Momentum Spectrum of Cosmic Radiation

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Abstract

If in a variable magnetic field charged high energy particles are injected at a certain point, they are *accelerated* but at the same time they *diffuse* outwards from the point of injection. Near the point of injection the density in a volume element of six-dimensional phase-space is increased by the acceleration process but decreased by the diffusion, and the momentum spectrum is obtained from the stationary state when these two effects are equal. The theory gives the power spectrum of (51) or (54), in which the value of the exponent depends in a very insensitive way on the rate of acceleration and the rate of diffusion.

The spectrum depends on the distance from the source of injection and the average energy density is orders of magnitude smaller than the density near the source of injection.

The theory is in agreement with the "local" theory of Cosmic Radiation, but it is difficult to reconcile it with galactic or extragalactic theories.

I. Introduction

There are good reasons to suppose that cosmic rays are accelerated by an electromagnetic process, as was first suggested by SWANN (1933), but there is no general agreement neither where the process is located, nor about the details of the mechanism. Different authors claim that cosmic radiation is generated in intergalactic space (BIERMANN, L., and BAGGE, E., 1949, BAGGE, E., 1950, HEIDMANN, J., 1955, COCCONI, G., 1956) interstellar space (FERMI, E., 1949, FAN, C. Y., 1951, MORRISON, P., OLBERT, S., and ROSSI, B., 1954, GINZBURG, V. L., 1953, BIERMANN, L., 1953, DAVIS, L., 1956), at the supernovae (BAADE, W., and ZWICKY, F., 1934, TER HAAR, D., 1950, SHKLOWSKI, J. S., 1953, SHKLOWSKY, J. S., 1954), or in the interplanetary space (RICHTMEYER, R. D. and TELLER, E., 1949, ALFVÉN, H., 1949, 1950, 1954). With assumptions which are not in definite conflict with astrophysical facts, many of the theories can account for the high energies, the isotropy, and the intensity of Cosmic Radiation.

However, none of the existing theories can account for the most peculiar of all the properties of cosmic radiation, *viz.* the fact that the momentum spectrum obeys a power law within a very wide range (10^{11}—10^{18} eV/c). The only existing theory of the momentum spectrum is due to FERMI (1949), but it is now clearly seen that the assumptions of his theory are not tenable (compare VI).

In the present paper the consequences of an accelerating mechanism, which was proposed some years ago, has been worked out. The mechanism is of a very general type and is now usually refered to as "magnetic pumping". It is the result of two simple effects: the betatron (or "cygnotron") acceleration and the scattering of particles from small inhomogeneities of the magnetic field. In order to show how this very general type of process works, two slightly different models are discussed in details. The energy of the particles increases and decreases due to the change in the magnetic field, but as energy is stored, either as motion parallel to the magnetic field (in model 1) or in an adjacent volume (in model 2) there is a net gain in energy. As a consequence of the process the accelerated particles diffuse in the magnetic field. The magnetic field may be frozen in into a medium with a high electric conductivity, which moves with the lines of force when the field changes.

The process does not accelerate all charged particles in space but only (or preferentially) particles which already have energies above a certain limit. This makes the *injection* of the particles very important. If we assume that the injection takes place at a certain point it is possible to work out the momentum spectrum of the accelerated particles. A power law is obtained with the exponent equal to between $n = 2$ and $n = 3$. The spectrum has a low-energy cut-off at an energy which is a function of the distance from the injector.

It is discussed in details in what way the exponent depends on the assumptions of the theory, and it is shown that the value of the exponent is very insensitive to the parameters of the theory. A comparison with Fermi's theory is made.

Conclusions about the location of the accelerating processes are drawn from the theory.

II. Acceleration of charged particles in varying magnetic fields

In order to study the acceleration of charged particles in varying magnetic fields we shall study two simple models.

Model 1

We assume that the charged particles move in a homogeneous magnetic field H_0 parallel to the z-axis, and that they are confined to a volume U_0 by two perfectly reflecting planes $z = 0$ and $z = z_0$. The magnetic field changes its strength to H_1 in a short time τ_0, remains at H_1 during the time $\tau_1 \gg \tau_0$, and then returns to H_0 in a short time $\tau_0' \ll \tau_1$. The variation of the field may be produced by motion of a conducting medium, e.g. interstellar gas, in which the field is frozen in. When the field varies, the x—y component $p_\perp$ and the z-component $p_{||}$ of the momentum $\vec{p}$ of a charged particle obeys the formulae

$$\frac{p_\perp^2}{H} = \text{const} \tag{1}$$

$$p_{||} \doteq \text{const} \tag{2}$$

Hence when the field changes, the momentum varies but at the end of the process there is no net gain or loss of momentum.

Suppose now that in the magnetic field there are a number of small-scale disturbances, which scatter particles passing them. These disturbances may be due to local currents in an ionized gas which fills the volume. The scattering produces a statistical distribution of the momentum vector, so that in average

$$p_\perp^2 = \frac{2}{3}p^2 \tag{3}$$

$$p_{||}^2 = \frac{1}{3}p^2 \tag{4}$$

If this equilibrium is disturbed it takes a certain time τ_i before it is restored. We assume that this "redistribution time" obeys

$$\tau_0,\ \tau_0' \ll \tau_i \ll \tau_1 \tag{5}$$

We shall show that under these conditions the process results in a net gain of momentum. We denote the momentum of a number of particles by p_0. The components $p_{\perp_0}$ and $p_{||_0}$ are given by

$$p_{\perp_0}^2 = \frac{2}{3}p_0^2 \tag{6}$$

$$p_{||_0}^2 = \frac{1}{3}p_0^2 \tag{7}$$

When the magnetic field changes from H_0 to H_1 the momenta are changed so that immediately after the field H_1 is established we have:

$$p_{\perp_1}^2 = kp_{\perp_0}^2 = \frac{2k}{3}p_0^2 \tag{8}$$

$$p_{||_1}^2 = p_{||_0}^2 = \frac{1}{3}p_0^2 \tag{9}$$

$$p_1^2 = p_{\perp_1}^2 + p_{||_1}^2 = \left(\frac{1}{3} + \frac{2k}{3}\right)p_0^2 \tag{10}$$

where

$$k = H_1/H_0 \tag{11}$$

After the time τ_i a redistribution has taken place so that the momentum components are:

$$p_{\perp_2}^2 = \frac{2}{3} p_1^2 = \frac{2}{3}\left(\frac{1}{3} + \frac{2k}{3}\right) p_0^2 \qquad (12)$$

$$p_{\|_2}^2 = \frac{1}{3} p_1^2 = \frac{1}{3}\left(\frac{1}{3} + \frac{2k}{3}\right) p_0^2 \qquad (13)$$

When the field goes back from H_1 to H_0, the momentum becomes

$$p_{\perp_3}^2 = \frac{1}{k} p_{\perp_2}^2 = \frac{2}{3k}\left(\frac{1}{3} + \frac{2k}{3}\right) p_0^2 \qquad (14)$$

$$p_{\|_3}^2 = p_{\|_2}^2 = \frac{1}{3}\left(\frac{1}{3} + \frac{2k}{3}\right) p_0^2 \qquad (15)$$

$$p_3^2 = p_{\perp_3}^2 + p_{\|_3}^2 = \left(\frac{5}{9} + \frac{2k}{9} + \frac{2}{9k}\right) p_0^2 = \alpha^2 p_0^2 \qquad (16)$$

After another time τ_i the momentum p_3 will be statistically distributed over $p_{\perp_3}$ and $p_{\|_3}$. If then the magnetic field once more changes from H_0 to H_1 and back again, a new acceleration takes place, but it starts from a momentum

$$p_3 = \alpha p_0 \qquad (17)$$

with

$$\alpha = \left(\frac{5}{9} + \frac{2k}{9} + \frac{2}{9k}\right)^{1/2} \qquad (18)$$

If for example $k = 10$, we have $\alpha = 1.6$.

If the process is repeated with a time interval $T \gg \tau_1$, the momentum will in average increase exponentially

$$p = p_0\, e^{t/\tau_a} \qquad (19)$$

with

$$\tau_a = \frac{T}{\ln \alpha} \qquad (20)$$

Suppose that the total number of particles in the momentum interval p_1 to $p_1 + dp_1$ is $F(p_1)dp_1$. After a certain time these particles will be found in the momentum range p_2 to $p_2 + dp_2$, so that a stationary state cannot be reached unless we have

$$F(p_1)dp_1 = F(p_2)dp_2. \qquad (21)$$

As

$$p_2 = p_1 e^{t/\tau_a}$$

$$dp_2 = dp_1 e^{t/\tau_a}$$

or

$$\frac{dp_1}{p_1} = \frac{dp_2}{p_2}$$

we get

$$F(p_1)p_1 = F(p_2)p_2 = C \qquad (21, a)$$

or

$$F = \frac{C}{p} \qquad (22)$$

where C is a constant. During the time T the momentum value p_1 will be passed by a number of particles which we denote by $\nu_0 T$. We have

$$\nu_0 T = \int_{p_1}^{p_1 e^{T/\tau_a}} F dp = CT/\tau_a \qquad (23)$$

which gives

$$F = \frac{\nu_0 T_a}{p} = \frac{N_0}{p} \qquad (24)$$

In order to get stationary conditions we must inject $N_0 = \nu_0 \tau_a$ low energy particles during the acceleration period τ_a (= time during which the momentum increases by a factor e). If the increase in momentum during one period is not infinitesimal, the injected particles must have an appropriate spectral distribution in order to give a smooth spectrum. Generally the injected low-energy particles will be accelerated up to high energies by our process.

Model 2

If the condition $\tau_i \gg \tau_0$ is not satisfied, the redistribution takes place during the change in the magnetic field, and the energy gain becomes smaller. If $\tau_i \ll \tau_0$ the momentum varies adiabatically with the magnetic field according to the formula (BLOCK, 1955, p. 85)

$$p^3/H = \text{const} \qquad (25)$$

and we obtain no net gain by the process. However, gain is obtained if there is an adjacent volume in which the magnetic field does not vary and particles accelerated when $H_0 \to H_1$ can be stored in this volume when the field goes back to H_0.

In order to illustrate this process by a simple model, let us assume that adjacent to the changing volume U_0 in model 1 there is a much larger volume U_2 limited by the planes $z = 0$ and $z = -z_2$ in which the magnetic field remains constant. The times τ_0 and τ_0' be so short that no appreciable number of particles are exchanged between U_0 and U_2 but the time τ_1 is so long that a complete mixing of the particles takes place.

Due to the frozen-in lines of force around which the particles spiral, the increase in the magnetic field by a factor k brings with it a reduction of the volume U_0 to $U_1 = \frac{1}{k} U_0$. (The change in the magnetic field can be thought to be produced by a radial oscillation of U). Let $F(p)$ be the momentum spectrum and consider the change in the number of particles in the interval p_2 to $p_2 + dp_2$. During the process $H_0 \to H_1$ the number of particles between p_2 and $p_2 + dp_2$ is increased by particles coming from a lower momentum range p_1 to $p_1 + dp_1$ and decreased by particles which are transferred to a higher momentum range p_3 to $p_3 + dp_3$. The net increase is

$$U_0 F(p_1) dp_1 - U_0 F(p_2)\, dp_2$$

The particles have now time to diffuse between U_1 and U_2. During the process $H_1 \to H_0$ the process is reversed but the volume is now U_1, so that the net increase is

$$U_1 F(p_3) dp_3 - U_1 F(p_2) dp_2$$

A stationary spectrum is obtained if the total net increase is zero. As

$$U_1 = \frac{1}{k} U_0$$

and according to (25):

$$\frac{dp_3}{dp_2} = \frac{dp_2}{dp_1} = k^{1/3}$$

we get

$$F(p_1) + k^{-1/3} F(p_3) = (k^{1/3} + k^{-1/3}) F(p_2) \cdot$$

$$p_3 = k^{1/3} p_2 = k^{2/3} p_1 \tag{26}$$

This equation is satisfied either by

$$F = \text{const}\, p^2 \tag{27, a}$$

or

$$F = \frac{\text{const}}{p} \tag{27, b}$$

The first solution corresponds to a reversible pumping, whereas the second solution gives a systematic increase in momentum. If particles with momentum p_0 are injected at a constant rate, a stationary spectrum has the form

$$\left.\begin{aligned} F &= \frac{N_0}{p_0^3} p^2 \text{ for } p \leqslant p_0 \\ F &= N_0/p \quad \text{for } p \geqslant p_0 \end{aligned}\right\} \tag{28}$$

where N_0 has the same significance as earlier. In order to establish the spectrum, some of the injected particles must loose momentum but as soon as a stationary state with this spectral distribution is reached, the injected particles are accelerated.

Discussion of the models

The study we have made shows that under very general conditions changing magnetic fields transfer energy to charged particles, a process which was first introduced by Swann and later has appeared in different forms in a number of theories. It should be observed that if the variations of a magnetic field cover an extended frequency spectrum, some parts of the spectrum accelerate high energy particles, whereas other parts tend to accelerate low energy particles. This depends upon how the inequality (5) is satisfied for different energies. The frequencies which tend to accelerate particles with thermal energy will be damped very quickly and we are only concerned with those frequencies which accelerate high energy particles specifically.

The models we have studied give a momentum spectrum p^{-n} with $n = 1$. This means that if we put

$$\xi = \ln p$$

the spectrum is

$$\Phi(\xi) = \text{const}$$

which is a result of the fact that the pumping speed is independent of ξ.

Like the observed cosmic ray spectrum, the spectrum we have obtained is a power spectrum, but the value of n differs considerably, the observational value being $n = 2.6$. The question is if the mechanism can be modified so as to give the observed value of n. In principle this is possible because τ_i in the first model and the time of mixing in the second model may depend on p with the result that the pumping speed becomes a function of ξ. In order to obtain the empirical spectrum, however, we must assume that the pumping speed *increases* as $p^{1.6}$. In other words, if the time needed to accelerate a particle from, say, 10^{16} to 10^{17} eV is T_{16} and the time for acceleration from 10^9 to 10^{10} eV is T_9, we should have $T_9/T_{16} = (10^{(16-9)})^{1.6} = 10^{11.2}$. This is completely impossible, because even if we assume that T_9 is very long, T_{16} would be unreasonably short.

The only possible alternative to obtain the observed value of n seems to be a process in which the *total* number of accelerated particles has a spectrum with $n = 1$ but that due to diffusion the particles are spread out in space in such a way that at a certain point the density is such as to give us the observed spectrum.

III. Diffusion of charged particles in a magnetic field

In the models we have discussed it is essential that there are field inhomogeneities which scatter the particles. As a scattering means a change $\overrightarrow{\Delta p_\perp}$ in $\overrightarrow{p_\perp}$ it necessarily causes a displacement $\vec{\Delta}$ of the centre of curvature of the particle. In fact we have

$$\vec{\Delta} = -\frac{C}{eH^2}\vec{H}\times\overrightarrow{\Delta p_\perp}$$

As $\overrightarrow{\Delta p_\perp}$ has a statistical distribution, $\vec{\Delta}$ has a random direction. If $\overrightarrow{\Delta p_\perp}$ is of the same order as $\overrightarrow{p_\perp}$ the displacement Δ is of the same order as the radius of curvature ϱ.

Thus we find that in the models we have considered the particles necessarily *diffuse* perpendicular to the magnetic field. The diffusion coefficient D could be put equal to

$$D = \frac{\varrho^2}{\tau_c} \tag{29}$$

where the "collision time" τ_c is a measure of the time between two scatterings which result in a displacement ϱ. The value of T_c can be computed only if we have a detailed knowledge of the disturbances in the field, which essentially means that we must know the magnetohydrodynamic turbulence spectrum.

If we introduce

$$\tau_g = \frac{2\pi\varrho}{c} \tag{30}$$

which when $v_\perp \approx c$ is the period corresponding to the gyro-frequency of the particle, the quantity

$$\vartheta = \frac{\tau_c}{\tau_g} \tag{31}$$

denotes the number of turns between two scatterings (for $v_\perp < c$ its significance is somewhat different).

We shall now study the case when in the model 1 we inject low energy particles at the z-axis. (With a small modification our discussion will also be applicable to model 2.) The particles are accelerated according to $p = p_0 e^{t/\tau_a}$. At the same time they diffuse away from the z-axis and as $\tau_1 \ll T$ the diffusion takes place in a field H_0. Hence we have also

$$\varrho = \varrho_0 e^{t/\tau_a} \tag{32}$$

If M particles with momentum p_0 are injected at the z-axis at the time $t = 0$, the space density n of the particles at the time t at a distance r from the z-axis is given by

$$n = \frac{M}{4\pi Y} e^{-r^2/4Y} \tag{33}$$

with

$$Y = \int_0^t D dt \tag{34}$$

(See for example CRANK: Mathematics of diffusion, Oxford 1956, p. 27 and 147).

As

$$t = \tau_a \ln\frac{\varrho}{\varrho_0}$$

and

$$dt = \frac{\tau_a}{\varrho} d\varrho$$

We have according to (29) and (34)

$$Y = \int_{\varrho_0}^{\varrho} \frac{\tau_a}{\tau_c} \varrho d\varrho \tag{35}$$

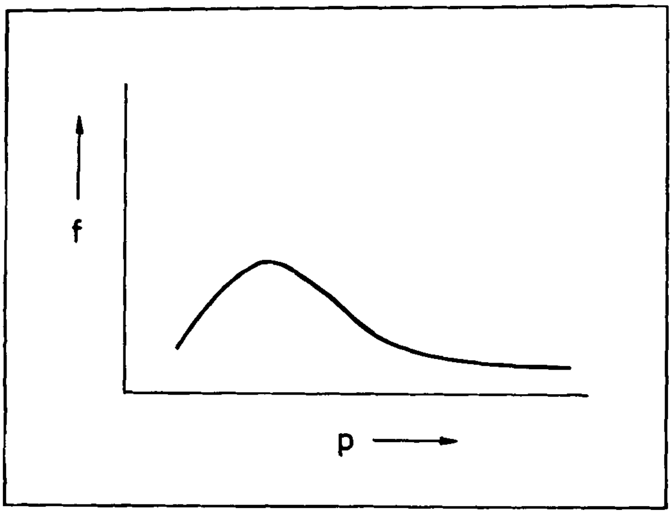

Fig. 1.

Y can be integrated only if we know the function τ_c. Let us put

$$\vartheta = \frac{\tau_c}{\tau_g} = \frac{1}{\gamma} \frac{c}{2\pi} \varrho^{\varphi} \tag{36}$$

where γ and φ are constants. φ gives the variation of ϑ with ϱ and is probably well within the limits $-1 < \varphi < +1$. Hence

$$\tau_c = \frac{1}{\gamma} \varrho^{1+\varphi} \tag{37}$$

and

$$Y = \frac{\gamma \tau^a}{1 - \varphi} (\varrho^{1-\varphi} - \varrho_0^{1-\varphi}) \tag{38}$$

or if $\varrho \gg \varrho_0$ approximately

$$Y = \frac{\gamma \tau_a}{1 - \varphi} \varrho^{1-\varphi}$$

or

$$Y = \frac{\gamma \tau_a}{1 - \varphi} \left(\frac{\varrho_0}{p_0}\right)^{1-\varphi} p^{1-\varphi} \tag{39}$$

IV. Cosmic ray spectrum

The energy spectrum f at a certain distance r from the z-axis is given by

$$f = n \cdot F \tag{40}$$

or from (33) and (39),

$$f = \frac{\text{const}}{p^{2-\varphi}} e^{-\frac{r^2}{Kp^{1-\varphi}}} \tag{41}$$

with

$$K = 4 \frac{\gamma \tau_a}{1 - \varphi} \left(\frac{\varrho_0}{p_0}\right)^{1-\varphi} \tag{42}$$

At the z-axis the spectrum is

$$f = \frac{\text{const}}{p^{2-\varphi}} \tag{43}$$

At a distance r from the z-axis the spectrum has the form as shown in fig. 1.

The spectrum is the same as for $r = 0$, for momenta large enough to make $4\,Y \gg r^2$, which according to (37) and (39) means

$$\frac{\varrho}{r} \gg \sqrt{\frac{1 - \varphi}{4} \frac{\tau_c}{\tau_a}} \tag{44}$$

For particles with a radius of curvature which is too small to satisfy the inequality, the density is reduced, so that we get a "knee" in the spectrum. The position of this depends on the distance from the region of injection. See fig. 1.

The spectrum becomes similar to the observed spectrum if $\varphi = -0.6$. As we shall see later this value is probably acceptable.

V. Acceleration depends on p.

The model we have discussed is very simple and it is of interest to investigate how sensitive the spectrum is to modifications of the model.

We have assumed that the increase in momentum is exponential, but in reality it is likely that τ_a increases with p because large-scale field variations which accelerate high energy particles are likely to be slower than the small scale variations near the z-axis by which the low energy particles are accelerated. Hence it is important to see how the spectrum changes if τ_a depends on p.

We put the logarithmic rate of increase in p equal to:

$$\frac{1}{p}\frac{dp}{dt}=\frac{1}{\tau_a} \tag{45}$$

which with τ_a=constant leads to (19). However, we assume that τ_a varies with p and for the case of simplicity we put

$$\tau_a=\mu\tau_\mu\left(\frac{p}{p_0}\right)^\mu \tag{46}$$

where τ_μ, p_0 and μ are constants.

Integrating

$$\frac{dp}{p^{1-\mu}}=\frac{1}{\mu\tau_\mu}p_0^\mu dt \tag{47}$$

we find

$$p=p_0\left(1+\frac{t}{\tau_\mu}\right)^{\frac{1}{\mu}} \tag{47 a}$$

Introducing (47) into (21) we find

$$F=\frac{\text{const}}{p^{1-\mu}} \tag{48}$$

Further we have

$$Y=\int_{\varrho_0}^{\varrho}\frac{\tau_a}{\tau_c}\varrho d\varrho \tag{49}$$

and if we introduce $\tau_c=\frac{1}{\gamma}\varrho^{1+\varphi}$ and $\tau_a=$ $=\mu\tau_\mu\left(\frac{p}{p_0}\right)^\mu$ we obtain

$$Y=\frac{\gamma\mu\tau_\mu}{\varrho_0^\mu}\int_{\varrho_0}^{\varrho}\varrho^{\mu-\varphi}d\varrho$$

or

$$Y=\frac{\gamma\mu\tau_\mu}{1+\mu-\varphi}\varrho_0^{1-\varphi}\left[\left(\frac{p}{p_0}\right)^{1+\mu-\varphi}-1\right] \tag{50}$$

For $\varrho \gg \varrho_0$ we obtain the spectrum near the z-axis as

$$f\approx\frac{F}{Y}=\frac{\text{const}}{p^{1-\mu}p^{1+\mu-\varphi}}=\frac{\text{const}}{p^{2-\varphi}} \tag{51}$$

The spectrum is independent of μ, which means that even if the acceleration goes slower at high momenta, this does not affect the spectrum. The result is very important because it shows that the theoretical spectrum is not a result of some tricky mechanism but is obtained under even more general conditions than we have assumed.

Another important limitation of our model is that it is two-dimensional. We shall not here enter into a detailed discussion of the motion of cosmic rays in interstellar space. The usual idea is that cosmic rays are confined in two dimensions by the magnetic field but are free to move in the third dimension (parallel to the field). This is correct for the motion of *one* charged particle but not for a multitude of particles because we have to take into account the fact that a large number of particles moving in one direction is equivalent to a current and causes a considerable magnetic field. Hence the streaming of particles will cause a series of complicated phenomena including setting up of oscillations. What the final result would be cannot be decided without a detailed discussion, but it is possible that even the outflow parallel to the magnetic field has the character of a diffusion. If this is the case we should consider a three-dimensional diffusion. This obeys the formula

$$n=\frac{M}{8(\pi Y)^{3/2}}e^{-r^2/4Y} \tag{52}$$

from which we derive the spectrum:

$$f\approx\frac{F}{Y^{3/2}}e^{-r^2/4Y} \tag{53}$$

or

$$f=\frac{\text{const}}{p^{2.5-1.5\varphi+0.5\mu}}e^{-r^2/K'p^{1-\varphi+\mu}} \tag{54}$$

with

$$K=\frac{4\gamma\mu_0\tau_\mu}{1-\varphi+\mu}\frac{\varrho_0{}^{1-\varphi}}{p_0{}^{1-\varphi+\mu}}$$

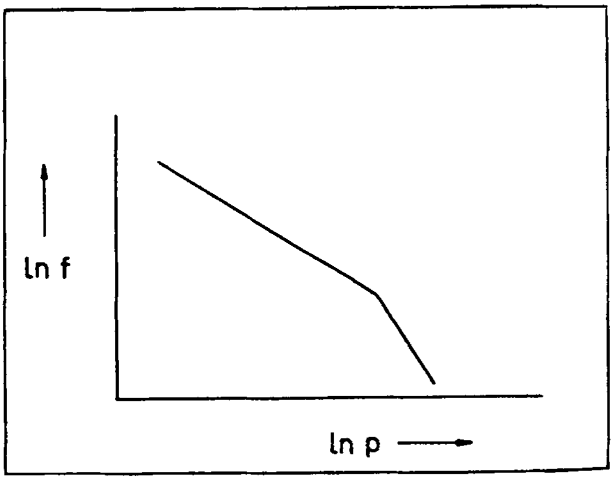

Fig. 2.

In this case the spectrum depends on μ but not in a very sensitive way.

If at two different momenta p' and p'' the number of turns per collision is ϑ' and ϑ'' and the times for increasing the energy by a factor e is τ_a' and τ_a'', we have

$$\frac{\ln(\vartheta'/\vartheta'')}{\ln(p'/p'')} \tag{55}$$

$$\frac{\ln(\tau_a'/\tau_a'')}{\ln(p'/p'')} \tag{56}$$

An agreement between the two dimensional formula and the observed spectrum $n = 2.6$ requires $\varphi = -0.6$. This means that if $p' = 10^{10}$ eV, $p'' = 10^{16}$ eV, we have $\vartheta'/\vartheta'' = 4{,}000$. If the high energy particle makes, say, 5 turns between the collisions, the low-energy particle should make 20,000 turns. The ratio is high but with our present knowledge of the turbulence spectrum it is not in conflict with any observational fact.

The three-dimensional formula is satisfied by, for example, $\varphi = +0.1$, $\mu = +0.5$. This means that the number of turns per collision is almost independent of the momentum but the rate of logarithmic acceleration decreases with increasing momentum, being 1,000 times more rapid for 10^{10} eV than for 10^{16} eV. This seems to be quite plausible.

If ϑ'' changes by a factor of 10, φ changes by 0.17. The power law exponent n changes by 0.17 in the two-dimensional model and by 0.25 in the three-dimensional model. If τ_a'' changes by a factor of 10, μ changes by 0.17. This does not effect n in the two-dimensiona formula but changes n by 0.08 in the three-dimensional formula. This demonstrates how insensitive the exponent n is to large variations in the collision frequency and in the time of acceleration.

VI. Absorption

If the gas density in our model is different from zero, the particles will loose energy by collisions. The process is rather complicated and different for protons and heavy nuclei, the latter being split up by collisions. Without entering into the details of the processes we may account for them roughly through an absorption formula:

$$n = n_0 e^{-t/\tau_b} \tag{57}$$

where τ_b is a time constant, which with the density in our galaxy is of the order 10^7 to 10^8 years. If $\mu \neq 0$ we have according to (47, a)

$$t = \tau_\mu \left[\left(\frac{p}{p_0}\right)^\mu - 1\right]$$

and we obtain for the spectrum (in the two-dimensional model)

$$F = \frac{\text{const}}{p^{1-\mu}} e^{-\frac{\tau_\mu}{\tau_b}\left[\left(\frac{p}{p_0}\right)^\mu - 1\right]} \tag{58}$$

$$f = \frac{\text{const}}{p^{2-\varphi}} e^{-\frac{r^2}{K' \cdot p^{1-\varphi+\mu}}} e^{-\frac{\tau_\mu}{\tau_b}\left[\left(\frac{p}{p_0}\right)^\mu - 1\right]} \tag{59}$$

If μ is not too small the spectrum decreases rapidly or almost gets a cut-off (see Fig. 2). This cut-off occurs at the momentum when the exponent surpasses the value 1.

The fact that no such cut-off is observed indicates that the time of absorption τ_b is larger than the time of acceleration even for the highest momenta in cosmic radiation. In case $\mu = 0$, we have

$$p/p_0 = e^{t/\tau_a}$$

or

$$t = \tau_a \ln \frac{p}{p_0}$$

and we obtain instead

$$F = \frac{\text{const}}{p} e^{-\frac{\tau_a}{\tau_b} \ln\left(\frac{p}{p_0}\right)}$$

or

$$F = \frac{\text{const}}{p^{1+\tau_a/\tau_b}} \tag{60}$$

This is the same formula as Fermi has obtained. If as Fermi assumes $\tau_a/\tau_b = 1.6$, this gives the desired power law for the *total* number of particles. Hence in Fermi's theory the cosmic radiation may fill up space uniformly, whereas in our theory an intensity gradient causing diffusion is essential.

One of the difficulties with Fermi's theory is that one must assume that by accident τ_a and τ_b are of the same order in spite of the fact that they are not at all physically related to each other. If e.g. the density in space were 10 times larger or smaller than assumed, the value of τ_b would decrease or increase 10 times, and the exponent would either be 1.16 or 17, instead of $n=2.6$. It is highly improbable that the ratio τ_a/τ_b should remain constant over a momentum range from 10^{10} up to 10^{17} eV and a variation in this ratio by only a factor 2 would be disastrous to the theory. This should be compared with the high degree of insensitivity of our spectrum to variations in τ_a or in ϑ. As shown by the numerical example in ch. V a variation of these quantities by a factor of ten gives only a hardly observable change in the exponent n.

VII. Is cosmic radiation a galactic or "local" phenomenon?

If we observe the cosmic radiation at a distance r from the injection, the spectrum shows a maximum for a momentum corresponding to a radius of curvature ϱ which is given by

$$r = \varrho \sqrt{\frac{4}{2-\varphi} \frac{\tau_a}{\tau}} \tag{61}$$

The observational spectrum also shows a maximum at about 10^9 eV corresponding to the "knee" in the latitude curve. It is not clear whether this maximum should be identified with the theoretical maximum, because there are also other phenomena which might produce a similar effect. However, we could state that the observational spectrum has no maximum *above* 10^9 eV. In a field of 10^{-5} gauss, this energy corresponds to $\varrho = 3 \cdot 10^{11}$ cm and the period is about 100 sec. We should expect τ_c to be larger, at least, say, 10^3 sec. With this value we obtain

$\tau_a = 10$ years $r < 3 \cdot 10^{14}$ cm
$\tau_a = 10^3$ " $r < 3 \cdot 10^{15}$ cm
$\tau_a = 10^5$ " $r < 3 \cdot 10^{16}$ cm

This shows that the distance from the injection could not be much in excess of the dimensions of our planetary system. The conclusion is that the injection is likely to take place near the sun or in any case within the planetary system.

The acceleration mechanism of the "local" theory (Alfvén, 1954) seems to be a reasonable injector of high energy particles. The process which has been discussed here is essentially of the same type as discussed in the cited paper but is able to bring the particles up to still higher energies.

This indicates that the cosmic radiation below say, 10^{14} eV is generated near the sun or in its environment. The highest portion of the spectrum ($> 10^{14}$ eV) may in part derive from other sources, e.g. supernovae, magnetically variable stars, double-stars, and colliding magnetized clouds. The radiation from these sources are superimposed but if all these sources accelerate particles according to the discussed mechanism the resultant spectrum should be essentially a power spectrum.

The present paper is a continuation of the research started by Alfvén and Åström (1958). It has been carried out at the Tata Institute of Fundamental Research, Bombay, during a stay at the invitation of Dr H. Bhabha. I have profited much from discussions with Prof. B. Peters and Dr K. S. Singwi.

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