Experiments on the Effect of Inhomogeneity and Obliquity of a Magnetic Field in Inhibiting Convection

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Abstract

Two experiments are described on the inhibition of convection in a magnetic field. In the first experiment a layer of mercury is placed in an inhomogeneous magnetic field and heated from below. Photographs show convection where the field is weak, but indicate no motion where the field is strong. The boundary between these two regions corresponds to a critical magnetic field in agreement with current theory. The boundary is well defined showing that the onset of convection is sensitive to variations in the magnetic field. In the second experiment with a homogeneous field, varied in direction from vertical to horizontal, it is the vertical component which controls the onset of motion. A strong horizontal field does not inhibit convection but causes motion in narrow cells elongated in the direction of the field.

Introduction

The theory of the onset of convection in a layer of an imcompressible fluid subject to thermal instability was originally treated by RAYLEIGH (1916) in the purely hydrodynamic situation and by THOMPSON (1951) and CHANDRASEKHAR (1952) when the fluid is electrically conducting and is placed in a homogeneous magnetic field. Papers which deal with experimental aspects of the magnetohydrodynamic problem are those of NAKAGAWA (1955), JIRLOW (1956) and LEVENGOOD (1956). Further references are given by CHANDRASEKHAR (1952) and by NAKAGAWA and FRENZEN (1955).

The theory shows that a layer of fluid heated from below first becomes unstable when the Rayleigh number, R, exceeds a certain critical value, R_0.

$$R = \frac{g\alpha\beta d^4}{\varkappa\nu}, \qquad (1)$$

where

d is the depth of the fluid layer
g, acceleration due to gravity
α, volume coefficient of thermal expansion
β, initial temperature gradient (directed downward to produce instability)
$\varkappa$, thermometric conductivity (the ratio of thermal conductivity to the product of the specific heat and the density)
ν, kinematic viscosity (ratio of coefficient of viscosity to density).

Motions with a cellular pattern depending on the boundary conditions ensue. If the fluid is electrically conducting and a magnetic field is introduced, electromagnetic forces will oppose these motions; R_0 will be increased to R_c. The ratio of R_c to R_0 has been given by CHANDRASEKHAR (1952) in terms of a parameter,

$$Q = \frac{B_z^2 \sigma d^2}{\varrho\nu}, \qquad (2)$$

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where

B_z is the vertical component of the magnetic field
σ, the electrical conductivity
ϱ, the density.

To appreciate the physical significance of the rigorous theory consider roughly a horizontal layer of fluid at rest, which is heated from below so that a uniform vertical temperature gradient, β, exists between its hot lower surface and its cold upper surface. Let small disturbing cellular motions be introduced. Rising currents carry the fluid to regions of greater temperature and less density. The reverse is true for those parts of the fluid with downward components of velocity. The temperature at any point is deviated by an amount $\triangle T$ from its initial value. This results in a gravitational driving force per unit volume, tending to maintain the cellular motion which is given by

$$F_g = \varrho g \alpha \triangle T. \tag{3}$$

This driving force is opposed by a viscous force given roughly by

$$F = \text{const } \nu \varrho v / d^2, \tag{4}$$

where v is a velocity characteristic of the cellular motion. Finally, the temperature deviation, $\triangle T$, which controls the driving force is related to this velocity by the usual equation for heat conduction in a steady state, namely

$$(\mathbf{v} \cdot \nabla) T = \varkappa \nabla^2 T. \tag{5}$$

For small disturbances this equation reduces to

$$v \beta = \text{const } \varkappa \triangle T / d^2 \tag{6}$$

as far as orders of magnitude are concerned.

Convection starts when the driving force just balances the viscous force. From equations (3), (4), (6) and (1), this condition reduces to $R = \text{const} = R_0$. For small velocities the relation between v and $\triangle T$ [eq. (6)] is linear. However, since the temperature deviation cannot exceed the total temperature range across the fluid layer, this relation cannot hold indefinitely. Thus, if for small cellular velocities the driving force exceeds the viscous force, a limiting convective velocity eventually will be reached.

Now suppose that the fluid is without viscosity, but has a finite electrical conductivity, σ. In the presence of a magnetic field, $\mathbf{B}$, the motion generates an electromotive force proportional to $\mathbf{v} \times \mathbf{B}$, which in turn gives rise to electric currents and an electrodynamic force per volume of the order

$$F_\sigma = \sigma B^2 v. \tag{7}$$

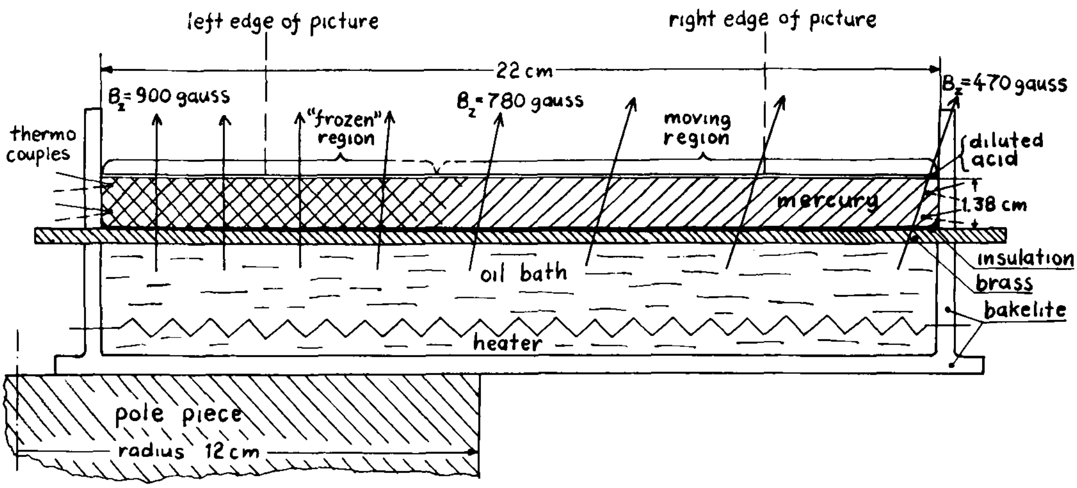

Fig. 1. Experimental arrangement used for investigation of thermal convection of mercury in an inhomogeneous magnetic field. Figure shows a cylindrical vessel composed of a lower part with a heat bath containing oil and an upper part containing a layer of mercury. Convective motion is observed at the upper surface of the mercury, which is kept clean by a thin layer of diluted acid. The directions of the magnetic field lines are indicated and the field strengths correspond to the situation of fig. 2 d. The temperature gradient is measured with thermo couples.

If magnetic and velocity fields are stationary, this force may be interpreted as a "viscous" force due to Joulean dissipation. Again equate the driving force to this "viscous" force as a condition for the onset of convection, and obtain from equations (3), (6), (7), (1), and (2)

$$\frac{\varrho g \alpha \beta d^2}{\varkappa \sigma B_z^2} = \text{const} = R/Q. \qquad (8)$$

It should be emphasized that the vertical component B_z and not the total field B has been introduced in formula (8). The horizontal component is not effective. With a purely horizontal field the convection takes place in narrow cells elongated in the direction of that field with no Joulean losses.

In general both viscous and Joulean dissipation will oppose the driving force and the onset of convection will be expressed in terms of R and Q as shown by the rigorous theory.

1. Experimental procedure

Two experiments were performed on thermal convection in a horizontal mercury layer, one with an inhomogeneous magnetic field and one with a homogeneous field forming oblique angles with the vertical direction.

Fig. 1 shows the experimental details in the first experiment. 80 watts were applied to the oil bath, which was separated from the mercury layer by a brass plate for eliminating horizontal temperature differences. Contact between mercury and all metals was prevented by a thin coating of insulating paint.

The mercury surface must be scrupulously clean in order to show convective motions. The mercury was passed through a bath of potassium hydroxide (15 %) and then through a bath of nitric acid (30 %) before each run. The mercury was always kept covered by a very thin layer of extremely dilute nitric acid which as far as could be observed had no effect on the motions of the mercury surface beneath it. Even with these precautions the surface became contaminated after an hour or so.

The temperature gradient in the mercury layer, which was 1.38 cm in depth, was measured by three pairs of copper-constantan thermocouples placed around the periphery of mercury layer. The vertical distance between the elements of a pair was 0.5 cm. The pairs were located where the applied magnetic field would have a maximum, a mean and a minimum value. Each couple had an independent circuit to avoid any possibility of short circuiting by the mercury. Temperatures were measured relative to an external constant temperature bath. The value of the temperature gradient was checked independently by the analysis of heating and cooling curves with the apparatus under similar conditions. Even so, the temperature gradient is by far the most uncertain of the experimental data.

When the mercury layer had reached a temperature of about 45° C and convective motion was clearly in evidence, magnetic fields of varying strength were applied. After the mass motion resulting from the change of the field had died down, a picture (exposure 5 sec) was taken. The method of investigating motion of the upper mercury surface was that devised by LEHNERT (1955). Clean grains of sand are sprinkled on the surface and illuminated from the side.

The pictures obtained by observing the surface on an inclined mirror show a region 13 × 9 cm in the experiment with an inhomogeneous field (Fig. 2). The cylindrical dish containing the mercury layer was placed between the pole pieces of a large electromagnet, but extended well out beyond edges of these poles into the region where the field became inhomogeneous. The central region of the pictures, as shown by Fig. 2h, comes where the field gradient was steepest.

The purpose of the second experiment was to study the effect of the direction of a homogeneous magnetic field. The dish of mercury was placed midway between the pole pieces and the magnet rotated about it. For varying values of this homogeneous field, the angle with the vertical at which the onset of convection starts was estimated. Fig. 3 was taken with a horizontal field of 4500 gauss with an exposure time of 5 seconds.

2. Experimental results

a. *Inhomogeneous magnetic field*

The pictures which map the velocity field as seen from above indicate from the length of the traces convective velocities from 0.5 to 2 mm/sec. Fig. 2a shows the convective pattern with no magnetic field and Fig. 2g

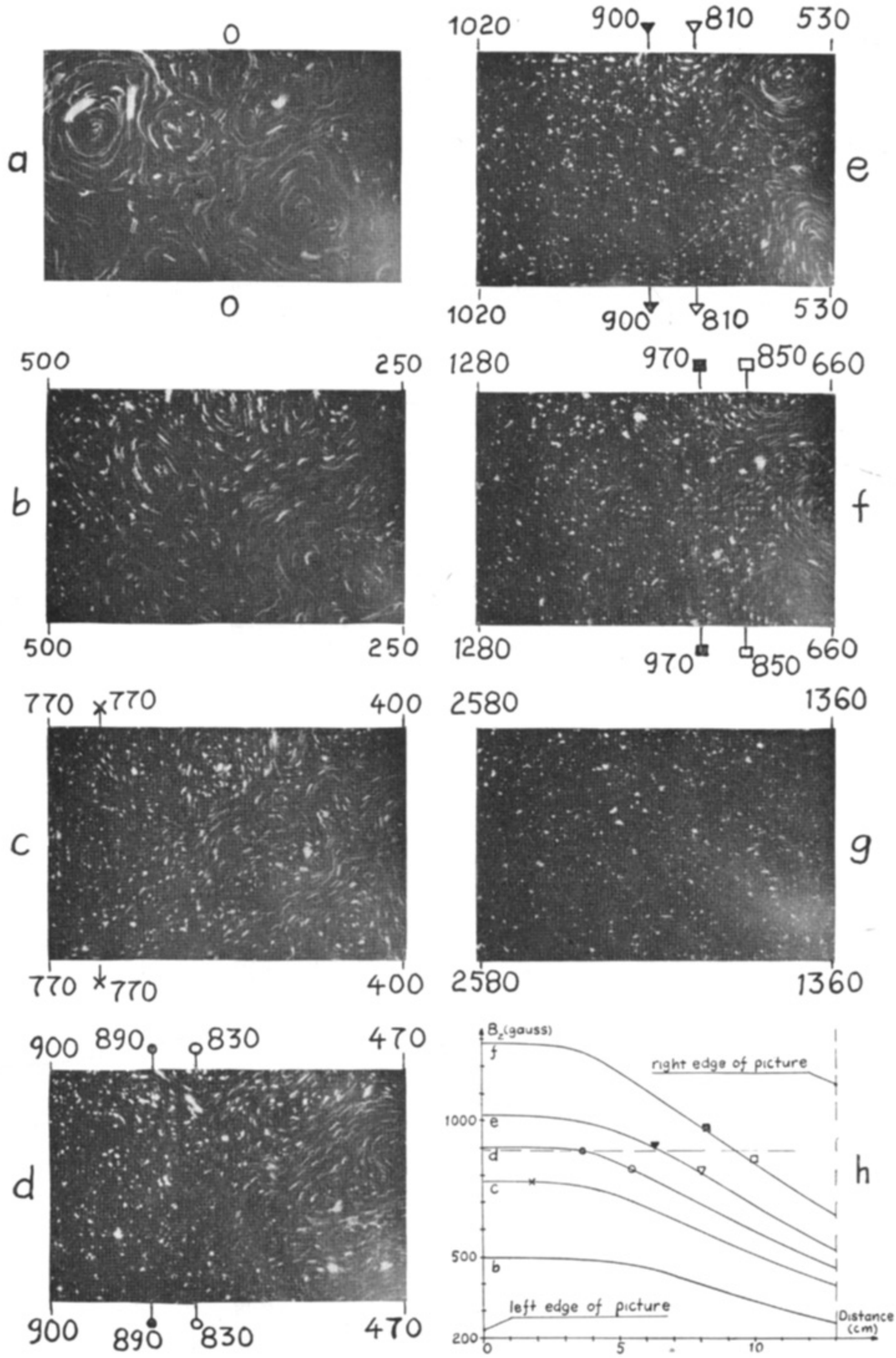

Fig. 2. Inhibition of thermal convection in mercury by an inhomogeneous magnetic field. Figs. a—g show mercury surface as seen from above. Lines of constant vertical magnetic field strength run from the top to the bottom of the pictures. Fig. h gives the distribution of the vertical field component B_z over pictures b—f from the left hand edge to the right hand edge. Fig. a is taken without magnetic field and data near the corners of figs. b—g indicate the range of field strengths in gauss. Solid marks in figs. c—h indicate lines to the left of which there is certainly no motion and open marks indicate lines to the right of which motion certainly exists. The broken horizontal line shows the estimated mean field strength 900 gauss for the onset of convection.

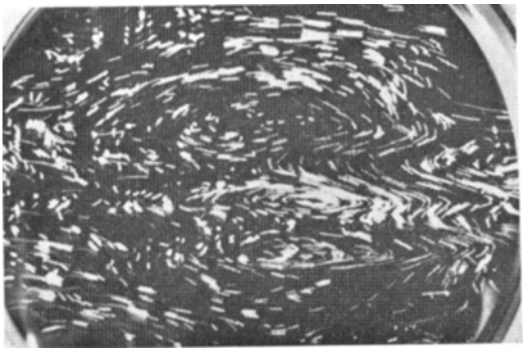

Fig. 3. Cellular convection in a layer of mercury in a magnetic field parallel to the free surface of the layer. The picture shows the surface as seen from above. The magnetic field (B = 4,500 gauss) runs from the right to the left and the cells are seen to be elongated in the field direction and to extend across the whole vessel. The flow pattern was observed to consist of alternating flow directions.

with a field strong enough to inhibit all convective motion or to "freeze" the surface. In Fig. 2b, although the maximum field is 500 gauss, there is no evidence of the inhibition of convection. The remaining pictures, Fig. 2c to f incl., clearly show the inhibiting effect of the field. Recall that the lines of constant field strength run from top to bottom of the pictures. This series of pictures was taken in quick succession keeping the attendant conditions as nearly constant as possible. As the magnetic field is increased the border line between stability and motion moves to the right. Estimates of the positions of a line to the left of which there is no motion (solid marks) and of a line to the right of which there is clearly motion (open marks) are recorded on the edge of the photographs together with the corresponding field strengths. These transition points are again indicated on the field strength curves of Fig. 2h. It may be seen that convection sets in within rather close limits for the magnetic field around a mean value of about 900 gauss.

In order to test quantitatively the theory, experimental values may be substituted in eq. (2) to find the parameter Q. Thus, in MKS rationalized units, with $d=0.0138$ meters, $B_z=0.090$ volt-sec./meter^2 (900 gauss), $\sigma=1.04\times10^6$ mho/meter, $\varrho=13550$ kilogram/meter^2, and $\nu=1.08\times10^{-7}$ meters^2/sec., Q becomes 1100. From Chandrasekhar's values for a fluid layer with one surface free and the other fixed the corresponding critical value R_c is 6000. With this value of R_c, $g=9.8$ meters/sec.^{2}, $\alpha=1.81\times10^{-4}$ per degree and $\varkappa=4\times10^{-6}$ meters^2/sec., eq. (1) yields a computed value of the temperature gradient of 40 degrees per meter. The range of experimental values was between 40 and 60 degrees per meter.

As seen from Figs. 1 and 2h, the difference between the total magnetic field and its vertical component is less than the range of values over which convection was estimated to have been inhibited. Therefore, from the results of this experiment one cannot say whether it is the total field or its vertical component (as used in eq. 2) which controls the onset of convection.

Appropriately, three regions of different states of motion might have been expected to have been found on the photographs. Following the magnetic field from high to low strengths, first might have come a "frozen" region with no motion, second a region of regular cellular convection and third a region of irregular non-stationary convection (turbulence). Only the first and third were observed. The absence of the second may be due to the fact that thermal conditions had not reached a steady state or that the cell size was too large to be uninfluenced by boundary conditions. Further, it should be remembered that the theory was developed for a homogeneous field. It might validly be applied to the inhomogeneous situation only if the size of the cells was small compared with the distance in which there was an appreciable change of field. There can be little doubt that the onset of convection alters the pattern of heat flow. With one region "frozen" and another adjacent to it in convective motion heat must flow across the boundary between them and indeed might alter the temperature of the bath below the mercury.

b. *Homogeneous, oblique magnetic field*

In the second experiment convection was studied in a homogeneous magnetic field forming oblique angles Θ with the vertical direction. The onset was estimated by visual observation of the free surface with an accuracy of about 10 % in the magnetic field strength. The estimated critical values of the total field B and the vertical component $B_z=B\cos\Theta$ are given in Table 1 for a range of different angles

Table 1. Critical values of the total magnetic field strength B and the vertical component $B_z = B \cos \Theta$ in a layer of mercury. The field forms the angle Θ with the normal direction to the free surface of the layer

Θ degrees	B gauss	B_z gauss	
0	936	936	mean: 799 ± 60
47	1080	734	
57	1440	785	
67	2160	844	
76	2880	696	
90	4500	no inhibition; see Fig. 3	

Θ. The computed value of B_z using experimental data, $d = 0.0130$ meters and a temperature gradient of 40° per meter, is 800 gauss. This agrees with the critical value given in Table 1. A picture was taken (exposure 5 sec.) with the field parallel to the free surface ($\Theta = 90°$) as shown by Fig. 3. Even with a field as strong as $B = 4500$ gauss, i.e., about five times the strength of a vertical field necessary to inhibit convection under the same attendant conditions, no inhibition could be observed. Convection took place in the form of elongated cells extended across the vessel and with stream lines running mainly parallel with the magnetic field. No measurement was made of the motion under the mercury surface. The presence of horizontal temperature gradients cannot be excluded.

The result is not fully understood at present. The dissipation for a convective pattern of cells which are "short" in the magnetic field direction is greater than that for a pattern of cells elongated in the field direction. But it is not obvious that this fact could be used as an argument for the observed effect, because one has to take into consideration the variation of the driving force with the cell shape. Further, one might as well have expected a motion in the form of long "rolls" rotating around an axis parallel with the magnetic field. As far as could be observed, no such motion was present in the experiment.

3. Conclusions

(1) The onset of convection in a horizontal layer of fluid heated from below with constant power input depends upon a critical field strength regardless of the distribution of the inhomogeneities of that field.

(2) The value of this critical field agrees with that calculated from current theory within the limits of experimental error.

(3) The boundary between moving and immovable regions is sharply defined and determined with an accuracy corresponding to a variation in field strength of about 10 percent.

(4) It is the vertical component of the magnetic field which is critical in limiting the onset of convection. A strong horizontal field does not inhibit convection.

(5) With a horizontal field the convection is limited to narrow horizontal cells elongated in the direction of the field. The motion observed was essentially along the axis and there was no evidence of motion around the axis of a cell.

At first sight the structure of sunspots might be connected with the influence of the magnetic field on convection as described in these experiments. The umbra, penumbra and the photosphere surrounding the spot would correspond to regions of thermal conduction, regular convection and turbulent motion, respectively. The needle-like structure of the granules in the penumbra would suggest a type of convective cells elongated in the directions of the diverging magnetic field of a sunspot. However, CHANDRASEKHAR (1952) has pointed out that the onset of convection is essentially different under terrestrial and astrophysical conditions. In the laboratory $1/\mu\sigma >> \varkappa$. Instability may give rise to convective motions sensitive to the magnetic field strength as shown in this experiment. In the photosphere where sunspots are seen, or in the interior of the sun under presently accepted models, $1/\mu\sigma << \varkappa$. Under this condition, instability can occur either as cellular convection or as an over-stability in the form of oscillations of increasing amplitude caused by magnetohydrodynamic waves. This latter form is insensitive to the magnetic field. Thus, the application of the results of these experiments to sunspots is of doubtful validity. However, a definite answer to the question of their applicability cannot yet be given.

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