Note on the Role of Mountains in the Energy Budget of the Atmosphere

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As a basis for discussing energy conversions in the atmosphere WHITE and SALTZMAN (1956) presented equations for the rate of change of horizontal kinetic energy and total potential and internal energy of an atmosphere in hydrostatic equilibrium. In the development the lower boundary was tacitly assumed to be a level surface. It is my purpose now to include the effects of orography.

Using the notation,

λ = longitude
ϕ = latitude
p = pressure
t = time
a = radius of the earth
u = $a \cos \phi d\lambda/dt$ = zonal component of velocity
v = $ad\phi/dt$ = meridional component of velocity
ω = dp/dt
T = temperature
α = specific volume
z = height of an isobaric surface above sea level
g = acceleration of gravity
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
k = $(u^2+v^2)/2$ = kinetic energy per unit mass
Φ = gz = potential energy per unit mass
Q = rate of heat addition per unit mass (including the generation of heat by friction)
D = rate of frictional dissipation of kinetic energy,

we have, as before,

$$\frac{\partial k}{\partial t} = -\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(k+\Phi)u + \frac{\partial}{\partial\phi}(k+\Phi)v\cos\phi\right] - \frac{\partial}{\partial p}(k+\Phi)\omega - \omega\alpha - D \qquad (1)$$

$$\frac{\partial}{\partial t}c_pT = -\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}c_pTu + \frac{\partial}{\partial\phi}c_pTv\cos\phi\right] - \frac{\partial}{\partial p}c_pT\omega + \omega\alpha + Q. \qquad (2)$$

If, now, we integrate (1) and (2) over a mass of atmosphere M, enclosed within a volume of horizontal area S, extending from the surface of the earth, where $z = h(\lambda, \phi)$, $p = p_0(\lambda, \phi, t)$ and $\omega = \partial p_0/\partial t + u(a\cos\phi)^{-1}\partial p_0/\partial\lambda + v a^{-1}\partial p_0/\partial\phi$, to the top of the atmosphere, where $p = 0$ and $\omega = 0$, we obtain

$$\frac{\partial}{\partial t}\int_M k\,dm = \int_L v_n(k+\Phi)\,dl - \int_S h\frac{\partial p_0}{\partial t}d\sigma - \int_M \omega\alpha\,dm - \int_M D\,dm \qquad (3)$$

$$\frac{\partial}{\partial t}\int_M (\Phi + I)\,dm = \int_L v_n c_p T\,dl + \int_S h\frac{\partial p_0}{\partial t}d\sigma + \int_M \omega\alpha\,dm + \int_M Q\,dm \qquad (4)$$

where,

I = c_vT = internal energy per unit mass
dm = $g^{-1}a^2\cos\phi d\lambda d\phi dp$ = element of mass
v_n = inward component of horizontal velocity at the lateral boundary, L.
dl = element of the lateral boundary with vertical coordinate p/g.
$d\sigma$ = $a^2\cos\phi d\lambda d\phi$ = projection of an element of the surface of the earth on sea level.

If we consider a closed region (i.e., the entire atmosphere) the first terms on the right of (3) and (4), which represent the horizontal transport of energy, vanish. In this case the integral introduced as a result of topography, $\int_S h\partial p_0/\partial t d\sigma$, which we henceforth denote by H, can be thought of as the rate of conversion of kinetic energy into potential and internal energy associated with the transfer of mass from the less elevated regions of the globe to the more elevated regions. A negative value indicates a release of kinetic energy as mass is removed from the more elevated regions. To the extent that the total mass of the atmosphere is constant and measured by $\int_S p_0 d\sigma$, we have $\int_S \partial p_0/\partial t d\sigma = 0$. Hence, for the whole earth, H depends on the correlation between the surface elevation h and the local rate of pressure change at the surface, $\partial p_0/\partial t$. In principle, even hills and smaller local irregularities are included in h, but in all probability effects due to these are small compared to those due to the large mountains ranges.

The terms representing the rate of change of kinetic and potential plus internal energy, on the left hand sides of (3) and of (4) respectively, as well as H, all vanish in the long time average if we exclude climatic changes from consideration. At any instant, however, all these terms generally have non-zero values. MUNK and MILLER (1950) estimate, for example, that from July to January the average increase of surface pressure over land (which occupies about three tenths of the surface of the earth) is 0.183 mb. This value is probably low, perhaps by an order of magnitude, judging from the more recent results of the STAFF MEMBERS, ACADEMIA SINICA (1957). If we take 1 mb as our estimate, and if we assume, further, that the mean height above sea level of the land areas experiencing this pressure change is 3 km (most of the mass change is concentrated over the Asiatic mainland, cf. STAFF MEMBERS, ACADEMIA SINICA, 1957) we obtain as a rough estimate of H, applicable between July and January, the value 3×10^{19} ergs/sec. This is two to three orders of magnitude less than the rate of release of kinetic energy due to baroclinic processes, measured by $\int_M \omega\alpha dm$ (cf., SALTZMAN and FLEISHER, 1959 and WIIN-NIELSEN, 1959), but may be of the same order as the actual rate of change of the total kinetic energy of the entire atmosphere between these two months. For elevated *open* regions (such as the Tibetan Plateau) the mountain term can have a larger value at any time reflecting temporary or seasonal local mass changes, but in this case it can no longer be considered as a measure of an energy "conversion" within the region, being related, instead, to simultaneous transports of energy across the lateral boundaries.

Additional considerations: If we allow that the elevation of the surface of the earth can vary with time, as in the case of ocean tidal oscillations and wave motions of ocean and sand, for example, an additional term, $+\int_S p_0 \partial h/\partial t d\sigma$, appears on the right hand side of (4). This term represents the creation of potential energy in the atmosphere through the work done by vertical motions of the earth's surface. Along with the work done by frictional stresses, this process can be viewed as a possible mode of energy transfer between the atmosphere and the lithosphere and hydrosphere. We note that the effects of water vapor have not been considered in this development. This and other general considerations concerning the energy budget are discussed by STARR (1948, 1951).

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