# Chandler's period in the mean sea level

By AXEL JESSEN, *Geodætisk Institut, Copenhagen*

(Manuscript received May 28, 1963)

ABSTRACT

The constants of the Chandlerian oscillation in the mean sea level are calculated for nine ports and the results compared with those of the International Latitude Service. The Chandler period is clearly observed and the amplitudes are large enough. The phase differences greatly depend upon the selected localities.

During the years 1894–1915 several attempts were made to prove the existence of Chandler's period in the mean sea level (m.s.l.). The method used is the harmonical analysis of a long series of observations, partly after heavy smoothing. The assumed period is 14 months, 431.24 days, and 434 days (PRZYBYLLOK, 1919; THOMPSON, 1915). The results are not quite satisfactory, largely because the reality of the oscillation is not proved.

In the present article another method is applied, the method of the phase-diagram. Here only an approximate value of the sought period is needed. If the right period turns out, then there is a high probability for the phenomenon being real.

The monthly m.s.l.'s are divided in groups of 7 years. In each group the mean of the 6 periods of 14 months is formed. This mean is an oscillation $A_0 + a\sin(360\,t/14 + \psi)$, where the constants $A_0$, $a$, $\psi$ are to be determined; $t$ = months. The annual period in the m.s.l. is eliminated. The phases $\psi$ are marked in a diagram as ordinates to the group numbers; an arbitrary multiple of 360 can be added to $\psi$. This diagram shows a specific trend: the phases are decreasing at a constant rate which indicates that the true period is $14 + \gamma$. The longer series of m.s.l. give a $14 + \gamma$ which lies remarkably close to the Chandler-period. Thus, at Helsinki, the groups 1879–85, 1886–92, ..., 1949–55 respectively give $\psi = 154$, 271, 136, 132, 324, 19, 325, 289, 202, 164, 87 degrees. The diagram suggests the phases 514, 631, 496, 492, 324, 379, 325, 289, 202, 164, 87, which by a least square adjustment leads to $\gamma = +0.312$ and the period 435.7 days (1 month = 30.44 days).

The monthly m.s.l.'s can also be divided in groups of 5 years. Then the constants of an oscillation with a 15-month period are to be determined. Thus, at Helsinki, the intervals 1880–84, ..., 1950–54 produce, respectively, the phases −213, 37, 52, 62, 214, 135, 376, 358, 428, 501, 604, 649, 670, 755, 893 degrees. A least square adjustment leads to $\gamma = -0.723$ and the period $15 + \gamma$ months = 434.6 days, which again is too close to the Chandler-period to be explained by pure coincidence.

The m.s.l. at 9 places is treated along these lines. The 15-month method is applied, being somewhat more flexible than the 14-month method. The m.s.l. is taken from the ASSOCIATION D'OCÉANOGRAPHIE PHYSIQUE (1940 f.). Table 1 gives some particulars. HL = m.s.l. based on high and low water; $n$ = number of readings per day; $c$ = integration of a continuous record (médimarémètre).

The next problem to consider is the phase-lag when compared with the international latitude observations (RESULTATE DES INTERN. BREITENDIENSTES, 1909 f.). These are treated in the same manner as the m.s.l. The 12 yearly group-combinations are used (the latitude Dec. 1922 is the mean of 2 group-combinations), and the observations at Mizusawa, Carloforte, Ukiah only are used, for the sake of homogeneity. The 15-month oscillation is determined separately at each station. Hereto the annual periods in the same 5-year intervals are added, and finally, by a least square adjustment transformed into the coordinates of the north-pole:

$$l = c_l + a_l \sin(30t + \alpha_l) + b_l \sin(24t + \beta_l),$$
$$m = c_m + a_m \sin(30t + \alpha_m) + b_m \sin(24t + \beta_m),$$

Table 1

| Station | | Record n | Unit of last digit | Remarks |
|---|---|---|---|---|
| Helsinki | 1879–1903 | 1 | cm | |
| | 1904–17 | 24 | mm | |
| | 1918–56 | 6 | mm | |
| Esbjerg | 1889–1959 | 24 | mm | (1) |
| Den Helder | 1865–83 | HL | mm | (2) |
| | 1884–1920 | 4 | mm | |
| | 1921–31 | HL | mm | (3) |
| | 1937–55 | 8 | mm | |
| Marseille | 1885–1958 | c | mm | (4) |
| Bombay | 1878–1930 | 24 | 0.01 f | (5) |
| | 1931–36 | HL | 0.1 f | |
| | 1937–56 | HL | 0.01 f | |
| Sydney | 1897–1954 | HL | 0.01 f | |
| Honolulu | 1905–36 | | 0.01 f | (6) |
| | 1937–56 | 24 | 0.01 f | |
| San Francisco | 1897–56 | 24 | 0.01 f | |
| Charleston | 1921–56 | 24 | 0.01 f | |

(1) The m.s.l. 1889–1937 differ slightly from those published by the Assocation d'Océanographie Physique (1940 f.).
(2) With shallow water corr.
(3) With shallow water corr.
(4) Marégraphe.
(5) Apollo Bandar.
(6) No information about the record.

$l = x$ and $m = -y$ substituting the traditional $x$ and $y$, and the longitude of the stations counted positive towards the east. Here approximately $b_m = b_l$ and $\beta_m = \beta_l - 90$, the Chandler-composant of the polar motion being circular. Using the latitudes $\varphi - 39°8'3.500''$ for Mizusawa, $\varphi - 39°8'8.800''$ for Carloforte, and $\varphi - 39°8'12.000''$ for Ukiah, the constants become, respectively, as shown in Table 2.

From the $\beta_l$, apart from those corresponding to nos. 5 and 6, the Chandler-period is calculated to $433.8 \pm 1.4$ days. The adjusted phase in interval no. 0 is $109° \pm 11°$.

The projection of the 15-month-composant, which approximates the Chandler-composant, on a meridian with longitude $\lambda$ is $b_l \sin(24t + \beta_l) \cos\lambda - b_l \cos(24t + \beta_l) \sin\lambda = b_l \sin(24t + \beta_l - \lambda)$. The phase-lag of m.s.l. at a station with longitude $\lambda$ is $180 + \beta_l - \lambda - \psi$ on the northern and $\beta_l - \lambda - \psi$ on the southern hemisphere, a maximum of the latitude corresponding to a minimum of the m.s.l.

When the circle of the Chandler-composant cuts the ellipse of the annual composant, there is a possibility for a discontinuity in the calculated phase; that accounts for the phase no. 5. The deviation of phase no. 6 can probably be accounted for by the great variation of the non-periodic composant, which invalidates the condition $c_l$ and $c_m$ = const. necessary for the harmonical analysis. The irregularity of nos. 5 and 6 is only due to the calculation; the course of the phases in m.s.l. show that there has been no temporary stop in the Chandler-composant during the time considered.

Table 3 gives for each of the 9 stations the phase $\psi$ and the amplitude $a$. The 5-year interval 1900–04 corresponds to no. 0. The units are: $\varphi$, $\lambda$, $\psi$ degrees, $a$ mm, Chandlers period days. The $\psi$ marked * have been omitted from the adjustment. $\psi_0$ is the adjusted phase in the interval no. 0; the phase-lag is calculated with this $\psi_0$.

The amplitudes of the 15-month m.s.l. oscil-

Table 2

The units: $c$, $a$, $b$ 0.001''; $\alpha$, $\beta$ degrees.

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| | 1900–04 | 1905–09 | 1910–14 | 1915–19 | 1920–24 | 1925–29 | 1930–34 |
| $c_l$ | 7 | 3 | − 2 | 10 | 85 | 62 | 51 |
| $c_m$ | 8 | − 8 | − 53 | − 85 | − 64 | − 65 | − 140 |
| $a_l$ | 92 | 66 | 87 | 119 | 137 | 53 | 134 |
| $a_m$ | 74 | 49 | 50 | 118 | 99 | 36 | 108 |
| $\alpha_l$ | 231 | 265 | 200 | 202 | 220 | 240 | 222 |
| $\alpha_m$ | 139 | 159 | 78 | 112 | 142 | 129 | 119 |
| $b_l$ | 119 | 172 | 201 | 137 | 75 | 40 | 64 |
| $\beta_l$ | 96 | 200 | 255 | 316 | 38 | 232 | 10 |

TABLE 3

| No. | Helsinki | | Esbjerg | | Den Helder | | Marseille | | Bombay | | Sydney | | Honolulu | | San Francisco | | Charleston | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ | $\psi$ | $a$ |
| −7 | | | | | −669 | 12 | | | | | | | | | | | | |
| −6 | | | | | −669 | 17 | | | | | | | | | | | | |
| −5 | | | | | −522 | 28 | | | | | | | | | | | | |
| −4 | −213 | 90 | | | −644* | 15 | | | −197 | 24 | | | | | | | | |
| −3 | 37 | 29 | | | −423 | 16 | | | −46 | 11 | | | | | | | | |
| −2 | 52 | 41 | −352 | 55 | −347 | 14 | −290 | 9 | 46 | 11 | | | | | | | | |
| −1 | 62 | 7 | −362 | 22 | −426* | 11 | −211 | 8 | 42 | 26 | | | | | | | | |
| 0 | 214 | 86 | −129 | 62 | −125 | 31 | 44 | 10 | 114 | 13 | −95 | 15 | | | −148 | 6 | | |
| 1 | 135 | 64 | −208 | 41 | −186 | 25 | 21 | 17 | 262 | 20 | −8 | 8 | 23* | 13 | 81 | 27 | | |
| 2 | 376 | 62 | 36 | 43 | 60* | 23 | 127 | 13 | 198 | 18 | −65 | 5 | 234 | 41 | 125 | 12 | | |
| 3 | 358 | 36 | 52 | 48 | 60 | 29 | 138 | 10 | 356 | 7 | 196* | 8 | 296 | 18 | 124 | 15 | | |
| 4 | 428 | 80 | 91 | 68 | 80 | 28 | 287 | 23 | 274 | 9 | 245 | 13 | 382 | 26 | 48* | 11 | | |
| 5 | 501 | 70 | 136 | 74 | 145 | 46 | 221 | 10 | 357 | 18 | 111* | 17 | 318* | 9 | 213 | 7 | 11 | 16 |
| 6 | 604 | 39 | 246 | 35 | 218 | 21 | 420 | 25 | 437 | 4 | 250 | 8 | 361* | 11 | 463 | 12 | 126 | 21 |
| 7 | 649 | 69 | 264 | 20 | 254 | 12 | | | 466 | 11 | 236* | 28 | 631 | 12 | 549 | 11 | 88 | 3 |
| 8 | 670 | 75 | 330 | 59 | 315 | 25 | 466 | 29 | 592 | 16 | 406 | 19 | 671 | 8 | 518 | 29 | 221 | 27 |
| 9 | 755 | 103 | 407 | 85 | 418 | 32 | 537 | 29 | 653 | 19 | 442 | 17 | 746 | 31 | 598 | 26 | 271 | 26 |
| 10 | 893 | 50 | 540 | 56 | 541 | 44 | | | 892* | 17 | 626 | 12 | 820 | 3 | 657 | 20 | 361 | 6 |
| 11 | | | 693 | 31 | | | | | | | | | | | | | | |
| $\varphi$ | 69 | | 55 | | 53 | | 43 | | 19 | | −34 | | 21 | | 38 | | 33 | |
| $\lambda$ | 25 | | 8 | | 5 | | 5 | | 73 | | 151 | | −158 | | −122 | | −80 | |
| $\psi_0$ | 160 ± 18 | | −210 ± 24 | | −200 ± 11 | | −66 ± 28 | | 111 ± 17 | | −109 ± 40 | | 85 ± 12 | | −69 ± 31 | | 43 ± 71 | |
| Phase-lag | 104 | | 131 | | 124 | | −10 | | 105 | | 67 | | 2 | | 120 | | −34 | |
| Ch.-period | 434.6 ± 1.1 | | 433.0 ± 1.3 | | 434.7 ± 0.6 | | 434.0 ± 1.9 | | 438.3 ± 1.1 | | 435.4 ± 2.1 | | 433.1 ± 0.6 | | 432.6 ± 1.4 | | 435.6 ± 2.9 | |

lation seem not to follow the amplitudes of the Chandler-composant. They are far greater than the theoretical amplitudes calculated by PRZYBYLLOK (1919), especially in the Baltic, where they attain unexpectedly large values, which may affect the conclusions drawn from the comparison of m.s.l. with precise levelling. The phase-lags (except at Marseille) seem to be larger on the eastern shores of an ocean than on the western shores.

## REFERENCES

ASSOCIATION D'OCÉANOGRAPHIE PHYSIQUE, Publication scientifique no. 5, 1940; 10, 1950; 12, 1953; 19, 1958; 20, 1959.

PRZYBYLLOK, E., 1919. Über die sogenannte Polflut in der Ost- und Nordsee. *Veröff. d. preuss. Geod. Inst.* Neue Folge no. 80.

RESULTATE DES INTERNATIONALEN BREITENDIENSTES (Results of the International Latitude Service), III, 1909; V, 1916; (VI) 1932; VIII, 1940.

THOMPSON, D'ARCY, W., 1915, On the mean sea level and its fluctuations. *Fishery Board for Scotland*, Scientifique Investigations, 1914, IV.