Measurement of small-scale oceanic motions with neutrally-buoyant floats

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ABSTRACT

The movement of water in the ocean was studied by following the motions of instrumented neutrally-buoyant floats. Measurements were made at various depths in the Caribbean Sea and in the Atlantic Ocean several hundred miles east of Bermuda.

There was a continuous vertical bobbing of the floats, and two floats at nearly the same depth separated at an uneven rate in time. Near the surface, vertical displacements of approximately 3 m took place during periods of 10 to 30 minutes, and movements of approximately 30 m took place over a period of tens of hours. The slow large-amplitude motions were probably forced by semidiurnal tides in the Caribbean. Vertical movements at depths near 1000 m in both operating areas amounted to approximately 10 m over periods of 1 to 2 hours.

Floats at their equilibrium depths do not faithfully follow the movements of the water environment. An approximate solution is derived for the response of a float to an internal wave. It was found that a resonance causes the float movements to exceed the water displacements at the shorter periods whereas the stiffness of the float tends to keep it from moving in deep water where the density gradient is small.

Introduction

Little is known about the movement of water beneath the surface of the ocean. Measurements with Swallow (1955) floats are still scarce, so that our notion of water velocities to be expected has been obtained from synoptic salinity and temperature measurements. The agreement between velocities calculated by using the method of dynamic heights relative to a layer of no motion and the velocities required to develop the known temperature and salinity structure of the ocean gives some faith in this indirect method of measurement (Wüst, 1957), but the nature of the data is such that the results represent the motion as averaged over a long period of time. If an actual measurement did not disclose the expected velocity, it could be considered a temporary fluctuation. On the other hand, such fluctuations may constitute the most important part of the velocity field both in order of magnitude and in the sense of determining the hydrodynamic response of the ocean to externally applied stresses.

Wüst's computations for the Atlantic Ocean show that deep currents having velocities in excess of 10 cm sec^{-1} are exceptional. It is not exceptional, however, to find deep tidal currents or currents associated with internal waves with velocities also of this magnitude. Other sources of fluctuating current, such as large scale storms, can be imagined to exist, and the existence of large fluctuating currents on a mean flow is easily established. Unfortunately, a series of long periods is associated with these movements, and any practical program of measurement will run into difficulties in obtaining a complete analysis of the motion. The problems encountered in making meaningful measurements under such conditions have been described by Stommel (1963).

This report will describe the fluctuations in water motion as obtained by following the movements of neutrally-buoyant floats at various locations in the ocean. It will be shown, however, that the path of a neutral float differs from that of water particles adjacent to it. On the other hand, two floats that are close to one another are expected to move together unless there are eddies present with a size comparable to the separation of the floats or unless the floats are at different depths in a shear flow. Measurements were made both of this change in separation with time and of the vertical displacements of two nearby floats. One can anticipate that the results would be influenced by the presence of shear flow, internal waves, horizontal diffusion, and convection. One cannot anticipate the extent of the influence or the possibility of distinguishing between the different physical mechanisms.

Navigational difficulties usually prevented obtaining accurate determinations of the individual float trajectories in the manner of Swallow. Nevertheless, the net movement during an experiment was often obtained.

Two floats were used to emphasize small-scale movements and to try to filter out the long period movements. The results, however, showed large movements with a period comparable to the time of the experiment. In retrospect, each test should have lasted five times longer, at least. But then there would have been no time to determine the effect of depth or geographical location. The reason for the movements may be questioned, but the extent of the motion is now known, and more definitive experiments can be designed. The present experiments should consequently be looked upon as attempts to follow water movements with floats instead of deducing them from density structure and to try to obtain rapid estimates of the motion to be expected at various locations. Interesting features can be isolated for more thorough study in the future. The results are presented graphically on the assumption that a well-trained eye is not as easily fooled by statistically suspicious data as is a machine.

Experimental procedure

Neutrally-buoyant floats are adjusted to have the same density as water at the desired test depth. They sink until they reach a stable position at the test depth and transmit pings of sound so that they can be tracked by a ship (Swallow, 1955). The floats used in the present experiments are equipped with pressure gauges and have transponding features. Two types of float are used. One periodically emits a rapid double ping, and the time interval between pulses in a pair is such as to specify the pressure or depth of the float. The second type of float hears these pulses and responds by transmitting its depth on its own pair of pings. The first float, in turn, hears the response and retransmits it at its own frequency, 10.5 kc. By listening at the frequency of the first, or master, float, one obtains a double pulse which specifies the depth of the master followed shortly after by another double ping which tells the depth of the second, or slave, float. The time between pairs of pings and the velocity of sound serve to determine the separation of the floats. Details of the equipment are described elsewhere (Pochapsky, 1961*a*).

The density of each float was adjusted relative to the known density of sea water in a tank on the ship. A pair of floats was then dropped into the sea, the drop of the second float being delayed so that the pair had an initial horizontal separation of approximately 10 m. Excess weight was often added to each float in order to reduce the time required to reach the test depth. This was automatically discarded by a pressure actuated knife.

Sound signals from the floats were detected with hydrophones over the side of the ship and recorded. An array of three hydrophones was used to determine the direction of the floats. A sound pinger in the water near the ship could excite the pinger on either float, and the time required to receive the excited signal was used to determine the range to a float. When the separation between the floats and ship increased to a few miles, the signals weakened sufficiently to make it necessary to suspend recording so the ship could return to the floats.

Floats could be recalled to the surface at will by means of a coded signal. Sometimes a noisy ship's screw or heavy rain found the code and put an unwished for end to an experiment. A small radio transmitter on each float simplified locating a surfaced unit.

Near land, radar fixes were used to establish the position of the ship. In later experiments away from land, the position was determined by dead reckoning with respect to a deep transponding float held on a mile length of polypropyl line anchored to the bottom.

Relationship between neutral float and water movements

Neutral floats are made of thick shells in order to withstand high pressures. This alone ensures that the compressibility of a float is less than that of water and that the float will reach a depth where it is in stable equilibrium. It also prevents the vertical movement of the float from duplicating that of the water. Consequently, the float will be exposed to different layers of water and follow the horizontal movements appropriate to the different layers. It will not, in general, follow a marked parcel of water.

The difference in the vertical movements is important in interpreting the results of the present float experiments. This can be treated analytically for the case of a float in an internal wave of single frequency, ω, and, although not rigorously applicable to the experiments, the results of such an analysis are quite useful.

First, let us consider the displacement, z_w, of water from its equilibrium position. The equation of motion can be written

$$g\left(\frac{d\varrho}{dz} + \varkappa_\eta \varrho^2 g\right) z_w - \frac{\partial p}{\partial z} = \varrho \ddot{z}_w \tag{1}$$

(ECKART, 1960; TOLSTOY, 1963). The quantities involved are: g, the acceleration of gravity; ϱ, the equilibrium density of water at the vertical position, z, measured upward; $\varkappa_\eta$, the adiabatic compressibility of water; z_w, the displacement of water from equilibrium and $\ddot{z}_w$, the corresponding acceleration; p, the non-hydrostatic pressure associated with the internal wave.

When $\partial p/\partial z = 0$, z_w can vary as $e^{i\omega_v t}$ and from Eq. (1)

$$\omega_v^2 = -\frac{g}{\varrho}\left(\frac{d\varrho}{dz} + \varkappa_\eta \varrho^2 g\right), \tag{2}$$

ω_v is known as the stability or Väisälä frequency.

Usually there is a pressure gradient and z_w varies as $e^{i\omega t}$. This gradient can be expressed by means of Eqs. (1) and (2) as

$$\frac{\partial p}{\partial z} = \varrho(\omega^2 - \omega_v^2) z_w. \tag{3}$$

The same reasoning as led to Eq. (1) can be used to derive the equation for the vertical motion of the float. Thus, the density of a float after a displacement z_F from position z_0 is

$$\varrho_{zF} = \varrho_{z0} - \left(\frac{\partial \varrho_F}{\partial p}\right)_\theta \varrho g z_F + \frac{\partial \varrho_F}{\partial \theta}\Delta\theta_F$$
$$= \varrho_{z0} - \varkappa_F \varrho_F \varrho g z_F - \varrho_F \alpha_F\left[(z_F - z_w)\frac{d\theta}{dz} - \frac{\theta\alpha}{c_p} g z_w\right],$$

where the last term corrects for the adiabatic change in temperature of the water. The additional quantities are: α, the thermal coefficient of volume expansion; θ, temperature; c_p, specific heat of water at constant pressure. Subscripts F signify quantities referring to the float rather than to the water.

The float equation is

$$g\left(\frac{d\varrho}{dz} + \varkappa_F \varrho_F \varrho g + \alpha_F \varrho_F \frac{d\theta}{dz}\right) z_F - g\alpha_F \varrho_F\left(\frac{d\theta}{dz} + \frac{\theta\alpha}{c_p} g\right) z_w - \frac{\partial p}{\partial z} - f(\dot{z}_F - \dot{z}_w) - h(\ddot{z}_F - \ddot{z}_w) = \varrho_F \ddot{z}_F, \tag{4}$$

where the first term is again the equilibrium density gradient of the water and the terms in f, a velocity drag coefficient, and h, an inertial coefficient, were added to take into account the forces generated when a float moves relative to the water.

When the pressure gradient, Eq. (3), is substituted in Eq. (4) and the motions are assumed to be of the form $e^{i\omega t}$, the relationship between the float and water amplitudes becomes

$$z_F = \frac{\varrho(\omega^2 - \omega_v^2) + (g^2\alpha_F/\alpha)\varrho^2(\varkappa - \varkappa_\eta) + g\alpha_F \varrho(d\theta/dz) - i\omega f + h\omega^2}{\varrho(\omega^2 - \omega_v^2) + g^2\varrho^2(\varkappa_F - \varkappa_\eta) + g\alpha_F \varrho(d\theta/dz) - i\omega f + h\omega^2} z_w. \tag{5}$$

Of the quantities involved, only f and h pose special consideration. f is a function of the velocity difference between the float and water. At relative velocities of 10^{-3} cm sec^{-1}, or less, Stokes' drag takes place, and f is approximately 2.5×10^{-4} dyne sec cm^{-4}. When the difference exceeds ~ 0.2 cm sec^{-1}, the drag begins to have a $\dot{z}^2$ dependence. f was measured to be approximately $3.10^{-2}\dot{z}^2$ cgs; Eq. (4) becomes non-linear at those velocities.

A moving float carries water along with it and responds to an accelerating force as though it has an increased mass. A sphere, for instance, has its mass augmented by an amount equal to one-half the mass of water which it displaces (LAMB,

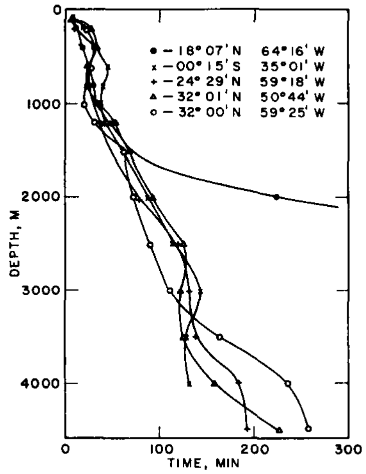

FIG. 1. Väisälä period at various depths and locations

TABLE 1. *The Väisälä frequency as a function of depth.*

Depth m	$\omega_v \times 10^3$	Response at $\omega = 0$	Period min
100	14.8	0.87	7
200	5.5	0.42	19
400	3.2	0.11	33
600	3.9	0.22	27
800	4.0	0.16	26
1000	3.7	0.20	28
1200	2.7	0.14	39
1500	1.7	0.049	62
2000	1.5	0.045	70
2500	1.0	0.013	105
3000	0.83	0.008	126
3500	0.83	0.011	126
4000	0.61	?	
4500	0.47		

1945). The quantity h was introduced in Eq. (5) to take this fact into consideration. For a sphere, h would be equal to $\frac{1}{2}$. The value is not known for cylindrical floats. Since the main effect of this quantity is to shift the reasonance frequency slightly, a compromise value of $h = \frac{1}{5}$ was used in the hope that this might best approximate the resonances of the cylinders.

Because of the importance of the Väisälä frequency, ω_v, in Eq. (5), values of the Väisälä period to be expected on the basis of published IGY data are shown in Fig. 1. These are given for various locations in the western North Atlantic Ocean. An average of these results at various depths was used in Eq. (5) to compute the response of a float as a function of depth, Fig. 2. The float response curves were drawn to try to approximate the motion of cylindrical floats with $h = \frac{1}{5}$; actually they were based on a spherical float 33 cm in diameter.

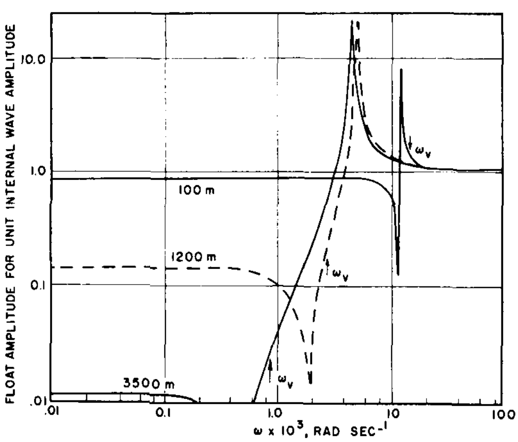

FIG. 2. Vertical amplitude response of a neutral float to an internal wave (Stokes' drag law) at three depths as a function of angular frequency. The Väisälä frequency as a function of depth is shown in Table 1.

As shown in Fig. 2, a neutrally-buoyant float follows the high frequency components of water movements at all depths. The ability of a float to follow low frequency vertical movements, however, falls off rapidly with increasing depth. Near the surface, the float displacement is 90 per cent of the water amplitude at low frequencies, but at a depth of 3500 m there is only a 1 per cent response.

The resonances of the float will be discussed later.

Observations

The movements of floats were followed at locations in the Caribbean Sea near the Anegada Passage and also at two locations several hundred miles east of Bermuda. A test was also run outside the Anegada Passage near Sombrero. The results are shown in Figs. 3 to 15. Most of these figures show the depths of two floats and their *radial* separation as a function of time over a period of approximately one day. Locations are noted on the figures. Depths ranged from a little less than 100 m to somewhat over 1000m. Float axes were vertical.

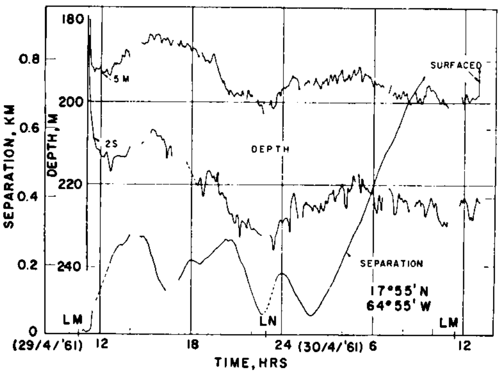

Fig. 3. Position of float pair.

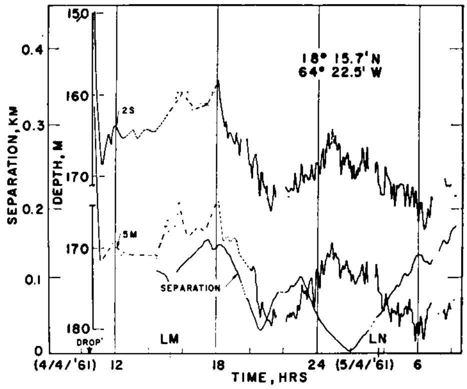

Fig. 6. Position of float pair.

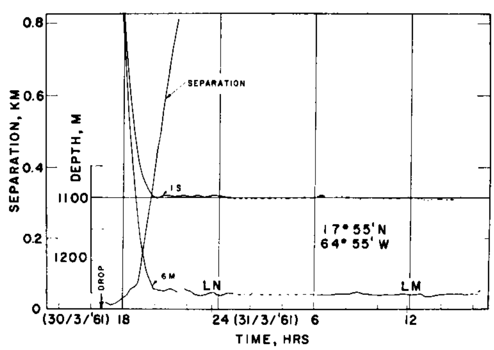

Fig. 4. Position of float pair.

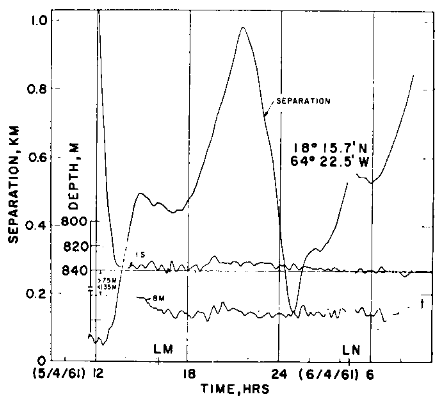

Fig. 7. Position of float pair.

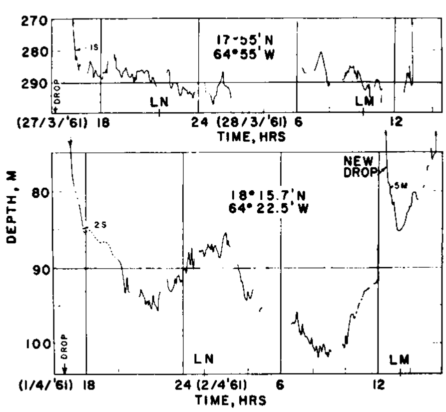

Fig. 5. Position of floats.

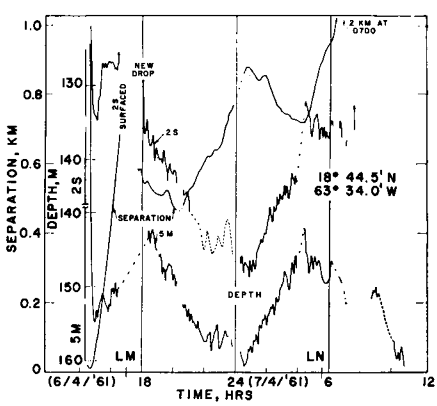

Fig. 8. Position of float pair.

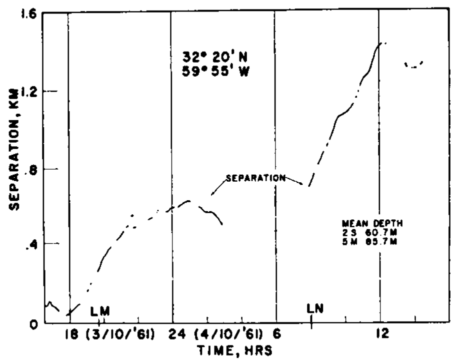

Fig. 9. Separation of float pair.

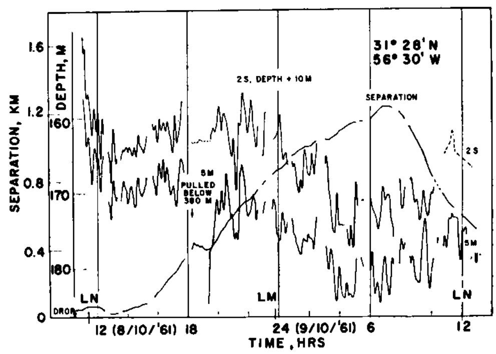

Fig. 12. Position of float pair

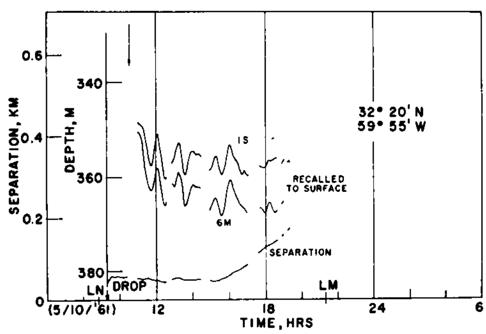

Fig. 10. Position of float pair.

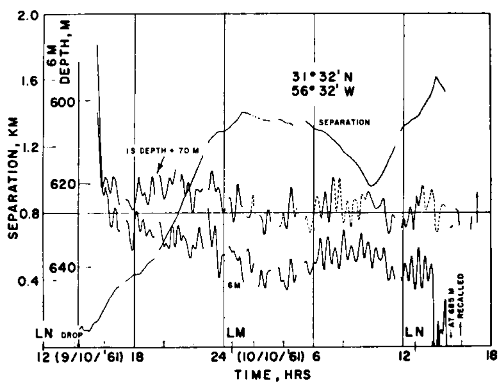

Fig. 13. Position of float pair.

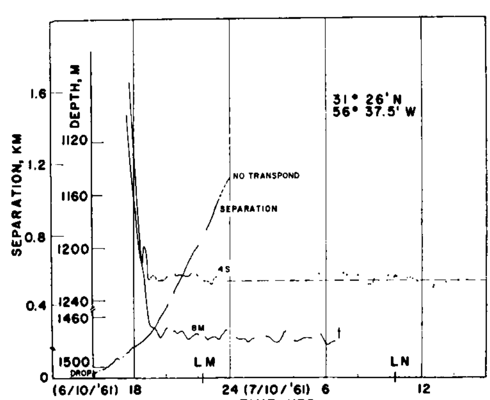

Fig. 11. Position of float pair.

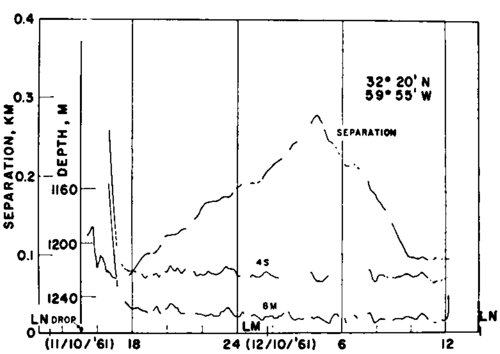

Fig. 14. Position of float pair.

TABLE 2. *Drift of neutrally-byoyant floats at various locations.*

Time and date, during 1961 from	to	Depth (m)	Average float velocity (cm sec^{-1})	Direction of motion (degrees)
17°55′ N, 64°55′ W (Near St. Croix, V. I.)				
0923/26/3	1310/27/3	116	11 ± 2	120
1518/27/3	1315/28/3	291	9.9 ± 1	260
1048/29/3	1300/30/3	188	20 ± 2	270
1640/30/3	1815/31/3	1110	3.5 ± 1	000
1640/30/3	1815/31/3	1450	6.7 ± 1	090
18°15.7′ N, 64°22.5′ W (Near Virgin Gorda, V. I.)				
1030/4/4	0840/5/4	171	21.1 ± 1	235
1143/5/4	0923/6/4	835	4.2 ± 1	035
1143/5/4	0923/6/4	~940	1.9 ± 1	005
18°44.5′ N, 63°34′ W (Sombrero)				
1745/6/4	1130/7/4	161	15 ± 1	165
31°30′ N, 56°30′ W (~400 miles E. of Bermuda)				
1800/6/10	0700/7/10	1480	< 5	?
1024/8/10	1315/9/10	175	< 8	W
1500/9/10	1600/10/10	637	< 10	N
32°22′ N, 59°52′ W (~250 miles E. of Bermuda)				
1503/11/10	1100/12/10	1250	2 ± 1	S
1435/12/10	1530/13/10	437	10 ± 1	190

The average of bathythermograph measurements at the Caribbean location shows the surface temperature, 27.8°C, fell slowly with depth to 26.9°C at 95 m and then more rapidly to 19.8°C at 198 m. The temperature at 274 m was 18.3°C. In the tests off Bermuda there was an isothermal surface layer 41 m deep. The temperature there, 27.5°C, then fell rapidly to 23.3°C at 61 m, 19.8°C at 128 m, and 18.3°C at 274 m.

Data were collected under a variety of favorable or adverse instrumental and sea conditions. Usually, the absolute depths are known to a ıttle better than 1 per cent. When the calibration of the pressure gauges shifted or other difficulties occurred, the resulting uncertainty is noted on the graphs. For the Caribbean experiments gauges were calibrated on land before and after sailing. At Bermuda calibration was done on board ship before and after drops. Gaps in the data take place whenever the ship had to steam closer to the floats. Dashed sections of the curves represent data collected under noisy or uncertain telemetering conditions. Depth and separation data were received at 5 to 10 sec intervals so that little of the curvature of the graphs was left to the imagination of the draftsman. In particular, the shapes of the separation curves are very accurate, and these curves could almost as well have been presented as a continuous sequence of data points. Errors in the separation arise from using a nominal value for the velocity of sound, 1525 m sec^{-1} and from a fixed error of approximately 1 msec

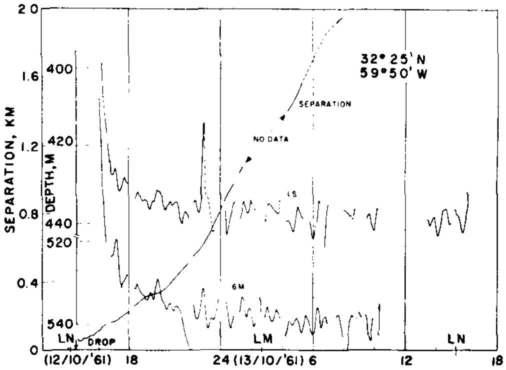

FIG. 15. Position of float pair.

in the time for sound to trip the slave float. Errors caused by fluctuations in sound transmission and pulse distortion are of the order of tenths of meters.

The position of the ship was established by means of radar in the Caribbean area and by dead reckoning relative to a deep anchored transponder off Bermuda. Establishing this position usually interfered with the acquisition of telemetered data. Detailed drift paths were not obtained, but the floats were located as accurately as possible at the drop and surfacing positions. No abrupt changes in the drift patterns were noticed during the experiments. The accuracy of the fixes was approximately ± 1 mile. Float drift velocities obtained in this fashion are given in Table 2.

Discussion of data

An inspection of Figs. 3 to 15 shows that restless water movements are the rule down to depths of at least 1500 m. Agitation was expected near the current-swept West Indies, but a comparable movement in the water east of Bermuda was not.

There are some features of the movements that are common to both locations. Floats in the top 600, or so, m of the sea carried out bobbing displacements of approximately 3 m during periods of 10 to 30 minutes. In addition there were slower vertical displacements of approximately 30 m which resemble internal tides. The originally closely spaced floats tended to separate in the horizontal direction but this movement was not done uniformly and there was a component of motion which tended to pull the floats together. The result was a pulsation in the separation during some tests of 0.1 to 1.0 km which appeared to have a periodicity related to that of the vertical movements. The average separation rate was approximately a kilometer per day, but there were striking exceptions.

In the deeper tests these high and low frequency components in the vertical movement were either not as obvious as in the shallow tests or they were absent. The spectra were broader, but there appeared to be strong components with periods of 1 to 2 hours. Total displacements were of the order of 10 m. Some of these deep floats tended to pull apart steadily: others moved relative to one another with a strong throbbing motion.

The high frequency bobbing motions of the floats had periods near the local water Väisälä frequency (Fig. 1). Unfortunately, the natural float frequencies lie in the range of the Väisälä frequencies and the float movements may amplify water displacements by as much as a factor of 20 at the float resonance if the damping follows Stokes' law, Table 1. Actually, the drag was probably comparable to the inertial forces during those portions of the cycle where the relative velocity between the float and water was large in the 2-meter excursions observed. Consequently, the float movement should be described by a non-linear equation of motion and the linear analysis given earlier can only be used to obtain an approximate description.

In order to gain some idea of the magnification of the water movement by a float, let it be assumed that the damping coefficient is a constant having four times the value obtained by Stokes' relation. Also, let all frequency components of the water motion have the same amplitude near the float resonant frequency ω_0. Calculation then shows that a float would amplify the net water movement for frequencies in a 10 per cent bandwidth about ω_0 by a factor of 1.7 at a depth of 100 m and 4.7 at 1200 m.

Contrary to the case at high frequencies, floats at low frequencies will move less than the water. At depths between 500 m and 1000 m, the ratio of amplitudes can be expected to be approximately $\frac{1}{2}$ (Fig. 2).

Although there are similarities in the results obtained in the Caribbean and at the location east of Bermuda, there are some important differences. One of the most important is in the long period vertical movements. The first reaction is that those movements are semidiurnal in the Caribbean and diurnal in the Atlantic. An examination of the separation between the floats at the former location shows that there is also a strong semidiurnal horizontal movement. Note that a displacement in which one float oscillates semidiurnally with respect to another would appear on the figures as a full-wave rectified wave having a strong 6-hour periodicity. This follows because the separation is the absolute value of the distance between floats and does not change sign with the amplitude as one float passes another. It is thus likely that the Caribbean results show the presence of internal tides, but the present data cannot be used to establish that fact with certainty for a number of reasons: (1) The tests were too short to show that the periods were actually tidal. (2) No fixed relationship existed between the phases of the waves in the various tests and the phase of the moon. Local lunar noon and midnight are designated on the figures as LN and LM. A fixed relationship, however, need not exist if a number of internal wave modes were present. (3) A measurement of wavelength was not obtained. The correlation in the vertical movement of a float pair shows only that the wavelength must be larger than something of the order of 10 km.

An argument to attribute the slow movements in the Atlantic tests to diurnal tides has a less firm basis than that for the Caribbean results. The inertial period is 39 hours at the latter location, but it is 22.6 hours at the former, and so inertial as well as tidal effects may be important in these short records of float movements east of Bermuda.

No large movements in the results suggested the "breaking" of internal waves. The sudden changes in depth which occurred at 1800 on October 8 and 1400 on October 10 (Figs. 12 and 13) were probably caused by marine life.

Over a period of a day the floats in the various trials were found to travel with average speeds ranging from a few up to 21 cm sec^{-1}. Two floats at nearly the same depth tended to travel as a pair, but much of the time there was a surprisingly high relative velocity between them. A relative velocity of 5 cm sec^{-1} between floats was not uncommon. Much of this difference in velocity probably resulted because the floats were at slightly different depths in water which was undergoing horizontal shear flow. If this interpretation is accepted, it can be concluded that vertical gradients of the horizontal velocity should not usually exceed a value of the order of 5×10^{-4} sec^{-1} in the ocean.

It is important to know whether the shorter period motions take part in turbulent mixing processes or whether they produce no change in the structure of the medium as would be true for internal waves. Arguments will be given for the possibility of having either, or both, but they are not convincing and will probably only reënforce personal preconceptions. Some comments can be made on whether the movements are locally generated or come from distant points.

Although floats were followed day and night in various sea states, the surface conditions had no pronounced effect on the magnitude of the motions. Energy supplied at the surface, however, must go through the mixed surface layer, and it is not unreasonable to expect it to be smoothed so as to be fairly steady in time before it reaches the depths studied in the tests. Horizontal flow at depth from various parts of the ocean could also provide either the motion or conditions to excite it. It is doubtful that turbulence is supplied mainly by convective flow because it decays rapidly. For example, if the length scale is taken to be comparable to the short period displacements of the floats, 3 m, and the time scale to one half the period, 600 sec, the rate of energy dissipation is

$$\varepsilon \sim \varrho A_v \left(\frac{du}{dz}\right)^2 = 10 \cdot \left(\frac{0.5}{300}\right)^2 = 3 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-3},$$

where u is a typical velocity and A_v, the eddy viscosity, is assumed to have a value of 10 cm^2 sec^{-1}. The total energy of a unit volume of water is near 10^{-1} erg cm^{-3} and so appreciable decay of this energy will occur in 3×10^4 sec or almost 8 hours. Turbulent motion transported in a 1-knot current would consequently be substantially weakened in a distance of 15 km. This sort of argument can also be applied to the horizontal shear of the water as obtained from the difference in velocities of two floats at differing depths. That shear, 10^{-3} sec^{-1}, produces a dissipation comparable with that attributed to turbulence. Now, however, the kinetic energy is that appropriate to velocities near 5 cm sec^{-1} and significant loss of this energy would take place only after a time of a month. If fluctuations in the float separation curves are attributed to internal waves, it is evident that such waves have a relatively high energy compared to the velocity gradients and so would decay at a rate comparable to that just computed for the horizontal shear. Because internal waves travel at velocities with magnitudes of the order of a knot, they would propagate for distances of the order of 1000 km before being damped appreciably. This statement does not hold for waves at the Väisälä frequency; these do not propagate but a slight decrease in frequency, say 10 per cent, will cause them to move rapidly.

Floats have one third the compressibility of water and so might be expected to have a resonant frequency well above the Väisälä frequency. This advantage is lost in a temperature gradient with the result that both resonances are comparable above depths of 500 m. One cannot, therefore, exclude the possibility that the resonant type float movements are induced by internal waves which originated far away. Support for this view can be obtained by noting the strong correlation between the vertical movements of floats that were less than 50 m apart in the results at 0200 on April 5, 1961, or at the beginning of the tests on October 8 and 9. This implies wavelengths near 100 m with periods near 1000 sec, or a phase velocity near 10 cm sec^{-1}.

A good example of what are probably internal waves is shown in Fig. 10. This promising test had to be stopped in order to evade hurricane Francis, which was 300 miles to the northeast at that time.

The case for attributing the dominant portions of the movements to internal waves would be strengthened if they could be shown to be much larger than required to account for commonly accepted values of the vertical transport coefficients. There was much more activity than anticipated on the basis of a brief, 3-hour, test reported earlier by the author (POCHAPSKY, 1961*b*). If recourse is again made to the mean free path model used in that report, the transport coefficient can be written

$$A_v = \tfrac{1}{2}\bar{v}l = 75\ \mathrm{cm}^2\ \mathrm{sec}^{-1}, \qquad (6)$$

where $\bar{v}$, an average vertical velocity, is now 0.5 cm sec^{-1} and $l = 300$ cm. This value for the eddy viscosity may be acceptable, but it seems a little excessive.

Turbulence can be established in a shear flow only if the velocity gradients are sufficiently high, and a criterion is that

$$\frac{A_s}{A_v}\,\frac{g\dfrac{\delta\varrho}{dz}}{\varrho\left(\dfrac{du}{dz}\right)^2} = \frac{A_s}{A_v}R_i < k. \qquad (7)$$

A_s/A_v is the ratio of the eddy diffusivity and eddy viscosity; R_i is the Richardson number; k is a number less than unity which theory suggests to have a value near 0.15; $\delta\varrho$ is the difference between the density of water moved adiabatically through a distance dz and the equilibrium density of the water displaced. Further details are found in a recent article by ELLISON and TURNER (1960). MUNK and ANDERSON (1948) showed that the well-known experiments of Jacobsen at Schultz's Grund and Randers Fjord lead to the ratio

$$\frac{A_s}{A_v} = \frac{(1+10\,R_i)^{\frac{1}{2}}}{(1+3.33\,R_i)^{\frac{3}{2}}}. \qquad (8)$$

For large values of R_i, this becomes

$$\frac{A_s}{A_v}R_i = 0.52 < k? \qquad (9)$$

In the Jacobsen experiments, R_i varied from 2.6 to 28 with a single exception of $R_i = 125$. The experimental value of k was higher than theory allows for turbulence to be present, but this may be a consequence of the high stability in the experimental waters.

In the present experiments, R_i ranged from 4 to 40 with an average of 16 for five tests in the Caribbean. There was no discernible dependence on depth at that location but R_i did tend to increase with increasing depth off Bermuda. At the latter location, R_i varied from 19 to 310 and averaged 115 for five runs. These values of the Richardson number were obtained from typical slopes in the separation curves of two floats. They ignore small fluctuations in the velocity gradients and times when the velocity gradients were near zero. Density gradients were obtained only from observed temperatures in the operating areas down to 250 m and from published data for deeper water.

If Eq. (9) is assumed to apply for the high R_i numbers of the present results, the viscosity coefficient will be 50 or more times larger than the diffusivity, and the diffusion coefficient will have a value near $1\frac{1}{2}$ $\mathrm{cm}^2\ \mathrm{sec}^{-1}$. This is a value to be expected.

Reservations must be made in the computations of the preceding paragraphs because of uncertainties in the density gradients and variations in the velocity gradients which are used in a simplified model of diffusion. These do not seem to be serious enough to reject the conclusion that the vertical movements of the floats could be accounted for by the presence of water eddies which take part in vertical transport. It must also be concluded that high values of the Richardson number are involved, and the consequence that the ratio of the transport coefficients must be large is troubling when it is realized that the density gradients were of the order of 10^{-8} instead of 10^{-5} to 10^{-6}, as was the case in Jacobsen's experiments.

It is inconceivable that turbulence can be present without generating internal waves, and the present experiments suggest that they are equally important in generating vertical motions. Further experiments should be designed with this in mind, and instrumentation must distinguish between a random addition of internal waves coming from all directions and the random movements of turbulent mixing.

It was originally hoped that observations on the relative horizontal movements of two floats might yield data on horizontal diffusion. In a homogeneously turbulent ocean, the velocity difference between two floats would depend on the separation according to KOLMOGOROFF's (1941) relation

$$\overline{(u_1 - u_2)^2} = \varkappa \varepsilon^{\frac{2}{3}} r^{\frac{2}{3}}.$$

Here, u_1 and u_2 are the velocities of the floats, ε is the rate of turbulent dissipation of energy per unit mass, and r is the separation between floats. No such relationship is implied in our results, possibly because of the strong internal wave movements and because the amount of data is limited.

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