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The Helicoidal Structures in the Cosmical Electrodynamics

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Abstract

The general-type solution of the equation $[\text{curl}(\text{curl}\,\boldsymbol{H}\times\boldsymbol{H})]=0$ are studied in the cylindrical systems, according to LUNDQUIST's and CHANDRASEKHAR's methods. In general, the magnetic fields are of helicoidal type in the cylindrical systems. Several examples are studied in the cosmical fields.

1. Introduction

CHANDRASEKHAR (1956a) has shown that a magnetic field none of whose components are azimuthal can always be expressed as the sum of a toroidal T field and a poloidal P field. In this paper, we study several configurations of magnetic fields which satisfy the following equations under the restriction that none of their components are azimuthal;

$$\text{curl}\,(\text{curl}\,\boldsymbol{H}\times\boldsymbol{H})=0. \tag{1}$$

In cylindrical co-ordinate systems $(\tilde{\omega}, \varphi, z)$, $\boldsymbol{H}$ with the above-mentioned restriction has a form

$$\boldsymbol{H}=-\tilde{\omega}\frac{\partial P}{\partial z}\mathbf{1}_{\tilde{\omega}}+\tilde{\omega}T\mathbf{1}_{\varphi}+\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2P)\,\mathbf{1}_z, \tag{2}$$

and the poloidal function P and the toroidal function T which satisfy (1) and (2) are the solutions of the following differential equations, respectively.

$$\frac{\partial^2P}{\partial\tilde{\omega}^2}+\frac{3}{\tilde{\omega}}\frac{\partial P}{\partial\tilde{\omega}}+\frac{\partial^2P}{\partial z^2}+\frac{1}{\tilde{\omega}^2}G\,(\tilde{\omega}^2P)=\Phi\,(\tilde{\omega}^2P), \tag{3}$$

$$\frac{d}{d(\tilde{\omega}^2P)}(\tilde{\omega}T)=2G\,(\tilde{\omega}^2P). \tag{4}$$

In the cosmical dimensions, such configurations in the cylindrical system seem to appear in the following examples: (i) Force-free discharges, (ii) Magnetic fields of spiral arm, (iii) Current jets, (iv) Isoroattion and the filamentary structures of diffuse nebulae. In these examples, P and T functions are reduced to the solutions of (3) and (4).

2. Magnetohydrostatic cases

The simultaneous equations governing the velocity field and the magnetic field in an incompressible, inviscid and conductive fluid are (in e.m.u.)

$$-\frac{\partial\boldsymbol{H}}{\partial t}=\text{curl}\left(\frac{1}{4\pi\sigma}\,\text{curl}\,\boldsymbol{H}-\boldsymbol{v}\times\boldsymbol{H}\right), \tag{5}$$

$$\frac{\partial\boldsymbol{v}}{\partial t}+\text{curl}\,\boldsymbol{v}\times\boldsymbol{v}=\frac{1}{4\pi\varrho}\,\text{curl}\,\boldsymbol{H}\times\boldsymbol{H}-\text{grad}\left(\frac{p}{\varrho}+\frac{1}{2}v^2+V\right), \tag{6}$$

$$\text{div}\,\boldsymbol{H}=0,\qquad \text{div}\,\boldsymbol{v}=0, \tag{7}$$

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where the various notations are the generally used ones (cf. CHANDRASEKHAR, 1956b).

Let us study some static cases. In this case, (6) becomes

$$\frac{1}{4\pi\varrho}\ \text{curl}\ \boldsymbol{H}\times\boldsymbol{H}=\text{grad}\left(\frac{p}{\varrho}+V\right). \quad (8)$$

Taking the curl of this, we obtain (1). As was pointed out by LUNDQUIST (1950, 1952), the solution of (8) satisfies

$$\frac{1}{4\pi\varrho}\ \text{curl}\ \boldsymbol{H}=\alpha\boldsymbol{H}+\frac{\boldsymbol{H}\times\text{grad}\left(\frac{p}{\varrho}+V\right)}{H^2}. \quad (9)$$

The two cases appear as the special solutions,

$$(i)\qquad \text{grad}\left(\frac{p}{\varrho}+V\right)=0, \quad (10)$$

$$(ii)\qquad \alpha=0. \quad (11)$$

(i) *Beltrami field*

In the first case, we find

$$\text{curl}\ \boldsymbol{H}=\alpha'\boldsymbol{H}\ \text{ or }\ \boldsymbol{j}\times\boldsymbol{H}=0,$$

where

$$\alpha'=4\pi\varrho\alpha.$$

In general, such a configuration in a vector field is called the Beltrami field. This case means that all the terms in the equation of motion (6) are zero, independently. LUNDQUIST (1950) and CHANDRASEKHAR (1956a) studied this problem for the axisymmetric case. The latter author stated in a separate paper (1957) that the magnetic field gets altered in such a way that its influence on the motions is progressively reduced and a stationary state is eventually reached. The writer is not sure that such a configuration appears in the independent system. For example, from where come the electric currents in the independent system?

However, if the electric discharge occurs along the magnetic lines of force in the solar atmosphere and $\boldsymbol{j}\times(\boldsymbol{H}_0+\boldsymbol{h})$ becomes larger than any other terms in (6), we can expect the following configuration;

$$\text{curl}\ \boldsymbol{h}\times(\boldsymbol{H}_0+\boldsymbol{h})=0, \quad (13)$$

and

$$\text{curl}\ \boldsymbol{H}_0=0. \quad (14)$$

Let us study this case along the method developed by CHANDRASEKHAR for the axisymmetric case. In cylindrical co-ordinate system, $\boldsymbol{H}$ and curl $\boldsymbol{H}$ are

$$\boldsymbol{H}=\boldsymbol{H}_0+\boldsymbol{h}=-\tilde{\omega}\frac{\partial P}{\partial z}\mathbf{1}_{\tilde{\omega}}+\tilde{\omega}T\mathbf{1}_{\varphi}+\left(\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2P)+H_0\right)\mathbf{1}_{\varphi}, \quad (15)$$

$$\text{curl}\ \boldsymbol{H}=\tilde{\omega}\frac{\partial T}{\partial z}\mathbf{1}_{\tilde{\omega}}-\tilde{\omega}\Delta_5P\mathbf{1}_{\varphi}+\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2T)\mathbf{1}_{\varphi}. \quad (16)$$

Then, (13) becomes

$$\text{curl}\ \boldsymbol{h}=\alpha(\boldsymbol{H}_0+\boldsymbol{h}), \quad (17)$$

or

$$-\tilde{\omega}\frac{\partial T}{\partial z}\mathbf{1}_{\tilde{\omega}}-\tilde{\omega}\Delta_5P\mathbf{1}_{\varphi}+\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2T)\mathbf{1}_z=\alpha\left\{-\tilde{\omega}\frac{\partial P}{\partial z}\mathbf{1}_{\tilde{\omega}}+\tilde{\omega}T\mathbf{1}_{\varphi}+\left(\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2P)+H_0\right)\mathbf{1}_z\right\}. \quad (18)$$

Comparing each component on both sides, we can obtain the following relations;

$$\Delta_5P=-\alpha T, \quad (19)$$

$$\frac{\partial T}{\partial z}=\alpha\frac{\partial P}{\partial z}, \quad (20)$$

$$\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2T)=\alpha\left(\frac{1}{\tilde{\omega}}\frac{\partial}{\partial\tilde{\omega}}(\tilde{\omega}^2P)+H_0\right) \quad (21)$$

Equation (21) can be reduced to

$$\tilde{\omega}^2T=\alpha\tilde{\omega}^2P+\frac{\alpha H_0}{2}\tilde{\omega}^2+K, \quad (22)$$

where K is a constant. But, K must be zero in this case, according to the requirement that T be regular on the z-axis. Then,

$$T=\alpha P+\frac{\alpha H_0}{2}. \quad (23)$$

Substituting (23) into (19), the differential equation which must be satisfied by P is

$$\Delta_5 P = -\alpha^2 P - \frac{\alpha^2 H_0}{2}. \tag{24}$$

This is the special case of (3). With suitable boundary conditions, we can expect the helicoidal structure of the magnetic field which will be studied in the next section. In fact, DUNGEY and LOUGHHEAD (1954) studied the stability of this type of field having the impression that some prominences show filaments being twisted, though they have not verified why such a configuration appears in such a system.

(ii) *Complex-lamellar field*

The second case is that

$$\text{curl } \boldsymbol{H} = \frac{4\pi\varrho}{H^2} \boldsymbol{H} \times \text{grad} \left(\frac{p}{\varrho} + V\right). \tag{25}$$

Taking the dot product with $\boldsymbol{H}$, we can get

$$\boldsymbol{H} \cdot \text{curl } \boldsymbol{H} = 0. \tag{26}$$

In general, such a configuration in a vector field is called the complex-lamellar field. The magnetic field which satisfies (26) can be expressed by the two scalar functions ψ and χ;

$$\boldsymbol{H} = \psi \text{ grad } \chi. \tag{27}$$

For axisymmetric case, this becomes

$$\boldsymbol{H} = \psi \frac{\partial \chi}{\partial \tilde{\omega}} \mathbf{1}_{\tilde{\omega}} + \psi \frac{\partial \chi}{\partial z} \mathbf{1}_z. \tag{28}$$

Comparing (28) with (2), we find

$$T = 0,$$

and also

$$H_{\tilde{\omega}} = -\frac{1}{\tilde{\omega}} \frac{\partial}{\partial z} (\tilde{\omega}^2 P) = \psi \frac{\partial \chi}{\partial \tilde{\omega}}, \tag{29}$$

$$H_z = \frac{1}{\tilde{\omega}} \frac{\partial}{\partial \tilde{\omega}} (\tilde{\omega}^2 P) = \psi \frac{\partial x}{\partial t}. \tag{30}$$

This is the case which is studied by FERRARO (1954) in the case of equilibrium of the magnetic star.

(iii) *Magnetic fields of spiral arm*

The Beltrami field and the complex-lamellar field are only the special solution of equation (8). In this part, we discuss the most simplest general-type solution of (8). This discussion is quite analogous with the method developed by CHANDRASEKHAR and PRENDERGAST (1956) and PRENDERGAST (1956).

Taking the curl of (8), we have the equation (1),

$$\text{curl } (\text{curl } \boldsymbol{H} \times \boldsymbol{H}) = 0 \tag{31}$$

As the simplest example, we consider the magnetic fields of the spiral arm. The force-free discharge discussed above can be studied quite analogously along this line. CHANDRASEKHAR and FERMI (1953) assumed the following equilibrium condition in the spiral arm,

$$p_{\text{grav.}} = p_{\text{kin.}} + p_{\text{mag.}} \tag{32}$$

They used this equation for somewhat different purpose. In that paper, they have supposed that the direction of the galactic magnetic field is roughly parallel to the direction of the spiral arm.

When we equate the gravitational pressure in the arm to the sum of the gas pressure and the magnetic pressure and the density in the arm is assumed to be constant, the similar equation to (32) reduced to (1). Using (1) and (16), the poloidal function which must satisfy (1) becomes (cf. PRENDERGAST (1956) eq. (15))

$$\frac{\partial^2 P}{\partial \tilde{\omega}^2} + \frac{3}{\tilde{\omega}} \frac{\partial P}{\partial \tilde{\omega}} + \frac{\partial^2 P}{\partial z^2} + \alpha^2 P = \varkappa, \tag{33}$$

and the toroidal function which also must satisfy (1) becomes

$$\frac{d}{d(\tilde{\omega}^2 p)} (\tilde{\omega}^2 T^2) = 2\alpha^2 \tilde{\omega}^2 P, \tag{34}$$

where we assume

$$G = \alpha^2 \tilde{\omega}^2 P \quad (\alpha = \text{constant}),$$

$$\Phi = \varkappa = \text{constant},$$

in the equations (3) and (4), respectively. It must be noticed that (33) has the same form with (24).

Then, (33) becomes

$$\frac{d^2P}{d\tilde{\omega}^2}+\frac{3}{\tilde{\omega}}\frac{dP}{d\tilde{\omega}}+\alpha^2P=\varkappa, \tag{35}$$

where we assume $\partial/\partial z=0$.
If we take $\varkappa=-\alpha^2H_0/2$, this equation is exactly the same as (24). The solutions of (34) and (35) which are regular on the z-axis are

$$P=\frac{\varkappa}{\alpha^2}+\frac{AJ_1(\alpha\tilde{\omega})}{\tilde{\omega}} \tag{36}$$

$$T=\alpha P. \tag{37}$$

Then, it is easily verified that

$$\text{curl } \boldsymbol{H}=4\pi\boldsymbol{j}=\alpha\boldsymbol{H}-\tilde{\omega}\varkappa\mathbf{1}_\varphi,$$

and

$$\boldsymbol{j}\times\boldsymbol{H}=-\frac{\tilde{\omega}\varkappa}{4\pi}\mathbf{1}_\varphi\times\boldsymbol{H}=$$
$$=-\frac{\varkappa}{4\pi}\left(\frac{\partial(\tilde{\omega}^2P)}{\partial\pi}+\frac{\partial(\tilde{\omega}^2P)}{\partial z}\right)=$$
$$=-\frac{\varkappa}{4\pi}\text{grad }(\tilde{\omega}^2P). \tag{38}$$

Rewriting (8) in the following form

$$\boldsymbol{j}\times\boldsymbol{H}=\text{grad } p+\varrho \text{ grad } V, \tag{39}$$

and substituting (38) into (39), we obtain

$$-\frac{\varkappa}{4\pi}\text{grad }(\tilde{\omega}^2P)=\text{grad } p+\varrho\text{ grad } V, \tag{40}$$

or

$$-\frac{\varkappa}{4\pi}(\tilde{\omega}P)=p+\varrho V \tag{41}$$

On our case, we take

$$\varrho V=\pi G\varrho\varrho_t R^2. \tag{42}$$

In the simplest case, the boundary conditions to be satisfied at the surface of the spiral arm are

$$\text{at } \tilde{\omega}=R, \begin{cases}\text{the pressure is zero,}\\ \text{the magnetic field is zero;}\end{cases} \tag{43}$$

where R is the radius of the spiral arm. Then, this is

$$\text{at } \tilde{\omega}=R, \begin{cases}p=0\\ P=0 \text{ and } \dfrac{\partial P}{\partial\tilde{\omega}}=0;\end{cases} \tag{44}$$

from which we can obtain the final form of P

$$P=\frac{\varkappa}{\alpha^2}\left(1-\frac{R}{\pi}\frac{J_1(\alpha\tilde{\omega})}{J_1(\alpha R)}\right). \tag{45}$$

The another requirement is

$$\left(\frac{\partial P}{\partial\tilde{\omega}}\right)_{\tilde{\omega}=R}=\frac{\varkappa R}{\alpha J_1(\alpha R)}\frac{1}{R}J_2(\alpha R)=0, \tag{46}$$

and this is satisfied, only if

$$J_2(\alpha R)=0. \tag{47}$$

The first root of (47) is

$$\alpha_1R=5.135. \tag{48}$$

When we take large α, the toroidal field becomes larger compared with the poloidal field and then it seems that this configuration becomes unstable, as was pointed out by ALFVÉN (1950a).

Substituting the final form of P (45) into (2), the magnetic field in the spiral arm is

$$\boldsymbol{H}=\frac{\varkappa\tilde{\omega}}{\alpha}\left(1-\frac{R}{\tilde{\omega}}\frac{J_1(\alpha\tilde{\omega})}{J_1(\alpha R)}\right)\mathbf{1}_\varphi+$$
$$+\left\{\frac{\varkappa R}{\alpha J_1(\alpha R)}J_2(\alpha\tilde{\omega})+\frac{2\varkappa}{\alpha^2}\left(1-\frac{R}{\pi}\frac{J_1(\alpha\tilde{\omega})}{J_1(\alpha R)}\right)\right\}\mathbf{1}_z. \tag{49}$$

The lines of force resemble a helix wrapped on the cylindrical surface.

The boundary conditions adopted here are, of course, very scanty. (cf. SPITZER (1956)). Moreover, this is not the pure magnetostatic problem. However, it may be noticed that SHAJN (1957) obtained the systematic deviation of the general field from the galactic plane about 18° in the solar vicinity.

(iv) *Current jets*

The current-jet theory of filaments is a more general case of the force-free discharge. It seems that the force-free discharge occurs in the high solar atmosphere, but the general case may occur in the chromospheric regions where $\boldsymbol{j}\times\boldsymbol{H}$ term can be balanced by the sufficient pressure gradient. This has been studied as the constriction of discharge by ALFVÉN (1950b). The current-jet theory assumes the form

$$-\operatorname{grad} p + \boldsymbol{j} \times \boldsymbol{H} = 0. \qquad (50)$$

Taking the curl of this equation, we find also

$$\operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) = 0. \qquad (51)$$

Thus, we consider the solution of (33) and (34) under the boundary conditions that the magnetic fields are zero except $b < \tilde{\omega} < a$. That is, the magnetic field is enclosed between two concentric cylindrical surfaces.

COWLING (1957) criticized the current-jet theory, because such a field implies surface currents flowing in opposite directions on $\tilde{\omega} = b$ and $\tilde{\omega} = a$ and such oppositely directed currents are difficult to set going. He probably says that this configuration is too artificial. However, COWLING's estimation is only a special solution of (1) and in general, such a simple configuration does not occur.

In this case, a poloidal function which satisfies (33) or (35) has a form

$$P = \frac{\varkappa}{\alpha^2} + \frac{AJ_1(\alpha\tilde{\omega})}{\tilde{\omega}} + \frac{BY_1(\alpha\tilde{\omega})}{\tilde{\omega}} \qquad (52)$$

From (34), we obtain a toroidal function

$$\tilde{\omega}^4 T = \alpha^2 \tilde{\omega}^4 P + K.$$

From the boundary conditions, we can determine A, B and α. It is easily shown that K must be zero. Substituting (52) and (53) into (2), we can obtain the magnetic field and the current system, which are expected to be far more complex than COWLING's model.

3. Lamb surface

Rewriting (5) into the following form (54) and taking the curl of this equation, we have

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{w} \times \boldsymbol{v} = \frac{1}{4\pi\varrho} \operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H} - \operatorname{grad}\left(\frac{p}{\varrho} + \frac{1}{2} v^2 + V\right), \qquad (54)$$

$$\frac{\partial \boldsymbol{w}}{\partial t} + \operatorname{curl}\ (\boldsymbol{w} \times \boldsymbol{v}) = \frac{1}{4\pi\varrho} \operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}), \qquad (55)$$

where

$$\operatorname{curl}\ \boldsymbol{v} = \boldsymbol{w}.$$

Then, at each regular point, we can determine a plane which we may call the Lamb surface and whose normal is parallel to the Lamb vector $\boldsymbol{w} + \boldsymbol{v}$. A necessary and sufficient condition for the existence of the Lamb surface is that the Lamb vector be complex-lamellar and non-vanishing (cf. TRUESEDELL), 1954). That is,

$$(\boldsymbol{w} \times \boldsymbol{v}) \cdot \operatorname{curl}\ (\boldsymbol{w} \times \boldsymbol{v}) = 0, \quad \boldsymbol{w} \times \boldsymbol{v} \neq 0. \qquad (57)$$

Substituting (55) into (57), we have

$$(\boldsymbol{w} \times \boldsymbol{v}) \cdot \left(\frac{1}{4\pi\varrho} \operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) - \frac{\partial \boldsymbol{w}}{\partial t}\right) = 0. \qquad (58)$$

Then, the Lamb surface exists, if

$$\operatorname{curl}(\boldsymbol{w} \times \boldsymbol{v}) = 0 \text{ or} \begin{cases} \partial \boldsymbol{w}/\partial t = 0 \text{ (steady vorticity)}, \\ \text{and } \operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) = 0. \end{cases} \qquad (59)$$

Another Lamb surface which is everywhere normal to the vector $\boldsymbol{j} \times \boldsymbol{H}$ exists, if

$$(\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) \cdot \operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) = 0. \qquad (60)$$

From (55), it appears that if curl $(\boldsymbol{w} \times \boldsymbol{v}) = 0$, then curl $(\boldsymbol{j} \times \boldsymbol{H}) = 0$ in the case of steady vorticity. That is, two types of the Lamb surfaces co-exist in the fluid in this case.

Then, we can imagine the line which is the intersection of two Lamb surfaces. On this line, a curvilinear Bernoulli theorem can exist.

$$\frac{p}{\varrho} + \frac{1}{2} v^2 + V = \text{constant}. \qquad (61)$$

In this case, we can determine the velocity and magnetic fields by the following equations,

$$\operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{H} \times \boldsymbol{H}) = 0, \qquad (62)$$

$$\operatorname{curl}\ (\operatorname{curl}\ \boldsymbol{v} \times \boldsymbol{v}) = 0, \qquad (63)$$

and

$$0 = \operatorname{curl}\left(\frac{1}{4\pi\sigma} \operatorname{curl}\ \boldsymbol{H} - \boldsymbol{v} \times \boldsymbol{H}\right) \qquad (64)$$

As we are concerned with the steady state, we can expect $\boldsymbol{v}$ and $\boldsymbol{H}$ progressively reduced to the following two cases.

(*i*) (62), (63) and curl $(\boldsymbol{v} \times \boldsymbol{H}) = 0$ $(\sigma \rightarrow \infty)$, (65)

(*ii*) (62), (63) and (64) (σ is finite). (66)

For the first case, we obtain FERRARO's isorotation (1937), as the special case. In general, $\boldsymbol{v}$ becomes parallel to $\boldsymbol{H}$ and in such a case, there is no interaction between the velocity field and the magnetic field and further two Lamb surfaces are parallel to each other. If $\boldsymbol{v}$ is not parallel to $\boldsymbol{H}$, then, in general, $\partial \boldsymbol{H}/\partial t$ does appear, the components of which do not satisfy (62) and (63) and $\boldsymbol{v}$ and $\boldsymbol{H}$ fields vary with the progress of time in such a manner that $\boldsymbol{v}$ and $\boldsymbol{H}$ become parallel.

As the concept of infinite conductivity comes mainly from very large linear dimension in cosmic field, some diffuse nebulae may be expected to reach finally this state. The equations (62) and (63) have the same form with (1) and we can expect that some filamentary nebulae have the helicoidal structure in their fine structure. SHAJN (1956) suggested the close relationships between the shape of these nebulae and the magnetic fields. SEVERRNY (1956) pointed out that the knots of eruptive prominences are moving along spirals as if it were a motion of an isolated charge. The latter type field will be studied in a separate paper.

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