Measurements of eddy fluxes of momentum in the surface layer of the Gulf Stream[1]

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(Manuscript received December 4, 1964)

ABSTRACT

Further observations are reported, in which the action of eddies in the Florida Current produces a surface flux of momentum towards higher mean velocities. The results are thus in contrast to what would be expected if the eddies acted as a frictional dissipation mechanism. Furthermore, associated with the momentum flux is a surface transfer of kinetic energy from the eddies to the mean flow.

The results from four locations along the length of the Florida Current are compared. The kinetic energy transfer between fluctuations and mean flow is found to be similar in magnitude at all locations and is restricted primarily to the zone of cyclonic shear of the mean surface velocity field. The flow of kinetic energy to the mean flow is sufficient to maintain the current against the dissipative actions of lateral eddy processes.

Both the techniques used to measure surface velocities and the sampling method are examined. Neither appears to introduce any biases which might account for the observed momentum fluxes.

Introduction

The Gulf Stream system between the Straits of Florida and Cape Hatteras, North Carolina (sometimes called the Florida Current), has a flow which is characterized by fluctuations having time scales of a few days and spatial scales of a hundred kilometers (VON ARX *et al.*, 1955; WEBSTER, 1961*a*). The interactions of these fluctuations with the mean flow at the surface has previously been measured at two sections across the current: off Miami, Florida, and off Onslow Bay, North Carolina (WEBSTER, 1961*b*). At both locations, an eddy flux of momentum towards higher mean velocities was observed. Associated with this momentum flux was a transfer of kinetic energy from the fluctuations to the time-averaged mean flow. The results were thus in contrast to what might have been expected if the regime were such that the fluctuations were dissipative: a flux of momentum towards lower mean velocities and a consequent drain of kinetic energy from the mean flow to the fluctuations.

The present paper presents the results of further measurements in the Florida Current which were designed to verify and extend the previous results. Those results had a large statistical uncertainty, partly because they were not collected with this particular use in mind, and partly because of the extreme difficulty in collecting enough measurements, even at the surface, to give a small statistical uncertainty. Thus, it seemed worthwhile to collect further observations, especially in view of the somewhat unexpected nature of the first results. Furthermore, the additional measurements (reported here) could be designed to test the techniques used for measuring surface velocities and to examine the methods of sampling and averaging.

Description of the observations

The observations which are here reported were made in two regions: across the Florida Current along latitude 30° N, between 79°10′ W, and 80°30′ W; and at a network of sections across the current off Cape Hatteras, North Carolina. The location of these regions is shown in Fig. 1 (locations 2 and 4) together with the locations of previous observations (1 and 3). Those at location 2, at 30° N, were collected

[1] Contribution No. 1570 from Woods Hole Oceanographic Institution.

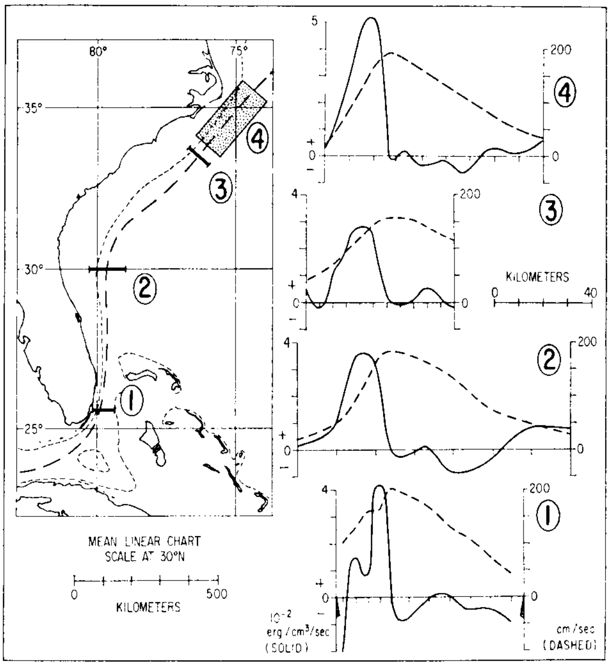

FIG. 1. A comparative diagram showing, on the right, observed mean downstream velocity, $\bar{v}$ (dashed line), and the average transfer of kinetic energy from eddies to mean flow, $\varrho \overline{u'v'}\, \partial\bar{v}/\partial x$ (solid line), at each of four locations along the length of the Florida Current. On the left is shown a chart with the positions of these locations.

during a seven-week period in October and November, 1961, from seventy crossings of the current made by the Research Vessel Crawford. The network of 20 crossings off Cape Hatteras, location 4 was made in October, 1962, also by the Crawford.

On each crossing, surface velocity measurements were made continuously with a geomagnetic electrokinetograph (G.E.K.) (VON ARX, 1950). The G.E.K. gives a continuous indication of the component of the surface velocity at right angles to the ship's track; since the ship in this case was crossing the current, the G.E.K. signal was approximately equal to the component of velocity in the direction of the mean current. At predetermined points on each crossing, the ship was maneuvered to provide two orthogonal components in order to specify the total horizontal surface water velocity vector. The G.E.K. indicates a velocity which is smaller than the true surface velocity due to the effect of the subsurface velocity field. Von Arx has denoted the ratio between the true surface speed and the surface speed as indicated by the G.E.K. as a factor k, which is almost a function of geographical position alone. A single factor for each location has been used with the present observations. The errors resulting from this approximation will affect the observed velocities by a factor which is a function of position only, and will not contribute to the apparent eddy momentum flux.

During each crossing, the position of the ship was regularly determined by Loran-A navigation. A comparison of the observed position with the anticipated position based on the ship's speed and course enabled an estimate to be made of the surface water velocity. The results are referred to as dead-reckoned (D-R) velocities. The error in measurement is reduced when the measurement is made over as long a time as possible; on the other hand, the time span between navigational position measurements imposes an averaging on the results which tends to filter out small-scale features of the velocity field. The results may be strongly influenced by the effect of wind on the ship. In order to exclude the most strongly wind-contaminated D-R velocities, none were computed for those instances where the corresponding wind was greater than Beaufort force 5 (11 meters/sec).

At the 30° N section, the ship went back and forth along the same line and G.E.K. fixes were made at every ten minutes of longitude between 79°15′ W and 80°25′ W. The dead-reckoned velocities were computed using the observed set of the ship over intervals having a length of 10 minutes of longitude, and centered on the points of the G.E.K. fixes.

Off Cape Hatteras, the crossings formed a network of parallel sections, centered on the mean axis as given by the U.S. Coast and Geodetic Survey Charts. In the region of these measurements, this axis runs in the direction 040° T, and passes through the point 34° N, 74°44′ 8 W. The most southwestern section crossed the axis at 33°44′.3 N, 76°03′.3 W; the most northeastern section crossed the axis at 34°24′.8 N, 74°21′.9 W. Because of the necessity of fitting these observations into a larger program of general physical oceanographic observations, it was not possible to complete more than twenty crossings in the time available.

Although observations were made in order to determine surface velocities by the dead-reckoning method, after those collected during periods of high winds were eliminated, not enough remained for useful computations.

Analysis of the observations

The offshore flux of downstream momentum by the action of eddies is defined as $\varrho\overline{u'v'}$, where a bar represents an average, defined below, a prime represents a deviation from the average, u and v are the horizontal velocity components in the cross-stream (x), and downstream (y) directions, and ϱ is the density of the water, here assumed to have the constant value of 1 g/cm³.

Associated with the momentum flux is a term

$$\varrho\overline{u'v'}\frac{\partial\bar{v}}{\partial x} \qquad (1)$$

which represents the transfer of kinetic energy from the perturbations (defined by the primed quantities) to the mean flow (defined by the barred quantities.). The full equation for the balance of kinetic energy has been given previously by WEBSTER (1961*b*).

The lateral shear of the mean surface velocity, $\partial\bar{v}/\partial x$ is determined from the slope of the smoothed profile of the mean downstream surface velocity, $\bar{v}$. The resultant transfer of kinetic energy from eddies to mean flow is determined by multiplying the eddy momentum flux, $\varrho\overline{u'v'}$, by this mean shear.

The standard errors of the means are given with the values tabulated. For large N, the standard error of the mean is defined as σ/N, where σ is the standard deviation of the sample from which the mean is calculated, and N is the number of observations.

Results from 30° N

The observations collected along the 30° N section were averaged as were the previous observations (WEBSTER 1961*b*), with the barred quantities defined as an average over time for all observations collected at each point during the interval of time from $t = -T/2$ to $t = T/2$:

$$\overline{(\quad)} = \frac{1}{T}\int_{-T/2}^{T/2} (\quad)\ dt.$$

Primed quantities are thus deviations from the time averages.

Table 1 gives the results of the measurements at this location. The transfer of kinetic energy from the perturbations to the mean flow has an average value across the section of 59×10^{-4} ergs/cm³/sec. The downstream surface flow has an average kinetic energy of 10.73×10^{3} ergs/cm³/sec. The calculated rate of energy transfer would, if no other actions were present, double the mean surface kinetic energy in 21 days.

Results off Cape Hatteras

Off Cape Hatteras, the observations were collected so that averages along zones parallel to the direction of the mean current could be computed. This is analogous to the zonal averages which are commonly used for similar computations in the atmosphere, and is in distinction to all the other results discussed here, where averages are obtained in time at a point. The change in the sampling method was made in order to verify that no systematic bias had been introduced into the earlier results as a conse-

TABLE 1. *30° North, G.E.K. observations.*

K = 1.55.

Position	$\bar{u}$ cm/sec		$\bar{v}$ cm/sec		$\overline{u'v'}$ cm²/sec²		$\frac{\partial \bar{v}}{\partial x}$ 10^{-5} sec⁻¹	$\overline{\varrho u'x'}\frac{\partial \bar{v}}{\partial x}$ 10^{-2} ergs/cm³/sec	
80°25′	− 2.4	± 10.1	21.1	± 10.5	+ 58	± 653	+ 1.84	+ 0.11	± 1.20
80°15′	− 5.1	± 6.0	50.6	± 12.8	+ 263	± 562	+ 4.41	+ 1.17	± 2.48
80°05′	+ 0.5	± 4.8	162.4	± 7.5	+ 443	± 875	+ 6.99	+ 3.11	± 6.12
79°55′	+ 11.9	± 4.5	173.1	± 6.5	+ 26	± 1016	− 3.15	− 0.08	± 3.20
79°45′	+ 11.2	± 4.5	134.2	± 6.2	+ 228	± 674	− 3.82	− 0.86	± 3.66
79°35′	+ 6.1	± 4.9	72.8	± 6.0	+ 106	± 625	− 3.05	− 0.33	± 1.90
79°25′	− 5.5	± 5.9	48.3	± 6.4	− 617	± 511	− 1.38	+ 0.84	± 0.70
79°15′	+ 6.1	± 7.1	29.0	± 7.3	− 650	± 496	− 1.20	+ 0.78	± 0.59

Average energy flux: 59 × 10⁻⁴ ergs/cm³/sec.

quence of the averaging method which had been used.

Thus, for the Cape Hatteras data, a barred quantity is defined as a space average. The observations were collected over a three-week period, but were assumed to be synoptic for the purposes of this analysis. All observations were plotted on a large-scale chart, and averages were formed using interpolated values from each of ten zones running downstream parallel to the mean axis of the current. These zones each have a width of 10 km, and are numbered from 1 to 10, going offshore. The center of zone 5 lies along the current axis.

Table 2 shows the results which were obtained for this location. The transfer of kinetic energy from the perturbations to the mean flow has an average value across the section of 82 × 10⁻⁴ ergs/cm³/sec. The downstream surface flow has an average kinetic energy of 13.10 × 10³ ergs/cm³/sec. The calculated rate of energy transfer would, if no other actions were present, double the mean surface kinetic energy in 18 days.

The results of the spatially-averaged measurements off Cape Hatteras are similar both in magnitude and in lateral scale to the other similar computations in which the observations were time-averaged. There is no apparent indication that the energy transfer results which were obtained previously had been substantially affected by the averaging methods used.

TABLE 2. *Cape Hatteras, G.E.K. observations.*

K = 1.30.

Position zone	$\bar{u}$ cm/sec		$\bar{v}$ cm/sec		$\overline{u'v'}$ cm²/sec²		$\frac{\partial \bar{v}}{\partial x}$ 10^{-5} sec⁻¹	$\overline{\varrho u'v'}\frac{\partial \bar{v}}{\partial x}$ 10^{-2} ergs/cm³/sec	
1	39.3	± 8.2	19.0	± 9.2	+ 47	± 553	6.38	+ 0.31	± 3.54
2	31.6	± 5.7	82.8	± 8.5	+ 438	± 753	7.19	+ 3.14	± 5.40
3	38.2	± 9.4	162.8	± 7.9	+ 974	± 1742	5.25	+ 5.12	± 9.16
4	41.6	± 9.8	188.0	± 10.3	+ 91	± 2014	− 0.23	− 0.02	± 0.46
5	34.7	± 8.3	158.0	± 7.9	+ 221	± 1606	− 2.80	− 0.62	± 4.48
6	31.3	± 6.5	132.1	± 7.2	+ 83	± 1056	− 2.82	− 0.24	± 2.99
7	24.7	± 5.3	102.6	± 6.5	+ 169	± 716	− 3.00	− 0.51	± 2.15
8	29.3	± 6.8	72.0	± 7.2	− 121	± 651	− 2.57	+ 0.31	± 1.67
9	25.1	± 5.5	51.2	± 6.6	− 81	± 431	− 1.91	+ 0.15	± 0.81
10	22.8	± 3.9	33.9	± 8.8	− 300	± 176	− 1.73	+ 0.53	± 0.31

Average energy flux = 81.7 × 10⁻⁴ ergs/cm³/sec.

TABLE 3. *30° North, D-R observations.*

Position	$\bar{u}$ cm/sec		$\bar{v}$ cm/sec		$\overline{u'v'}$ cm²/sec²		$\frac{\partial \bar{v}}{\partial x}$ 10^{-5} sec⁻¹	$\varrho \overline{u'v'} \frac{\partial \bar{v}}{\partial x}$ 10^{-2} ergs/cm³/sec	
80°05′	−9.9	±6.3	122.4	±17.9	+479	±825	+5.60	+2.68	±4.6
79°55′	−1.8	±5.6	151.7	±21.5	−145	±1024	−1.22	+0.18	±1.25
79°45′	−10.3	±5.2	131.0	±19.0	+160	±861	−1.17	−0.28	±1.52
79°35′	−11.2	±4.6	94.8	±14.0	+198	±568	−1.96	−0.39	±1.11
79°25′	−7.2	±4.3	67.8	±10.5	−134	±373	−1.68	+0.22	±0.63

Average energy flux: 48×10^{-4} ergs/cm³/sec.

Possible G.E.K. bias

The surface kinetic energy exchanges between fluctuations and mean flow measured with the G.E.K., at four locations in the Florida Current, shown in Fig. 1, fit a consistent pattern. There exists a possibility, however, that the G.E.K. may introduce a systematic bias into the observations which would itself produce an apparent momentum flux of the sort which is described here. In order to examine this possibility, an independent computation of the eddy momentum fluxes was made using dead-reckoned surface velocities collected from the 30° N section.

Table 3 summarizes the results of a computation using D-R velocities instead of G.E.K. velocities as in all other examples. Fewer observations are available than with the G.E.K. on the same section, and the measurements do not cover as wide a section across the current. In that respect then, the D-R and G.E.K. results are not exactly comparable. In addition, the method of surface current measurement by means of dead reckoning imposes a spatial averaging which is here evident in the lower and broader downstream velocity profile shown in the upper half of Fig. 2. However, over the region in which both methods are available, there is substantial agreement between them. A graphical comparison of the eddy kinetic energy transfer, shown in the lower half of Fig. 2, does not indicate any discrepancy between the character of the results as obtained by the two different methods.

If all G.E.K. velocities are affected by a constant factor, there will not be any consequent apparent eddy momentum flux, since both u' and v' will be modified in a similar manner and although the magnitude of $\overline{u'v'}$ might be affected, its sign will not. Thus, for example, G.E.K. cable droop (KNAUSS & REID, 1957), which changes the apparent surface velocity by a factor which is dependent upon the speed at which the G.E.K. cable is towed through the water will not make any contribution to the correlation between surface eddy velocity components.

Conclusions

In each of the four locations along the length of the Florida Current where surface eddy fluxes of momentum have been measured, a flow of momentum towards higher mean velocities has been observed. The individual measurements in nearly every case do not exceed the range of possible uncertainty. Nevertheless, when all the observations are considered as a group, an unmistakable pattern emerges, as can be seen in Figure 1.

The largest contribution to the kinetic energy exchange between the fluctuations and the mean flow, (1), occurs at each section in a zone about 20 kilometers wide; this zone is the region of cyclonic lateral shear. Outside the cyclonic shear region, there is no evidence for a significant kinetic energy exchange between the fluctuations and mean flow in either direction. In the cyclonic shear region, the lateral gradient of mean velocity, the correlation between velocity components, and the magnitude of the eddy velocities reach maximum values. Consequently, the momentum flux there reaches its greatest value, and the associated surface kinetic energy transfer is several times greater than is observed elsewhere.

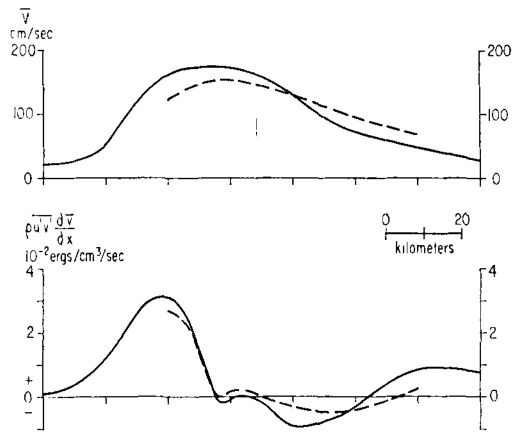

FIG. 2. A comparison between results from G.E.K. velocities (solid line curves) and dead-reckoned velocities (dashed line curves) for the 30° N. section. *Above:* Mean downstream velocity, $\bar{v}$. *Below:* Kinetic energy transfer between eddies and mean flow, $\overline{\varrho u'v'}\,\partial\bar{v}/\partial x$.

The instantaneous cyclonic shear is usually much larger than is indicated by the mean velocity profile, while the anticyclonic shear is probably fairly represented by the mean profile. The shifting lateral position of the cyclonic shear region produces a mean value of lateral shear which at its maximum approximates the local Coriolis parameter; however, the maximum cyclonic shear on an individual crossing may reach several times this value (WEBSTER, 1961*a*). As the cyclonic shear region moves back and forth, the large fluctuations of surface velocity at the points through which it passes produce the large eddy velocities whose correlation is evident in the resulting eddy momentum flux.

Table 4 shows some comparisons between surface observations collected at the various sections along the length of the Florida Current. The average transfer of kinetic energy from fluctuations to mean flow, ($\overline{\varrho u'v'}\,\partial\bar{v}/\partial x$) shows an increase northward. There may be a danger of making too much of this trend, in view of the large uncertainty attached to each of the values. However, the trend can be noted from another point of view by considering the regeneration time, T, which is the time required for the surface fluctuations acting alone to build up the observed surface kinetic energy in the absence of any friction. This time shows a decrease northward which may imply that the fluctuations in the current play a more important role in the maintenance of the mean current northward from the Straits of Florida.

The velocity maximum for each section, $\bar{V}$, appears to indicate a decrease northward of the maximum mean downstream surface velocity. That this does not necessarily imply a deceleration of the current as a whole is shown in the last column of Table 4, which is the average kinetic energy of the mean downstream surface motion. This value does not indicate any clear downstream trend. The similarity in the scale of the mean downstream velocity profiles shown in Fig. 1 adds further to the conclusion that between the Straits of Florida and Cape Hatteras, the Florida Current at the surface does not undergo any major change, either in scale or in kinetic energy. There is no evidence here for an acceleration of the current downstream.

What is the evidence for the role of fluctuations in maintaining the mean flow of the Florida Current? It is possible to answer this question in the surface layer only. Before the kinetic energy and momentum balance for the entire current can be determined realistically, current measurement techniques must be significantly improved. The present surface results

TABLE 4

	Average $\overline{\varrho u'v'}\,\dfrac{\partial\bar{v}}{\partial x}$		T days	$\bar{V}$ max cm/sec		$\frac{1}{2}\varrho\bar{v}^2$ 10^3 ergs/cm^3
1 Miami	3	±27	329	203	±9	8.55
2 Jacksonville	59	±84	21	173	±7	10.73
3 Onslow Bay	79	±30	11	157	±6	7.30
4 Cape Hatteras	82	±98	18	188	±10	13.10
2 Jacksonville (D–R)	48	±81	33	152	±22	13.75

seem to indicate a clear pattern, however. In the Straits of Florida, the current is closed in by boundaries on both sides, and the role of friction is apparently more dominant than further downstream; the contribution of kinetic energy by the eddies in the onshore shear region may not be sufficient to maintain the mean current against the dissipative effects at the boundaries. There, the current may be driven primarily by downstream pressure gradients.

Downstream from the Straits of Florida, where the current no longer is so strongly affected by nearby shorelines, the flow of kinetic energy from fluctuations to mean flow in the onshore shear region exceeds the opposite transfer by lateral eddy processes in the remainder of the current. That is, in the surface layer, where the dominant frictional effect would be associated with lateral eddy motions, the fluctuations provide more kinetic energy for the mean flow than is dissipated.

From whence do the fluctuations receive their kinetic energy? The present results indicate that the source cannot be the kinetic energy of the mean flow. Furthermore, Oort (1964) has made a study in which he computed the correlation between the downstream eddy velocity components and associated temperature and density fluctuations. He concluded from his results that the eddies do not draw on the available kinetic energy but must be externally forced. Although his results, perhaps even more than those reported here, are subject to a large uncertainty, it is clear that an understanding of the source of kinetic energy for the eddies is essential to an understanding of the energetics of the stream as a whole.

Acknowledgements

This work was supported by the National Science Foundation under grant NSF-G 19251. The observations at sea could not have been collected without the cooperation of the officers and crew of the R/V Crawford, and of the many scientists at the Woods Hole Oceanographic Institution who took part in the cruises.

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