On the maintenance of the kinetic energy of mean zonal flow in the southern hemisphere

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ABSTRACT

Wind data reported at the eight pressure levels (850, 700, 500, 400, 300, 200, 100 and 50 mb) for 121 Southern Hemisphere plus 22 Northern Hemisphere equatorial stations during the calendar year 1958 has enabled us to study the rate of generation of the kinetic energy of the mean zonal flow. The results indicate that the kinetic energy of the mean zonal motion is maintained against frictional dissipation to a large extent through a conversion of transient eddy kinetic energy through the action of the horizontal wind. The generation of zonal kinetic energy by mean meridional motion through the action of the coriolis force cannot be measured well enough, but is probably small as in the Northern Hemisphere. The standing eddy transformation integral appears to be unimportant—a result which is not true for the Northern Hemisphere. If the conversion of the kinetic energy of the transient eddies into the kinetic energy of the mean zonal flow were to cease, the atmosphere of the Southern Hemisphere would be in solid rotation in about 2½ weeks. This assumes a continuation of a normal rate of dissipation during this period.

1. Introduction

In a previous article of this journal KUO (1951) derived the equation of the balance of zonal kinetic energy. Because of the fragmentary observational data available, his computations of the conversion of transient eddy kinetic energy to the kinetic energy of the zonal flow were restricted to the North American continent. STARR (1953) made similar computations and in addition included the term involving the conversion of the kinetic energy of the mean meridional motion to the kinetic energy of mean zonal flow through the coriolis transformation. His computations were for the entire Northern Hemisphere.

In this paper some of the terms involved in the balance equation will be evaluated for the entire Southern Hemisphere and comparison will be made for analogous studies for the Northern Hemisphere. The data employed in this study has been fully discussed in another paper OBASI (1963). To avoid repetition the interested reader is directed to that paper.

The following notation will be employed:

$$\bar{x} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} x\,dt = \text{time average of } x.$$

$$[x] = \frac{1}{2\pi}\int_0^{2\pi} x\,d\lambda = \text{zonal average of } x.$$

$$x' = x - \bar{x} = \text{departure from time average.}$$

$$x^* = x - [x] = \text{departure from zonal average.}$$

Using the above notations, the equation of balance of zonal kinetic energy for the hemisphere of a spherical earth is given by

$$\frac{a^2}{g}\int_{p_G}^{0}\int_0^{\pi/2}\int_0^{2\pi}\frac{\partial}{\partial t}\left(\frac{[\bar{u}]^2}{2}\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (1)$$

$$= -\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\phi}[\overline{u'v'}]\cos^2\varphi + \frac{\partial}{\partial P}[\overline{u'\omega'}]\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (2)$$

$$-\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi + \frac{\partial}{\partial P}[\bar{u}^*\bar{\omega}^*]\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (3)$$

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$$+\frac{a}{g}\iiint[\bar{u}]^2[\bar{v}]\sin\varphi\, d\lambda\, d\varphi\, dp \qquad (4)$$

$$-\frac{a}{2g}\iint[\bar{u}]^2[\bar{v}]\cos\varphi\, d\lambda\, dp\Big| \qquad (5)$$

evaluated at the equator

$$+\frac{a^2}{g}\iiint f[\bar{u}]\,[\bar{v}]\cos\varphi\, d\lambda\, d\varphi\, dp \qquad (6)$$

$$-\frac{a^2}{g}\iiint[\bar{u}]\,[T_\lambda]\cos\varphi\, d\lambda\, d\varphi\, dp. \qquad (7)$$

In the above equation

a = radius of the earth
g = acceleration due to gravity
λ = longitude
φ = latitude
P = pressure
P_G = pressure at the ground
t = time
u $= a\cos\varphi(d\lambda/dt)$ = eastward component of the wind
v $= a(d\varphi/dt)$ = northward component of the wind
ω $= dp/dt$ = individual pressure change
Ω = angular velocity of the earth
f $= 2\Omega\sin\varphi$ = Coriolis parameter
T_λ = eastward component of viscous force per unit mass

In the long term average the left-hand side of the equation vanishes since there is no progressive increase or decrease in the kinetic energy of the mean zonal flow for the hemisphere. We now evaluate each of the terms involved in the right-hand side of the equation.

2. Evaluation of the integral

$$\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}\overline{[u'v']}\cos^2\varphi + \frac{\partial\overline{[u'\omega']}}{\partial p}\right)\cos\varphi\, d\lambda\, d\varphi\, dp.$$

This term represents the conversion of transient eddy kinetic energy to the kinetic energy of the zonal flow. At present there is no available data of vertical motion in the Southern Hemisphere. This data may be indirectly obtained from enthalpy study of the Southern Hemisphere (see for example JENSEN, 1961). However preliminary studies in the Northern Hemisphere, as well as theoretical considerations, indicate that

$$\frac{a^2}{g}\iiint[\bar{u}]\frac{\partial}{\partial p}\overline{[u'\omega']}\cos\varphi\, d\lambda\, d\varphi\, dp \qquad (2.1)$$

is much smaller than

$$\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}\overline{[u'v']}\cos^2\varphi\right)\cos\varphi\, d\lambda\, d\varphi\, dp. \qquad (2.2)$$

Because of this reason and since vertical motions are unavailable, we shall evaluate only the latter term. The integral

$$\frac{a^2}{g}\iiint\left(\frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}\overline{[u'v']}\cos^2\varphi\right)d\lambda\, d\varphi\, dp \qquad (2.3)$$

$$= -\frac{a^2}{g}\iiint\overline{[u'v']}\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda\, d\varphi\, dp \qquad (2.4)$$

$$+\frac{a^2}{g}\iiint\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\overline{[u'v']}\cos^2\varphi\right)d\lambda\, d\varphi\, dp. \qquad (2.5)$$

The integral (2.5) is

$$\int\frac{2\pi a}{g}[\bar{u}]\,\overline{[u'v']}\cos\varphi\, dp \qquad (2.6)$$

evaluated at the equator. All the three integrals, namely (2.3), (2.4) and (2.6) were evaluated by finite difference approximation.

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Table 1 shows the integrand of (2.3). When the computations are performed by levels and then integrated over the entire hemisphere, we obtain a conversion of transient eddy kinetic energy to zonal kinetic energy. This value is 9.63×10^{20} ergs/sec.

When use is made of vertically-averaged relative angular velocity and vertically averaged gradient of transient eddy momentum flux, the integral (2.3) gives 9.89×10^{20} ergs/sec.

Use of the 500-mb data alone for the relative

TABLE 1. *Values of the term* $\frac{[\bar{u}]}{\cos\varphi}\frac{\partial}{\partial\varphi}\overline{[u'v']}\cos^2\varphi$ *in winter 1958.*

Multiply by 4.09×10^{17} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.58	2.27	20.95	41.25	29.91	11.46	1.30	—	78.74
75–70	17.98	26.63	32.80	59.46	33.20	13.60	9.79	—	168.28
70–65	−3.74	40.46	21.88	16.55	33.67	10.69	4.92	−0.94	121.00
65–60	−2.33	−9.59	−39.25	−69.92	−29.91	−23.33	−10.66	−0.81	−201.68
60–55	15.58	−6.97	−153.49	−151.89	−95.33	−26.66	−21.53	−4.81	−482.75
55–50	23.29	−58.38	−247.29	−219.62	−164.84	−52.98	−41.86	−12.24	−838.32
50–45	3.22	−67.53	−169.50	−203.22	−175.60	−74.49	−49.77	−19.97	−840.32
45–40	0.22	−61.36	−194.30	−232.10	−143.02	−66.12	−22.64	−23.10	−806.05
40–35	−6.24	−19.54	−191.20	−267.50	−129.08	−49.72	−3.20	2.69	−681.24
35–30	−7.60	−5.66	−24.31	−12.29	4.11	−18.61	−3.28	−2.04	−80.67
30–25	−0.39	31.72	108.81	175.45	37.27	10.74	−1.06	−0.02	359.25
25–20	0	49.37	194.81	147.76	97.88	17.13	1.62	0.34	506.76
20–15	2.52	25.54	124.64	94.23	37.52	13.85	0.31	0.63	300.16
15–10	−1.14	10.16	45.87	32.93	4.74	1.04	−4.43	−0.41	83.25
10–5	0.16	0.51	5.69	−3.12	−8.12	−5.48	−5.02	−5.80	−35.10
5–0	−2.06	0.55	−13.45	−14.37	−13.61	−7.19	−1.68	−6.14	−70.79
Hemisphere	9.63×10^{20} ergs sec^{-1}								

angular velocity and transient eddy momentum shear, assuming these to be representative of the average for the atmosphere resulted in the quantity 10.02×10^{20} ergs sec^{-1}.

Table 2 shows the integrand of (2.4). When the computation is performed by levels and then integrated throughout the mass of the hemisphere we obtain 9.41×10^{20} ergs sec^{-1}.

The use of vertically-averaged momentum transport and a vertically-averaged shear in

TABLE 2. *Values of the term* $\overline{[u'v']}\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0	0	0.35	−0.31	−1.20	−0.39	0.07	—	−1.22
75–70	0.21	0.29	1.39	2.30	0.69	0.74	0.94	—	8.88
70–65	0.25	0.98	3.06	4.67	3.77	2.35	1.75	1.70	20.75
65–60	−0.03	0.42	1.73	3.51	3.34	1.49	1.66	1.40	17.15
60–55	−0.57	−0.89	0.02	0.10	0.78	−0.01	0.56	0.76	2.48
55–50	−1.96	−0.96	6.25	4.06	0.60	0.07	−0.03	0.11	9.02
50–45	−2.34	2.11	13.68	12.33	3.09	1.16	1.03	0.09	32.67
45–40	−1.68	2.17	−1.30	−3.87	7.52	3.70	2.92	1.29	16.28
40–35	−0.72	−1.90	−18.47	−14.21	4.45	4.57	3.57	2.12	−12.32
35–30	0.07	−1.04	−12.15	−7.43	5.70	4.88	3.34	1.83	2.67
30–25	0.34	3.63	22.67	19.45	11.97	5.06	2.53	1.56	72.59
25–20	0.25	3.69	31.06	28.83	12.16	4.97	2.20	1.40	89.46
20–15	0.34	0.55	17.11	21.64	11.93	5.04	2.09	1.77	66.54
15–10	0.28	−1.17	5.28	9.46	6.61	2.67	0.90	0.04	26.34
10–5	−0.62	−1.21	−0.95	1.88	2.95	0.58	0.17	−0.55	2.44
5–0	−0.80	−0.97	−3.97	−0.70	0.40	−0.03	0.02	−0.18	6.01
Hemisphere	9.41×10^{20} ergs sec^{-1}								

TABLE 3. *Values of the terms shown below at the equator in winter 1958.*

Units are in $m^3\ sec^{-3}$.

Pressure in mb	$[\bar{u}]\,[\overline{u'v'}]$	$[\bar{u}]\,[\bar{u}^*\bar{v}^*]$
50	−8.78	−0.44
100	−5.56	−3.00
200	−44.09	−8.13
300	−18.24	−0.32
400	5.19	0.47
500	−4.94	−1.10
700	0.36	−1.54
850	4.26	−0.07
Vertical integral 10^{20} ergs sec^{-1}	−0.27	−0.06

evaluating the same integral gives 9.57×10^{20} ergs sec^{-1}.

If one uses the 500-mb data alone for the momentum flux and shear, assuming them to be representative of the average for the hemisphere one obtains 9.61×10^{20} ergs sec^{-1}.

Table 3 shows the values of

$$\frac{2\pi a}{g}[\bar{u}]\,[\overline{u'v'}]\cos\varphi$$

at the equator. Computations by levels of this expression gives a vertical integral of 0.27×10^{20} ergs sec^{-1}. If one uses the vertically-averaged transient eddy momentum flux and the vertically-averaged zonal motion at the equator, one obtains 0.13×10^{20} ergs sec^{-1}. If one assumes the 500-mb level data to be representative of the entire column of the atmosphere above the equator, one obtains 0.20×10^{20} ergs sec^{-1}.

Assuming that the integration by levels is much more accurate one obtains 9.63×10^{20} ergs sec^{-1} for integral (2.3) and 9.68×10^{20} erg sec^{-1} for the sum of integrals (2.4) and (2.5). A difference of 0.08×10^{20} erg sec^{-1} is the difference in the truncation error.

B. Summer

Table 4 shows the integrand of (2.3). When one integrates over the entire mass of the hemisphere one obtains 9.72×10^{20} ergs sec^{-1}.

If use is made of the vertically-averaged relative angular velocity and vertically-averaged shear of transient eddy momentum flux, the integral (2.3) becomes 9.04×10^{20} ergs sec^{-1}.

If one assumes that the 500-mb relative velocity and momentum flux shear are representative of the average for the entire atmosphere, the integral results in 8.53×10^{20} ergs sec^{-1}.

Table 5 shows the integrand of (2.4). Using this table which gives the integrand by level

TABLE 4. *Values of the term* $\dfrac{[\bar{u}]}{a\cos\varphi}\dfrac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi$ *in summer 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	1.02	0.46	1.26	1.72	0.90	0.26	−0.22	—	5.15
75–70	1.86	1.52	1.08	1.64	1.36	0.81	−0.10	—	7.72
70–65	1.38	0.64	−4.77	−5.72	−0.69	−0.61	−0.08	−0.19	−11.15
65–60	0.75	−4.16	−15.60	−15.33	−5.10	−4.02	−1.85	−0.07	−48.02
60–55	−3.03	−8.27	−29.53	−24.49	−11.51	−7.82	−5.17	−2.10	−99.50
55–50	3.15	−9.20	−42.13	−33.34	−15.98	−8.34	−7.51	−6.41	−141.08
50–45	−0.71	−5.69	−39.16	−34.42	−17.07	−4.85	−1.92	−5.35	−118.12
45–40	−0.13	−0.54	−30.30	−23.46	−9.12	−2.45	−0.73	0.31	−67.63
40–35	0.06	−5.01	−22.34	−1.62	−1.96	−2.94	−1.18	2.37	−30.78
35–30	−0.52	−8.62	6.22	13.48	1.71	−1.60	−0.40	0.73	13.10
30–25	−0.48	−4.27	26.23	14.52	1.82	−0.91	0.47	0	38.47
25–20	−0.37	2.09	26.47	11.42	6.66	−0.11	−0.28	0.19	44.76
20–15	0.03	3.19	15.54	7.07	2.85	0.58	0	0.27	29.35
15–10	0.88	1.11	5.70	2.72	1.23	0.58	−0.27	0.06	11.52
10–5	2.39	0.11	−0.54	0.25	0.19	−0.38	−0.45	−0.89	−1.59
5–0	−0.26	0.08	−0.63	−0.02	−0.14	−0.81	−0.44	−0.84	−4.79
Hemisphere	9.72×10^{20} ergs sec^{-1}								

TABLE 5. *Values of the term* $\overline{[u'v']} \cos^2 \varphi \frac{\partial}{\partial \varphi}\left(\frac{[\bar{u}]}{a \cos \varphi}\right)$ *in summer 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.07	0.03	0.58	1.02	0.27	0.26	0.08	—	2.25
75–70	0.20	0.23	1.09	2.29	0.99	0.79	0.23	—	6.10
70–65	−0.07	0.48	0.56	1.64	1.66	0.93	1.03	0.61	8.73
65–60	−0.61	0.06	−0.98	0.55	0.68	0.36	0.48	1.19	2.79
60–55	0.76	0.22	−1.19	−0.89	−0.51	−0.99	−1.10	0.07	6.24
55–50	0.15	1.18	4.08	2.61	−0.87	−0.67	−0.55	−1.07	2.44
50–45	0.94	2.76	9.73	6.22	0.68	0.95	2.74	0.80	27.41
45–40	1.12	1.64	9.74	16.23	5.87	0.46	5.07	3.58	57.65
40–35	0.96	1.61	10.14	15.00	8.20	5.32	4.81	3.70	59.99
35–30	0.63	3.51	12.90	13.74	8.47	5.50	4.01	1.97	57.91
30–25	0.18	7.58	24.30	15.03	8.74	5.25	3.17	1.12	69.82
25–20	−0.01	8.50	18.48	9.27	5.83	4.43	1.74	0.59	50.95
20–15	0.01	5.61	9.16	4.12	2.93	2.99	0.93	0.13	26.83
15–10	−0.07	2.33	2.35	0.57	1.49	2.05	0.32	−0.05	9.63
10–5	0.95	0.50	0.45	−0.35	0.43	0.94	0.18	0.10	2.13
5–0	−1.70	−0.07	0.55	−0.16	0.02	0.25	0.01	0.13	0.23
Hemisphere	9.86×10^{20} ergs sec^{-1}								

one obtains a value of 9.86×10^{20} erg sec^{-1}. If use is made of the vertically-averaged transient eddy flux of momentum and vertically-averaged shear of relative angular velocity, the integral (2.4) becomes 8.99×10^{20} ergs sec^{-1}.

Use of the 500-mb data alone for the transient eddy flux of momentum and relative velocity shear gives a value of 8.58×10^{20} ergs sec^{-1}.

Table 6 shows the values of

$$\frac{2\pi a}{g}[\bar{u}]\,\overline{[u'v']} \cos \varphi$$

at the equator. Computation by level of this expression gives a vertical integral of 0.03×10^{20} ergs/sec. If one uses a vertically-averaged transient eddy flux of momentum and a vertically-averaged zonal motion at the equator, one obtains -0.03×10^{20} ergs sec^{-1}. The use of the 500-mb data alone, assuming this to be representative of the average of the atmospheric column over the equator, gives a value of 0.10×10^{20} ergs sec^{-1}.

Assuming the computation by levels to be more representative, one obtains a value of 9.72×10^{20} ergs sec^{-1} for the left hand side of (2) and a value of 9.89×10^{20} ergs sec^{-1} for the right hand side. The difference -0.17×10^{20} ergs sec^{-1} is the difference in the truncation errors.

C. APPROXIMATION OF THE TERM FOR THE NORTHERN HEMISPHERE

$$\frac{a^2}{g}\iiint \overline{[u'v']} \cos^2 \varphi \frac{\partial}{\partial \varphi}\left(\frac{[\bar{u}]}{a \cos \varphi}\right) d\lambda\, d\varphi\, dp.$$

In order to have better appreciation for the generation of zonal kinetic energy by the transient eddies, we include in this section the results obtained in the Northern Hemisphere.

This integral has been evaluated by using Buch's yearly values of $[\bar{u}]$ and $\overline{[u'v']}$ for the Northern Hemisphere.

TABLE 6. *Values of the terms shown below at the equator in summer 1958.*

Units are in $m^3\ sec^{-3}$.

Pressure in mb	$[\bar{u}]\,\overline{[u'v']}$	$[\bar{u}]\,[\bar{u}^*\bar{v}^*]$
50	0.39	−0.05
100	−2.13	−0.87
200	−5.12	−12.08
300	0.70	−0.71
400	−0.71	−1.75
500	0	−0.36
700	−1.59	0.69
850	0	6.03
Vertical integral 10^{20} ergs sec^{-1}	−0.03	−0.00

When the integration was performed by levels a value of 4.84×10^{20} ergs sec^{-1} was obtained. This value is to be compared with the mean of summer and winter for the Southern Hemisphere, namely, 9.63×10^{20} ergs sec^{-1}.

If use is made of the vertically-averaged transient eddy momentum flux and vertically-averaged shear of the relative angular velocity, one obtains 4.55×10^{20} ergs sec^{-1}, the corresponding value for the Southern Hemisphere is 9.28×10^{20} ergs sec^{-1}.

If one uses the 500-mb data alone, and assumes this to be representative of the mean for the atmosphere, then one obtains a value of 4.95×10^{20} ergs sec^{-1} for the Northern Hemisphere, as compared to 9.09×10^{20} ergs sec^{-1} for the Southern Hemisphere.

Using the following sources of data:

(*a*) the first six months of the year 1950,

(*b*) the second six months of the year 1950,

(*c*) One month of data (January 1949) published by MINTZ, STARR (1953) evaluated an integral approximately similar to (2.4) for the Northern Hemisphere. Using vertically averaged winds and transports, the data (*a*) gives 4.2×10^{20} ergs sec^{-1} the data (*b*) gives 4.6×10^{20} ergs sec^{-1} and (*c*) gives 10.5×10^{20} ergs sec^{-1}.

Evaluation of the integrand at individual levels before integration yields for data (*a*) 3.9×10^{20} data (*b*) 5.5×10^{20} and data (*c*) 9.8×10^{20} ergs sec^{-1}.

Starr also used more elaborate methods in evaluation of the integral, namely by days and levels, but the results were similar to the above values.

3. Evaluation of the integral

$$\frac{a^2}{g}\iiint [\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi + \frac{\partial}{\partial P}[\bar{u}^*\bar{\omega}^*]\right)\cos\varphi\, d\lambda\, d\varphi\, dp.$$

This term represents the conversion of standing eddy kinetic energy to the kinetic energy of zonal motion. From BURGER'S (1958) scale analysis we conclude that

$$\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi \quad \text{and} \quad \frac{\partial}{\partial p}[\bar{u}^*\bar{\omega}^*]$$

are of the same order of magnitude. Although it is realised that by using the continuity equation one can obtain the $\bar{\omega}$ and $\bar{\omega}^*$ and so compute the vertical transport of zonal momentum by the standing eddies, because of the labour

TABLE 7. *Values of the term* $\frac{[\bar{u}]}{\alpha\cos\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	1.74	1.42	0.01	−0.47	−0.52	−0.56	−0.11		0.43
75–70	4.11	1.70	0.78	−0.09	−0.42	−0.31	−0.11		4.05
70–65	2.00	0.46	1.70	0.23	0.01	0.64	0.44	−0.15	5.42
65–60	−5.38	−0.40	−0.28	−0.95	0.20	1.35	0.27	0.41	−2.57
60–55	−5.76	−5.34	−6.04	1.02	0.62	0.03	−0.37	0.21	−12.87
55–50	6.29	1.46	4.94	3.55	3.12	2.31	1.24	0.34	23.83
50–45	5.11	0.87	0.63	−1.93	3.30	0.10	0.41	−0.13	7.07
45–40	−0.24	−0.31	−6.66	−1.80	−4.24	−0.33	−0.67	−0.93	−16.87
40–35	−1.96	0.23	−4.18	−1.53	−4.05	−1.44	0.15	−0.06	−13.10
35–30	−0.55	1.94	10.03	0.23	−0.01	0.05	1.66	0.28	14.91
30–25	−0.07	3.12	7.01	2.80	2.40	−0.29	−0.35	0.00	13.43
25–20	0.00	0.63	−6.96	0.23	0.31	−0.66	−0.68	0.12	−7.87
20–15	−0.10	−0.35	0.21	−0.19	−0.26	0.06	−0.01	0.05	−0.39
15–10	0.07	−0.49	2.76	0.08	0.10	0.03	−0.41	1.47	5.28
10–5	−0.01	−0.13	−0.08	0.01	0.00	−0.15	−0.43	−2.00	−5.66
5–0	−0.06	−0.14	0.07	0.01	0.14	0.18	0.54	0.22	1.80
Hemisphere	-0.44×10^{20} ergs sec^{-1}								

TABLE 8. *Values of the term* $[\bar{u}^*\bar{v}^*] \cos^2 \varphi \dfrac{\partial}{\partial \varphi}\left(\dfrac{[\bar{u}]}{a \cos \varphi}\right)$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.16	0.02	0.00	0.00	0.12	0.11	−0.06		0.35
75–70	0.72	0.30	0.04	−0.06	−0.07	−0.20	−0.42		−0.04
70–65	0.79	0.50	0.33	−0.08	−0.31	−0.43	−0.44	−0.78	−1.09
65–70	0.00	0.23	0.35	−0.11	−0.26	−0.10	−0.26	−0.15	−0.58
60–55	−0.35	−0.19	0.00	0.00	−0.07	0.00	−0.14	0.20	−0.25
55–50	−0.64	0.10	0.15	−0.26	−0.04	−0.08	−0.02	0.16	−0.34
50–45	−2.74	−0.26	−0.53	−0.47	−0.34	−0.27	−0.03	−0.05	−4.15
45–40	−2.71	−0.13	−0.02	0.01	−0.41	−0.38	−0.02	−0.12	−3.43
40–35	−1.05	0.07	−1.19	−0.25	0.09	−0.16	0.05	0.04	−2.16
35–30	−0.18	0.09	−0.50	−0.15	0.27	−0.02	−0.32	−0.06	−1.18
30–25	0.05	−0.78	−0.50	0.21	0.34	0.01	−0.42	−0.13	−1.52
25–20	0.09	−1.97	−0.65	−0.08	0.07	0.18	−0.02	0.03	−1.76
20–15	0.04	−1.52	0.67	−0.07	0.13	0.39	0.35	0.22	1.31
15–10	−0.01	−1.07	−0.89	−0.06	0.06	0.13	0.08	0.01	−1.34
10–5	−0.03	−0.37	−1.40	−0.05	0.06	−0.11	−0.11	−0.16	−2.54
5–0	−0.02	−0.34	−1.01	−0.02	0.00	−0.05	−0.05	0.02	−1.43
Hemisphere	-0.53×10^{20} ergs—sec^{-1}								

involved and the probable insignificance of the standing eddies in the Southern Hemisphere, no effort has been made to compute

$$\frac{\partial}{\partial p}[\bar{u}^*\bar{\omega}^*].$$

The integral

$$\frac{a^2}{g}\iiint\left(\frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi\right)d\lambda\, d\varphi\, dp \quad (3.1)$$

$$= -\frac{a^2}{g}\iiint[\bar{u}^*\bar{v}^*]\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda\, d\varphi\, dp \quad (3.2)$$

$$+\frac{a^2}{g}\iiint\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi\right)d\lambda\, d\varphi\, dp. \quad (3.3)$$

Integral (3.3) is equivalent to the value of

$$\int\frac{2\pi a}{g}[\bar{u}][\bar{u}^*\bar{v}^*]\cos\varphi\, dp$$

at the equator.

Three different methods were again used in evaluating the above integrals.

(*a*) Computing the integrand at each level and finally integrating throughout the entire mass of the hemisphere.

(*b*) Using the vertically-averaged standing eddy transport of momentum and the vertically-averaged relative angular velocity.

(*c*) Using the 500-mb data alone and assuming this to be representative of the mean for the atmosphere of the hemisphere.

A. WINTER

Table 7 shows the integrand of (3.1). When use is made of method (*a*) one obtains -0.44×10^{20} ergs sec^{-1}. Method (*b*) gives -0.13×10^{20} ergs sec^{-1}, while (*c*) gives -0.27×10^{20} ergs sec^{-1}.

Using the same methods for integral (3.2) (see Table 8) method (*a*) gives -0.53×10^{20} ergs sec^{-1}; (*b*) gives -0.16×10^{20} ergs sec^{-1} and (*c*) gives -0.23×10^{20} ergs sec^{-1}.

The integral (3.3) gives for these three methods the values 0.06, 0.05 and 0.04×10^{20} ergs sec^{-1} respectively.

B. SUMMER

Table 9 shows the integrand of (3.1). When use is made of method (*a*) one obtains -0.05×10^{20} ergs sec^{-1}. Method (*b*) gives -0.31×10^{20} ergs sec^{-1}, while (*c*) gives -1.51×10^{20} ergs sec^{-1}.

TABLE 9. *Values of the term* $\frac{[\bar{u}]}{a \cos \varphi} \frac{\partial}{\partial \varphi} [\bar{u}^* \bar{v}^*] \cos^2 \varphi$ *in summer 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	−0.09	−0.06	−0.40	−0.27	−0.29	−0.11	0.09		−1.14
75–70	0.69	0.22	−0.23	0.05	−0.33	−0.43	0.07		−0.41
70–65	1.93	0.46	1.21	1.34	0.22	−0.78	−0.07	0.17	3.40
65–60	−0.80	−0.83	0.56	−0.45	1.55	−1.15	0.67	0.06	−0.09
60–55	−2.53	0.57	0.40	1.49	2.08	2.27	0.68	−0.09	6.89
55–50	−1.53	1.28	2.50	5.72	3.00	4.26	0.44	−0.37	17.36
50–45	0.43	1.74	9.43	9.97	1.21	5.83	1.21	−0.34	32.34
45–40	0.15	0.60	4.26	0.81	−1.75	0.01	0.58	0.21	5.39
40–35	0.00	−1.52	−9.36	−6.73	−2.73	−1.14	0.18	−0.10	−21.60
35–30	−0.14	0.88	−9.34	−10.50	−2.85	−1.48	0.54	0.19	−22.99
30–25	−0.49	−0.24	−10.20	−3.53	−2.69	−1.29	−0.53	0.01	−19.81
25–20	0.48	−2.20	3.87	0.66	−0.54	−0.18	−0.18	−0.01	2.11
20–15	1.08	−0.44	0.56	0.07	0.35	−0.02	0.00	0.07	1.58
15–10	0.40	0.17	−1.11	0.55	0.80	0.07	−0.05	0.12	0.96
10–5	−0.65	0.17	−1.91	−0.09	0.20	0.00	−0.16	0.21	−1.97
5–0	−0.24	−0.17	−0.76	−0.01	−0.05	−0.09	0.03	0.53	−0.01
Hemisphere	-0.52×10^{19} ergs—sec^{-1}								

Using the same methods for integral (3.2) (see Table 10), method (*a*) gives -0.07×10^{20} ergs sec^{-1}; (*b*) gives -0.16×10^{20} ergs sec^{-1} and (*c*) gives -1.61×10^{20} ergs sec^{-1}.

The integral (3.3) gives for these three methods the values 0.00, -0.10 and 0.02×10^{20} ergs sec^{-1} respectively.

The yearly (summer plus winter) means of the interaction between the standing eddies and the mean zonal flow are respectively -0.25, -0.22 and -0.89×10^{20} ergs sec^{-1} with methods (*a*), (*b*) and (*c*). These results indicate that the standing eddies do not play a significant role in the maintenance of the mean zonal flow.

TABLE 10. *Values of the term* $[\bar{u}^* \bar{v}^*] \cos^2 \varphi \frac{\partial}{\partial \varphi} \left(\frac{[\bar{u}]}{a \cos \varphi} \right)$ *in summer 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.03	0.01	−0.19	−0.37	−0.17	−0.20	0.04		−0.97
75–70	0.04	0.02	−0.34	−0.53	−0.40	−0.45	−0.16		−2.06
70–65	−0.03	0.09	−0.18	−0.27	−0.61	−0.68	−0.59	−0.08	−3.14
65–60	−0.28	0.02	0.00	−0.07	−0.30	−1.16	−0.85	−0.10	−3.88
60–55	−0.12	−0.02	0.01	0.00	0.04	−0.86	−0.19	0.05	−1.56
55–50	0.47	−0.11	−0.11	−0.16	0.14	−0.09	0.00	−0.03	−0.06
50–45	0.70	−0.37	−0.72	−0.81	−0.11	−0.15	−0.13	0.04	−1.76
45–40	0.46	−0.28	−1.04	−2.35	−0.60	−1.20	−0.46	0.16	−6.08
40–35	0.33	−0.21	−0.71	−1.51	−0.37	−1.09	−0.53	0.17	−4.67
35–30	0.26	−0.28	−0.03	−0.05	0.20	−0.61	−0.57	0.07	−1.64
30–25	0.05	−0.56	2.09	1.80	1.01	−0.05	−0.38	−0.04	3.69
25–20	0.00	−0.07	3.08	2.07	1.36	0.23	−0.01	−0.03	6.72
20–15	−0.01	0.53	1.97	1.59	1.08	0.21	0.14	0.00	5.59
15–10	−0.42	0.36	1.76	0.96	0.59	−0.12	0.09	0.00	3.22
10–5	−0.41	0.02	1.64	0.43	−0.04	−0.04	0.05	0.03	1.95
5–0	−0.02	−0.01	1.51	0.18	−0.22	0.00	0.03	0.20	1.96
Hemisphere	-0.70×10^{19} ergs—sec^{-1}								

Buch's yearly data of $[\bar{u}^*\bar{v}^*]$ and $[\bar{u}]$ for the Northern Hemisphere give for the integral (3.1) the value 0.88×10^{20} ergs sec^{-1} for method (a), 0.80×10^{20} ergs sec^{-1} for method (b) and 1.02×10^{20} by method (c).

We therefore arrive at the important conclusion that while the standing eddies play an insignificant role in the maintenance of the kinetic energy of zonal motion in the Southern Hemisphere, the generation of the zonal kinetic energy by these eddies in the Northern Hemisphere is of significance.

The intergrals 4 to 6 involve the mean meridional motion. Owing to the difficulty in the measurement of $[\bar{v}]$ because of the presence of spurious effects, it was decided that the values could not be used in further computations.

4. Evaluation at the equator of the integrals

$$\frac{a}{g}\iint [\bar{u}]\,[\overline{u'v'})\cos\varphi\, d\lambda\, dp \qquad (4.1)$$

and

$$\frac{a}{g}\iint [\bar{u}]\,[\bar{u}^*\bar{v}^*]\cos\varphi\, d\lambda\, dp. \qquad (4.2)$$

These two integrals evaluated at the equator are a measure of interhemispheric exchanges of zonal kinetic energy due respectively to transient and standing eddies and are therefore a measure of a certain interaction between the two hemispheres.

A. Winter

Integral (4.1) gives 0.27×10^{20} ergs sec^{-1} while integral (4.2) gives 0.06×10^{20} ergs sec^{-1}. For further details about the stress integrals, see Table 3.

B. Summer

Integral (4.1) gives -0.03×10^{20} ergs sec^{-1} while integral (4.2) gives 0.00×10^{20} ergs sec^{-1}. For further details about the stress integrals, see Table 6.

5. Nature and importance of transient eddies

The total relative mean zonal kinetic energy of the Southern Hemisphere is 18.93×10^{26} ergs during the winter. The corresponding summer value is 9.87×10^{26} ergs.

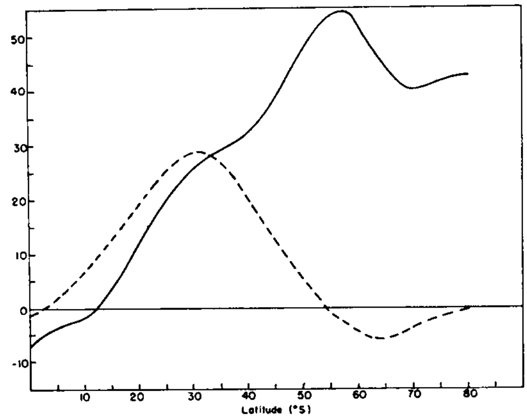

Fig. 1. The full curve gives the relative angular velocity in 10^{-7} sec^{-1} as function of latitude. The dashed curve gives the transient eddy flux of relative angular momentum in 10^{25} g cm^2 sec^{-2}. The curves are for winter 1958.

Discussions of the previous sections have shown that the transient eddies are the major producers of zonal kinetic energy. If these eddies cease to produce zonal kinetic energy, the atmosphere of the Southern Hemisphere will be in solid rotation with the earth, in about $2\frac{1}{2}$ weeks, assuming normal rates of dissipation. Similar computations by Starr (1953) for the Northern Hemisphere show that it will take about 2 weeks for the atmosphere of that hemisphere to be in solid rotation with the earth. Because of the obvious importance of these eddies, it is of interest to examine further some of their properties.

The solid curves of Figs. 1 and 2 give the distribution with latitude of the angular velocity relative to the earth, averaged with respect to pressure and time. Fig. 1 denotes the situation in winter while Fig. 2 denotes the condition in summer.

From these profiles the effect of true lateral friction would be to retard the zones of most rapid rotation and to increase the angular velocity of the less rapidly rotating ones, so as to cause the whole to assume a more nearly uniform angular velocity. This means that lateral friction would then cause a flow of angular momentum southward and also northward away from the zone of most rapid rotation.

The dashed curves in the two figures show the transient eddy angular momentum transports in summer and winter. These curves show

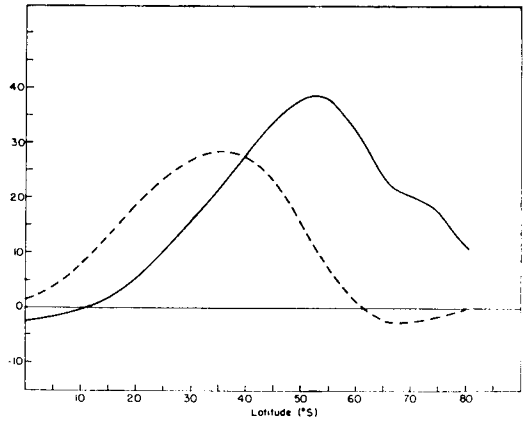

FIG. 2. The curves show the situation for summer 1958. The full curve gives the relative angular velocity in 10^{-7} sec^{-1} as function of latitude. The dashed curve gives the transient eddy flux of relative angular momentum in 10^{25} g cm^2 sec^{-2}.

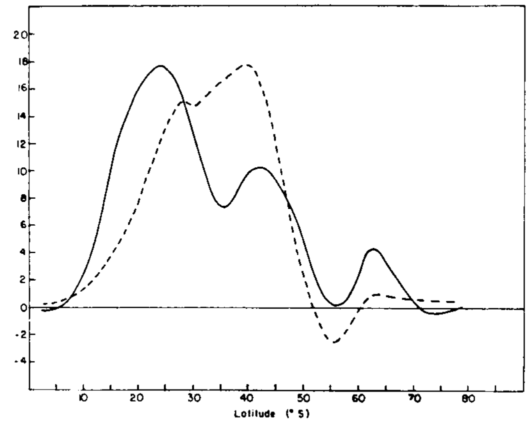

FIG. 3. Production of zonal kinetic energy using mean wind and transient eddy transport of relative angular momentum. The full curve represents the situation in winter while the dashed curve represents the summer condition. The units are in 10^{19} ergs sec^{-1}.

TABLE 11

$$I \textit{ denotes } \frac{a^2}{g}\iiint \frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}([\text{eddy mom.}]\cos^2\varphi)\,d\lambda\,d\varphi\,dp.$$

$$II \textit{ denotes } \frac{a^2}{g}\iiint [\text{eddy mom.}]\cos^2\varphi\,\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda\,d\varphi\,dp.$$

$$III \textit{ denotes } \int \frac{2\pi a}{g}\,[\bar{u}]\,[\text{eddy mom.}]\cos\varphi\,dp \textit{ at the equator.}$$

W = Winter
S = Summer } 1958 Southern Hemisphere.
M = Winter and Summer mean
N = Yearly mean using Buch's data 1950 for the Northern Hemisphere.

Units are in 10^{20} ergs sec^{-1}.

Method		Transient eddies I	II	III	Standing eddies I	II	III	All eddies I	II	III
Mean profiles	W	9.89	9.57	0.13	−0.13	−0.16	0.05	9.76	9.41	0.18
of zonal	S	9.04	8.99	−0.03	−0.31	−0.16	−0.10	8.73	8.83	−0.13
wind and momentum	M	9.46	9.28	0.05	−0.22	−0.16	−0.02	9.12	9.12	0.03
transport	N	4.35	4.55	—	0.80	0.80	—	5.15	5.35	—
Integration	W	9.63	9.41	0.27	−0.44	−0.53	0.06	9.19	8.88	0.33
by levels	S	9.72	9.86	0.03	−0.05	−0.07	0.00	9.67	9.79	0.03
	M	9.67	9.63	0.15	−0.25	−0.30	0.03	9.42	9.33	0.18
	N	5.84	4.84	—	0.88	0.87	—	6.72	5.71	—
Using 500 mb data	W	10.02	9.61	0.20	−0.27	−0.23	0.04	9.75	9.38	0.24
only	S	8.53	8.58	0.00	−1.51	−1.61	0.02	7.02	6.97	0.02
	M	9.27	9.09	0.10	−0.89	−0.92	0.03	8.38	8.17	0.13
	N	6.10	4.95	—	1.02	0.85	—	7.12	5.80	—

that there exists a strong *poleward* eddy transport of angular momentum towards the regions of maximum rotation. This state of affairs is contrary to classic eddy viscosity concepts and is compatible with them only if one assumes negative virtual viscosity coefficients.

The net influence of this observed property of the transient eddies is to increase the kinetic energy of zonal motions. Except for the equatorial boundary term, the rate of generation of the kinetic energy of the zonal motion is the mass integral of the product of eddy flux of momentum into the shear of relative angular velocity. This quantity can then easily be measured from Figs. 1 and 2. The areas under the curves of Fig. 3 measure the production of zonal kinetic energy. The full curve is for winter and the dashed curve for summer. In each case the total area for the hemisphere is positive by a wide margin.

9. Summary of results

Table 11 shows certain terms in the balance equation of the rate of generation of the kinetic energy of mean zonal flow. The preeminent importance of the transient eddies as major sources of the kinetic energy of the mean zonal flow is apparent.

In comparing the results in Table 11 with previous computations given by STARR and others, it appears that the rate of conversion of eddy kinetic energy to the kinetic energy of mean zonal flow is about twice as much in the Southern Hemisphere as in the Northern Hemisphere.

This result suggests then that the conversion rate of eddy available potential energy to eddy kinetic energy will be twice as much in the Southern Hemisphere as in the Northern Hemisphere. The verification of this has to await the study of the enthalpy budget.

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