On inertially-controlled flow patterns in a β-plane ocean

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ABSTRACT

Possible flow patterns in a rectangular, β-plane, ocean are generated by assuming that the ocean basin can be divided into an interior region in which the dynamical balance is of the Sverdrup type (1947) and a boundary region where the dynamical balance is governed by inertial forces. The flow pattern which was derived by CARRIER & ROBINSON (1962) is shown to be the only possible pattern for a one-layer and a two-layer ocean when friction is not taken into account. The impcrtance of friction is discussed and possible alternatives are mentioned for cases in which friction is included.

1. Introduction

This paper contains a derivation of the flow pattern in a β-plane ocean where it is assumed that the flow in the major regions of the ocean is determined by a balance between coriolis acceleration, pressure gradients and wind stress,[1] and that in the remaining regions potential vorticity is conserved. No attempt will be made to deduce the equations in a rigorous manner—e.g., the questions which arise in connection with the derivation of the β-plane equations from those for flow on a sphere are not considered. Nor is the very essential role of friction treated in a detailed fashion. These questions and others of equal importance have been considered in some detail by previous authors (cf. MORGAN (1956) and CARRIER and ROBINSON (1962)). The present purpose is to give a simple derivation of the results which can be obtained by the application of ideas and concepts which have attained some degree of acceptance through continued usage and consequent familiarity. It will be seen that many of the results which have been derived through the use of relatively sophisticated mathematical technique can be easily understood from simple physical arguments.

In particular, the flow patterns for a homogeneous ocean and for a two-layer ocean are derived by using the balance of forces outlined in the first sentence. The results show that in a homogeneous ocean basin the interior flow must be toward the west and that the circulation pattern for a basin with a zonal wind stress given by $\tau \sim -\cos \pi y'$, $0 \leqslant y' \leqslant 1$ (where y' is a non-dimensional coordinate of latitudinal distance) requires the existence of a zonal jet in the interior part of the ocean. These same results have been derived by CARRIER and ROBINSON (1962) with the use of boundary layer theory. The details which Carrier and Robinson obtain for the flow pattern are not derived here but these details are not necessary for an understanding of how the circulation pattern can arise. Thus details are sacrificed for the advantages of simplicity.

The real advantages of the present method are seen more clearly in the discussion of the two-layer ocean. This case cannot be treated easily by the method used by Carrier and Robinson. The flow pattern for the two-layer ocean is the same as that of the homogeneous ocean but certain minor restrictions on the velocity amplitudes in the boundary layer must be added.

2. Homogeneous ocean basin

The quasi-geostrophic equations which govern steady flow in the interior of a homogeneous ocean basin are

$$-fv = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial z}, \tag{1}$$

[1] This will be called quasi-geostrophic flow. It is the flow in the interior of the ocean first derived by Sverdrup (1947).

$$fu = -\frac{1}{\varrho}\frac{\partial p}{\partial y} + \frac{\partial \tau_y}{\partial z}, \quad (2)$$

$$\frac{1}{\varrho}\frac{\partial p}{\partial z} = -g, \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (4)$$

$$\frac{d}{dt}(h-z) = 0 \quad \text{at} \quad z = h, \quad (5)$$

where u, v, w are velocity components positive to the east (x), north (y), and upward (z) respectively, $f = f_0 + \beta_y$ is the familiar linearized form of the vertical component of the earth's rotation vector, τ_x and τ_y are the x and y components of the stress vector (horizontal friction is neglected in the interior), ϱ is the (constant) density and p is the pressure. Equation (5) defines the free surface height, h.

If the equations be integrated over the depth from $z = 0$ to $z = h$, the pressure gradients can be expressed in terms of gradients of h. Also, the vertical velocity, w, can be expressed in terms of the horizontal components of velocity at the top and bottom of the ocean through equation (5) and the equations reduce to the form

$$-fV = -\frac{g}{2}\frac{\partial h^2}{\partial x} + \tau, \quad (6)$$

$$fU = -\frac{g}{2}\frac{\partial h^2}{\partial y}, \quad (7)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \quad (8)$$

U and V are now horizontal transports defined by $\int_0^h u dz$, $\int_0^h v dz$ respectively. It has been assumed that the stresses at the bottom of the ocean vanish, that τ_y evaluated at $z = h$ vanishes also, and that $\tau_x = \tau$ at $z = h$. Thus the circulation is the result of a zonal wind-stress acting at the surface, $z = h$. Equations (6)–(8) are a rigorous consequence of equations (1)–(5) when the above stress conditions are invoked.

Now cross-differentiating equations (6) and (7) yields

$$\beta V = -\frac{\partial \tau}{\tau y}$$

or

$$\boxed{V = -\frac{1}{\beta}\frac{\partial \tau}{\partial y}.} \quad (9)$$

Sustituting V into equation (8) then provides the relation

$$\boxed{\frac{\partial U}{\partial x} = \frac{1}{\beta}\frac{\partial^2 \tau}{\partial y^2}.} \quad (10)$$

Equations (9) and (10), originally derived by Sverdrup (1947), are two equations which are necessary to deduce the flow in the interior. The expression for V yields the direction and magnitude of the meridional transport uniquely once $\partial\tau/\partial y$ is known. Equation (10) does not give unique information about U even when $\partial^2\tau/\partial y^2$ is known because the zonal transport is determined only to within an arbitrary function of y. It is therefore necessary to introduce additional constraints.

It will be assumed that quasi-geostrophic balance is valid throughout the interior and up to either the eastern or the western boundary. Depending on which of the latter conditions is chosen, the direction of zonal flow is determined. For example, if $\partial^2\tau/\partial y^2 > 0$ and if equations (9) and (10) obtain up to the *eastern* boundary of a rectangular ocean (where $U = 0$), then $\partial U/\partial x > 0$ and $U < 0$ throughout the interior. A western boundary layer must be added to close the flow. If, on the other hand, $\partial^2\tau/\partial y^2 > 0$ and equations (9) and (10) are assumed valid up to the *western* boundary, then $\partial U/\partial x > 0$ and $U > 0$ throughout the interior. In this case an eastern boundary layer closes the flow.

There are, therefore, eight possible flow patterns determined by three independent parameters—viz., the sign of $\partial\tau/\partial y$, the sign of $\partial^2\tau/\partial y^2$, and the validity of equations (9) and (10) up to either the eastern or the western boundary. It is possible to consider flows with boundary layers on both sides of the ocean but such flows simply add an unnecessary complication to the present considerations.

The eight flow patterns in the interior are shown in Figs. 1*a* to 1*h*. In Figs. 1*a* to 1*d* quasi-geostrophic flow is assumed right up to the eastern boundary and a western boundary layer is necessary to close the flow. For example, in Fig. 1*a* the net meridional transport in the interior is northward and a southward flow

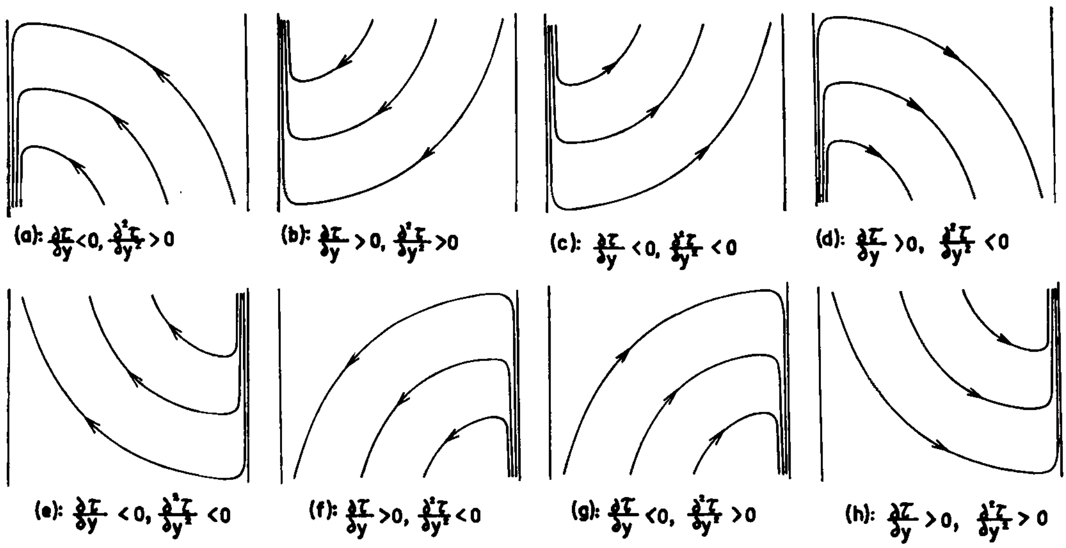

FIG. 1. The eight possible flow patterns in a homogeneous or two-layer ocean are shown. The patterns are determined by the signs of $\partial\tau/\partial y$ and $\partial^2\tau/\partial y^2$ and by the closure of the flow with either a western (Figs. *a* to *d*) or an eastern (Figs. *e* to *h*) boundary current. Some of the patterns cannot satisfy the constraint imposed by conservation of potential vorticity in the boundary layer. (See text.)

must be added in the western boundary layer for purposes of mass conservation. Figs. 1*e* to 1*h* contain eastern boundary layers. The patterns within each group 1*a*–1*d* and 1*e*–1*h* are determined by the signs of $\partial\tau/\partial y$ and $\partial^2\tau/\partial y^2$.

Thus far quasi-geostrophic flow in the interior is the only dynamical constraint which has been considered. The flow in the boundary layers is assumed to be inertially controlled and this assumption provides an additional constraint on the flow. Using boundary layer considerations one can easily derive the relation

$$\boxed{V \cdot \nabla\left(f + \frac{\partial v}{\partial x}\right) = 0}^{1} \qquad (11)$$

in the boundary layer regions. Equation (11) states that absolute vorticity is constant along a particle path. (The vertical component of vorticity is consistently approximated by $\partial v/\partial x$). The principal simplification involved in equation (11) is the lack of importance of the wind-stress term. Physically one expects this to be the case—the boundary layer region is so narrow that the effect of the wind-stress in the boundary layer is negligible compared to its effect in the interior. The boundary layer flow is driven by the interior flow.

Without going into the details of the boundary layer solution, one can expect that the meridional velocity essentially vanishes at the outer edge of the boundary layer and has a maximum amplitude at the wall. Thus if the flow is northward (southward) in the western boundary layer the relative vorticity, $\partial v/\partial x$, is negative (positive). For the eastern boundary layer northward (southward) flow implies positive (negative) relative vorticity.

One can now apply the boundary layer constraint to the flow pattern. Since $\partial v/\partial x + f$ is constant along a streamline, it is necessary that the relative vorticity, $\partial v/\partial x$, compensate for the change in the coriolis parameter, f, along a streamline. Consider, e.g., Fig. 1*a*. The flow comes in from the interior and turns southward so that f decreases. In order that $\partial v/\partial x + f$ remain constant it is necessary that $\partial v/\partial x$ increase and since $\partial v/\partial x$ changes from zero to a large positive value, the pattern in Fig. 1*a*

[1] The velocities can be considered as averages in the vertical ($V = vh$) for the present purpose. h can be considered as essentially constant for the homogeneous ocean.

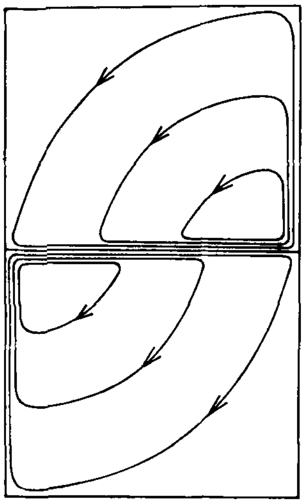

FIG. 2. A flow pattern which is consistent with all constraints for both the one- and two-layer oceans when the wind stress is of the form $\tau \sim -\cos \pi y'$, $0 \leq y' \leq 1$.

is consistent with the boundary layer constraint.[1]

Similarly, in Fig. 1*b* f increases and $\partial v/\partial x$ decreases along a streamline to yield a consistent picture. On the other hand, in Fig. 1*c* the flow leaves the boundary layer and along a streamline $\partial v/\partial x$ changes from a positive value to zero so that $\partial v/\partial x$ decreases. However, f also decreases and $f + (\partial v/\partial x)$ cannot remain constant but must decrease. Therefore, Fig. 1*c* represents an impossible flow pattern. Similar reasoning shows that the circulations in Figs. 1*a*, 1*b*, 1*e* and 1*f* are consistent with the necessary constraints and that those in Figs. 1*c*, 1*d*, 1*g* and 1*h* are not.

One is therefore led to the conclusion that *only those patterns with westward flow in the interior satisfy the necessary conditions.* This means that, if one tries to form a complete circulation pattern for a given zonal wind-stress distribution, it is necessary to put all the eastward flow into regions which are governed by dynamics which are not quasi-geostrophic.

As a particular example consider the wind-stress distribution $\tau \sim -\cos \pi y'$ where the non-dimensional coordinate y' ranges from 0 to 1. Here, $\partial \tau/\partial y > 0$ throughout, $\partial^2 \tau/\partial y^2 > 0$ for $0 \leq y' < \frac{1}{2}$ and $\partial^2 \tau/\partial y^2 < 0$ for $\frac{1}{2} < y' \leq 1$. Thus for this example the only possible circulation with inertial balance in the boundary regions is one with Fig. 1*b* for the southern half-basin and Fig. 1*f* for the northern half-basin. This pattern is shown in Fig. 2 with a zonal jet to provide the eastward flow. The mid-latitude jet is not unique—it is possible that several jets could exist. However, the pattern shown is the simplest one consistent with the ideas introduced above.

It should be noted that the qualitative structure in the zonal jet is determined by the dynamics of the interior solution and the boundary layers along the coasts. For example, in the southern half-basin the interior flow generates the western boundary layer. Thus the southern half of the zonal jet receives water from the western boundary layer and in turn gives water off to the interior. In the northern half-basin the eastern boundary layer weakens northward and gives water off to the interior region. Hence, the northern half of the zonal jet is generated by the interior solution and, when it is fully formed, it turns northward and generates the eastern boundary layer.

This example is essentially the one considered analytically by CARRIER and ROBINSON (1962) although they were forced to linearize the wind-stress in order to obtain an analytical solution. However, the linearization which they introduced was simply convenient and did not involve any changes in the qualitative flow pattern.

As was mentioned earlier, circulations with boundary layers at both the eastern and western boundaries are also possible. The simplest way of looking at such flows is to picture the total circulation as a superposition of a combination of the patterns described above and one or more free inertial solutions of the type considered by FOFONOFF (1954). The latter involve symmetric boundary layers at the eastern and western boundaries. The amount of recirculation of the free inertial type which one can add is completely arbitrary because such flows are independent of any driving mechanism and therefore the amplitude is undetermined. The present problem is non-linear in the boundary layers but linear in the interior so that one can imagine superposition in the interior with a non-linear behavior in the

[1] This argument was first used by MORGAN (1956) to show that a western boundary current could be generated by an interior solution but an eastward boundary current could not.

inertial regions. It would appear that the only change that would occur in treating the full non-linear problem in the boundary layer is one of detail although one should carry out the analysis to check such a speculation. In the work of Carrier and Robinson the wind-stress is linearized and the boundary layer problem becomes a linear one. The principle of superposition is therefore valid and, in fact, their solutions are simply sums of wind-driven circulations and free inertial flows.

3. Two-layer ocean

Consider next an ocean consisting of two immiscible layers with the less dense water in the upper layer. If the lower layer is assumed to be at rest, the equations which determine the flow in the upper layer away from coastal boundaries are the same as (6), (7) and (8) with g replaced by $g' = g(\Delta\varrho/\varrho_L)$ (where $\Delta\varrho$ is the (positive) density difference and ϱ_L is the density of the water in the lower layer). The depth, h, is now the variable depth of the upper layer. The flow in the interior is therefore determined exactly by equations (9) and (10) as before so that the eight flow patterns in Fig. 1 again represent the possible circulations in the interior. The essential difference between the two models is that the depth h is variable. Although a variable h does not change the results for the interior flows, it plays a very important role in the boundary layers.

A rigorous boundary layer expansion at the eastern or western boundaries yields the following set of equations:

$$-fv = -g'\frac{\partial h}{\partial x}, \tag{12}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g'\frac{\partial h}{\partial y}, \tag{13}$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \tag{14}$$

where either the velocities can now be considered as vertical averages ($U = uh$, $V = vh$) or the velocities are assumed independent of the vertical. Equation (12) is simply the geostrophic equation. Again the essential simplification in (12) is the lack of importance of the stress—the non-linear terms drop out in the boundary layer expansion but the inclusion of those terms leads to no difficulties.

Now if (12) and (13) are cross differentiated and if (14) is used, one deduces the relation

$$V \cdot \nabla\left(\frac{\dfrac{\partial v}{\partial x} + f}{h}\right) = 0. \tag{15}$$

This is the familiar expression for conservation of potential vorticity, i.e., $(\partial v/\partial x + f)/h$ is constant along a streamline. Furthermore, if one multiplies (12) by u and (13) by v and adds the equations, one finds

$$V \cdot \nabla\left(\frac{v^2}{2} + g'h\right) = 0. \tag{16}$$

Equation (16) is the Bernoulli equation which states that the total energy per unit mass, $v^2/2 + g'h$, is conserved.

At the junction between the boundary layer and the interior the potential vorticity is simplified since $\partial v/\partial x < f$ and the energy expression is also simplified since $v^2/2 < g'h$. Hence, one writes

$$\frac{\dfrac{\partial v}{\partial x} + f}{h} = \frac{f_i}{h_i}, \tag{17}$$

$$\frac{v^2}{2} + g'h = g'h_i, \tag{18}$$

where f_i and h_i are the values of f and h on a streamline at the point where the streamline enters or leaves the boundary layer.

Now one can solve for h/h_i in (18)

$$\frac{h}{h_i} = 1 - \frac{v^2}{2g'h_i} \tag{19}$$

and substitute (19) into (17) to derive

$$\boxed{\frac{\partial v}{\partial x} = f_i - f - \frac{v^2}{2g'h_i}f_i} \tag{20}$$

along a streamline. Equation (20) is the constraint which is imposed by the boundary layer dynamics on the flow patterns of Fig. 1.

Consider again the various cases. In Fig. 1*a*, the interior flow is northwestward and turns south in the boundary layer. Since southward flow in the boundary layer implies that the relative vorticity, $\partial v/\partial x$, is positive, it is necessary that the right-hand side of equation (20) remain positive. Now $f_i > f$ so that the change in f has the proper sign. However, $(v^2/2g'h_i)f_i$, is a positive quantity and the contribution of this term tends to decrease the right hand side of (20). Thus the pattern in Fig. 1a is possible provided that $(v^2/2g'h_i)f_i$ is not too large.[1]

In Fig. 1b the flow is northward in the western boundary layer so that $\partial v/\partial x$ must be negative. Now $f_i < f$ and one sees that both the second and third terms on the right hand side of equation (20) contribute with the appropriate sign. Hence, the flow in Fig. 1b is completely consistent.

In Fig. 1c it is necessary that $\partial v/\partial x$ be positive in the boundary layer. Here, $f_i < f$ (since the flow comes *out* of the boundary layer *into* the interior). Thus the combination of terms in (20) yields a negative value for $\partial v/\partial x$ and this flow cannot be realized.

In Fig. 1d $\partial v/\partial x$ must be negative. However, $f_i > f$ and it is necessary for the term $(v^2/2g'h_i)f_i$ to be sufficiently large in amplitude to overcome the effect of the change in planetary vorticity.[1] This pattern is also possible therefore but it requires very large velocities in the boundary layer.

Thus of the four circulations with boundary layers on the western edge, that of Fig. 1b is completely consistent with the inertial constraints, that of Fig. 1c is impossible, that of Fig. 1a requires that the velocities in the boundary layer be limited in amplitude and that of Fig. 1d requires very large velocities in the boundary layer.

Similar reasoning for the eastern boundary layer cases gives the following results:

Fig. 1 e: $\dfrac{\partial v}{\partial x} < 0, f_i < f.$ completely consistent

Fig. 1 f: $\dfrac{\partial y}{\partial x} > 0, f_i > f,$

$\dfrac{v^2}{2g'h_i}f_i$ must not be too large

[1] The magnitude of the term $v^2/2g'h_i$ can be expressed in terms of a Froude number. "Not too large" and "not too small" can be interpreted respectively as cases in which the Froude number is less than one and greater than one.

Fig. 1 g: $\dfrac{\partial v}{\partial x} < 0, f_i > f,$

$\dfrac{v^2}{2g'h_i}f_i$ must be very large

Fig. 1 h: $\dfrac{\partial v}{\partial x} > 0, f_i < f,$ impossible

FOFONOFF, however, has pointed out to me that there is an additional constraint based on mass transports which must also be satisfied and when one includes this constraint, the two flow patterns 1d and 1g are not possible. The argument is straightforward and simple and can be seen from the following:

Consider Fig. 1d. The mass transport in the boundary current can be computed from the geostrophic relation (12) by multiplying (12) by h, making use of the stream function definition,

$$hv = \frac{\partial \psi}{\partial x}, \quad hu = -\frac{\partial \psi}{\partial y} \tag{21}$$

and integrating across the boundary layer from $x = 0$ to $x = x_i$ at any value of y. Thus one finds

$$f_i \psi_i = \frac{g'}{2}(h_i^2 - h_0^2), \tag{22}$$

where h_0 is the value of h at $x = 0$.

Now the northward mass transport given by (22) must equal the eastward transport which one derives by integrating with respect to y the geostrophic relation

$$fuh = -\frac{g'}{2}\frac{\partial h^2}{\partial y} \tag{23}$$

at the off-shore edge of the stream from the value y to the northern boundary (denoted by subscript n) of the basin. Using (21), one can integrate (23) to get

$$\int_y^{y_n} \beta \psi dy + f_i \psi_i = -\frac{g'}{2}(h_n^2 - h_i^2). \tag{24}$$

Then subtracting (22) from (24) yields

$$\int_y^{y_n} \beta \psi dy = \frac{g'}{2}(h_0^2 - h_n^2). \tag{25}$$

Since $\psi > 0$, by definition, it is necessary that

$$h_0^2 > h_n^2. \tag{26}$$

But by the Bernoulli relation (16), along the streamline $\psi = 0$ one has

$$\frac{v_0^2}{2} + g'h_0 = g'h_n. \tag{27}$$

Therefore $$h_n > h_0. \tag{28}$$

Since h is positive (26) and (28) are contradictory. Hence, Fig. 1*d* is not a possible pattern.

The above argument also rules out the pattern of Fig. 1*g*, and more generally any decelerating western boundary current or accelerating eastern boundary current.

Hence for the specific wind stress distribution discussed for the homogeneous ocean one is again led to Fig. 2 as the only possible pattern. The only restriction imposed by the additional considerations of the two-layer ocean is that $(v^2/2g'h_i)f_i$ not be too large along the eastern boundary of the northern half-basin. (It should be noted that the same restriction applies to the homogeneous case also but since g' is replaced by g and since h_i is much greater for the homogeneous ocean, the term $(v^2/2gh_i)f_i$ is always very small. Essentially one assumes that such is the case when one considers h as constant for the homogeneous basin.)

4. Additional considerations

The results derived in the previous sections are based on certain simplifications and approximations which must be reconsidered at this point.

It is clear, for example, that a driven system which is bounded by coasts must have some dissipative mechanism to offset the driving force.[1] In particular, if one forms the steady state vorticity equation for the homogeneous ocean one finds that

$$V \cdot \Delta(f + \zeta) = \text{curl}_z \tau + \text{friction}.$$

If friction is neglected, the total vorticity of a particle increases by a multiple of $\text{curl}_z \tau$ when the particle makes a complete circuit and returns to its original point. Thus one must either introduce a discontinuity in the streamline pattern or one must include friction in order to achieve a steady state. All streamlines must effectively pass through a frictional region in order to satisfy the steady state vorticity balance.

Furthermore, if one integrates the vorticity equation over the basin, one finds that the vorticity input given by the integral of $\text{curl}_z \tau$ must be balanced by vorticity diffusion at the boundaries. This vorticity diffusion will occur at the bottom and/or along the sides.

No attempt will be made here to consider the role of friction in a detailed manner. It suffices to note that friction *must* be introduced and that it requires a considerably more detailed investigation than is possible with the simple ideas presented here.

It should be pointed out, however, that MOORE (1961) has looked into the problem of attaching a frictional sub-layer onto the inertial boundary layers along the sides. He found that for the homogeneous ocean a frictional sub-layer can be added to the inertial boundary layer on the western boundary of the southern half-basin. However, with the Navier-Stokes form for the frictional term it is impossible to find a solution with a frictional sub-layer on the eastern boundary of the northern half-basin. In fact, he draws an exact mathematical analogy between the present problem and the flow past a cylinder. The western boundary layer corresponds to the boundary layer along the front half of the cylinder. The eastern boundary layer corresponds to flow along the back half of the cylinder and no boundary layer solution exists—a wake region is formed. In both problems the determining factor seems to be that a boundary layer solution cannot be found when the flow is against the pressure gradient.

A further point which one must consider is that the thermocline structure is quite unrealistic. The interface between the two layers deepens northward up to the zonal jet, where it becomes rapidly shallower. North of the jet it deepens again. This pattern of the thermocline is, of course, associated with westward flow in the interior. Even in those areas where the wind stress is directed eastward the slow interior flow is westward. It is difficult to accept such a picture of the circulation, particularly since much of the slow geostrophic flow of the oceans of the world is eastward.

There are several alternative arguments which

[1] In an unbounded system a momentum balance could be achieved by advection to infinity.

one can invoke and which might lead to results that differ markedly from those given here.

In the first place, it is possible that a hydraulic jump occurs in the western boundary current so that the dynamics of that region are changed drastically. In fact, since the thermocline is observed to come to the surface near the inshore edge of the Gulf Stream, a jump is bound to occur (because $v^2 > 2g'h$ as $h \to 0$) and the results of the inertial theory cannot be applied there.

Secondly, the basic idea underlying the present investigation, viz., that of inertio-frictional boundary layers with slowly varying interior solutions, may not be valid. An alternative possibility is one with rapidly oscillating flows in the interior of the northern half-basin with something like standing Rossby waves to give the oscillatory pattern. MOORE (1961) has constructed such a model with oscillatory solutions.

In addition it is conceivable also that no steady-state solution exists for the complete non-linear set of equations and that locally the motions are time dependent. This possibility can perhaps best be explored through numerical models on an electronic computer.

A final point is that the two-layer ocean model subsumes a parametric density distribution. In the real ocean the supply of warm, upper-layer water is probably controlled by the overall heat balance. Hence, it is quite possible that the amount of upper-layer water available to the system controls the circulation pattern. The combined dynamic-thermodynamic model is another possibility which can probably be explored only with numerical techniques.

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