Further statistics on the exchange of kinetic energy between harmonic components of the atmospheric flow

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ABSTRACT

The transfers of kinetic energy between harmonic components of the 500 mb geostrophic flow over the Northern Hemisphere have been measured for an ensemble of daily maps covering a nine-year period, based on a truncation at zonal wave number, $n = 15$. The results show (1) that, in the mean, all waves ($n = 1$–15) transfer their energy to the zonally-averaged motion ($n = 0$) and, of more physical significance, the aggregate of all waves in the group $n = 2$–15 transfer energy to support the asymmetric polar vortex comprised of wave numbers zero and one, and (2) that, among the waves themselves, waves of $n = 2$ and 5–10 are sources of kinetic energy and all the rest are sinks. The energy source at $n = 2$ seems to be a significant new result indicating a strong forced conversion of energy on the scale of the major continents and oceans. Seasonal variations are discussed.

Daily measurements of the rates of transfer of kinetic energy to the zonal current from the eddies of different wave number, n (denoted by M) (SALTZMAN & FLEISHER, 1960*a*), and between the different eddy scales themselves (denoted by L) (SALTZMAN & FLEISHER, 1960*b*), have now been extended to cover a nine-year period within the years 1955 to 1964.

In view of the longer period of record, we feel enough confidence has been added to the averages to present a resolution into three-month and half-year "seasonal" averages in addition to the annual average, and also into individual wave numbers instead of the groups of wave numbers given previously. The measurements are still only for the 15° to 80° N zonal band at the 500-mb level and for $n = 1$–15. They are based on the same assumptions used in the previous studies (e.g., only the horizontal, geostrophic, components of the motion are considered).

The new nine-year averages are given in Table 1 along with probable errors (computed using half the total number of cases) for the ensemble of daily values over the entire year, the warmer and colder half years, and the three-month seasons. The six-month means are also shown graphically in Fig. 1, and the annual budget is represented schematically in Fig. 2.

From the M values, it can be seen that all waves tend to feed their energy into the zonal current, with maxima at $n = 2$ and 7 in the annual, colder six-month, and winter means. A minimum in M occurs at $n = 4$, as it did in the previous study covering the year 1951. For all wave numbers, the mean values of M appear to be significantly different from zero. The total gain of kinetic energy by the zonal current, measured by $\sum_{n=1}^{15} M(n)$, is given in the last column of Table 1.

The redistribution of kinetic energy among the individual waves, measured by L, shows, in general, a net gain by the long waves $n = 1$, 3, and 4, and the short waves $n = 11$–15. The large loss from $n = 2$ appears to be an important new finding that suggests a large forced conversion of potential energy on this scale associated with the continent–ocean structure. The loss from the cyclone band $n = 5$–10 is probably compensated by the normal free baroclinic development processes within the troposphere.

From a synoptic viewpoint, we note that the so-called "polar vortex" is no more than the sum of an axially-symmetric component of motion corresponding to $n = 0$ and an axially-asymmetric component corresponding to $n = 1$. Thus, the results here show that the whole polar vortex tends to derive an important

TABLE 1. *Mean values of M and L and their probable errors ε, in units of 10^{-3} ergs/cm² sec mb.*

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
Year																	
M		5	26	21	11	14	21	22	15	12	8	5	4	2	2	1	169
$\varepsilon(M)\pm$		*3*	*5*	*5*	*4*	*4*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*0*	*0*	*12*
L		87	-53	29	12	-3	-37	-39	-35	-14	-11	4	5	12	17	26	0
$\varepsilon(L)\pm$		*21*	*24*	*23*	*23*	*24*	*22*	*23*	*20*	*17*	*14*	*11*	*9*	*8*	*7*	*6*	—
Gain	169	82	-79	8	1	-17	-58	-61	-50	-26	-19	-1	1	10	15	25	
Colder months (Oct.–Mar.)																	
M		12	42	27	12	19	23	31	18	14	8	6	4	3	2	1	222
$\varepsilon(M)\pm$		*5*	*9*	*10*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*22*
L		148	-107	47	25	-4	-38	-57	-55	-21	-20	1	4	12	27	38	0
$\varepsilon(L)\pm$		*38*	*43*	*43*	*42*	*44*	*39*	*40*	*36*	*30*	*25*	*20*	*17*	*15*	*12*	*11*	—
Gain	222	136	-149	20	13	-23	-61	-88	-73	-35	-28	-5	0	9	25	37	
Warmer months (Apr.–Sept.)																	
M		-2	10	14	11	10	20	14	11	10	7	4	3	2	1	1	116
$\varepsilon(M)\pm$		*2*	*4*	*4*	*4*	*3*	*3*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*0*	*0*	*11*
L		26	1	13	-1	-2	-35	-22	-15	-8	-3	7	5	12	8	14	0
$\varepsilon(L)\pm$		*17*	*19*	*19*	*19*	*20*	*20*	*20*	*18*	*16*	*13*	*10*	*8*	*7*	*6*	*5*	—
Gain	116	28	-9	-1	-12	-12	-55	-36	-26	-18	-10	3	2	10	7	13	
Winter (Dec.–Feb.)																	
M		15	54	25	11	19	22	28	17	11	7	5	4	2	1	1	222
$\varepsilon(M)\pm$		*7*	*15*	*17*	*12*	*11*	*9*	*7*	*6*	*4*	*4*	*3*	*2*	*1*	*1*	*1*	*35*
L		210	-154	61	22	-18	-16	-73	-53	-15	-35	-10	1	9	26	45	0
$\varepsilon(L)\pm$		*63*	*71*	*68*	*69*	*70*	*61*	*65*	*59*	*49*	*40*	*32*	*27*	*24*	*19*	*18*	—
Gain	222	195	-208	36	11	-37	-38	-101	-70	-26	-42	-15	-3	7	25	44	
Spring (Mar.–May)																	
M		2	20	17	21	12	22	27	12	11	8	5	4	2	1	1	165
$\varepsilon(M)\pm$		*6*	*9*	*9*	*8*	*7*	*7*	*6*	*5*	*3*	*3*	*2*	*1*	*1*	*1*	*1*	*22*
L		73	-27	14	16	28	-67	-28	-53	-24	-5	9	9	10	13	32	0
$\varepsilon(L)\pm$		*36*	*45*	*45*	*42*	*46*	*46*	*46*	*41*	*33*	*32*	*23*	*20*	*16*	*15*	*12*	*32*
Gain	165	71	-47	-3	-5	16	-89	-55	-65	-35	-13	4	5	8	12	31	
Summer (Jun.–Aug.)																	
M		-6	9	7	3	4	13	8	10	10	6	3	3	2	1	1	74
$\varepsilon(M)\pm$		*2*	*4*	*4*	*4*	*4*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*1*	*0*	*11*
L		15	4	9	-3	-1	-12	-23	-8	-7	-1	6	3	7	6	5	0
$\varepsilon(L)\pm$		*17*	*17*	*16*	*18*	*18*	*19*	*19*	*18*	*15*	*12*	*10*	*8*	*7*	*6*	*5*	—
Gain	74	21	-5	2	-6	-5	-25	-31	-18	-17	-7	3	0	5	5	4	
Autumn (Sep.–Nov.)																	
M		8	23	35	10	22	28	27	20	16	9	7	4	4	3	1	217
$\varepsilon(M)\pm$		*4*	*8*	*9*	*8*	*8*	*8*	*6*	*5*	*4*	*3*	*2*	*2*	*2*	*1*	*1*	*23*
L		54	-37	36	13	-22	-51	-32	-26	-12	-3	10	5	20	24	21	0
$\varepsilon(L)\pm$		*37*	*39*	*43*	*44*	*43*	*40*	*40*	*33*	*28*	*23*	*20*	*16*	*15*	*11*	*10*	—
Gain	217	46	-60	1	3	-44	-79	-59	-46	-28	-12	3	1	16	21	30	

part of its energy from the higher wave numbers ($n \geq 2$) by the non-linear transfer processes measured by $\sum_{n=2}^{15} M(n)$ and $L(1)$.

As should be expected from these geostrophic calculations, the results over the spectral region studied are in accord with the theorems on energy transfer in two-dimensional, non-divergent flows presented by FJØRTOFT, 1953: e.g., the loss of energy from intermediate scales is accompanied by a gain of energy by larger

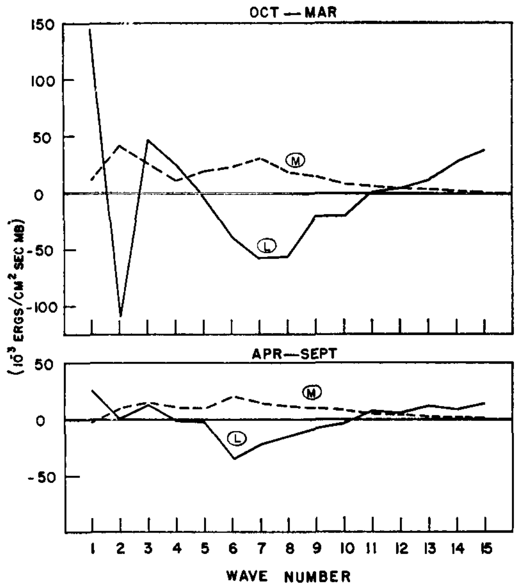

FIG. 1. Energy transfer spectral functions, L (solid line) and M (dashed line), for the cold and warm half years, based on a nine-year record. Lines are drawn between discrete values only for visual aid.

scales as well as by smaller scales. The consistent gain of energy by the higher wave numbers, $n = 11$–15, gives the appearance of a Kolmogoroff-type cascade constrained by the truncation at $n = 15$. In this connection, it is important to recognize that the transfer spectrum is a function of the particular truncation point chosen. For example, if we could extend the calculation to include infinitely high wave numbers (and, even more so, if we included *vertical* transfer processes) we would thereby encompass dissipative energy transfers associated with eddy viscosity, and, as a result, this would markedly affect the entire transfer spectrum. In our case, we have arbitrarily truncated at $n = 15$ in the belief that this represents a rough limit of the scales describable on hemispheric synoptic charts. Accordingly, we consider the aggregate of all scales $n \geq 16$ as a sort of viscous sink for the surplus energies acquired in the long-time average by eddies in the group $n = 1$–15, through the processes measured by L and M and by conversion from potential energy. In truth, however, the region $n \geq 16$ is itself rich in energetical detail involving all subsynoptic phenomena, and, in fact, certain portions of this region may even be significant *sources* of kinetic energy for the synoptic motions (e.g., organized cumulus convective motions).

Inspection of the error estimates ($\varepsilon = 2\sigma/\sqrt{N/2}$ where σ is the standard deviation and N is the number of days) shows much more variability in L than in M. In fact, for most wave numbers, L varies on a daily or longer period basis between positive and negative values so that, from time to time, the dominant kinetic energy source appears to shift from one wave number to another.

The month-to-month variation in kinetic energy transfer by those waves demonstrating a large and distinct annual cycle is shown in

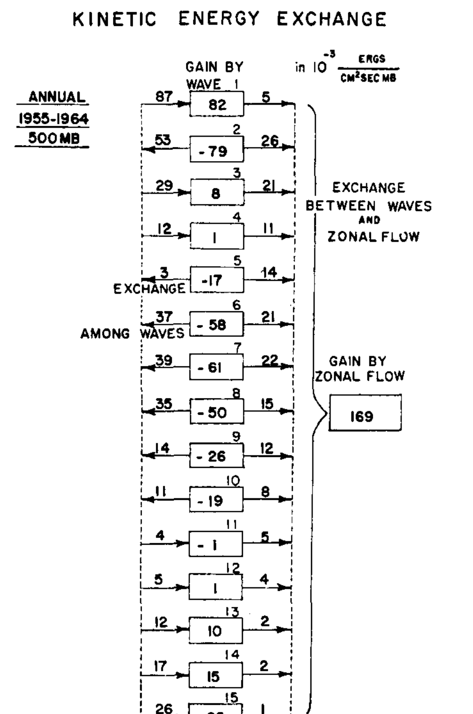

FIG. 2. Annual means of L and M represented in the form of a budget. L values are in the first open column and M values in the second open column with the plus sign shown by an arrow pointing toward the right. The net gain of kinetic energy by individual waves $[L(n) - M(n)]$ and by the zonal flow $\sum_{n=1}^{15} M(n)]$ is shown by the figures within boxes. A negative value within a box thus represents an exported quantity of kinetic energy which, it is assumed, was generated within the given wave number principally by conversion from available potential energy and which is in excess of any amount consumed by friction.

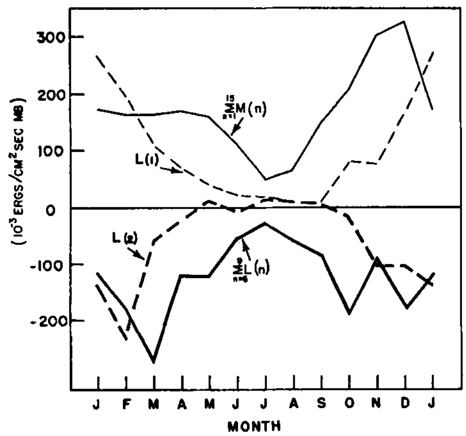

Fig. 3. Monthly means of $\sum_{n=1}^{15} M(n)$, $L(1)$, $L(2)$, $\sum_{n=6}^{9} L(n)$.

Fig. 3. It is interesting to note the degree to which wave numbers 1 and 2 appear to complement each other. The transfer *into* wave number 1 and the transfer *out* of wave number 2 both have large values in the colder months and near zero values in the warmer months, wave number 2 in particular having little net transport between April and October. Although there is a similar annual cycle in the transfer *into* the zonal flow, $\sum_{n=1}^{15} M(n)$, and the transfer *out* of the group of wave numbers 6–9, the rate of transfer remains substantial even in summer when a minimum rate of transfer appears in July. A unique characteristic of the individual month of February, included in the winter statistics, is the tendency for a large positive value of L in wave number 4.

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