Further studies of energy exchange between the zonal flow and the eddies

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ABSTRACT

The present study describes the results of calculations of transports of sensible heat and momentum and of exchanges of available potential energy and kinetic energy between the zonal flow and the eddies in the wave number regime. The study is based on five levels of data for the months of April, July, October 1962 and January 1963, while only two levels of data (850 and 500 mb) were available for the months of January 1959 and April, July, October 1961.

The heat and momentum transport calculations in the wave number regime show that a very large fraction of the total heat and momentum transport is carried out by the very long waves during winter while a smaller percentage of the total heat and momentum transport is accounted for by these waves during the remaining part of the year.

The exchange from zonal to eddy available potential energy shows a marked variation during the year with maxima during the winter and minima during the summer. The spectral distributions are such that we find a maximum for wave numbers 2 or 3 during the winter. This maximum disappears during summer, when we find the maximum at much smaller wave lengths. This energy exchange is positive in all months thus far investigated not only for the total conversion but also for the individual wave numbers.

The interchange from eddy to zonal kinetic energy has a different behavior. It differs from the annual variation of the potential energy exchanges in that we find the maximum value during fall. The spectra are very irregular with no clearly defined maxima and minima. This energy exchange is positive for all months investigated so far except for January 1963, but the average values are always much smaller than the average exchanges between the two forms of available potential energy. It is demonstrated that a very large fraction of the total (negative) interchange during January 1963 is accounted for by wave number 3.

The results of our calculations are compared with those of other investigators showing fair qualitative agreement.

1. Introduction

In a recent paper (WIIN-NIELSEN, BROWN & DRAKE (1963), hereafter referred to as (I), we have given the necessary formulas to compute the energy exchanges from the zonal flow to the eddies for both available potential energy and kinetic energy in the wave number regime. As interesting by-products we also obtained the transport of heat and relative momentum as a function of wave number. The computational procedures for these calculations were based on observed height data for isobaric surfaces and were described in I. We have furthermore, given the results of our calculations for a single winter month, January 1962.

When we want to obtain representative values for the energy exchanges and transformations for the general circulation of the atmosphere, it is naturally not sufficient to use the data for a single month. There are, for instance, great variations in the circulation pattern from one January to the next in addition to the marked seasonal variations which occur throughout the year. In order to obtain stable mean values and to obtain a better insight into seasonal and annual variations it is necessary to repeat the calculations for many months representing the different seasons and years. It is the purpose of

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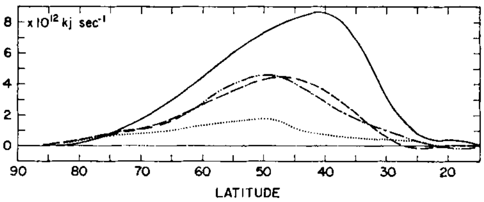

FIG. 1. The total northward transport of sensible heat across a latitude circle as a function of latitude. Units: 10^{12} kj sec^{-1}. Full drawn curve for January 1963, dashed curve for April 1962, dotted curve for July 1962 and dashed-dotted curve for October 1962.

this paper to present results of additional calculations representing altogether ten months: January, April, July and October for the years 1961 and 1962 and January and February 1963. For the months in 1962 and 1963 we have routine analyses available from the National Meteorological Center for the five levels 850, 700, 500, 300 and 200 mb, while the operational analyses prior to 1962 only included the two levels 850 and 500 mb. All analyses extended from the pole to approximately 15° N. Even though the computations for the months in 1961 are less representative than those from the later months they have nevertheless been included.

There have been no changes in the methods of calculation since the results described in I. The presentation will follow the same lines as in I but due to space limitations we will not present the results in as great detail as before.

2. The heat transport

It is the purpose of this section to describe the results of the calculations of the eddy transport of sensible heat in the wave number regime. In order to demonstrate the seasonal variation of the eddy heat transport we have prepared Fig. 1, which shows the total transport of sensible heat across latitude circles for the four months of April, July and October 1962 and January 1963. It is evident from Fig. 1 that there is a marked seasonal variation with a maximum transport of sensible heat in the winter and minimum in the summer. The two curves for April and October are remarkably similar. The position of the maximum heat transport is further to the north in the summer than in the winter. The eddy transport of sensible heat is predominantly positive. Only in the low latitudes of the region are there small southward transports.

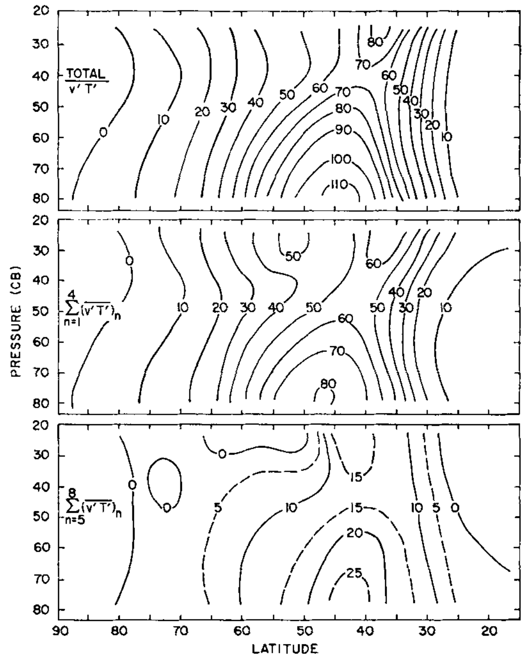

FIG. 2. The transport of sensible heat as a function of latitude and pressure for January 1963. Units: 10^9 kj sec^{-1} cb^{-1}. The upper part of the figure shows the total heat transport, the middle part gives the heat transport by the waves with wave numbers 1, 2, 3 and 4, while the lower part shows the heat transport by waves with wave numbers 5, 6, 7 and 8.

We are next going to consider the eddy transport of sensible heat in the wave number regime. The transports were computed for the first 15 wave components, but we shall, as in I, combine the contributions from the waves with wave numbers 1 to 4, wave numbers 5 to 8 and finally wave numbers 9 to 15. It was demonstrated in I that the last group of wave numbers gave only a small contribution to the total eddy transport of sensible heat. This result holds also for all the other months under investigation. To save space we have therefore selected not to present the results for the waves with wave numbers in the last category.

Fig. 2 shows the results for January 1963. The upper part of the figure is the total eddy transport of sensible heat in the unit 10^9 kj sec^{-1} cb^{-1} as a function of latitude and pressure. The middle part of Fig. 2 shows the contribution to the total transport from the waves with wave numbers 1 to 4, while the lower part gives the contribution from the next group of wave

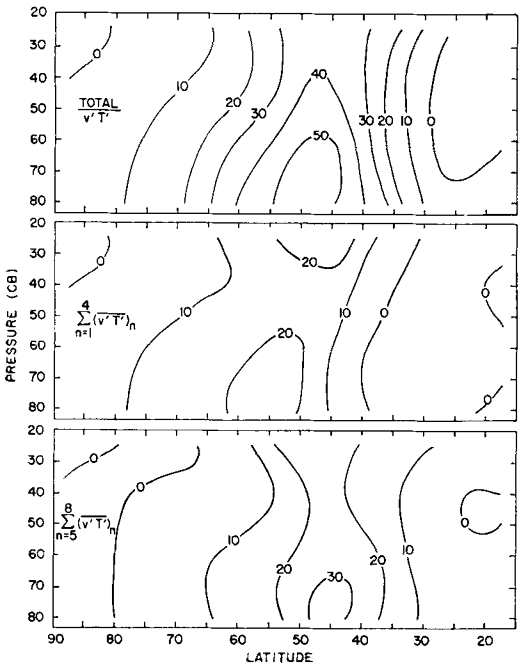

FIG. 3. Heat transport for April 1962. For arrangement see Fig. 2.

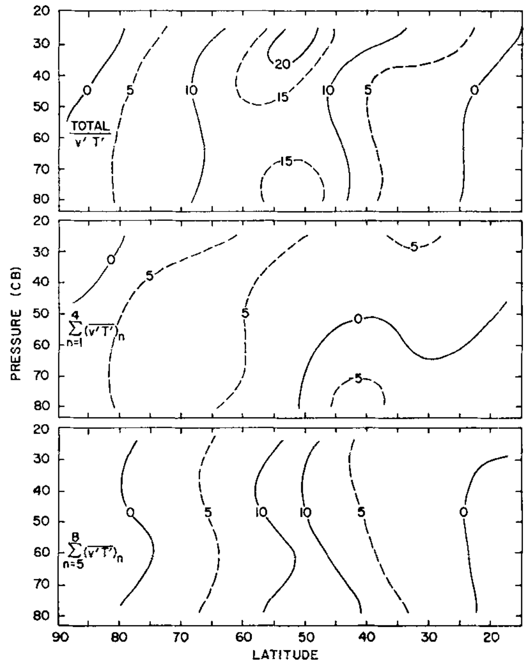

FIG. 4. Heat transport for July 1962. For arrangement see Fig. 2.

components. Fig. 2 in this paper should be compared with Figs. 1, 2 and 3 in I because they apply to the month of January. One notices first of all that there exists a great similarity between the two figures. The absolute maximum occurs at low levels in middle latitudes. There is a saddle point in the distribution of the eddy transport of heat around the 400 mb level in middle latitudes in both months. Above the saddle point we have a secondary maximum at the 200 mb level, but this maximum is further to the south in January 1963 than in January 1962. The eddy transport of heat is larger in January 1963, and a larger fraction of the transport is accounted for by the four lowest wave numbers, while the transport by waves of wave numbers 5 to 8 is very similar and of the same magnitude in the two months.

We consider next the distributions for the month of April 1962, displayed in Fig. 3. It is evident by comparison with Fig. 2 that the total heat transport in the month of April is smaller than for the winter month of January. By comparing the middle and upper parts of Fig. 3 with the corresponding parts of Fig. 2 it is furthermore seen that the very long waves ($n = 1$ to 4) explain a much smaller fraction of the total eddy transport of heat in April 1962 than they do in both Januaries under investigation. The intermediate waves ($n = 5$ to 8) explain on the other hand more than half of the total transport. The maximum heat transport for the intermediate waves occurs further to the south than the maximum for the very long waves. This statement holds not only for April 1962, but for all the months which have been investigated.

The next month to be considered is July 1962. The results from this month are given in Fig. 4. The eddy transport of heat has decreased even further compared to the winter and spring months. We find now the absolute maximum in the eddy transport of sensible heat in the upper troposphere, but the absolute values in the maximum are not much higher than the values found in the secondary maximum in the low levels in middle latitudes. Only a small fraction of the total heat transport found in middle latitudes is now explained by considering the contribution from the very long waves, while the intermediate waves have gained in importance in comparison with the winter and spring months.

To complete the presentation of our examples

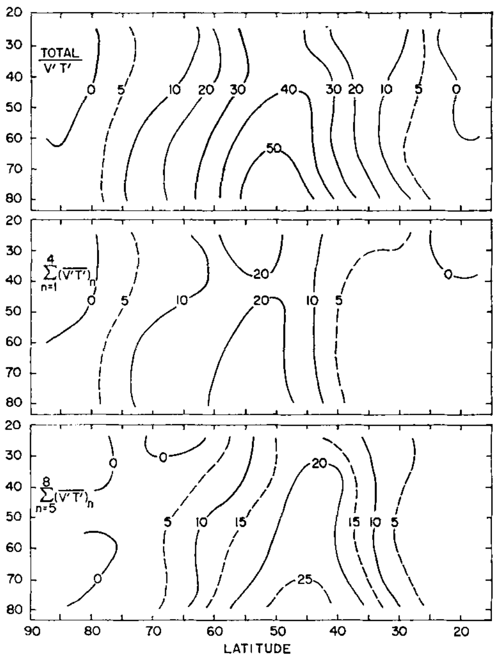

FIG. 5. Heat transport for October 1962. For arrangement see Fig. 2.

of the eddy heat transport for the months representing the different seasons we finally refer to Fig. 5 which gives the results for October 1962. The statements which have been made in connection with Fig. 3 which showed the results for April 1962 can be repeated with respect to Fig. 5. There is a remarkable similarity between the two figures. Any further discussion seems unnecessary.

With respect to the eddy transport of sensible heat we have demonstrated by examples for four months of the year 1962 supplemented by the month of January 1963 that there exists a marked seasonal variation in the heat transport. Even more important is the information which has been obtained with respect to the relative contributions from atmospheric motions on the different scales. While the ultra-long waves account for a substantial fraction of the total heat transport during the month of January, they play a less important role as the year progresses and account for very little of the total heat transport in the middle latitudes during the summer. During the fall the ultra-long waves increase in relative importance with respect to the fraction of the total heat transport carried out by these waves. The intermediate waves act oppositely in that a larger fraction of the total heat transport is carried out by these waves during the summer compared to the winter months.

The seasonal variation which has been found in the eddy transport of sensible heat is naturally first of all due to the seasonal variation in the intensity of the atmospheric circulation. It is well known that the amplitudes in both temperature and height waves are considerably smaller on all wave numbers during the summer season than during winter. For this reason alone one would expect to find a marked variation. The sign of the eddy transport of sensible heat indicates the position of the temperature wave relative to the height wave on an isobaric surface. If the eddy transport of heat is positive for a given wave component, the temperature wave is lagging the height wave while a negative sign indicates the opposite arrangement. It is a result of the quasi-geostrophic (or for that matter any) baroclinic instability theory that the baroclinically unstable disturbances will transport heat northward. The empirical results of the heat transport calculations obtained in this study for the waves with wave numbers 5 to 8 which corresponds to the unstable waves, at least in middle latitudes, are in agreement with the predictions of the theory.

With respect to the very long waves ($n = 1$ to 4), we also find a northward transport for all months at all levels in the high latitudes, but in April and July 1962, we observe weak southward transports in the lower latitudes at lower elevations. These results indicate that the structure of the very long waves is such that the temperature wave is lagging behind the height wave or, equivalently, the wave has a westward slope with decreasing pressure during most of the year, while the very long waves tend to be vertical or with a slight eastward slope in the lower latitudes during spring and summer. An analysis of the structure of the very long waves based on the normal maps for January and July (WIIN-NIELSEN, 1961*a* and *b*) indicates a westward slope in January and an eastward slope in July at 50° N. At the same latitude we find a slight westward slope in July 1962. The difference might be explained on the grounds that the particular month under investigation is very anomalous, but it might also mean that there is a great difference between the so-called "standing" and "tran-

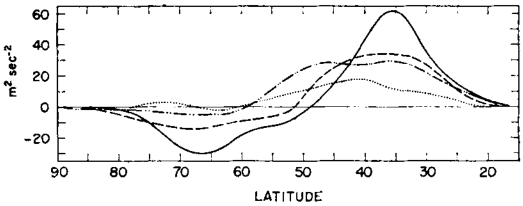

FIG. 6. The average northward transport of relative momentum across latitude circles as a function of latitude. Units: $m^2\ sec^{-2}$. Full drawn curve for January 1953, dashed curve for April 1962, dotted curve for July 1962 and dashed-dotted curve for October 1962.

sient" eddies as pointed out by SALTZMAN & FLEISHER (1962) and MURAKAMI (1960). This question can only be resolved by finding the eddy transport of heat in the standing and transient eddies. The results of such an investigation will be reported later.

3. The momentum transport

The results of the momentum transport calculations for the four different months will be given in this section. We are first going to consider the momentum transport averaged in the vertical direction. It should be kept in mind that all the figures referred to in this section will give the transport of relative momentum as measured by $\overline{u'v'}$ in the unit $m^2\ sec^{-2}$.

Fig. 6 shows the vertical average of the momentum transport for the months of April, July and October 1962 and January 1963. The main characteristics of the four curves are the same, giving a positive (northward) transport of momentum in the low latitudes and somewhat smaller negative (southward) transports in the high latitudes. There is a marked seasonal variation with the largest transports in the winter season and the smallest transports during the summer. The maximum northward transport is in its southernmost position during the winter. A comparison of the curve for January 1963 in Fig. 6 with the corresponding curve for January 1962, given as Fig. 7 in I, shows that the magnitudes of the maximum and minimum momentum transports are about the same in the two months, but that the zero transport occurs about 10 degrees of latitude further to the south during January 1963.

As we did for the eddy transport of heat, we are next going to consider the momentum transport in the wave number regime. We shall make the same grouping of the different wave components as we did in the previous section and shall consider each of the months separately.

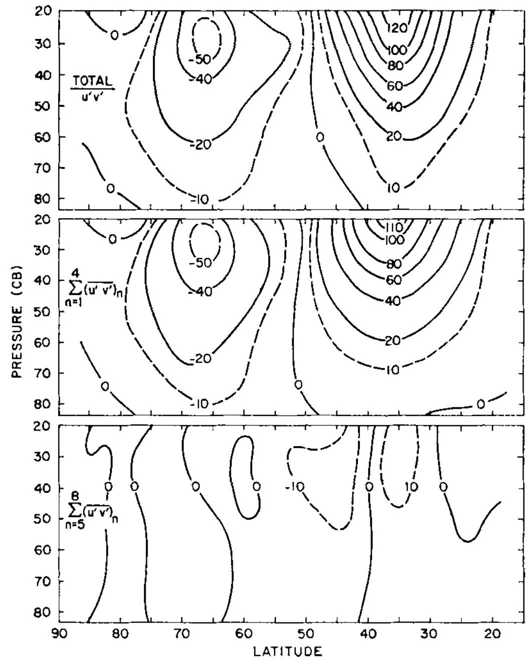

FIG. 7. The transport of relative momentum as a function of latitude and pressure for January 1963. Units: $m^2\ sec^{-2}$. The upper part of the figure shows the total momentum transport, the middle part gives the momentum transport by waves with wave numbers 1, 2, 3 and 4, while the lower part show the momentum transport by waves with wave numbers 5, 6, 7 and 8.

In Fig. 7 we show the total transport of momentum by all wave components as a function of latitude and pressure in the upper part. The middle part of the figure gives the momentum transport for the very long waves ($n = 1$ to 4) in a similar coordinate system, while the lower part gives the momentum transport by the intermediate waves ($n = 5$ to 8). We are not presenting the transport by the short waves ($n = 9$ to 15) because of the small contribution to the total transport.

One notices first of all the great similarity between the upper part of Fig. 7 of this paper and Fig. 7 of I. The magnitude and position of the maximum and minimum momentum transport vary little. However, when it comes to the transport by the very long waves ($n = 1$

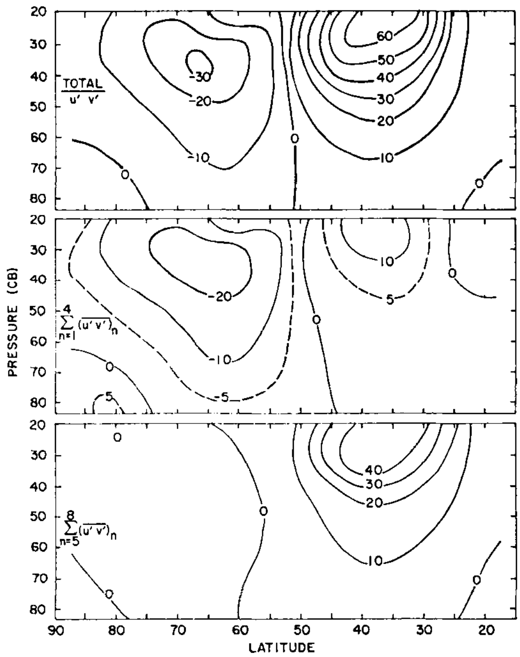

Fig. 8. Momentum transport for April 1962. For arrangement see Fig. 7.

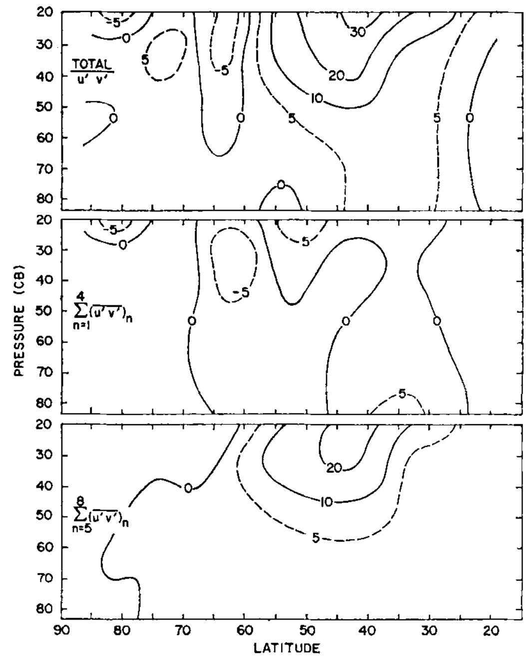

Fig. 9. Momentum transport for July 1962. For arrangement see Fig. 7.

to 4) given in the middle part of Fig. 7 and in Fig. 8 of I, we find a large difference. In January 1962 we found that roughly 50 % of the total momentum transport in middle latitudes was accounted for by the very long waves. During January 1963 the percentage is much higher, which shows the dominance of the very long waves for this particular month. This statement is substantiated by a comparison of the lower part of Fig. 7 in this paper and Fig. 9 of I. The calculations for the two different Januaries of the momentum transport are in agreement with the corresponding calculations of the eddy transport of sensible heat in showing the great importance of the very long waves in the budgets of heat and momentum during the winter season.

The dominating role of the very long waves disappears for the other seasons under investigation. Fig. 8 shows the results for April 1962 in an arrangement similar to Fig. 7. The maximum northward momentum transport at about 35° N is only half of the value found in January, but only a very small fraction of this momentum transport is carried out by the very long waves as seen from the central part of Fig. 8. A much larger fraction, about $\frac{2}{3}$, is accounted for by the intermediate waves with wave numbers 5 to 8. In the higher latitudes we find naturally that the very long waves account for most of the momentum transport, but this is primarily due to the fact that the waves corresponding to the low wave numbers are relatively short waves at these high latitudes.

Proceeding to July 1962 we find a continuation of the trend which we observed from January to April. The results of the momentum transport calculations for this month are shown in Fig. 9. The maximum positive transport in the high troposphere in middle latitudes is now roughly one half of the magnitude found in a similar location during April, but hardly anything is accounted for by a consideration of the contribution from the very long waves. As in the month of April we find that the intermediate waves contribute roughly $\frac{2}{3}$ of the total momentum transport in the region of the maximum.

During the month of October 1962 there is a return towards the winter situation. The total momentum transport (Fig. 10) has increased roughly by a factor of 2 compared to July 1962

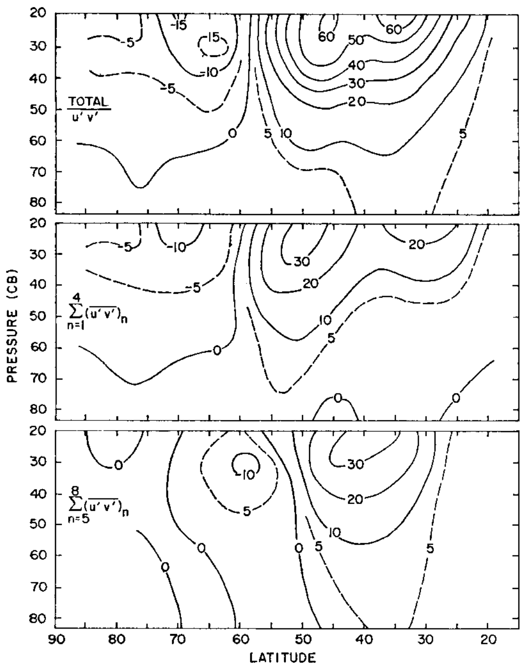

Fig. 10. Momentum transport for October 1962. For arrangement see Fig. 7.

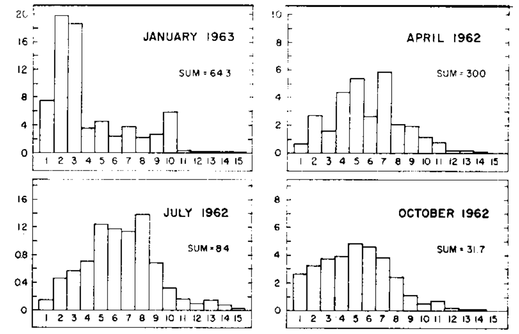

FIG. 11. The energy exchange from zonal to eddy available potential energy as a function of wave number for January 1963 (upper left), April 1962 (upper right), July 1962 (lower left), and October 1962 (lower right). Unit: 10^{-4} kj m^{-2} sec^{-1}.

and a larger fraction of the total momentum transport is accounted for by the very long waves. The contribution from the intermediate waves is by and large as great as the contribution from the very long waves.

The main result of the momentum transport calculations is therefore in many respects simi- ar to the sensible heat transport calculations. The very long waves play an important role in satisfying the momentum budget during the winter season, while their contribution is of smaller importance during the remaining part of the year. It should further be noticed that the sign and magnitude of the momentum transport give us important information on the horizontal structure of the atmospheric waves. Just as a positive value of the heat transport at a certain level and a certain latitude tells us that the axis of the wave component slopes towards west with altitude, it is known that a positive value of the momentum transport indicates a southwest to northeast tilt of the trough (or ridge) line at a given level and latitude. The results obtained in this section show therefore that the atmospheric disturbances of both very large and intermediate scale have a southwest-northeast tilt in the low and middle latitudes while the opposite tilt is observed in the high latitudes. It is furthermore observed that there is no qualitative difference between the very long and intermediate waves in this respect. It is known that the quasi-geostrophic theory incorporating variations in the meridional direction (CHARNEY, 1959; PHILLIPS, 1956) predicts the tilt observed in these calculations. Any theory for the ultra-long waves which might be developed must also account for a similar tilt of these disturbances. At the present time we have only rudimentary theories for the motion and maintenance of the very long waves, and none of these theories include variations in the meridional direction.

4. Exchange of potential energy from the zonal mean to the eddies

The energy exchange from the zonal mean to the eddies for available potential energy has been computed using the same procedure and the same kind of data as in I.

The results for the same four months which have been described in the preceding two sections are given in Fig. 11. Each of the four sections of the figure gives the energy transfer $C(\overline{P},P')$ as a function of wave number for the months of April, July, October 1962 and January 1963. The unit is in all cases 10^{-4} kj m^{-2} sec^{-1}, but it will be noticed that the vertical scale is different on the figures.

The January 1963 results should be compared with Fig. 16 of I in which the results for January 1962 are given. The outstanding similarity

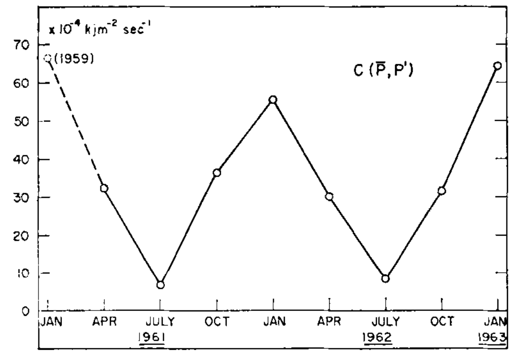

FIG. 12. The variation of the monthly averages of total energy exchange $C(\bar{P},P')$ as a function of time for the years 1961, 1962 and January 1963.

between the two figures is the dominating role of wave numbers 2 and 3. The contribution from any of the two wave numbers is about twice as large in January 1963 compared to January 1962, while the contribution from waves with $n \geq 4$ is smaller in January 1963 than in January 1962. The total energy exchange in January 1963 is therefore not very much larger than in the other winter months, 64.3×10^{-4} kj m^{-2} sec^{-1} compared to 55.5×10^{-4} kj m^{-2} sec^{-1}.

As one would expect from the results of the eddy transport of sensible heat the dominating role of the very long waves disappears in the other months being investigated. Although some of the spectra are a little irregular, they tend to have a maximum somewhere in the intermediate waves. In April 1962 we find the maximum at wave number 7, in July 1962 at wave number 8 and in October 1962 at wave number 5. The total transfer is about equal in April and October 1962: 30.0 and 31.7×10^{-4} kj m^{-2} sec^{-1}, respectively, while the total conversion for July 1962 is as low as 8.4×10^{-4} kj m^{-2} sec^{-1}.

So far we have only discussed results obtained from data from the years 1962 and 1963. Data has also been available to us from January 1959 and April, July and October 1961. These data were, however, only for the two levels, 850 and 500 mb, and one cannot expect to get representative results from these levels alone. In order to compare results obtained with the 5 levels of data with those for only 2 levels we have made our calculations for January 1962 twice, once with 5 levels of data and the second time with only 2 levels of data. When both models were

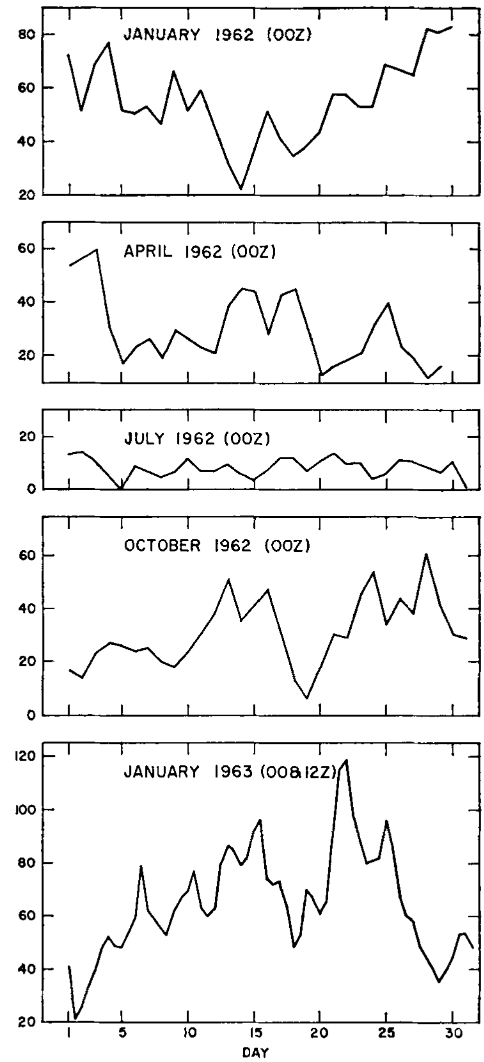

FIG. 13. The variation of the daily values of the total energy exchanges $C(\bar{P},P')$ as a function of time for the five months of January 1962, April 1962, July 1962, October 1962 and January 1963. Unit: 10^{-4} kj m^{-2} sec^{-1}.

made to simulate the total depth of the atmosphere, we found for the 5 levels of data 55.5×10^{-4} kj m^{-2} sec^{-1} while the 2 levels of data gave 70.9×10^{-4} kj m^{-2} sec^{-1}. For this particular month we would therefore have to apply a correction factor of 0.78 to the 2 level calculations to obtain agreement with the 5 level calculations. This correction factor was then arbitrarily applied to all the 2 level calculations. The assumption is probably by and large correct

TABLE 1.

	Jan. 1962	April 1962	July 1962	Oct. 1962
850/700	8.91	5.00	1.11	5.45
700/500	15.22	9.25	2.20	9.56
500/300	16.20	8.95	3.02	9.51
300/200	2.09	0.61	0.32	0.59

in the case of the calculations of $C(\bar{P},P')$ because the contribution from the different layers always is of the same sign (positive) and roughly in the same ratio. Fig. 12 shows the variation of $C(\bar{P},P')$ as a function of time during the years 1961, 1962 and January 1963, where the results from January 1959 have been substituted for the unknown results for January 1961. One observes a marked seasonal variation with maxima in the winter and minima in the summer.

The significance of the different monthly mean values could be given by the standard deviations from the monthly averages. We prefer to give the daily values of $C(\bar{P},P')$ as a function of time for the different months. These are given in Fig. 13. It is seen that all values of $C(\bar{P},P')$ are positive with rather large variations during the individual months.

In order to illustrate the contribution from the different layers to the conversion $C(\bar{P},P')$ we present Table 1 which gives the contribution for the 4 layers 850–700, 700–500, 500–300 and 300–200 mb for the 4 months for which the calculations have been made during 1962 (unit: 10^{-4} kj m^{-2} sec^{-1}).

5. Exchange of kinetic energy from the zonal mean to the eddies

The presentation of the results concerned with the kinetic energy exchange between the zonal average and the eddies will in the beginning follow the presentation given in the preceding section.

The results for the four months of April, July, October 1962 and January 1963 are given in Fig. 14 where the mean energy exchange is given as a function of wave number. One notices first of all that the spectra are very irregular. Fig. 14 should be compared with Fig. 18 in I. During the month of April 1962 we find a very small positive value of the total $C(K',\bar{K})$. The main positive contributions are

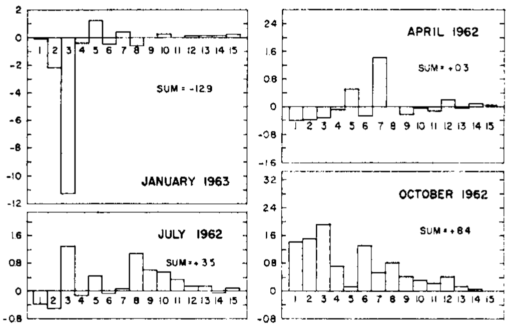

FIG. 14. The energy exchange from eddy to zonal kinetic energy as a function of wave number for January 1963 (upper left), April 1962 (upper right), July 1962 (lower left) and October 1962 (lower right). Unit: 10^{-4} kj m^{-2} sec^{-1}.

coming from the waves with wave numbers 5 and 7, while the very long waves ($n = 1$, 2 and 3) give negative contributions in the average. The spectrum for July, 1962 shows a somewhat larger total value of $C(K',\bar{K})$. While the contribution from waves with wave numbers 1 and 2 is still negative, we find positive (or very small negative) contributions from the other wave numbers. The contribution from waves with wave numbers larger than 8 is now significantly larger than in the months of January 1962 and April 1962, which seems to indicate that the wave lengths of the most active waves are somewhat shorter during summer than in the remaining part of the year. October 1962 shows the largest value of $C(K',\bar{K})$ with positive contributions from all wave numbers, and with very significant contributions from the very long waves.

It is apparent from the irregular behavior of the spectra that in order to obtain significant spectra and total values of the energy exchange $C(K',\bar{K})$ for the different seasons it will be necessary to consider an ensemble of the different months. Only by creating large ensembles can we be sure that we obtain representative values. The months represented in this paper should therefore merely be considered as examples.

Returning to Fig. 14 we shall next consider the spectrum for January 1963. It is first of all seen that the total energy exchange $C(K',\bar{K})$ is negative in the average for this month: -12.9×10^{-4} kj m^{-2} sec^{-1}. Furthermore, wave number 3 accounts for most of the total value, a result which is very different from the other months

TABLE 2.

	Jan. 1962	Apr. 1962	July 1962	Oct. 1962	Jan. 1963	Weights
850	+ 1.93	+ 0.49	+ 1.01	+ 0.83	− 0.15	0.225
700	+ 3.18	+ 1.21	+ 1.51	+ 1.55	− 0.26	0.175
500	+ 5.06	+ 2.45	+ 2.03	+ 4.28	− 3.48	0.200
300	− 1.86	− 0.19	+ 3.01	+ 13.38	− 25.03	0.150
200	− 2.95	− 1.88	+ 8.77	+ 20.38	− 33.27	0.250
Total	+ 1.00	+ 0.31	+ 3.54	+ 8.42	− 12.85	

under consideration. We shall consider a little later the special case of wave number 3.

Fig. 15 shows the variation of $C(K',\bar{K})$ as a function of time from January 1962 to January 1963. The curve is probably not typical since only one month is represented for each data point. It is unfortunately not possible to make a longer record in this case by using a correction factor as we did for the energy exchange $C(\bar{P},P')$. The reason for this inability is illustrated in Table 2 in which we give the contribution from each level to the total transfer $C(K',\bar{K})$. The numbers in Table 2 are given as the energy exchange which one would have if the total depth of the atmosphere was represented by the single level. The weights which have to be applied to the numbers for each level in order to compute the total energy conversion are given in the last column of Table 2.

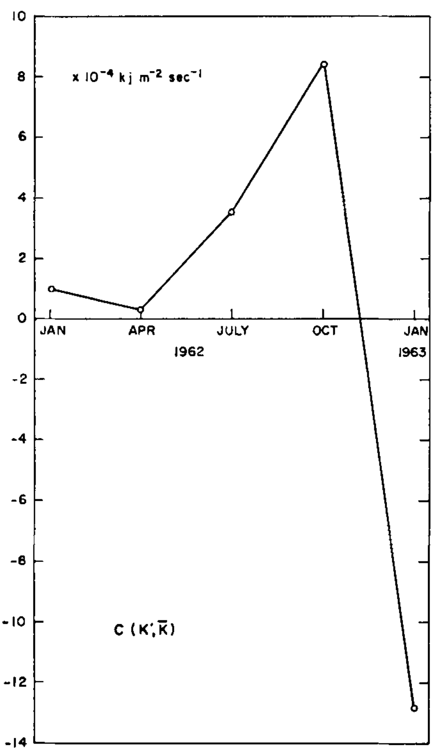

FIG. 15. The variation of the monthly averages of the total energy exchange $C(K',\bar{K})$ as a function of time from January 1962 to January 1963.

It is seen from Table 2 that we sometimes have a change in sign for levels above 500 mb and sometimes not. There is furthermore no single proportionality factor between the levels. These are the reasons that we cannot use a single correction factor.

We can, however, give the results for the different months during the time when only two levels of data were available. These results are given in Fig. 16 for the months of January 1959, April, July and October 1961.

The results for January 1963 for the energy exchange $C(K',\bar{K})$ stand out as a special case simply because the sign of the energy exchange is opposite that of the other months. It is of some interest to investigate the distributions of the

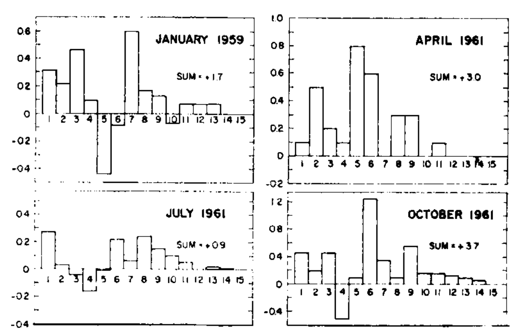

FIG. 16. The energy exchange from eddy to zonal kinetic energy as a function of wave number for January 1959 (upper left), April 1961 (upper right), July 1961 (lower left) and October 1961 (lower right). Unit: 10^{-4} kj m^{-2} sec^{-1}. Calculations based on data from 850 and 500 mb.

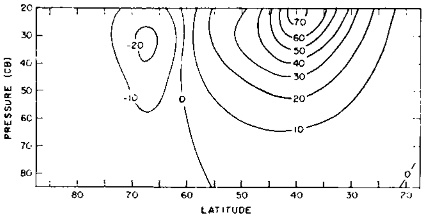

Fig. 17. The momentum transport by wave number 3 as a function of latitude and pressure for January 1963. Unit: m^{-2} sec^{-2}.

factors which enter into the calculation of the energy conversion, the distribution of the zonal wind $\bar{u}$ and of the momentum transport $\overline{u'v'}$ according to (3.7) of I. It is furthermore clear from Fig. 14 that wave number 3 gives by far the largest negative contribution to the energy exchange $C(K,\bar{K})$. Although the average of the energy exchange is obtained by making calculations on each single day and then forming the mean, it is possible to illustrate the distributions by considering the time-average of the momentum transport and the time-average of the zonal wind. These distributions are given in Fig. 17 and Fig. 18, where Fig. 17 contains the momentum transport of the wave with wave number 3 only. The sign of the energy exchange $C(K',\bar{K})_3$ is determined by the correlation between $(\overline{u'v'})_3$ and $\partial[\bar{u}/(\cos\varphi)]/\partial\varphi$. A simple inspection of Figs. 17 and 18 shows that the two factors on the average are negatively correlated. The main reason for this is the existence of the double maximum in $\bar{u}$ and the position of the maximum and minimum momentum transports. The maximum momentum transport is located in a region where $\partial[\bar{u}/(\cos\varphi)]/\partial\varphi$ is negative and the minimum momentum transport in a region where $\partial[\bar{u}/(\cos\varphi)]/\partial\varphi$ is positive. Fig. 19 gives the energy exchange $C_3(K',\bar{K})$ as a function of time for January 1963. A computa-

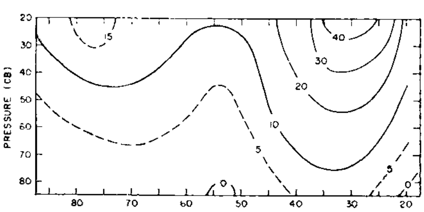

Fig. 18. The time average of the zonal winds as a function of latitude and pressure for January 1963. Unit: m sec^{-1}.

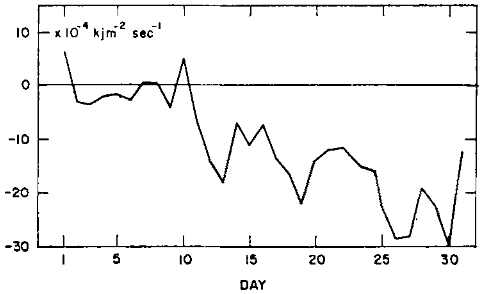

Fig. 19. The daily values of the energy exchange $C(K',\bar{K})$ for wave number 3 as a function of time for January 1963.

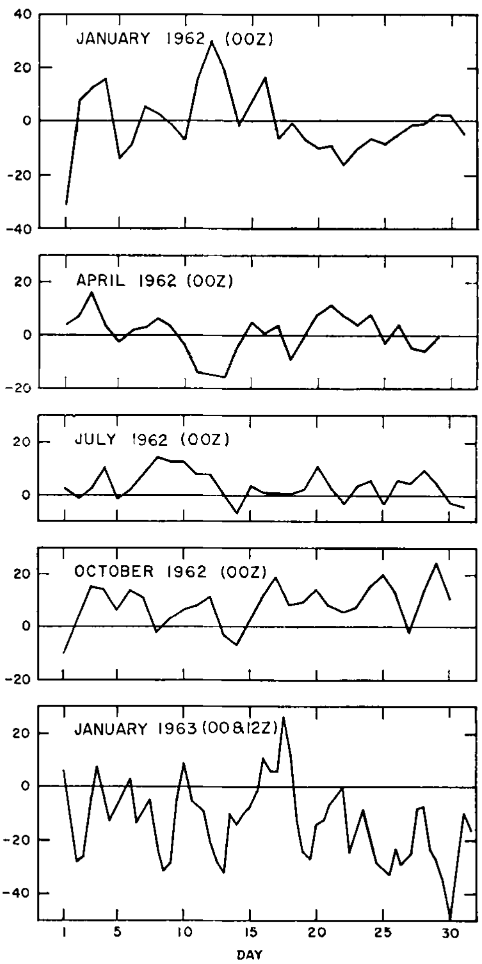

Fig. 20. The variation of the daily values of the total energy exchange $C(K',K)$ as a function of time for the five months of January 1962, April 1962, July 1962, October 1962 and January 1963. Unit: 10^{-4} kj m^{-2} sec^{-1}.

tion of $C(K',\bar{K})_3$ from Figs. 17 and 18 gives a value of -9.47×10^{-4} kj m^{-2} sec^{-1}. This value is computed from the time average of the momentum transport and the time average of the mean zonal wind and should be compared with the value which is obtained by forming the time average of the daily values of $C(K',\bar{K})$. The latter value (see Fig. 14) is -11.28×10^{-4} kj m^{-2} sec^{-1} which shows that for this month we find most of $C(K',\bar{K})$ in the so-called standing wave for wave number 3.

Fig. 20 shows the energy exchange $C(K',\bar{K})$ as a function of time for the five months of January, April, July and October 1962 and January 1963. The records indicate clearly a large standard deviation of $C(K',\bar{K})$.

6. Comparison with other results

The energy exchange $C(\bar{P},P')$ has been computed earlier by Peixoto (personal communication) who, however, only included transient eddies in his calculations. The data were for the year 1950 and his results are given for the year, the six winter months (Oct.–March) and the six summer months (April–September). This result for $C(\bar{P},P')$ for the winter 1950 turned out to be 12.5×10^{-4} kj m^{-2} sec^{-1}, while our result using the two winter months, January and October 1962, is 43.6×10^{-4} kj m^{-2} sec^{-1}. The somewhat larger value obtained in this study may be due to differences between the two sets of data, but is at least partly explained by the fact that Peixoto only included transient eddies. For the summer of 1950 Peixoto obtained 7.3×10^{-4} kj m^{-2} sec^{-1} compared to our result 19.2×10^{-4} kj m^{-2} sec^{-1}, using April and July 1962. A remark similar to the one made above applies in this case.

Taking again January and October 1962 as representing the winter months and April and July 1962 as representing the summer months, we find a value for the winter 1962 for $C(K',\bar{K})$ equal to 4.7×10^{-4} kj m^{-2} sec^{-1}, while the value for the summer is 1.9×10^{-4} kj m^{-2} sec^{-1}. These values may be compared with those obtained by SALTZMAN & FLEISHER (1960) who computed $C(K',\bar{K})$ from 500 mb only and got 2.3×10^{-4} kj m^{-2} sec^{-1} for the winter months of 1950 and 0.7×10^{-4} kj m^{-2} sec^{-1} for the summer months of 1950. STARR (1954) has computed $C(K',\bar{K})$ for the year 1950 using strings of data along latitude circles and got 1.8×10^{-4} kj m^{-2} sec^{-1}.

We may also compare our results with those obtained in the numerical experiments of the general circulation by PHILLIPS (1956) and SMAGORINSKY (1963). These experiments are supposedly modeled in such a way that they correspond more closely to the annual mean value of the energy generation and dissipation. We should therefore compare these values with our annual mean values of energy exchanges. Taking an average for the year 1962 based on the months of January, April, July and October we find for $C(\bar{P},P')$: 31.4×10^{-4} kj m^{-2} sec^{-1} and for $C(K,\bar{K})$: 3.3×10^{-4} kj m^{-2} sec^{-1}. The corresponding values quoted as mean values from PHILLIPS (1956) are 38.4×10^{-4} kj m^{-2} sec^{-1} and 16.2×10^{-4} kj m^{-2} sec^{-1}, respectively, while Smagorinsky gets 29.8×10^{-4} kj m^{-2} sec^{-1} and 12.5×10^{-4} kj m^{-2} sec^{-1}. There is a fair agreement between the values for $C(\bar{P},P')$, while the values for $C(K',\bar{K})$ obtained in the numerical studies are larger by a factor of 3 to 4 than those obtained from observational studies. Several explanations for this discrepancy are possible. One possible explanation is that the frictional dissipation $D(K')$ is smaller in the model used in the numerical calculations than actually exists in the atmosphere. Another more likely explanation is that the numerical models do not allow for a mechanism by which the eddy available potential energy P' can be destroyed by diabatic processes (i.e. $G(P') < 0$) as is found from observational studies (WIIN-NIELSEN & BROWN, 1960, and BROWN, 1963). This means that most of the energy which is transferred to eddy available potential energy by the exchange process $C(\bar{P},P')$ must be converted to eddy kinetic energy by the process $C(P',K')$ in the numerical models, while a large fraction of the eddy available potential energy is removed, at least in winter, due to the negative sign of $G(P')$. An indication that this explanation is partly correct is that the energy conversion $C(P',K')$ in the numerical study of PHILLIPS (1956) is 38.0×10^{-4} kj m^{-2} sec^{-1}, while SMAGORINSKY (1963) gets 25.3×10^{-4} kj m^{-2} sec^{-1}. These numbers should be compared with those obtained from observational studies of which we may quote WIIN NIELSEN (1959) who obtained $C(P',K') = 15.0 \times 10^{-4}$ kj m^{-2} sec^{-1} for January 1959.

7. Summary and conclusions

The heat transport, the momentum transport, the energy exchanges between zonal and eddy available potential energy and the energy exchanges between zonal and eddy kinetic energy have been evaluated from atmospheric data for different months in the wave number regime in order to gain some insight into the seasonal variations of the different quantities.

It has been found that the very long atmospheric waves play a very important role in the transport and exchange processes during the winter season, but, relatively speaking, a less important role during the summer season. The marked variation from winter to summer suggests that further investigations of the dynamics of the very long waves must be made. The present theories of these waves are mainly confined to the steady state theories. Although further extensions of the steady state solutions are desirable, it is also necessary to develop theories for the transient, very long waves. The present investigation probably does not contain time series which are long enough to divide the transport and exchange processes into the parts due to the standing eddies and the transient eddies. An investigation of these aspects of the processes will be reported later.

Some preliminary insight into the seasonal variation of the energy exchange processes has been gained during this study, but it is evident that reliable estimates of representative values must be based on larger ensembles of data from the different seasons. This is especially true for the processes which show the greatest variation in time and the largest amount of irregularity in the spectra such as $C(K',\bar{K})$. We hope to present such estimates when the data collections allow the calculations.

It has been demonstrated during this study that there may be large differences between a calculation based on data from the five levels which have entered into this calculation and a calculation based on data from only two levels from the lower part of the troposphere. In this connection it is worthwhile to point out that the existing estimates of the generation of available potential energy and the energy conversion between available potential energy and kinetic energy are based on the minimum of two levels. It is consequently of great importance to try to extend these calculations to a greater vertical resolution.

8. Acknowledgements

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