Standard errors of cosmic ray data corrected for atmospheric effects

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ABSTRACT

A formula is obtained for the standard error of cosmic ray data which have been corrected for atmospheric effects. It is used to estimate standard errors of the nucleonic and meson components corrected by different methods. The relative standard error is found to decrease with increasing counting rate up to an approximate value of 10^6 counts per time interval if simultaneous meteorological measurements are available for each recording interval. Above that counting rate the relative standard error is essentially constant.

Mathematical treatment

In order to estimate standard errors of cosmic ray data an investigation was carried out by one of the authors (Dyring 1962). This work deals with uncorrected neutron monitor and meson telescope data as well as pressure corrected neutron monitor data. It now seems urgent to us to extend the study to meson telescope data which are corrected for atmospheric effects.

There exist different methods to correct cosmic ray data for atmospheric effects. We will here discuss simple pressure correction (nucleonic and meson component) and two more complex corrections according to Duperier and Dorman (meson component).

The basic equations are:
for simple pressure corrections

$$N_c = N_r (1 - \alpha_0 \delta P) \tag{1}$$

for corrections according to Duperier (1949)

$$N_c = N_r (1 - \alpha\delta P - \beta\delta H - \gamma\delta T) \tag{2}$$

and for corrections according to Dorman (1958)

$$N_c = N_r \left(1 - a\delta P - \sum_{i=1}^{n} b_i \,\delta T_i\right). \tag{3}$$

N_r stands for the recorded intensity, N_c for the corrected intensity. δP, δH, and δT are deviations of surface pressure, height of the 100 mb level, and mean temperature between 100 and 200 mb from the values adopted as normal. In the Dorman case the atmosphere is divided into n layers. δT_i is the deviation of the mean temperature of the ith layer from the normal one, and b_i is the product of the density of the temperature coefficient and the thickness of the ith layer. Eqs. (1), (2), and (3) can be generalized to

$$N_c = \sum_{i}^{k} N_r \nu_i A_i. \tag{4}$$

The variance of x is by definition

$$D^2(x) = E(x^2) - E^2(x), \tag{5}$$

where $E(x)$ stands for the true mean of the variable x. We now form the variance of N_c by means of eqs. (4) and (5) under the assumption that all variables are independent.

$$D^2(N_c) = \sum_i D^2(N_r \nu_i A_i) + 2D^2(N_r) \sum_{i \neq j} E(\nu_i)\, E(A_i)\, E(\nu_j)\, E(A_j), \tag{6}$$

where

$$D^2(N_r \nu A) = E^2(N_r \nu)\, D^2(A) + E^2(N_r A)\, D^2(\nu) + E^2(\nu A)\, D^2(N_r) + D^2(N_r)\, D^2(\nu)\, D^2(A).$$

Each variable in eq. (6) may be written as

$$x = E(x) + \varepsilon_x, \tag{7}$$

where ε_x is assumed to be approximately normally distributed with $E(\varepsilon_x) = 0$. Accordingly eq. (6) can be written

$$D^2(\varepsilon_{N_c}) = \sum_i D^2(N_r \nu_i A_i) + 2D^2(\varepsilon_{N_r}) \sum_{i \neq j} \nu_i A_i \nu_j A_j, \quad (8)$$

where

$$D^2(N_r \nu A) = N_r^2 A^2 D^2(\varepsilon_\nu) + N_r^2 \nu^2 D^2(\varepsilon_A) + A^2 \nu^2 D^2(\varepsilon_{N_r}) + D^2(\varepsilon_{N_r}) D^2(\varepsilon_\nu) D^2(\varepsilon_A).$$

As already remarked eq. (8) is valid only if ε_{N_r}, ε_ν and ε_A are independent. If there is some dependence between the quantities ε_{A_i}, we have to add a third term to the right member of eq. (8), and then we get

$$D^2(\varepsilon_{N_c}) = \sum_i D^2(N_r \nu_i A_i) + 2D^2(\varepsilon_{N_r}) \sum_{i \neq j} \nu_i A_i \nu_j A_j + 2[N_r^2 + D^2(\varepsilon_{N_r})] \sum_{i \neq j} \nu_i \nu_j E(\varepsilon_{A_i} \varepsilon_{A_i}). \quad (9)$$

Estimates of errors affecting the corrected intensity values

In the calculations based on eq. (8) we have used the values of ν_i, and $D(\varepsilon_{\nu_i})$ shown in Table 1, which is valid for sea level stations. The ν-values for the Duperier case were taken from Table 5 in LINDGREN and LINDHOLM (1961). The accuracy $D(\varepsilon_{A_i})$ of the atmospheric parameters must also be known. The aerological data are obtained by radiosondes. In the methods of both Duperier and Dorman the accuracy of the temperature recordings is important. The temperature values to be used in the corrections can be obtained in different ways. DORMAN *et al.* (1958) suggest to use the temperature at the mean pressure of the layer. Another method often used is to calculate the mean of the temperatures at the lower and upper surfaces of the layer. However, it must be stressed that these methods can both give big errors for single observations especially in layers near the surface of the earth and around the tropopause. Although for a series of data over a long period these errors tend to group symmetrically around the mean = 0, one can avoid these difficulties by calculating the true mean temperature, or, which is practical, by using the height difference between the upper and lower surfaces of the layer. This difference will cover all existing temperature variations.

Several authors have studied the accuracy of radiosonde data. RAAB and RODSKJER (1950) and NYBERG (1952) have made special investigations where more than one radiosonde are launched by each balloon. From the simultaneous recordings estimates of $D(\varepsilon_T)_P$ and $D(\varepsilon_P)$ are obtained for certain pressure layers. These errors include both recording errors and radiosonde errors. For practical applications in cosmic ray corrections the temperature error must be assumed to be a function of both $D(\varepsilon_T)_P$ and $D(\varepsilon_P)$ which is expressed by (NYBERG, 1952):

$$D^2(\varepsilon_T) = D^2(\varepsilon_T)_P + 0.01\, D^2(\varepsilon_P). \quad (10)$$

Table 2 shows the results of RAAB *et al.* and

TABLE 1.

Component	Method	A_i		$\nu_i \pm D(\varepsilon_{\nu_i})$	
Nucleonic	Pressure	A_1	1	ν_1	1
	$k=2$	A_2	P (mb)	$-\nu_2 = \alpha_0$	-0.71 ± 0.02 (%/mb)
Meson	Pressure	A_1	1	ν_1	1
	$k=2$	A_2	P (mb)	$-\nu_2 = \alpha_0$	-0.15 ± 0.02 (%/mb)
Meson	Duperier	A_1	1	ν_1	1
	$k=4$	A_2	P (mb)	$-\nu_2 = \alpha$	-0.12 ± 0.01 (%/mb)
		A_3	H (km, 100 mb)	$-\nu_3 = \beta$	-5.0 ± 0.5 (%/km)
		A_4	T (°C, 200–100 mb)	$-\nu_4 = \gamma$	0.05 ± 0.01 (%/°C)
Meson	Dorman	A_1	1	ν_1	1
	$k=11$	A_2	P (mb)	$-\nu_2 = a$	-0.14 ± 0.01 (%/mb)
		A_3	T_1 (°C, 1000–900 mb)	$-\nu_3 = b_1$	
		—	—	—	-0.030 ± 0.005 (%/°C)
		A_{11}	T_9 (°C, 200–100 mb)	$-\nu_{11} = b_9$	

TABLE 2.

Raab & Rodskjer (1950)		Nyberg (1952)		
Level, mb	$(D\varepsilon_T)$, °C	Level, mb	$D(\varepsilon_T)$ night, °C	$D(\varepsilon_T)$ day, °C
1000–600	0.7	1000–850	0.6	0.6
600–350	0.9	850–750	0.7	1.1
350–150	0.85	700–500	0.6	0.95
		500–300	0.8	1.2
		300–200	1.1	1.2

NYBERG when eq. (10) is used. It must be stressed that these investigations have been carried out by specialists and under careful control. Thus the results are certainly underestimates compared with radiosonde data obtained during ordinary working conditions at the aerological stations. TREFALL and NORDÖ (1959) have estimated the error variance for some aerological parameters by comparing the records of two adjacent aerological stations on Spitzbergen. Their results (Table 3) seem to us to be more useful for cosmic ray applications as they contain all sources of errors. No results are obtained for the lower part of the atmosphere and we have tried to extrapolate these values (Table 3). These estimates are used in the following calculations.

It must also be stressed, that the aerological observations are carried out only twice a day (at some stations four times a day) and that it is tempting to make a linear interpolation between the points of measurement. Such a treatment will also introduce errors. For long periods the distribution of these errors tends to be symmetrical around the mean = 0.

The discussion above has given us reasons to believe that the estimates of $D(\varepsilon_T)$ and $D(\varepsilon_H)$ used in this paper are by no means too large. We rather think that they are underestimates. However, more accurate estimates are hard to find.

Instead of using temperatures there is a possibility to use the heights to the pressure layers obtained from aerological maps. BERGGREN (1959) has studied the accuracy of these data. At the 500 mb level $D(\varepsilon_H)$ is estimated to 16 m and at the 300 mb level to 22 m.

Table 3 also shows estimates of $D(\varepsilon_P)$ and $D(\varepsilon_H)$. There is no reason to assume that there

TABLE 3. *References: TN = Trefall & Nordö (1959), D = Dyring (1962).*

Level or component	Estimate	Ref.
1000–500 mb	$D(\varepsilon_T) = 1.0$°C	
500–400 mb	1.3°C	
400–300 mb	1.6°C	TN
300–100 mb	2.2°C	TN
100 mb	$D(\varepsilon_H) = 0.072$ km	TN
Surface	$D(\varepsilon_P) = 0.2$ mb	D
Nucleonic	$D(\varepsilon_{N_r}) = 1.2\, N_r^{-\frac{1}{2}}$	D
Meson	$= 1.14\, N_r^{-\frac{1}{2}}$	D
1000–100 mb	$E(\varepsilon_{T_i}\varepsilon_{T_j}) = 0$	
H: 100 mb; *T:* 200–100 mb	$E(\varepsilon_H \varepsilon_T) = 0.1$ km °C	TN
T: 1000–100 mb	$E(\varepsilon_P \varepsilon_{T_i}) = 0$	
H: 300–100 mb	$E(\varepsilon_P \varepsilon_H) = 0$	

exist any correlations between the errors of the cosmic ray intensity values and the errors of the constants nor between these errors and the errors of the atmospheric parameters. However, it is not clear that the errors of the different atmospheric parameters are uncorrelated. By use of the Spitzbergen data mentioned above TREFALL and NORDÖ (1959) have obtained the correlations between ε_{H_1} (100 mb) and ε_{T_1} (100 mb) as well as between ε_{H_1} (100 mb) and $\varepsilon_{H_1-H_2}$ (200 and 300 mb). Their results are not surprising as the height of an isobaric level is a function of the temperature distribution in the underlying atmosphere. Trefall and Nordö did not find any significant correlation between ε_{H_2} (200 mb) and $\varepsilon_{H_1-H_2}$ (100–200 mb) nor between ε_{H_3} (300 mb) and $\varepsilon_{H_1-H_3}$ (100–300 mb) where the atmospheric layers are not overlapping. This indicates that there are no correlations between the different ε_{T_i} terms appearing in eq. (8) when it is applied to the Dorman case. We also assume that the error of the surface pressure and the error of the parameters obtained from radiosonde measurement are uncorrelated.

The estimated values of $E(\varepsilon_{A_i}\varepsilon_{A_j})$ are also shown in Table 3. The standard deviations of δP and δT_i are obtained from meteorological records:

$D(P) = 12$ mb Stockholm (59° N, 18° E), mean over 10 years.

$D(T) = 6$°C Larkhill (51° N, 2° W), mean over 4 years.

TABLE 4.

δP, mb	Nucleonic C_1	Nucleonic C_2	Meson C_1	Meson C_2
30	1.91	38.0	1.39	36.1
20	1.74	18.0	1.36	16.1
10	1.58	6.0	1.33	4.1
0	1.44	2.0	1.30	0.1
−10	1.30	6.0	1.27	4.1
−20	1.18	18.0	1.24	16.1
−30	1.07	38.0	1.21	36.1

Concerning the height of the 100 mb level a value of $D(H_t) = 400$ m has been found corresponding to $D(T_t) = 6°$C.

$D(T_t)$ has the same order of magnitude at all levels of the atmosphere up to 100 mb (LARKHILL, 1952). For long periods the distribution of P and T_t tends to be symmetrical and thus we can roughly assume that 60–70 per cent of all values are inside the limits of plus and minus the standard deviation.

Results

The standard error of N_c is estimated by means of eq. (8) and the values in Tables 1 and 3. The calculations have been carried out for the following values of δP, δH and δT_t:

$$\begin{array}{lll} \delta P = 0 \text{ mb}, & \pm 10 \text{ mb}, & \pm 20 \text{ mb}, \\ \delta H = 0 \text{ m}, & \pm 400 \text{ m}, & \pm 800 \text{ m}, \\ \delta T = 0°\text{C}, & \pm 6°\text{C}, & \pm 12°\text{C}. \end{array}$$

We have assumed the temperature change to be the same through the whole atmosphere up to 100 mb.

The results for the different methods of correction come out in the form

$$D^2(\varepsilon_{N_c}) = C_1 N + C_2 10^{-6} N^2$$
$$C_1 \text{ and } C_2 \text{ constants.} \tag{11}$$

The values of C_1 and C_2 for the three cases represented by eqs. (1), (2), and (3) are listed in Tables 4 and 5. Concerning Table 4 it should be added that the usual correction of the nucleonic intensity by a logarithmic formula leads to standard errors which do not differ appreciably from those in Table 4.

It is evident from eq. (9) that a positive correlation between the ε_{T_t} values of the different atmospheric layers would increase the values in Table 4 (Dorman method). Assuming a correlation coefficient of 0.5 this increase would amount to the constant value of approximately $4 \cdot 10^{-6} N^2$.

TABLE 5.

δP, mb	δT, °C	δH, km	Duperier C_1	Duperier C_2	Dorman C_1	Dorman C_2
20	12	0.8	1.45	33.7	1.46	9.0
	6	0.4	1.39	19.7	1.41	6.6
	0	0	1.36	15.3	1.37	5.8
	−6	−0.4	1.31	19.7	1.33	6.6
	−12	−0.8	1.27	33.7	1.29	9.0
10	12	0.9	1.42	29.7	1.42	6.0
	6	0.4	1.38	16.7	1.38	3.6
	0	0	1.33	14.3	1.34	2.8
	−6	−0.4	1.29	16.7	1.29	3.6
	−12	−0.8	1.24	29.7	1.25	6.0
0	12	0.8	1.39	28.7	1.39	5.0
	6	0.4	1.34	15.7	1.34	2.6
	0	0	1.30	11.3	1.30	1.8
	−6	−0.4	1.26	15.7	1.26	2.6
	−12	−0.8	1.21	28.7	1.22	5.0
−10	12	0.8	1.36	29.7	1.35	6.0
	6	0.4	1.31	16.7	1.31	3.6
	0	0	1.27	14.3	1.26	2.8
	−6	−0.4	1.22	16.7	1.22	3.6
	−12	−0.8	1.18	29.7	1.18	6.0
−20	12	0.8	1.33	33.7	1.29	9.0
	6	0.4	1.28	19.7	1.27	6.6
	0	0	1.24	15.3	1.23	5.8
	−6	−0.4	1.19	19.7	1.19	6.6
	−12	−0.8	1.15	33.7	1.14	9.0

The relative standard errors of N_c calculated according to eq. (8) are shown in Figs. 1 and 2 as functions of the counting rate.

Conclusions

Estimates are obtained of standard errors for cosmic ray data which have been corrected for atmospheric effects. If we assume that all the ε's involved in the calculations (eq. 7) are approximately normally distributed (a reasonable assumption) the true corrected intensity has a probability of $\frac{2}{3}$ to be inside the limits formed by plus and minus the standard error of the recorded values.

We have tried to choose the parameters in the calculations so as to make the results applicable to the instrumentation in common use

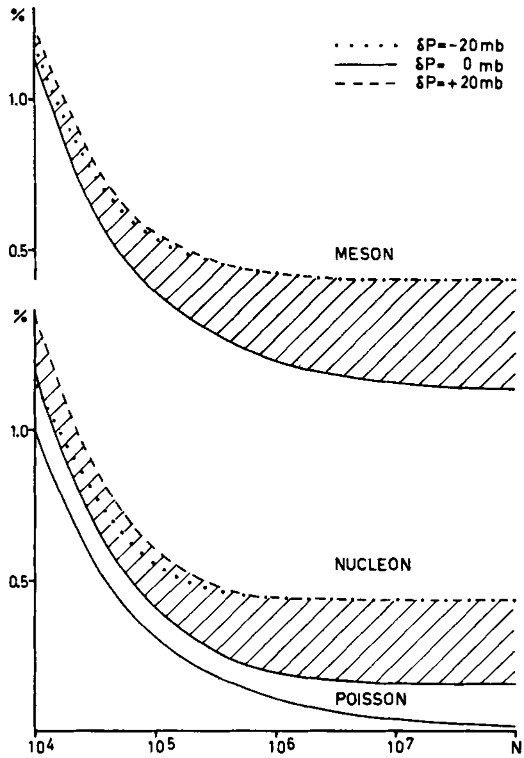

FIG. 1. Relative standard errors for pressure corrected cosmic ray data as functions of the number of counts per recording interval (N).

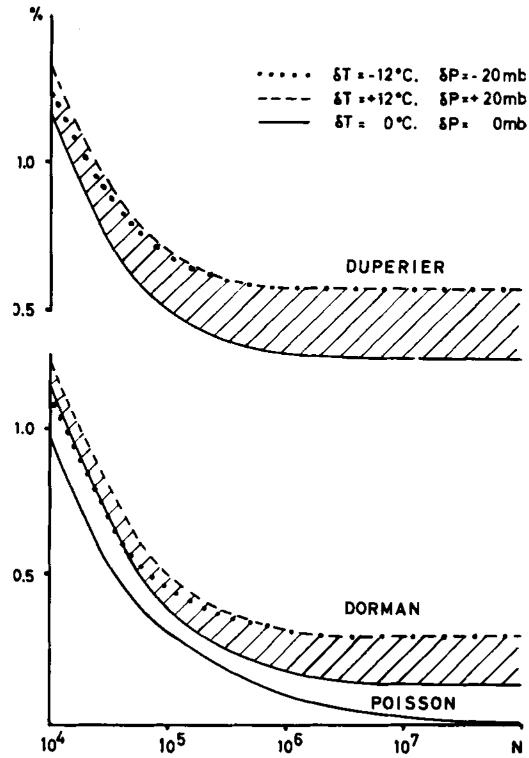

FIG. 2. Relative standard errors for corrected meson data as functions of the number of counts per recording interval (N).

at present. Of course, every cosmic ray and meteorological station has its own characteristics as concerns the recording accuracy. Until now very little has been made to establish these characteristics. If we want to have a more detailed error calculation for cosmic ray data special studies have to be made. This may be of importance for a critical cosmic ray analysis. However, we believe that the standard errors obtained in this report can be used as fairly good estimates for most cosmic ray purposes.

The results can be summarized as follows:

(1) The standard error of the corrected cosmic ray intensity is of course a function of the recorded intensity and the errors of the atmospheric parameters. Besides, it is also dependent on the magnitude of the deviations of the atmospheric parameters from the normal values.

(2) The results shown in this report are valid for single observations. In cosmic ray analyses sums or means over a period are often used.

The error variance of a mean over M intervals is

$$D^2(\varepsilon_{\text{mean}}) = M^{-2} \sum_{i=1}^{M} D^2(\varepsilon_i).$$

From Figs. 1 and 2 we can easily see that

$$D(\varepsilon_{\text{mean}}) > D(\varepsilon)\, M^{-\frac{1}{2}}$$

if $D(\varepsilon)$ is calculated for $\delta P = 0$, $\delta T = 0$.

The actual value is dependent on the distribution of the intensity values and the atmospheric parameters during the period.

(3) Figs. 1 and 2 show that the relative standard error tends to a constant level with increasing counting rate. The standard error of the corrected intensity can be said to consist of two components, the standard error of the uncorrected intensity and a standard error which is due to the uncertainty in the corrections. The former component tends to zero with increasing counting rate, whereas the latter is

approximately constant. For that reason the relative standard error is roughly constant from about 10^6 counts per recording interval and upwards. Thus, for each interval length there exists a critical counting rate which it is meaningless to pass in order to decrease the standard error, as long as no improvements are made in the meteorological recording methods. This counting rate depends on the accuracy of the meteorological values which are inserted in the correction formulas. The use of interpolated values moves the critical counting rate towards lower values and increases the plateau value of the relative standard error.

It must be observed that our calculations have been carried out under the assumption that simultaneous meteorological measurements are available for each recording interval. If we want to study short time variations of cosmic rays we must also have corresponding meteorological data at our disposal. Otherwise the statistical advantages of a high counting rate will be seriously affected by uncertainties in the atmospheric corrections. In the case of simple pressure correction the use of a good barograph can solve our problem. Concerning corrections for other atmospheric effects (meson component) the use of short time intervals is more doubtful as long as the frequency of aerological measurements is only two or four per day. Today we know very little about the magnitude of the errors which are introduced by interpolation of atmospheric parameters between two points of measurement.

(4) Standard errors calculated directly from a Poisson distribution of the corrected intensity values are definitely underestimates.

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