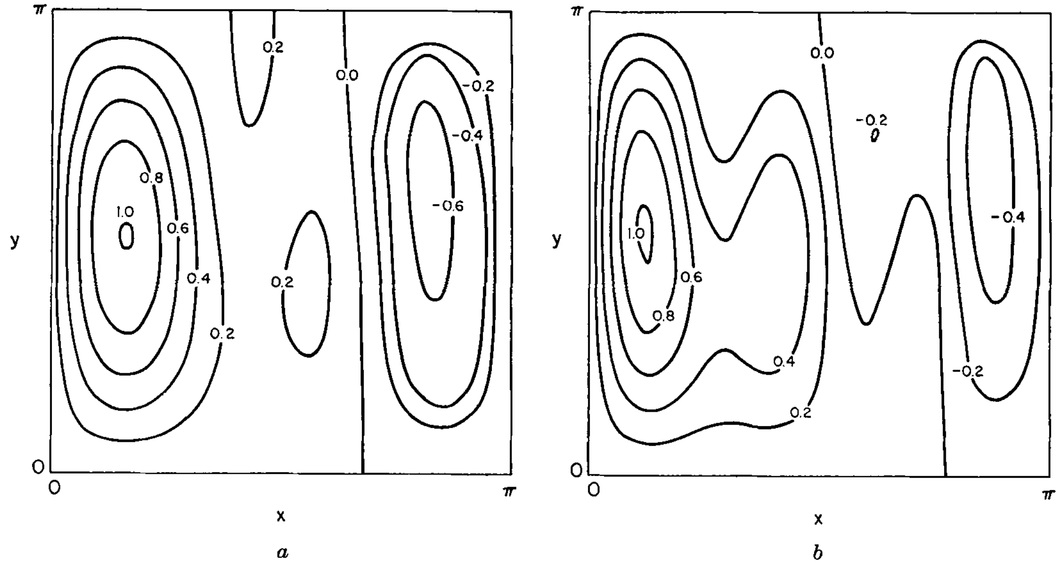


FIG. 2*a*. Same as Fig. 1*a* but with $\varepsilon = 0.03$, $R = 0.01$. $\psi_{max} = 1.56$.

FIG. 2*b*. Same as Fig. 1*b* but with $\varepsilon = 0.03$, $R = 0.01$. $\psi_{max} = 1.68$.

is satisfied. Now, the oscillations acquire an amplitude sufficiently large so that at least one of the two processes is balanced. In the present case the Rossby number is closer (since it varies inversely as the *square* of the wave number) to the value required for dominance of non-linear processes ($R \approx 0.06$) than is the value of ε which would be required for frictional processes to dominate ($\varepsilon \approx 0.25$). Thus non-linear effects are exaggerated. Figure 1 shows an example of frictional control. In any pattern which we call the Sverdrup solution and in which the representation is inadequate either non-linear terms or frictional terms will be exaggerated.

The 4×4 and 6×6 cases reproduce the qualitative behavior exhibited by the 2×2 case. The solutions are again divided into frictionally controlled ones (large ε), Sverdrup types (small ε and small R), non-linear ones (small ε and large R) and limit cycles (small ε and intermediate R). These particular types are not all shown. Figures 1 and 2 show Sverdrup solutions which differ in the manner described in the preceding paragraphs.

(B) LIMIT CYCLE SOLUTIONS

A typical limit cycle solution for the 4×4 case is shown in Fig. 3 with the coefficient of $\sin x \sin y$ plotted against the coefficient of $\sin 2x \sin y$ for the case $\varepsilon = 0.03$, $R = 0.06$. The cycle, shown in three parts for a range of values of t, is repeated after approximately 1200 units in t (about one year in real time). It differs from the 2×2 case in that the oscillations occur between values in phase space which are much closer together than were the corresponding values in the 2×2 case. Thus the amplitude of $\sin x \sin y$ varies between values of 1 and 2 whereas in the 2×2 case it varied between values of -1 and 30. This verifies a second conjecture put forth in A, viz., that as the representation is increased oscillations would take place over a much more confined region of phase space. Furthermore, the values of the Rossby number for which the oscillatory solutions occur decrease with increasing representation. In the 2×2 case the oscillatory solutions occurred for values of R centering about 0.25 ($= 1/N^2$, where N is the maximum wave number of the representation). In the 4×4 case the oscillations take place around $R = 0.06$. It should be kept in mind that in a continuum the oscillatory behavior could conceivably disappear. This possibility can be partially tested by going to a system with many more degrees of freedom.

An additional result of A which has been verified here is that the occurrence of a limit cycle depends on the initial conditions. Thus

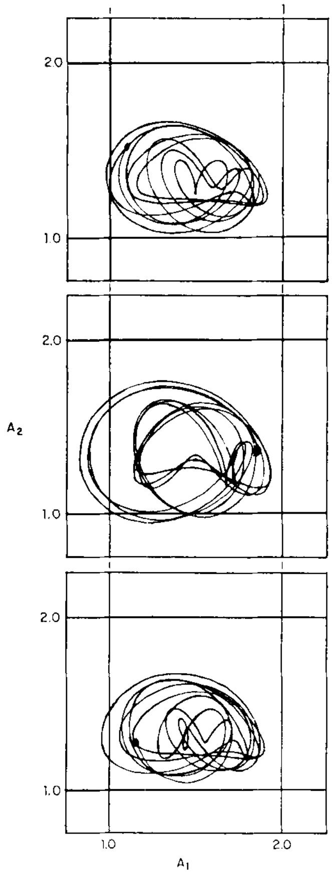


FIG. 3. Part of the limit cycle which occurs for the 4×4 case with $\varepsilon = 0.03$, $R = 0.06$. The three figures show the portions which occur in the ranges $2400 \leqslant t \leqslant 2600$, $2600 \leqslant t \leqslant 2800$, $2800 \leqslant t \leqslant 3000$ respectively. The pattern is repeated after approximately 1200 units in t.

consider the 4×4 case shown in Fig. 3, i.e., $\varepsilon = 0.03$ and $R = 0.06$. When R is reduced to 0.05 and the system is started from rest with a windstress of the form $-\sin x \sin y$, the steady flow pattern shown in Fig. 4*a* results. If this pattern is taken as the initial condition for the case $\varepsilon = 0.03$ and $R = 0.06$ the steady flow pattern shown in Fig. 4*b* is produced. If the case $\varepsilon = 0.03$, $R = 0.06$ is started from rest, the limit cycle shown in Fig. 3 occurs. A time average of the limit cycle circulation produces the mean circulation shown in Fig. 4*c*. This circulation is much more intense than the steady one in Fig. 4*b*. The maximum transport has increased from 1.85 for the steady case to 4.85 for the average periodic flow. The point of maximum transport is shifted to the north and east. This is characteristic of the circulations with stronger non-linear effects.

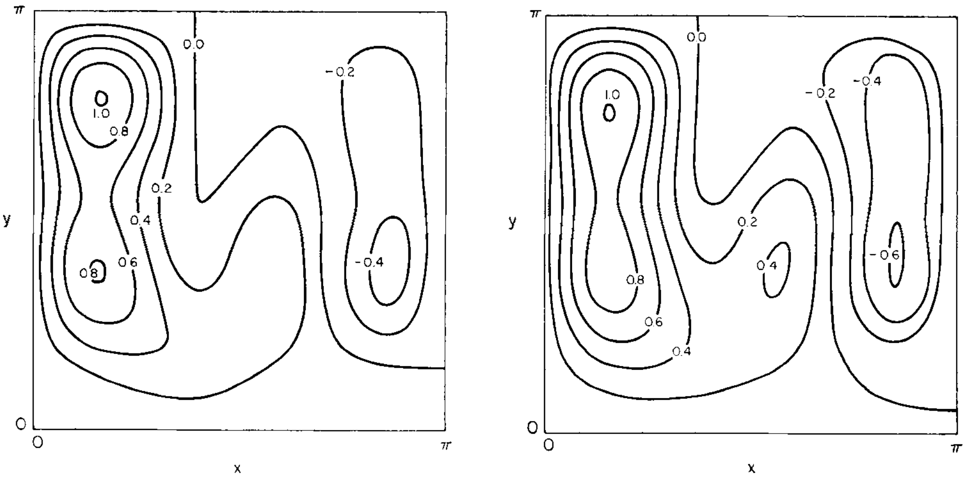
(c) FLUCTUATING WINDS

To test the hypothesis of generating a mean circulation by fluctuating winds several types of computations were carried out. These computations are discussed for the 4×4 case with $\varepsilon = 0.03$, $R = 0.05$ since it is helpful to keep a particular case in mind.

For the first type of computation the following procedure was adopted. The steady solution (shown in Fig. 4*a*) for $\varepsilon = 0.03$ and $R = 0.05$ with a wind-stress curl of the form $-\sin x \sin y$ is taken as an initial condition. Then a wind-stress curl of the form $A \sin 4x \sin 4y \sin \gamma t$ (A is the amplitude of the fluctuating wind-stress relative to the steady wind-stress and γ is the frequency) is superimposed on the steady wind and the time integration is begun. It was observed that after ten periods of the fluctuating wind, the time-averaged solution settled down to a fairly steady value. This average solution was plotted and the value of the maximum transport was determined.

It was found that the results depend very strongly on both the frequency, γ, and amplitude, A, of the fluctuating wind. For example, when $A = 1$ (the amplitudes of steady and fluctuating wind-stresses were equal) there is little effect on the transport. When $A = 2$, the maximum transport, $\psi_{\max}$, is increased by as much as 30 % over the steady value for some values of the frequency, γ. For other values of γ there is a slight decrease in the transport and for still others there is no change. The results are shown in Fig. 5 where the maximum transport is plotted as a function of γ for the case $\varepsilon = 0.03$, $R = 0.05$ and $A = 2$, and in Fig. 6 where the mean circulation pattern is shown for the case where $\gamma = 0.17$. Also shown in Fig. 5 are points giving the value of $\psi_{\max}$ for a few values of γ when $A = 1$.

The range of values of γ correspond to periods of between one and two weeks for the fluctating wind. The graph in Fig. 5 shows a strong dependence of the transport on the frequency with maxima for $A = 2$ occurring for $\gamma = 0.17$ (corresponding to a period of about 12 days) and $\gamma = 0.205$ (corresponding to a period of about 10 days). These results are for the specific form of the fluctuating wind-stress ($= A \sin 4x \sin 4y$



a *b*

FIG. 4*a*. The streamline pattern for the 4×4 case with $\varepsilon = 0.03$, $R = 0.05$, and driven by a steady wind-stress of the form $-\sin x \sin y$. $\psi_{max} = 1.57$.

FIG. 4*b*. The steady streamline pattern for the 4×4 case with $\varepsilon = 0.03$, $R = 0.06$ when the initial conditions are given by the pattern of Fig. 4*a*. $\psi_{max} = 1.85$.

$\sin \gamma t$) and as can be seen from the figure, are strongly dependent on the amplitude, A.

The amplitude and frequency of the fluctuating wind which have been used here are realistic values for storms. However, the distribution in space and time is not realistic. It would be more appropriate to use a moving wind system with some randomness in both the time and space scales. This can be treated more easily by finite-difference methods.

By comparing Figs. 4*a* and 6 one sees that the addition of the fluctuating wind leads to a mean circulation which does not appear to be

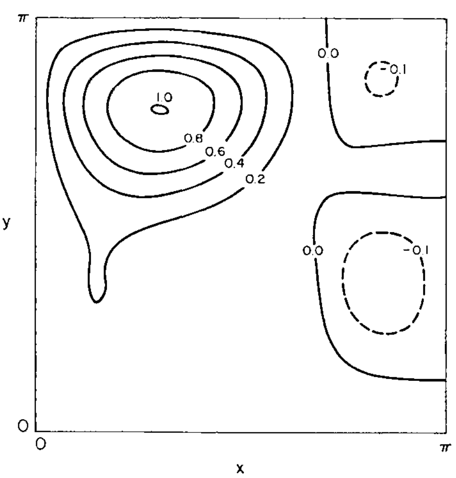


FIG. 4*c*. The time-averaged streamline pattern of the limit cycle for the 4×4 case $\varepsilon = 0.03$, $R = 0.06$ which is obtained when the system is started from rest. The flow is very intense in the north-western corner of the basin. $\psi_{max} = 4.85$.

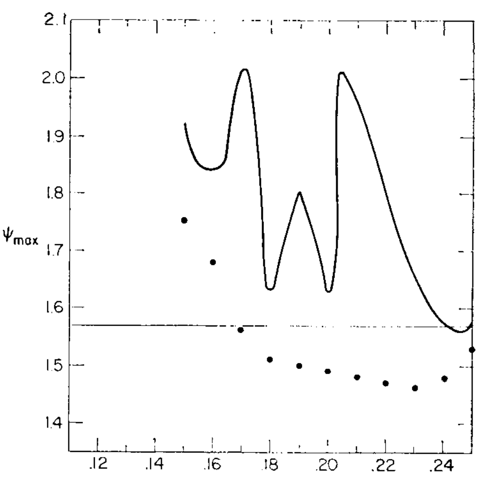


FIG. 5. The maximum value of the transport function, ψ_{max}, for the 4×4 case with $\varepsilon = 0.03$, $R = 0.05$ and with a fluctuating wind of the form $A \sin 4x \sin 4y \sin \gamma t$ superimposed on the mean wind is plotted as a function of γ for the cases $A = 2$ (solid line) and $A = 1$ (points). The light horizontal line at $\psi_{max} = 1.57$ is the value of ψ_{max} when only the mean wind-stress acts on the system.

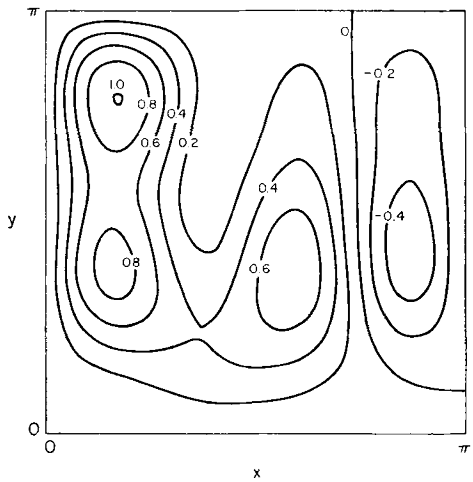


FIG. 6. The time-averaged streamline pattern corresponding to Fig. 5 when $\gamma = 0.17$.

significantly different from that which exists with the steady wind alone. Some smoothing of the pattern of Fig. 4*a* has appeared as a result of the fluctuating winds.

It will be interesting to see the effect of fluctuating stresses on the circulation when many more degrees of freedom (in a finite-difference calculation) are included. The mean square amplitudes and some other statistical feature of the transient circulation should yield useful information. With the present truncated system the results have been confined to those which exhibit qualitative effects such as the change in the transport through the "Gulf Stream".

A second calculation was made to test in a mild way the dependence of the result on the fact that there is total reflection of the waves in this truncated system, i.e., no "window" is available to allow smaller-scale motions to dissipate their energy to even smaller scales. To allow smaller-scale motions to dissipate more rapidly the following test was made.

In the equation for the Fourier coefficient of the term $\sin 4x \sin 4y$ the value of friction (ε) is doubled. The system is driven as before by a steady wind of the form $-\sin x \sin y$. When it has attained a steady state a fluctuating wind of the form $A \sin 4x \sin 3y \sin \gamma t$ is superimposed and the system is averaged over ten periods of the fluctuating wind-stress. The results are compared in Table 1 to the same case with the normal form of friction in all of the equations. The conclusion is that doubling the frictional dissipation in the highest-order equation can change the average transport, in some cases increasing it, in some decreasing it. Hence introducing a "window" in the smaller scales provides no consistent change in the generation of mean circulation by a fluctuating wind.

A final calculation was made with only fluc-

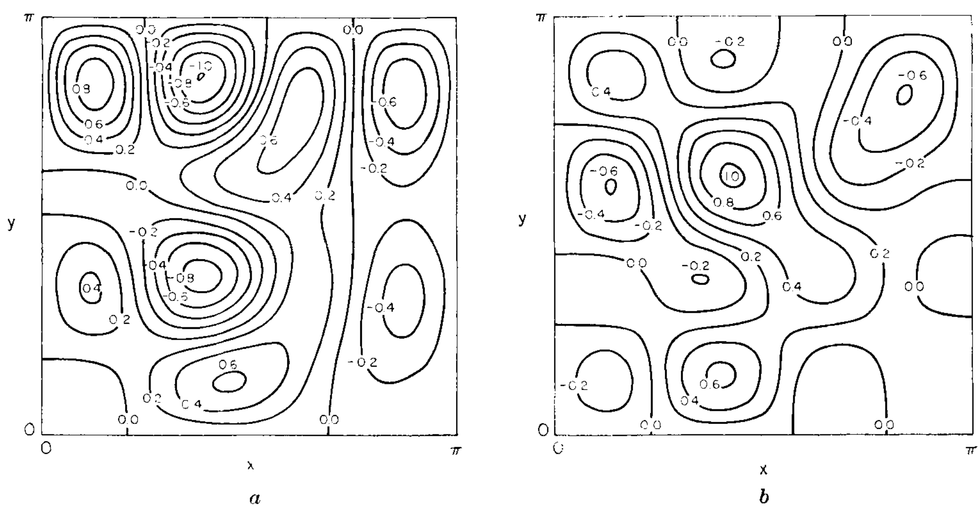


FIG. 7*a*. The time-averaged streamline pattern for the 4×4 case with $\varepsilon = 0.03$, $R = 0.05$ which results from a fluctuating wind-stress (no mean) of the form $2 \sin 4x (\sin 3y + \sin 4y) \sin \gamma t$ with $\gamma = 0.1625$. $\psi_{max} = 0.75$.

FIG. 7*b*. Same as Fig. 7*a* but with $\gamma = 0.2$. $\psi_{max} = 0.45$.

TABLE 1

	ψ_{max} (normal friction)	ψ_{max} (2 in $\sin 4x \sin 4y$ component)
0.1625	2.08	3.57
0.2	2.11	2.01
0.25	1.70	2.0

Table 1 shows the effect of doubling ε in the equation for the Fourier amplitude of $\sin 4x \sin 4y$. With no fluctuating wind and a mean wind in the $\sin 4x \sin 3y$ equation $\psi_{max} = 1.57$ with normal friction and $\psi_{max} = 1.61$ with 2ε in the $\sin 4x \sin 4y$ equation. When a fluctuating stress in the $\sin 4x \sin 3y$ equation is superimposed on the system, the values of ψ_{max} are those recorded above.

tuating wind-stresses in the 4×4 system. Here a fluctuating wind-stress of the form[1] $2(\sin 4x \sin 4y + \sin 4x \sin 3y) \sin \gamma t$ is imposed on a system *initially at rest*. The calculation is carried on until the time-averaged system settles down to more or less steady values. The results for $\varepsilon = 0.03$, $R = 0.05$ are shown in Fig. 7 for two values of γ.

A considerable amount of small-scale structure is apparent in both figures. The north–south asymmetry with stronger spatial variations in the northern half of the basin reflects the non-linearity of the flow. It is interesting that the two cases which differ only in the value of the frequency of the oscillating wind, should show such large differences in their mean circulations. Of course, in the absence of a mean wind the flows show no similarity to the observed circulation in, say, the North Atlantic. The maximum transport in the two cases is one half (for $\gamma = 0.1625$) and one third (for $\gamma = 0.2$) of that which results from a single gyre, steady wind with the same value of the Rossby number. The maximum value of the stream function occurs at different locations in the two cases and both are different from the location of ψ_{max} in Fig. 4*a*.

The ramifications and some speculations based on the present results are included in the final section of the paper.

[1] This form is taken because a single component would always produce a circulation with nodal lines in the same locations as those of the wind. In the β-plane model it is not possible to eliminate imposed nodal lines in y because the non-linear terms generate only harmonics of the basic wind-stress.

4. Conclusions and speculations

This investigation has shown an increase in the mean oceanic transport as a result of fluctuating winds. Since the actual wind systems acting on the oceans are made up of mean winds with fluctuations which are larger than the mean there is some justification for pursuing the model further. Certainly one cannot simply ignore the fluctuating winds nor is it clear how one can parameterize the effect in terms of a steady model. However, the foregoing investigation raises several questions.

Will the results be verified in systems with a considerably more detailed representation or are these results essentially due to the high truncation of the system? The latter is certainly a possibility because it is well known that systems with a discrete number of modes can behave in a manner which is qualitatively different from the behavior of a continuum.

Would a system with a more random fluctuating wind-stress be selective in ordering those fluctuations which tend to increase the transport or would the mean behavior also be more random in its response? Can a barotropic ocean select preferably those moving storm systems which tend to increase the transport through the Gulf Stream? How does the ocean filter out most of the transient behavior in its interior and maintain a Gulf-Stream transport which is relatively constant? And finally, would the added degrees of freedom in a baroclinic system enhance the present result or is it essentially a barotropic phenomenon?

The questions listed above refer to the large-scale circulation. There are also questions which deal with mechanism on a smaller scale and the influence of such mechanisms on the large-scale flow. In our basic model we have incorporated a viscosity coefficient, K, which is supposed to parameterize the effects of some of the smaller-scale motions on the large-scale flow. We also treated explicitly some of the smaller-scale motions, those which are driven by a fluctuating wind. What does such a separation of small-scale effects imply?

Ordinarily, an eddy viscosity coefficient (or a bottom drag coefficient as we have used) is meant to incorporate the effects of smaller-scale motions which are not in themselves critical in determining the large-scale flow. The implication is that instabilities of the large-scale flow result in small-scale turbulence near boundaries or in local regions of high shear. The resultant turbulent eddies act to brake the flow and eventually dissipate themselves into motions of smaller scales until molecular viscosity damps out the very smallest scales which are generated. Throughout such a process the assumption is, of course, that the small scales act as a sink for such properties as the momentum and vorticity of the large-scale flow.

However, when some ordering mechanism exists in the system, it is possible that smaller-scale motions which are forced or which can be generated locally may feed energy, momentum or vorticity into larger-scale motions so that significant changes take place in the large-scale flow as a result. In such a case it is obviously not possible to parameterize the effect by an eddy viscosity since the transfer of properties takes place in a direction opposite to that which is implied by an eddy viscosity. Yet the use of an austausch coefficient for those transient motions which are caused by, say, instabilities of the Gulf Stream or of the interior flow of the ocean may be appropriate and worthwhile—especially since there is no obvious or tractable way of accounting for such motions in a general circulation model.

The dilemma is a real one but there is some hope of arriving at an answer if there is an appropriate separation of scales. In particular, if the scales of the parameterized processes are smaller than those of the transient processes which can send energy to larger scales, then there is some hope for parameterizing the first and treating the second explicitly. If on the other hand the scales are of the same order, there seems to be no obvious way of treating one and parameterizing the other. A good deal of effort, both theoretical and observational, will be necessary to settle the issue.

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