Tellus
SERIES A
DYNAMIC
METEOROLOGY
AND OCEANOGRAPHY
PUBLISHED BY THE INTERNATIONAL METEOROLOGICAL INSTITUTE IN STOCKHOLM

Baltic sea level low-frequency variability

By EVGUENI A. KULIKOV[1,2]*, IGOR P. MEDVEDEV[1,2] and KLAUS PETER KOLTERMANN[2], [1]*P.P.Shirshov Institute of Oceanology, Russian Academy of Sciences, Moscow, Russia*; [2]*Natural Risk Assessment Laboratory, Lomonosov Moscow State University, Moscow, Russia*

(Manuscript received 5 August 2014; in final form 11 December 2014)

ABSTRACT

The low-frequency sea level spectrum in the Baltic Sea has been analysed based on long-term time series of sea level data (15–124 yr) from three tide gauge stations in the Baltic Sea and two stations in the North Sea. The principal periodicities detected in the spectrum are seasonal and tidal oscillations including the pole tide with a period of about 14 months. Cross-spectral analysis has been applied to estimate the frequency response of sea level oscillations in the Baltic Sea relative to the North Sea. It is demonstrated that the basic factor in the formation of the low-frequency sea level spectrum in the Baltic Sea is the barotropic water exchange through the Danish straits. The limited throughput of these straits plays the role of a natural low-pass filter for the sea level variations: high-frequency sea level variations from the North Sea are effectively damped, while the low-frequency signal can pass through into the Baltic Sea almost undisturbed. A simple model of the barotropic water exchange used in the study allows us to estimate the parameters of the filter. It is shown that the cutoff frequency is about 0.014 cpd (74 d period): the energy of sea level oscillations at this frequency is reduced by one half after their penetration into the Baltic Sea. This study contributes to quantifying extreme sea level events in the Baltic Sea in height and time to improve their predictability.

Keywords: Baltic Sea, sea-level, tide gauges, extreme events, barotropic water exchange, natural low-pass filter

1. Introduction

The Baltic is a large semi-enclosed shallow sea. Its area is 393,000 km^2 with an average depth of 54 m and a maximum depth of 459 m (Leppäranta and Myrberg, 2009). The Baltic Sea is connected to the North Sea by the narrow and shallow Danish straits (the Great Belt, the Little Belt and the Öresund). The flow rate through these straits reaches values exceeding river runoff by about 20 times (Stigebrandt, 1980). In fact, this barotropic water exchange is the main factor forcing the low-frequency variability of Baltic sea level. As Carlsson (1997, p.1) pointed out, 'These connections act as low-pass filters for the sea level variations, high-frequency variations of the sea level in the Kattegat are effectively damped while low frequencies can pass almost undisturbed into the Baltic Sea'.

In Samuelson and Stigebrandt (1996), it was suggested to divide the Baltic sea level variability into two types: (1) 'external' variations generated by the sea level changes outside the Baltic (in the Kattegat and the North Sea) and fresh water supply, and (2) 'internal' variations caused by forcing from the atmosphere (air pressure, wind) and water density changes in the Baltic Sea. At the same time, the internal sea level variability determines the sea level spectrum in the range of periods less than 2 d – they are substantially related to their Eigen free oscillations (seiches) in the Baltic Sea. In turn, changes in the sea level caused by external factors dominate the spectrum at periods longer than 1 month.

In Kulikov and Medvedev (2013), long-term datasets of hourly sea level observations at 22 tide gauges of the coast of Russia, Finland, Latvia, and Estonia were used to study the Baltic sea level spectrum. According to the classification of Samuelson and Stigebrandt (1996), the analysed frequency range corresponds to the 'internal' sea level variability. It is shown that the spectrum of the sea level variability was formed mainly by varying air pressure and winds. At the same time, resonance properties of the Baltic basin play an important role in defining the spatial structure of the sea level variability and the generation of free oscillations (Kulikov and Fain, 2008).

As the continuation of the study by Kulikov and Medvedev (2013) in this paper, we present the results of

*Corresponding author.
email: KulikovE@ocean.ru

Citation: Tellus A 2015, **67**, 25642, http://dx.doi.org/10.3402/tellusa.v67.25642

the spectral analysis of low-frequency Baltic sea level variability, covering the range of periods from 2 d to 20 yr, defined as 'external' oscillations. Special attention is paid to the role of the water exchange between the Baltic and the North seas in the formation of the spectrum of sea level.

It should be mentioned that in this study we do not analyse in detail the secular changes in the level of the Baltic Sea (trends). There are several papers where this problem has been well studied (e.g. Ekman, 2009).

2. Data and basic characteristics of sea level variability

To study the low-frequency part of the Baltic sea level variability, it is necessary to use long-term continuous observations. There are about 30 tide stations on the Baltic coast with durations of the measurement exceeding 100 yr. At Stockholm, the sea level has been recorded since 1754, and this record is the world's longest continued sea level recording (Ekman, 1988).

In this paper, the variability of the Baltic sea level has been analysed using hourly measurements from five tide gauges (Table 1, Fig. 1): Station Gorniy Institute (St. Petersburg, Russia), Göteborg, Stockholm (Sweden), Cuxhaven (Germany) and Wladyslawowo (Poland). In Stockholm, continuous hourly measurements have been recorded for 124 yr of observations. At stations, Gorniy Institute and Wladyslawowo hourly data have been presented in the form of relatively short series of 30 and 15 yr, respectively. Sea level data in the Kattegat were analysed by observations from Göteborg with a record of 40 yr. The Cuxhaven station is an 'external station' in relation to the Baltic Sea area as it is located on the North Sea coast of Germany. Duration of the series for this station is 81 yr. Data for stations of Cuxhaven, Göteborg and Stockholm were taken from the site of the University of Hawaii Sea Level Center (UHSLC) (http://uhslc.soest.hawaii.edu). All measurements were reduced to the same time scale (GMT). The sea level was adjusted to the zero of the Baltic System of Heights (0 BSH) (Lazarenko, 1961). Sea level data were thoroughly checked for errors and spikes; the gaps in the records were filled with interpolated data. We should emphasise the high quality of the analysed sea level records: the percentage of gaps in the observational period did not exceed 1% for all stations.

Sea level variability in the Baltic Sea becomes apparent at different time scales. Figure 2 shows plots of sea level changes in Stockholm based on different sampling rates and averaging scale.

According to Carlsson (1997) on longer time scales – centuries – the post-glacial uplift is important as the sea bottom becomes dry land. Most of long-term Baltic sea level records demonstrate significant trends (Fig. 2a). The highest rate of vertical uplift is observed in the northern Gulf of Bothnia. In the southern part of the Baltic Sea, the earth's crust subsidence resulted in the positive trend of changes of the level. After subtracting the trend, the sea level record appears as a superposition of different types of variability, with the dominant one being the seasonal cycle in sea level (Fig. 2b). In the Baltic Sea, annual and semi-annual components are dominating the seasonal signal. This kind of variability is mainly of meteorological origin; hence, their amplitude and phase are subject to significant changes from year to year (Ekman and Stigebrandt, 1990; Medvedev, 2014). In the frequency range of synoptic meteorological processes (with periods of 2–30 d), no periodic components such as sea level or tidal oscillations are not observed: sea level variability is predominantly influenced by meteorological factors (atmospheric pressure and wind stress) and therefore is mostly of random character (Fig. 2c). A regular component – seasonal variation – is also visible on the graph. In Fig. 2d, we see tidal oscillations and some meteorologically forced variations. They have their largest amplitudes of up to 3–4 m as storm surges and seiches in the Gulf of Finland.

3. Spectral analysis of the sea level

Spectral analyses of stochastic processes are commonly used in order to estimate the distribution of sea level variance (energy) over frequency. Depending on the nature of the oscillation, the spectrum can be either of a continuous nature of the energy distribution (continuum), which is typical for processes of turbulent noise character, or in the form of sharp delta-like peaks (discrete spectrum), which corresponds to regular harmonic components with fixed frequencies. For instance, tidal oscillations appear in the spectrum in the form of sharp peaks at the major tidal harmonic frequencies K_1, O_1, M_2, S_2, etc. (Magaard and

Table 1. Hourly sea level datasets characteristic

No	Station	Latitude	Longitude	Country	Time range
1	Cuxhaven	53.87	8.72	Germany	1917–1987
2	Göteborg	57.68	11.79	Sweden	1967–2006
3	Stockholm	59.32	18.08	Sweden	1889–2012
4	Gorniy Institute	59.93	30.28	Russia	1977–2006
5	Wladyslawowo	54.80	18.42	Poland	1992–2006

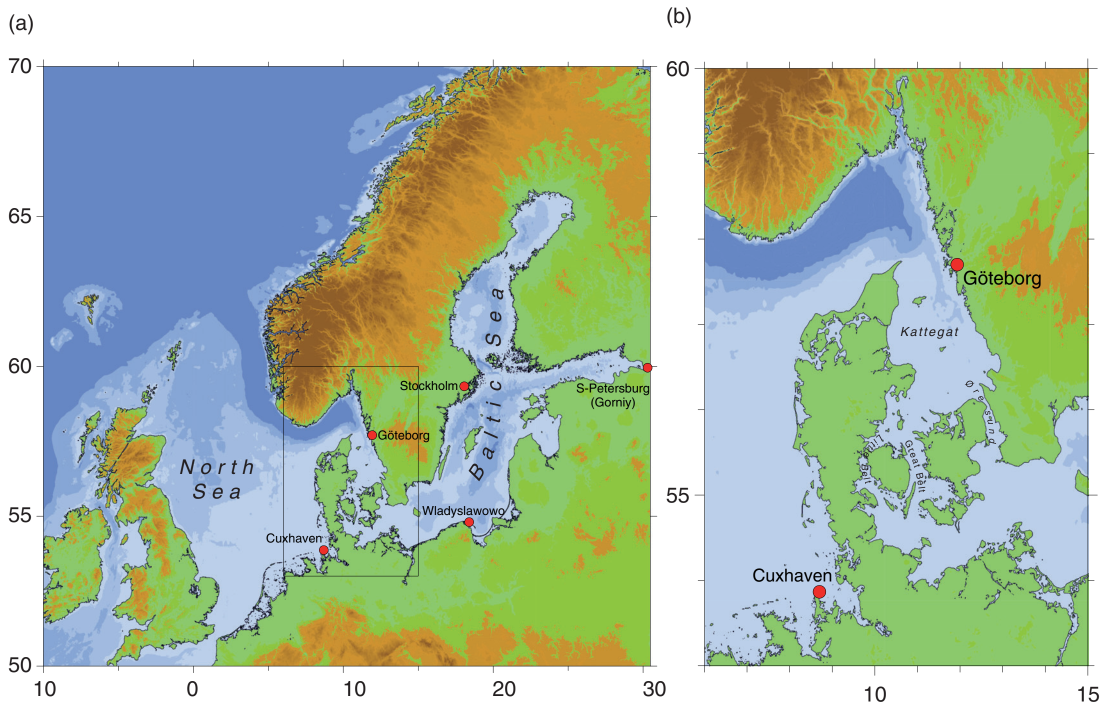

Fig. 1. Map of the Baltic and North seas. (a) Five tide gauge stations for long-term sea level observations. Their results have been used in this study. (b) The map of Danish straits (the Oresund, the Great Belt and the Little Belt) shows with red marks the sea level sites used.

Krauss, 1966). In turn, changes in sea level caused by the influence of alternating atmospheric pressure and wind fields at the sea surface are basically random in nature and have a noise spectrum as a continuous function of frequency.

Results of the spectral analysis of recorded data of the sea level of hourly measurements are presented for frequencies from 10^{-4} to 0.5 cpd (Fig. 3). In order to adequately display the spectral density distribution for such a wide range of frequencies, the calculation of the spectrum was carried out using a fast Fourier transform algorithm applied to the entire record and subsequent averaging of the periodogram in frequency domain. Smoothing was performed using a Gaussian-weighted moving window, where its width increases with frequency so that the magnitude of the relative resolution remains constant for all frequencies. Figure 3 shows the spectra for the two longest series of observations of the level at stations of Göteborg (1979–2006) and Stockholm (1889–2012). Dependence of the spectrum $S(f)$ on frequency f is presented in the graphs in linear-logarithmic scale, where function $f{\cdot}S(f)$ correctly displays the energy distribution of sea level oscillations over the logarithm of the frequency.

The energy of the low-frequency oscillations is distributed predominantly in the range of 10^{-3}–10^{-1} cpd. Excluding seasonal peaks in the sea level spectrum in Göteborg, the energy maximum lies within the synoptic period range of 5–50 d, while in Stockholm most energy is concentrated in the intra-annual period range of 30–300 d. The total oscillation energy is 505 and 395 cm^2, respectively. Such difference in spectra inside and outside the Baltic Sea corresponds to the effect of the low-pass filtering mentioned above.

The seasonal component provides a significant contribution to the total energy of sea level oscillations. In Fig. 3a and b besides the main peak Sa, corresponding to the annual harmonic, the semi-annual Ssa component also is clearly visible. Averaged over a long time period (more than 100 yr), the amplitudes of the annual and semi-annual components reach 14 and 5 cm in the Baltic Sea, respectively. The maximum amplitudes of the annual component are observed in the Gulf of Bothnia and in the Gulf of Finland, the semi-annual – in the central part of the Baltic Sea (Ekman, 1996). In Göteborg, the annual cycle dominates, and the amplitude of the semi-annual cycle appears much weaker.

Seasonal sea level variability is mainly associated with the annual cycle of atmospheric pressure gradients, wind and, to a lesser degree, density changes due to changes in water temperature and salinity. The contribution of the gravitational tide to the seasonal cycle of the sea level is very low. Its amplitude is close to the static response to the annual gravitational forcing, and does not exceed several millimetres. The harmonic characteristics of the main factors vary from year to year (for example, runoff or atmospheric pressure fluctuations). Consequently, the amplitude and

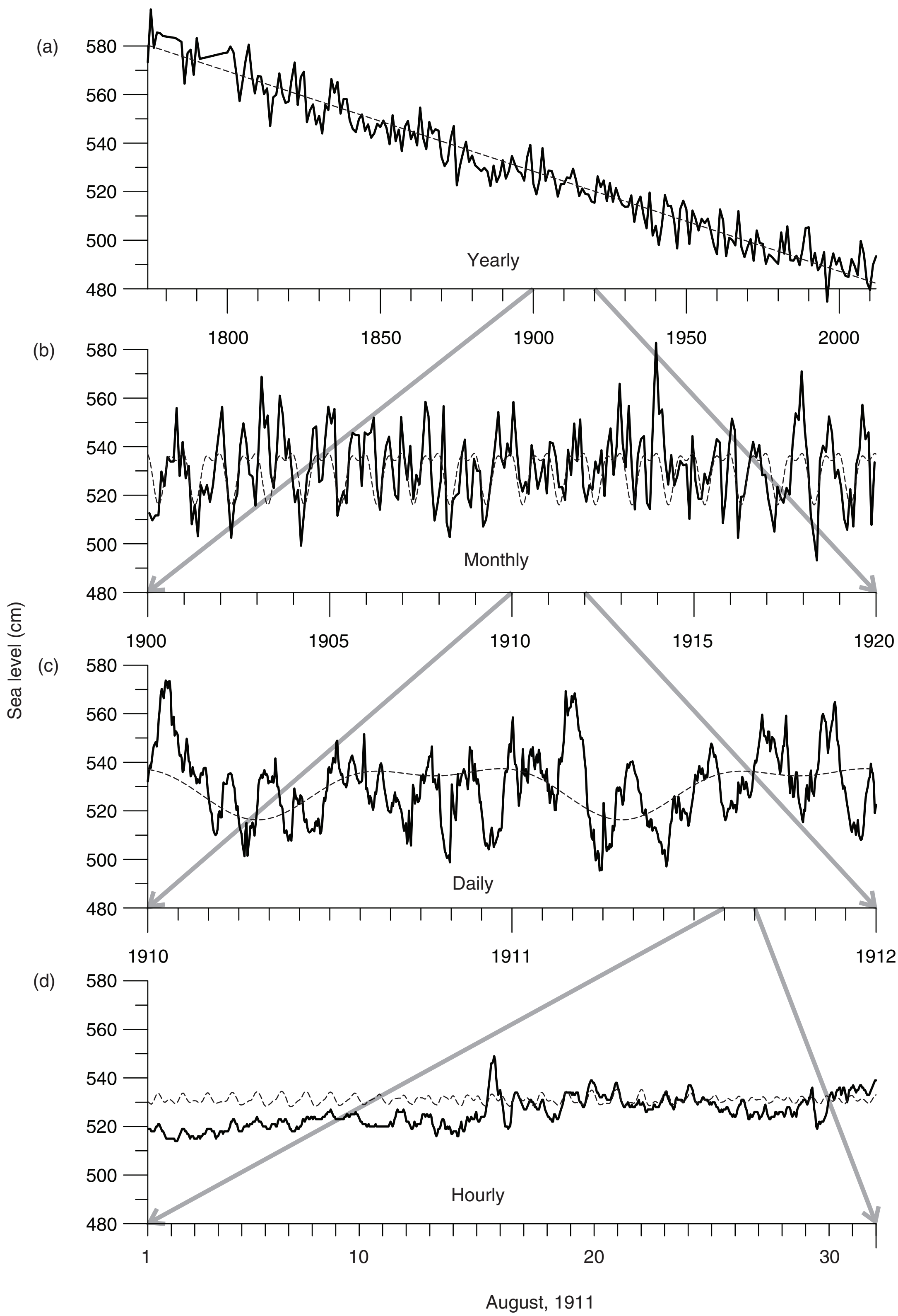

Fig. 2. Sea level records in Stockholm (a) annual mean sea level data, dashed line shows a linear trend; (b) monthly mean sea level data, dashed line shows climatologic seasonal cycle; (c) daily mean sea level, dashed line shows climatologic seasonal cycle; (d) hourly sea level, dashed line shows the predicted tide. Data are referenced to 0-BSH + 500 cm.

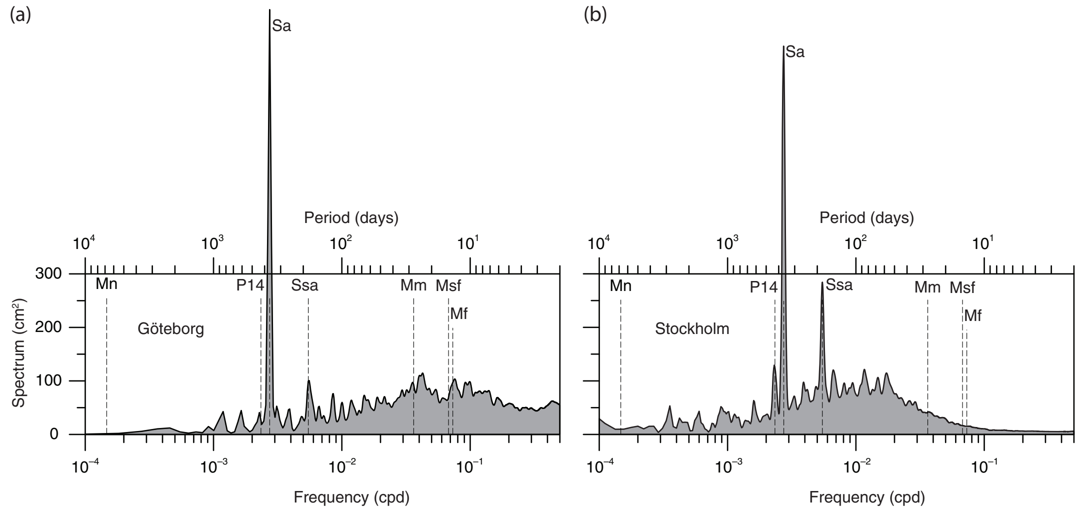

Fig. 3. Spectra of the sea level variations f·S(f): (a) in Göteborg (the Kattegat, Sweden), and (b) in Stockholm (the Baltic Sea, Sweden). Dashed lines show frequencies of tidal harmonics: a 18.6-yr nodal one (Mn), a month one (Mm), half-month ones (Msf, Mf), and seasonal ones (Sa, Ssa – annual and semi-annual) and a 14-month pole tide (P14).

the phase of the seasonal sea level variability show appropriate changes. Figure 4 shows wavelet diagrams of sea level variability in Stockholm. Annual oscillations are the dominant feature of the sea level wavelet diagrams and are defined better than semi-annual oscillations. Temporal variations of the intensity of generating forces are the reason of the modulation of the intensity of the annual signal.

In the spectrum of the Stockholm sea level oscillations (Fig. 3b), calculated for a 124-yr series, it is possible to distinguish a frequency peak of about 0.86 cpy (14-month period). This harmonic corresponds to the Chandler frequency (Chandler Wobble) – the frequency of a free nutation of the Earth axis (Maximov, 1970). G. Darwin called the corresponding wave in the ocean the 'pole tide', which to a large extent is similar to long-period tidal oscillations (Darwin, 1898). Medvedev et al. (2014) specified the main features of the Baltic Sea pole tide on the basis of long-term tide gauge measurements. Data from 71 stations were used to calculate the pole tide sea level variation RMS (root mean square), which made it possible to describe the pole tide spatial variability. The spectral analysis of long observation series revealed time-related changes of the oscillation in amplitude and period. The pole tide in the Baltic Sea seemed to be anomalously high. At the same time, a distinct increase of the tidal amplitudes in the northeast direction from 1.5 to 4.5 cm was noted in the Baltic Sea, its maxima are located in the Gulfs of Bothnia and Finland. In Fig. 4, a pole tide component (dashed line Fig. 4)

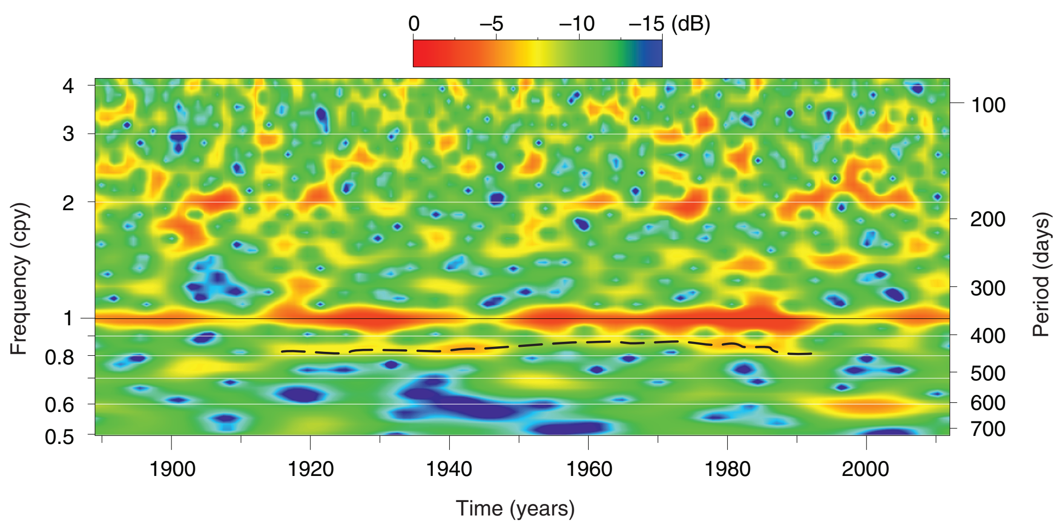

Fig. 4. Wavelet diagrams of the sea level variations in Stockholm. Dashed lines show pole tide (P14).

is discernible close to the annual frequency. The pole tide had significant temporal variations in amplitudes and periods during the 19th and 20th centuries and a distinguished prominent intensification for the period from 1920 to 1950 and from 1970 to 1990.

Medvedev et al. (2013) have shown that a characteristic property of the tidal waves of the Baltic Sea is a marked predominance of diurnal tidal harmonic amplitudes over semidiurnal ones. This feature seems to be surprising, as in the Atlantic Ocean and its marginal seas semidiurnal tides significantly dominate. They argue that it is probable because of the resonant character of diurnal tides in the Baltic Sea. It is obvious that the main reason of such dominance is related to the effect of the low-pass filter mentioned by Carlsson (1997) that causes stronger damping of semidiurnal tides in comparison with diurnal tides (lower frequency) penetrating to the Baltic basin. This abnormal damping of semidiurnal tidal oscillations may be also amplified by the effect of non-linear (quadratic) flow resistance in the entrance of straits introduced by Stigebrandt (1980).

In general, the tidal height is small (5–8 cm); maximum heights are recorded at the head of the Gulf of Finland – up to 20 cm (Medvedev et al., 2013). In the North Sea, tidal oscillations are much stronger: at the station of Cuxhaven, spring tides reach heights of 3.2 m.

In the spectra of the analysed series of the level observations, we cannot allocate significant peaks corresponding to the long-period tidal components Mm, Msf and Mf. In the Baltic Sea, the amplitude of these harmonics is very small, and it is difficult to detect them in background noise by the spectral method. In the spectrum, it is also problematic to select the long-term tidal harmonic Mn with 18.6-yr period (nodal tide). In Wróblewski (2001), its amplitude for the Baltic Sea is estimated at approximately 0.6–0.9 cm, which is close to the value for the static tide.

4. Barotropic water exchange between the Baltic and North seas, and its influence on formation of the sea level spectrum

As noted in Stigebrandt (1980), the dominating factor that forms the low-frequency variability of the Baltic sea level is the water exchange between the Baltic and the North Sea through the Danish straits. Long-term constant flows arising from time to time in the narrow straits (the Little Belt, the Great Belt and the Sound) cause significant water outflow or inflow and can lead to noticeable changes in the volume of the Baltic Sea. So Carlsson (1997) argues that during intense inflows/outflows through these straits, the mean sea level in the Baltic Sea can change by almost a meter within a few weeks. The characteristic quantity of this flow intensity is about 10^5 m^3 s^{-1} (Gustafsson and Andersson, 2001), with peak values up to 10^6 m^3 s^{-1}.

The unbalance of the sea levels inside and outside the Baltic appears to occur for different reasons: tidal movements of water masses, wind or atmospheric pressure gradients, river runoff, precipitation, etc. In narrow and shallow straits, the resulting compensation flow is mainly controlled by two physical components: the force of friction (hydraulic resistance) in the near-bottom turbulent boundary layer, and a hydrostatic pressure gradient formed along the channel. The low capacity of the Danish straits is a natural factor which isolates the Baltic basin from impact of short-period sea level changes in the Kattegat. For example, the penetration into the Baltic of tidal oscillations with amplitudes in the North Sea of more than 1 m is difficult; on average, here they are not more than a few centimetres. At the same time, seasonal and inter-annual sea level changes in the North and Baltic seas do not differ significantly. Figure 5 shows daily mean of synchronous sea level data in Cuxhaven and Stockholm. It is evident that oscillations in Stockholm

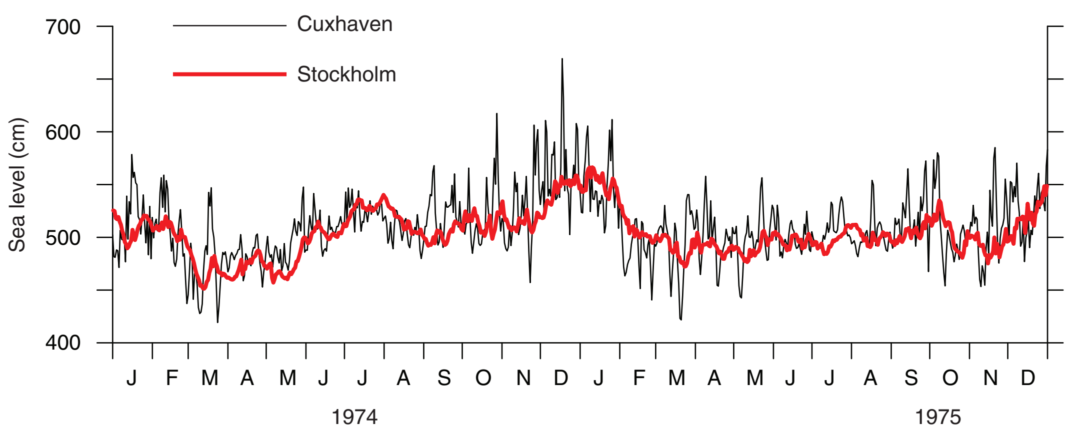

Fig. 5. Synchronous daily mean sea level records in Cuxhaven (the North Sea) and Stockholm (the Baltic Sea). Data are referenced to 0-BSH + 500 cm.

look like a smoothed record for Cuxhaven. In the graph, we see the result of noticeable low-pass filtering: oscillations with periods less than 1 month, that were well manifested in the North Sea (Cuxhaven tide gauge), practically disappear after passing the Danish straits, and they are barely visible in the Stockholm record.

In order to calculate parameters of this natural filter, we estimate the spectral frequency response between the sea level records in the North and Baltic seas. Pairs of synchronous records have been selected: Cuxhaven and Stockholm (1967–1987), Göteborg and Stockholm (1983–2006), and Cuxhaven and Gorniy (1977–1987). Additionally, we analysed pairs of records from the Baltic tide gauges of Stockholm and Wladyslawowo (1992–2006). In the process of calculations of cross-spectral characteristics, the series of the sea level observations at Cuxhaven and Göteborg, located in the North Sea, have been regarded as the *input* signal with respect to the internal (Baltic) tide gauges at Stockholm and Gorniy. The records at these stations were considered as the *output* signals of a linear system under consideration. Spectral analysis was carried out by a fast Fourier transform and the method of spectral density smoothing over the segments using the Kaiser-Bessel window, in order to improve the quality of the calculations and to reduce the Gibbs effect. The window length was set at 16.384 hours, which makes it about 2 yr. For selected pairs of records, the number of degrees of freedom varied from 18 to 48. Figure 6 presents the results of calculating the cross-spectral characteristics: coherence, frequency response and phase spectrum for four pairs of level records (a, b, c, d). The top row shows the coherence function, where the 95% confidence level is marked. Below we show the normalised frequency response and the phase. At low frequencies, the coherence is quite high, and only for periods of less than 5 d the correlation between the sea level oscillations becomes statistically poor. The so-called *external* (low-frequency) oscillations of the Baltic and North seas correlate well, while the smaller coherence for the periods of less than 5 d means that in this frequency range, the internal oscillations (e.g. seiches), which are not related to the level oscillations in the North Sea, play the greater role in the Baltic sea level spectrum. Note that for the Stockholm–Wladyslawowo

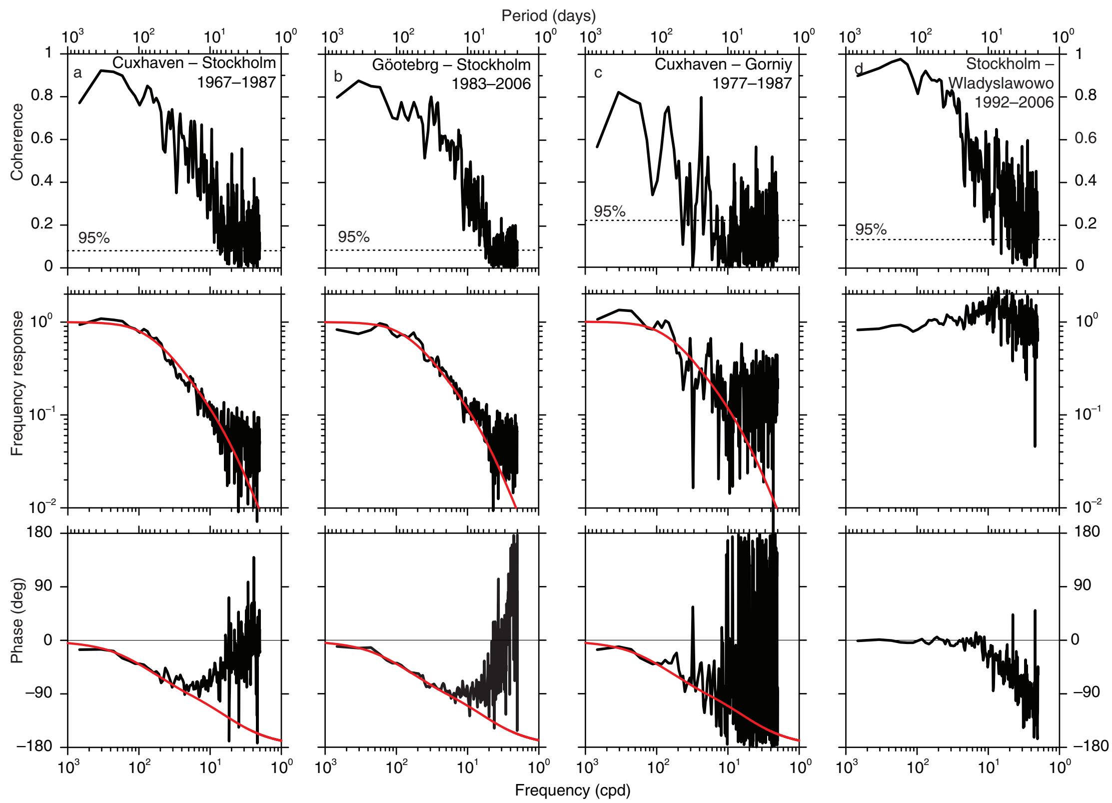

Fig. 6. Cross-spectral characteristics calculated for four pairs of synchronous sea level records: Cuxhaven – Stockholm (a), Göteborg – Stockholm (b), Cuxhaven – Gorniy (c), Stockholm – Wladyslawowo (d). Graphs of coherence are given in the upper panel; dashed line shows the 95% confidence interval. The central panel presents graphs of the normalised frequency magnitude response function. The red line traces the approximation of the response changes by analytical relationship [eq. (7)]. A frequency phase response function is given in the bottom panel. The red line shows the approximation of the phase changes by analytical relationship [eq. (8)].

pair the coherence still remains quite high at low periods. This is natural as both records belong to the same internal basin.

Frequency response is an analogue of the regression coefficient of random variables, and within the frame of the stochastic processes theory, it establishes a measure of a coherent link in a linear system between the input and output signals which represent a ratio of the amplitude of the output signal (response) harmonics to the amplitude of the input signal of specified frequency. For three pairs of stations (Fig. 6a, b, and c), we see the same behaviour of the amplitude ratio with respect to frequency: in the basin of the Baltic Sea, the magnitude of frequency response to external forcing (sea level changes in the North Sea) gradually decreases with increasing frequency. At the frequency of 0.1 cpd, the magnitude of the frequency response is reduced by an order. The calculated phase functions for these three pairs of stations (a, b, c) show a significant time lag in the response of the Baltic sea level relatively to the corresponding sea level changes in Göteborg and Cuxhaven. For the period of 100 d, the phase shift is about -40°; that is, sea level changes of the Baltic Sea are delayed for more than 10 d relatively to the sea level of the North Sea. For the period of 20 d, the phase is -90° (the delay of 5 d). Note that at higher frequencies (>0.1 cpd), the estimates of the frequency response and phase are statistically less significant (low coherence) and, therefore, their scatter is much higher.

For the pair of Baltic stations, Stockholm–Wladyslawowo low-frequency (frequencies <0.1 cpd) oscillations occur synchronously. The magnitude of the response Wladyslawowo/Stockholm varies around 1. It means that these slow sea level changes display the increase or decrease of the Baltic Sea water volume.

It is evident that the frequency and phase characteristics calculated for pairs of records in the system of the North–Baltic seas characterise the throughput of the Danish straits depending on the sea level time-scale variability and the ensuing water exchange.

Some simple models of the sea level response to the barotropic forcing of the open ocean are examined in Stigebrandt (1980) for partially isolated waters (bay, fjord, gulf, etc.). In order to describe the relationship between the sea levels of the Baltic and North seas, we use a model that Stigebrandt calls 'Helmholtz resonator'. Miles (1974) was the first who described the seiche Helmholtz mode. However, the Stigebrandt model additionally takes into account the energy dissipation due to turbulent friction in a channel.

Consider an arbitrary shaped basin (semi-enclosed bay) connected to the open sea by a narrow channel (see Fig. 7). Within the frame of linear equations of motion for a homogeneous fluid in the approximation of shallow water, we will determine the connection of the sea level oscillations in the bay with the sea level changes in the open sea. We introduce a limitation of slow movements of fluid where the typical time scale of the sea level variations (T) significantly exceeds the traveltime of a gravitational wave across the basin and channel. In this case, it is reasonable to use the 'quasi-stationary' approximation: the velocity of the sea level changes in the Gulf is related to the slope of the sea surface along the channel, which in turn depends on the flow velocity and hydraulic resistance. We can neglect the Coriolis force in narrow channels, ignoring the cross-channel sea level difference. The problem geometry is shown in Fig. 7: the surface of the bay is A, the channel length is L, its depth is D, and its width is W. Accordingly the cross-sectional area $S = D \cdot W$.

We assume that the typical time scale of the sea level adjusting within the bay area is much smaller than the typical variability time scale of the flow velocity in the channel. In this case, we can suggest the sea level to be constant over the bay area. It should be noted that this assumption is debatable the rotation effect is taken into account, it will allow topographic Rossby waves to appear in the internal waters. They are much slower than the gravity ones and may affect the rate of reaching equilibrium in the bay. However, it is well known that such waves belong to the class of quasi-geostrophic motions in which sea level deviations are relatively small (near rigid lid approximation); so for this case we ignore this effect. Manifestation of the topographic Rossby waves in the Baltic Sea is considered in detail in Fuks (2005).

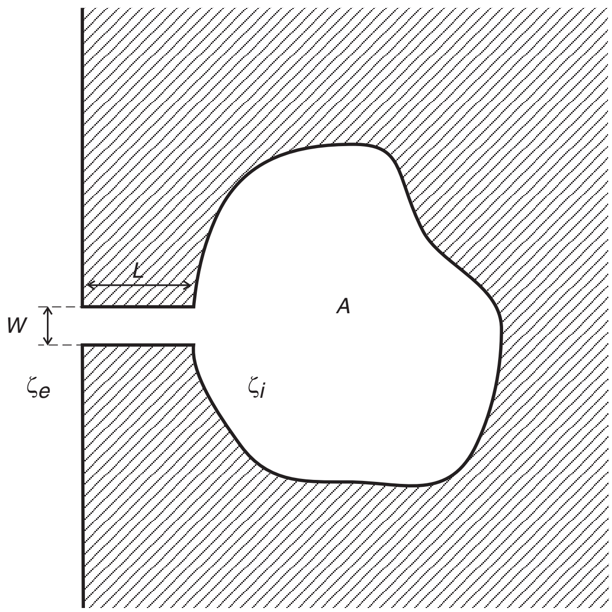

Fig. 7. Geometry of the problem: A is the surface of the bay, L is the channel length and its width is W.

Denote the level from the open sea as ζ_e, and in the bay as ζ_i. Equation for water balance can be written in the form:

$$A\frac{d\zeta_i}{dt} = uS, \tag{1}$$

where u is the flow velocity in the channel. Note that the flow velocity is assumed constant along the channel. This is true if the length of the channel L is much smaller than the scale of a gravitational wave with a period of the order of the time-scale sea level changes $(L << T\sqrt{gD})$.

The momentum equation for the channel flow taking into account linear hydraulic resistance can be written as:

$$\frac{du}{dt} = -g\frac{d\zeta}{dx} - ru, \tag{2}$$

where r is a coefficient of resistance. Integrating eq. (2) along the channel, we get the relation between the sea-surface slope along the channel and the flow velocity:

$$\frac{du}{dt} = g\frac{(\zeta_e - \zeta_i)}{L} - ru. \tag{3}$$

In order to estimate the water exchange between the Baltic and the North Sea, Stigebrandt (1980) recommends using the formula for calculating hydraulic resistance with a quadratic dependence on the flow velocity: bottom friction force is defined as $\tau_b = C_b u^2$. Unfortunately, such a non-linear model is incompatible with conventional spectral analysis for linear systems. Therefore, in this study we constrain the model by using the simple linear relation [eq. (3)] to calculate the hydraulic resistance.

Equations (1) and (3) can be transformed into an ordinary differential equation describing a damped oscillator:

$$\frac{d^2\zeta_i}{dt^2} + 2\delta\frac{d\zeta_i}{dt} + \Omega^2\zeta_i = \Omega^2\zeta_e, \tag{4}$$

where $\Omega^2 = \frac{gS}{AL}$, Ω is a resonant frequency of undamped oscillations, $\delta = r/2$ is a damping coefficient.

Let's rewrite the eq. (4) into a dimensionless form. To do this, we replace Ωt by τ:

$$\frac{d^2\zeta_i}{d\tau^2} + 2\gamma\frac{d\zeta_i}{d\tau} + \zeta_i = \zeta_e, \tag{5}$$

here $\gamma = \frac{\delta}{\Omega}$ is a dimensionless damping coefficient.

Equation (5) defines a substantially linear physical system linking the input ζ_e and output ζ_i signals. The frequency response function of such a system is well known – it is usually used to describe the effect of resonance.

Let's suppose that at the outside part of the channel we observe a periodic level change $\zeta_e = a_e e^{i\omega t}$ $\omega = \frac{2\pi}{T}$ or $\zeta_e = a_e e^{i\sigma\tau}$, where $\sigma = \omega/\Omega$ is the dimensionless frequency of the constraining force. The forced solution will be of the form $\zeta_i = a_i e^{i\sigma\tau}$, where the input/output frequency response function is calculated from eq. (5):

$$a_i = \frac{1}{(1-\sigma^2) + 2i\gamma\sigma} a_e = R(\sigma)e^{i\varphi}a_e, \tag{6}$$

where the amplitude response could be expressed as:

$$R(\sigma) = \frac{1}{\sqrt{(1-\sigma^2)^2 + 4(\gamma\sigma)^2}}, \tag{7}$$

and the phase response can be written as:

$$tg\varphi = \frac{2\gamma\sigma}{1-\sigma^2}. \tag{8}$$

The system behaves differently depending on the value of the damping coefficient: the resonance properties are manifested only when $\gamma < 1$; in this case, the solution of the homogeneous differential eq. (6) is in the form of damped oscillations, and the maximum frequency response corresponds to the resonant frequency, $\sigma_0 = \sqrt{1-\gamma^2}$. When $\gamma > 1$, we observe aperiodic damping, while for $\gamma = 1$ the so-called critical damping.

In relation to the Baltic Sea, the use of this model involves replacing the system of the Danish straits with one 'equivalent' strait of the same throughput. Calculated frequency response and phase spectrum (Fig. 6) corresponds to aperiodic (supercritical) attenuation ($\gamma > 1$) linear system of the North –Baltic Sea. The approximation of amplitude variations [eq. (7)] and phases [eq. (8)] of the frequency response in the frequency range 0–0.1 cpd give an estimate of the attenuation coefficient $\gamma \approx 2.0$, while the undamped resonance frequency $\Omega \approx 0.31$ rad/day (a period of about 20 d). In fact, the dependence of eq. (7) is the response function of the low-pass filter referred to Carlsson (1997). The frequency 'cutoff' of the filter is equal to 0.014 cpd (74 d period): the energy level of oscillations of such frequency is halved after their penetration into the Baltic Sea.

The most important result this model provides is the calculation of the flow velocity in the Danish straits based on the difference in mean sea levels in the Baltic and North seas. In Stigebrandt (1980), a quasi-steady approximation has been used. It was assumed that the flow is controlled only by the sea level difference between the Kattegat and the south-western part of the Baltic Sea. In fact, in this model it means that the acceleration in the eq. (3) is small. In the channel, neglecting the 'dynamics', we can write the flow velocity as $u \approx g\frac{(\zeta_e - \zeta_i)}{rL}$, and the formula for estimating the water flow Q is given by:

$$Q \approx \frac{(\zeta_e - \zeta_i)}{2\gamma}\Omega \cdot A. \tag{9}$$

A typical value of the daily mean sea level difference according to the analysis of sea level data from Göteborg and Stockholm amounts to $\sqrt{\left\langle (\zeta_e - \zeta_i)^2 \right\rangle} \approx 0.3$ m; it corresponds to the average water flow through the straits, calculated by the formula [eq. (9)], $Q \approx 10^5$ m^3/s. This estimate coincides with the data from Jacobsen (1980).

5. Summary

Sea level variations are known to predominantly reflect the barotropic fluid motion caused by tidal forces, and the influence of wind stress and atmospheric pressure gradients at the sea surface. In the internal seas, a significant impact on the sea level changes may be caused by runoff, precipitation and water density changes due to seasonal temperature and salinity variations.

In Samuelson and Stigebrandt (1996), it was proposed to split the Baltic sea level changes into two types: 'external' and 'internal' variations. It refers both to oscillations generated in the Baltic Sea and the 'induced' ones caused by changes in the sea level of the North Sea. It is the external contributions (low-frequency variations) which have been the subject of this study. They are determined by the barotropic water exchange between the Baltic and the North seas.

The description of the low-frequency spectrum of the Baltic sea level oscillations primarily requires the analysis of the energy distribution over the frequencies. In the spectrum of the frequency range from 10^{-1} to 10^{-4} cpd, one can clearly distinguish the following periodic components: seasonal ones (annual and semi-annual – Sa and Ssa) and the pole tide (P14) with a period of about 14 months.

Low-frequency aperiodic oscillations are mainly caused by stochastic effects of alternating wind and atmospheric pressure fields at the sea surface. Due to the sea level differences inside and outside the Baltic Sea, an intensive water exchange through the Danish straits can take place. Limited throughput of these straits creates a natural low-pass filter. We have shown that the North Sea oscillations with a period of 10 d penetrate into the Baltic Sea damped 10-fold. The frequency of the filter 'cutoff' can be estimated as 0.014 cpd (a 74-d period); its coefficient of the energy attenuation of oscillations is ½.

Regular periodic sea level oscillations (astronomical tides, seasonal changes, pole tide) are constantly present in the Baltic Sea. The remarkable property of these components is their predictability. Extreme events (e.g. Neva flooding) caused by the influence of alternating atmospheric pressure and wind fields are basically random in nature and statistically not predictable. But a significant part of sea level changes during extreme events could be mathematically predicted from known parameters of regular periodic components. The total amplitude of regular periodic fluctuations in the level of the Baltic Sea can reach 70–80 cm: spring tide, 10 cm (Medvedev et al., 2013); pole tide, 5 cm (Medvedev et al., 2014); the annual variations, 30–35 cm; semi-annual, 25–30 cm (Medvedev, 2014). This range of regular periodic oscillations is comparable to the critical height of sea level rise during floods in the Neva Bay (160 cm above zero Kronstadt tide gauge). Thus, understanding the mechanisms of regular periodic fluctuations of the sea level and knowledge of their maximum possible amplitudes occupy a significant place in the forecast extreme level rise in the Gulf of Finland.

6. Acknowledgements

This research was supported by the Russian Foundation for Basic Research (Grant 12-05-00733, 13-05-41360 and 14-05-31461), the Russian Science Foundation (Grant 14 37-00038) and the Grant of the Ministry of Education and Science (Contract No 11.G34.31.0007).

References

Carlsson, M. 1997. *Sea Level and Salinity Variations in the Baltic Sea – An Oceanographic Study Using Historical Data*. PhD Thesis. Department of Oceanography, Göteborg University.

Darwin, G. H. 1898. *The Tides and Kindred Phenomena in the Solar System*. J. Murray, London.

Ekman, M. 1988. The world's longest continued series of sea level observations. *Pure Appl. Geophys.* **127**, 73–77.

Ekman, M. 1996. A common pattern for interannual and periodical sea level variations in the Baltic Sea and adjacent waters. *Geophysica*. **32**, 261–272.

Ekman, M. 2009. *The Changing Level of the Baltic Sea During 300 Years: A Clue to Understanding the Earth*. Summer Institute for Historical Geophysics, Åland Islands.

Ekman, M. and Stigebrandt, A. 1990. Secular change of the seasonal variation in sea level and of the pole tide in the Baltic Sea. *J. Geophys. Res.* **95**(C4), 5379–5383.

Fuks, V. R. 2005. Gradient-eddy waves in the Baltic Sea. *Russ. Meteorol. Hydrol.* **9**, 47–51.

Gustafsson, B. G. and Andersson, H. C. 2001. Modeling the exchange of the Baltic Sea from the meridional atmospheric pressure difference across the North Sea. *J. Geophys. Res.* **106**(C9), 19731–19744. DOI: 10.1029/2000JC000593.

Jacobsen, T. S. 1980. *Sea Water Exchange of the Baltic, Measurements and Methods. Belt Project*. National Agency of Environmental Protection, Copenhagen, Denmark.

Kulikov, E. A. and Fain, I. V. 2008. Numerical modeling of the Baltic sea-level variability. *Comput. Technol.* **13**(2), 39–46 (in Russian).

Kulikov, E. A. and Medvedev, I. P. 2013. Variability of the Baltic Sea level and floods in the Gulf of Finland. *Oceanology*. **53**(2), 145–151.

Lazarenko, N. N. 1961. The Baltic Sea level fluctuations. *Proc. State Oceanogr. Instit*. **65**, 39–127 (in Russian).

Leppäranta, M. and Myrberg, K. 2009. *Physical Oceanography of the Baltic Sea*. Springer-Verlag, Berlin, Heidelberg, New York.

Magaard, L. and Krauss, W. 1966. Spectra of the Baltic sea level oscillations in 1958. *Kiel. Meeresforsch*. **22**, 155–162.

Maximov, I. V. 1970. *The Geophysical Forces and Water in the Oceans*. Gidrometeoizdat, Leningrad (in Russian).

Medvedev, I. P. 2014. The Baltic seasonal sea-level oscillations. *Russ. Meteorol. Hydrol*. **39**(12), 775–787.

Medvedev, I. P., Rabinovich, A. B. and Kulikov, E. A. 2013. Tidal oscillations in the Baltic Sea. *Oceanology*. **53**(5), 526–538.

Medvedev, I. P., Rabinovich, A. B. and Kulikov, E. A. 2014. Pole tide in the Baltic Sea. *Oceanology*. **54**(2), 121–131.

Miles, J. W. 1974. Harbor seiching. *Ann. Rev. Fluid Mech*. **6**, 17–35.

Samuelson, M. and Stigebrandt, A. 1996. Main characteristics of the long-term sea level variability in the Baltic sea. *Tellus A*. **48**, 672–683.

Stigebrandt, A. 1980. Barotropic and baroclinic response of a semi-enclosed basin to barotropic forcing from the sea. In: *Fjord Oceanography* (eds. H. J. Freeland, D. M. Farmer and C. D. Levings) Plenum, New York, pp. 141–164.

Wróblewski, A. 2001. Lunar nodal tide in the Baltic Sea. *Oceanologia*. **43**(1), 99–112.