Cosmic Rays in the Terrestrial Magnetic Dipole Field

By E.-Å. BRUNBERG, The Royal Institute of Technology, Stockholm

(Manuscript received November 5, 1955)

Abstract

The results of scale model experiments with electron orbits in a magnetic dipole field are given in the form of diagrams on big globes. By using these globes it is possible to find the asymptotic directions of cosmic ray particle orbits as a function of momentum and direction of incidence at a cosmic ray recording station. Globes, representing 16 stations are depicted.

When studying cosmic ray intensity variations it is of fundamental importance to know the deflection of cosmic rays in the earth's magnetic field. The deflection of particles in a magnetic dipole field has been studied mathematically by STÖRMER (1904—1937), LEMAÎTRE and VALLARTA (1936) and many others. Scale model experiments have been performed at the Royal Institute of Technology in Stockholm by MALMFORS (1945) and BRUNBERG and DATTNER (1953). These experiments give a solution to the problem with an accuracy good enough for cosmic ray purposes.

The experiment was carried out in a vacuum chamber where the deflection of an electron beam in a magnetic dipole field could be measured. The parameters were chosen in such a way that the electron orbits in the model corresponded to primary particles with momenta from 10 GeV/c to infinity.

The final direction of an orbit far away from the terrella or as we say, the asymptotic direction, may be represented by a unit vector. The direction of this vector is defined in the geomagnetic system by two angles, Φ_N, the north latitude angle and Ψ_E, a longitude angle, reckoned eastwards from the meridional plane through the point where the orbit starts on the terrella (Fig. 1).

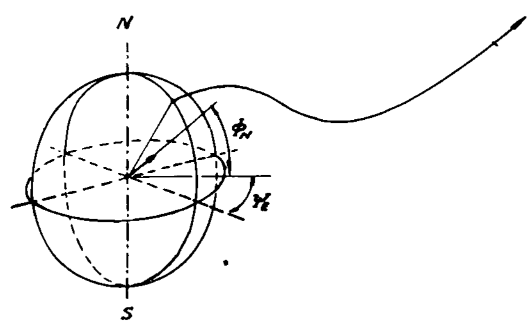

Fig. 1. The asymptotic direction given by the angles Ψ_E and Φ_N.

The asymptotic direction has been determined as a function of momentum, initial direction (i. e. particles are shot out with different zenith- and azimuthangles on the terrella) and the latitude and longitude of the starting point. The best method to visualize the results of measurements is to plot the asymptotic directions on big globes (Fig. 4—32). The momentum range is 10 GeV/c $\rightarrow \infty$, but a few orbits are also plotted in the range 4—10 GeV/c. The lower values are calculated from the measurements of MALMFORS (1945).

Assume that particles are shot out in different directions from a certain point on the terrella, for example a point with the same latitude and longitude as Stockholm. Each curve on the globe is then a locus of the afore-mentioned vector, which gives the asymptotic direction for different particle momenta. A certain, initial direction on the terrella has a corresponding curve on the globe.

As an example, take the curve marked zenith. This curve gives the asymptotic directions for particles which initially move in the zenith direction from the point Stockholm. The asymptotic direction for infinitely large particle momenta is of course the same as the initial direction but with decreasing momentum there is a deflection, which can be read off by means of the curve. (The numbers indicate momenta of values, 4, 5, 6 ... 10, 12 ... 30, 35, 40, 45 ... 90, 100 GeV/c). Curves marked 16° N, 32° N, 48° N give in the same way the asymptotic directions for particles starting in the 16° N, 32° N and 48° N directions and the same applies to curves marked E, W and S. For particles whose original directions do not coinside with N, E, S or W, asymptotic directions can be interpolated by means of the *point curves*, which have been drawn on most globes. Each of these curves is a locus of the asymptotic direction for different azimuth directions. The zenith angle (16°, 32° or 48°) and momentum (20 GeV/c) are constant. On the Ahmedabad globe the momentum 25 GeV/c has been chosen.

The *point-dash* curves are the loci of the asymptotic directions for different zenith angles. The azimuth direction (N, E, S or W) and momentum are here constant.

We must remember that the errors in the interpolated curves may be greater than in the curves of the four main directions.

It is necessary to point out that the measurements were made in the earth's geomagnetic system and all directions have been converted into the geographic system. This is the reason why directions, which represent infinitely large momenta on the curves 16° S, 32° S and 48° S are not on the geographic, meridional circle through the starting point, but on a geomagnetic meridional circle. The formulae used for converting the angles Ψ_E and Φ_N of the geomagnetic system to Ψ and Φ of the geographic system are:

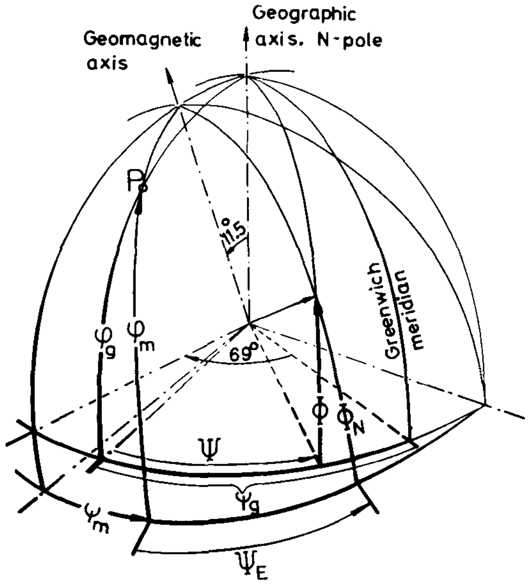

Fig. 2. Conversion of the angles Ψ_E and Φ_N of the geomagnetic system into Ψ and Φ of the geographic system.

For a station on or near the geomagnetic equator the east and west curves coincide with the zenith curve. In this case curves have been drawn alongside the zenith curve on the globe and marked with momenta for different zenith angles in the east- and west directions. The appropriate direction is however read on the zenith curve.

$$\sin \Phi = \sin \Phi_N \cdot \cos 11^\circ.5 - \cos \Phi_N \cdot \cos (\Psi_E + \psi_m) \cdot \sin 11^\circ.5$$

$$\sin (\Psi + \psi_g + 69^\circ) = \frac{\cos \Phi_N}{\cos \Phi} \cdot \sin (\Psi_E + \psi_m)$$

where ψ_m is the geomagnetic and ψ_g the geographic latitude of the cosmic ray intensity recording station (Fig. 2).

Consequently we must remember:

I. The figures 16° S, 16° N, 16° E etc., which tell us the direction of an orbit at the point Stockholm, are all valid in the geomagnetic system.

II. Orbits marked 16° N, 32° N and 48° N start in directions pointing towards the geomagnetic south pole.

III. All asymptotic directions are given in the earth's geographic system by longitude angles Ψ and latitude angles Φ (corresponding to Ψ_E and Φ_N in the geomagnetic system).

So far we have looked at particles starting from a point on the terrella within an infinitely small solid angle. But if we want to know the

Table I

Station	Geographic Latitude φ_g	Geographic Longitude ψ_g	Geomagnetic Latitude φ_m	Geomagnetic Longitude ψ_m
Ahmedabad	23°N	73°E	14°	144°
Cheltenham	39°N	77°W	50°	— 9°
Christchurch	44°S	173°E	— 48°	253°
Climax	39°N	106°W	48°	— 44°
Friedrichshafen[1]	48°N	10°E	49°	89°
Godhavn	69°N	54°W	80°	35°
Hobart	43°S	147°E	— 51°	224°
Huancayo	12°S	75°W	— 1°	— 6°
Kampala	0°	33°E	— 2°	102°
Kiruna	68°N	20°E	65°	116°
Kodaikanal	10°N	78°E	1°	147°
London	51°N	0°	54°	82°
Nagoya	35°N	137°E	25°	203°
Rom	42°N	12°E	42°	92°
Stockholm	59°N	18°E	58°	107°
Svalbard	78°N	15°E	74°	131°

[1] For Stuttgart, Freiburg and Innsbruck the same globe may be used.

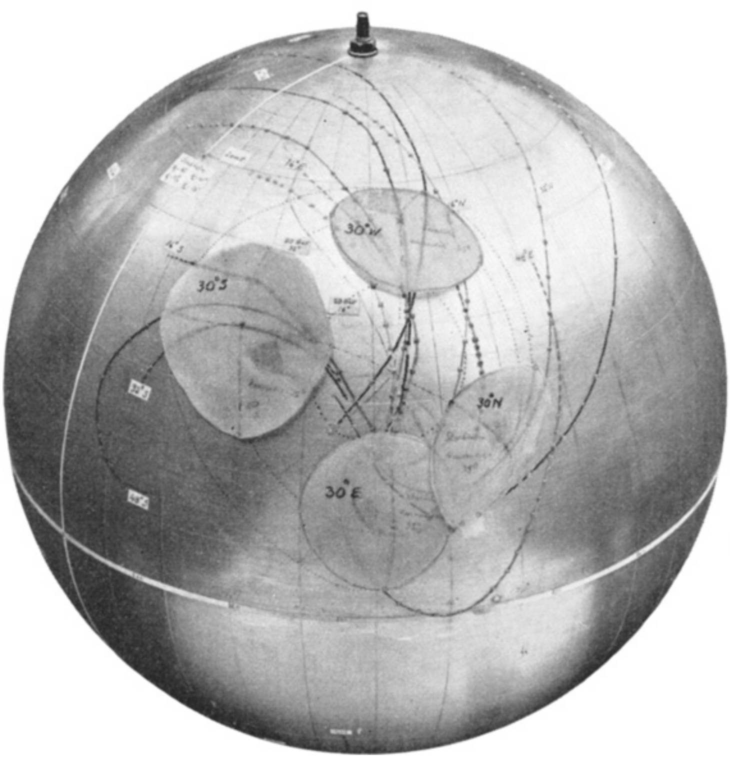

Fig. 3. The Stockholm-globe.

asymptotic directions of all particles starting within a finite solid angle we have to intergrate a great number of small solid angles inside the big one. We then get a surface on the globe which represents all the possible directions for the given solid angle within which the particle started and for a given momentum. The boundary of this surface is in most cases the boundary of the definite solid angle at the starting point, which has been projected on the surface of the globe by means of the particle orbits.

The preceding discussion applies to particles which start from a point on the terrella, but the same argument can be used in the cosmic ray case. The asymptotic directions of those particles we measure with our G—M telescopes and ionization chambers are easily read off on the globe if the primary particle momentum is known.

The exact appearance of that part of the primary spectrum, which causes the diurnal variation is still unknown. But using the results from two-directional measurements with G—M telescopes it can be shown that the effective center of gravity of this spectrum is around 20 GeV/c, in any case at latitudes above 50° degrees and for zenith angles less than 30°—40°, BRUNBERG and DATTNER (1954). Thus 20 GeV/c will be used as an average momentum, when calculating the deflection in the earth's magnetic field.

As an example the Stockholm-globe is shown with four different telescopes. Each telescope is supposed to measure within a cone with a full apex angle of 35° (Fig. 3).

The telescopes are tilted 30° from the vertical and measure in N, S, E and W directions. The solid angle covered by a telescope is projected on the surface of the globe and represented by pieces of paper, stuck to the globe.

From this picture a phase difference of about 2.6 hours can be expected between cosmic ray intensity variations measured with the north and south pointing telescope. It is also evident that the north pointing telescope precedes the south pointing telescope. The east and west pointing telescope on the other hand will give approximately the same phase, but they scan different parts of space. At present 16 globes representing 16 different stations around the earth have been completed, able 1. Drawings of the globes are given in fig. 4—32.

I should like to thank Mrs N. Sedlacek for calculating and drawing the curves and Mr A. Spångberg for photographic work. A grant from Knut and Alice Wallenberg Foundation has made the work possible.

REFERENCES

BRUNBERG, E Å., and DATTNER, A., 1953: Experimental Determination of Electron Orbits in the Field of a Magnetic Dipole. *Tellus*, **5**, 2.

BRUNBERG, E. Å., and DATTNER, A., 1954: On the Interpretation of the Diurnal Variation of Cosmic Rays. *Tellus* **6**, 1.

LEMAÎTRE, G., and VALLARTA, M. S., 1936: On the Geomagnetic Analysis of Cosmic Radiation. *Phys. Rev.* 49, p. 719

MALMFORS, K. G., 1945: Determination of Orbits in the Field of a Magnetic Dipole with Application to the Theory of the Diurnal Variation of Cosmic Radiation. *Arkiv för Matematik, Astronomi och Fysik* **32** A, No. 8.

STÖRMER, C., 1904: *Sur le mouvement d'un point material portant une charge d'électricité sous l'action d'un aimant élémentaire.* Videnskabs-selskabet i Christiania.

— 1934—1937: On the Trajectories of Electric Particles in the Field of a Magnetic Dipole with Applications to the Theory of Cosmic Radiation. *Astrophysica Norvegica, Oslo*, Vol. I, No. 1, June 1934; Vol. I, No. 4, April 1935; Vol. II, No. 1, June 1936; Vol. II, No. 4, Jan. 1937.

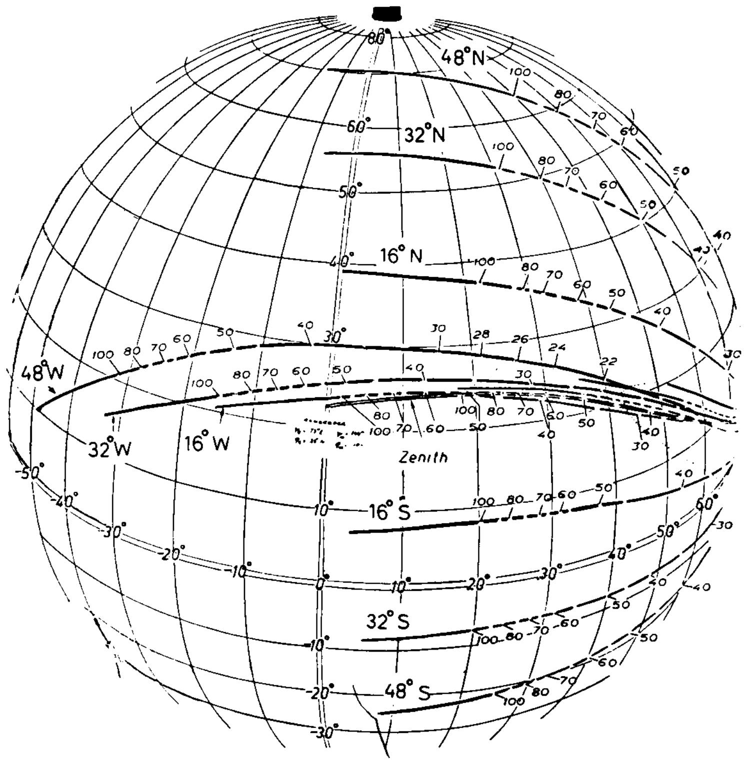

Fig. 4. Ahmedabad

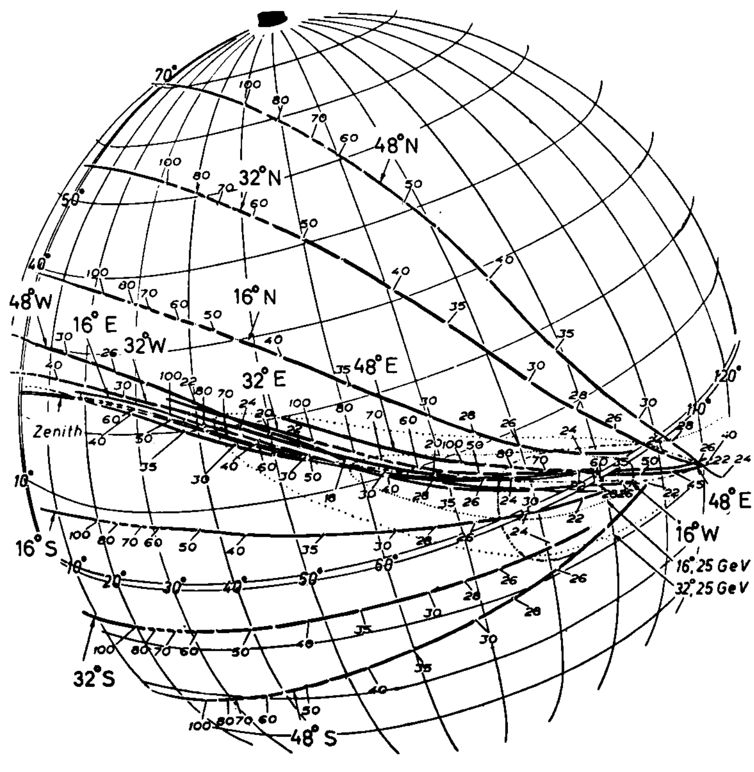

Fig. 5. Ahmedabad

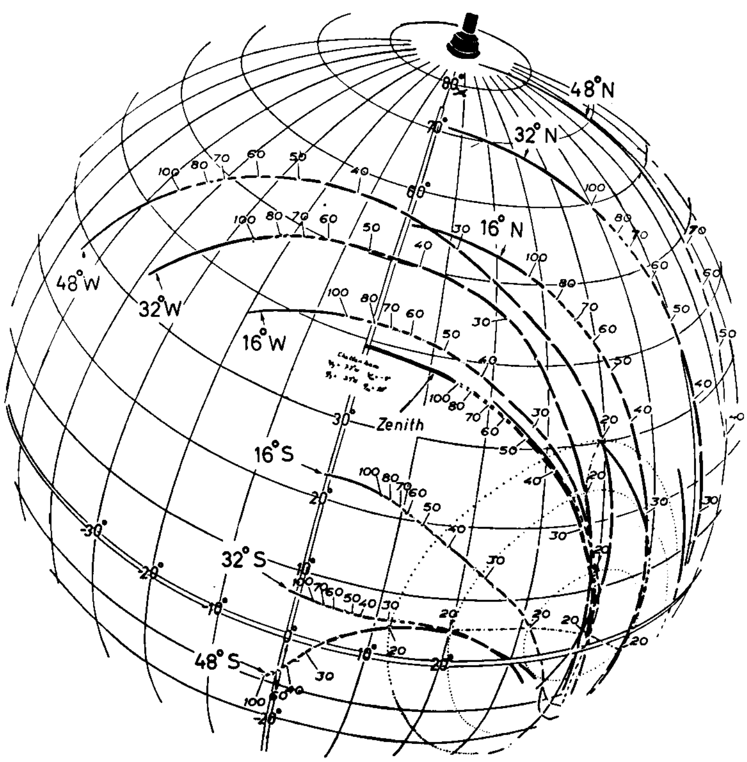

Fig. 6. Cheltenham.

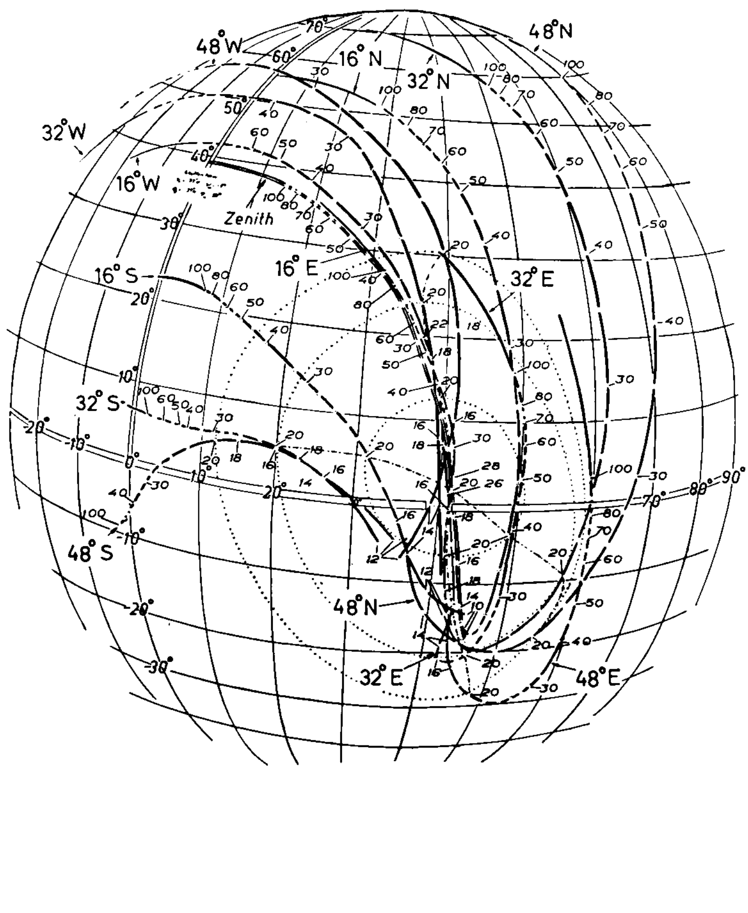

Fig. 7. Cheltenham.

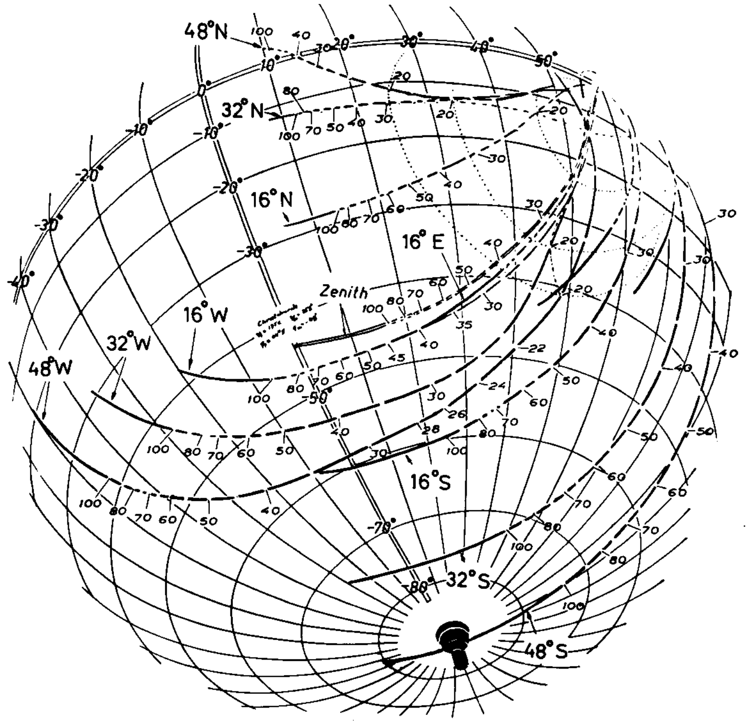

Fig. 8. Christchurch.

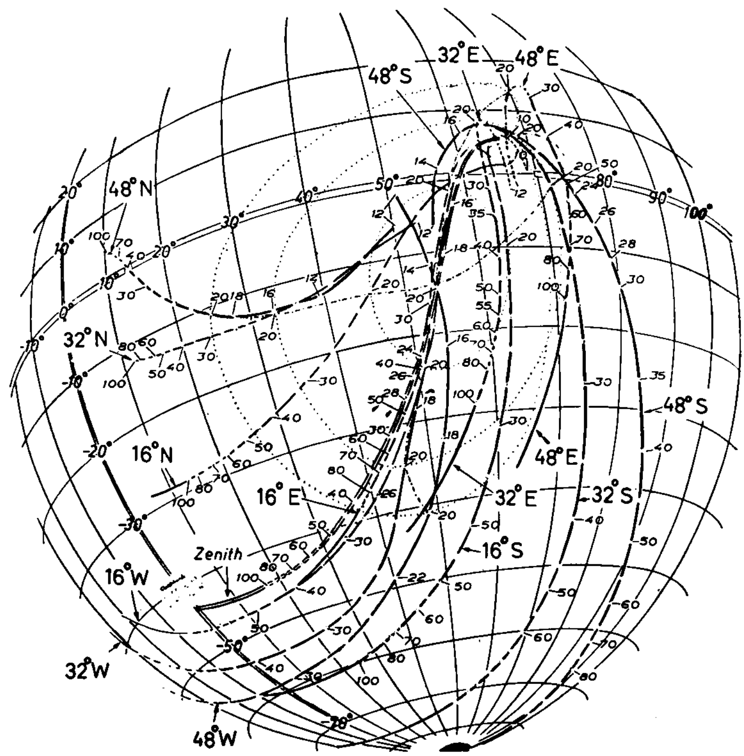

Fig. 9. Christchurch.

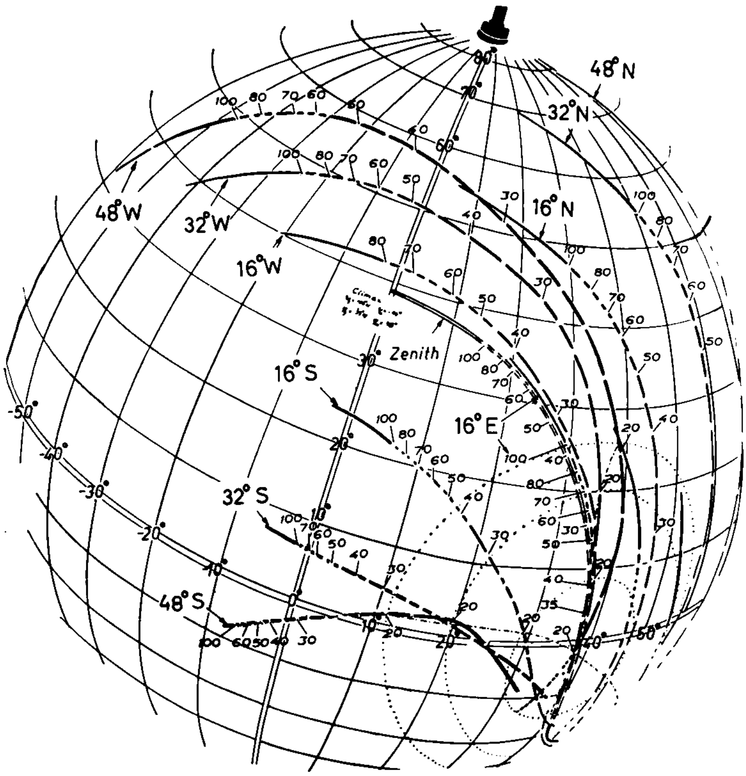

Fig. 10. Climax.

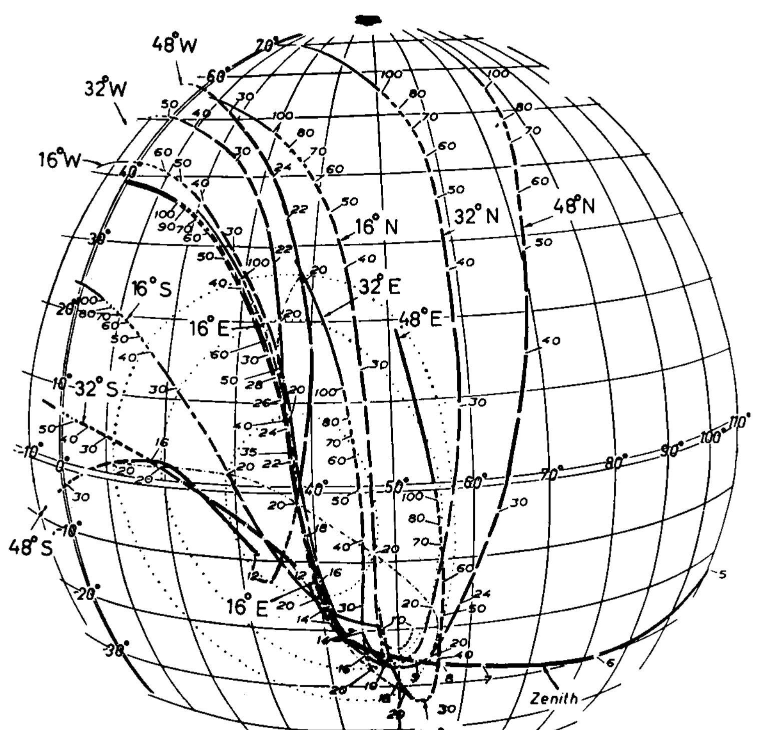

Fig. 11. Climax.

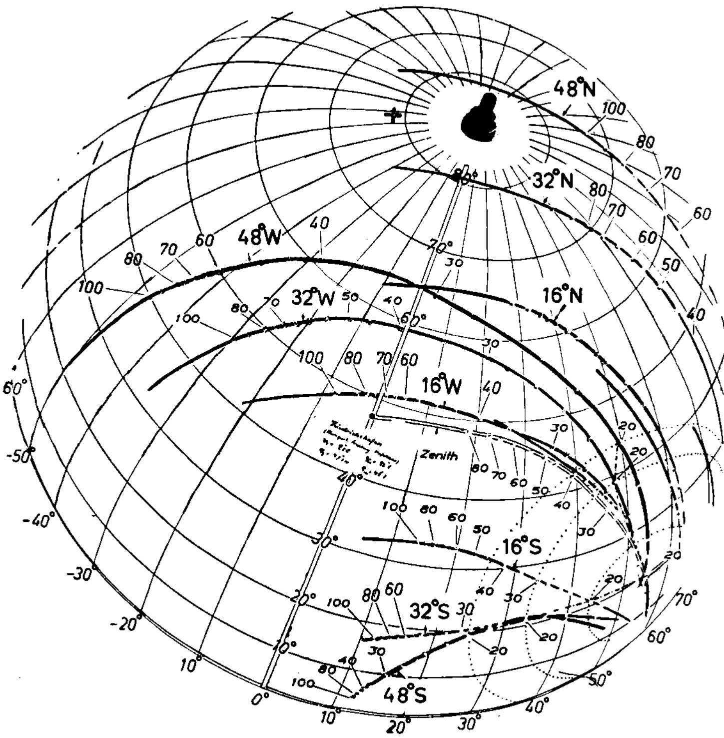

Fig. 12. Friedrichshafen.

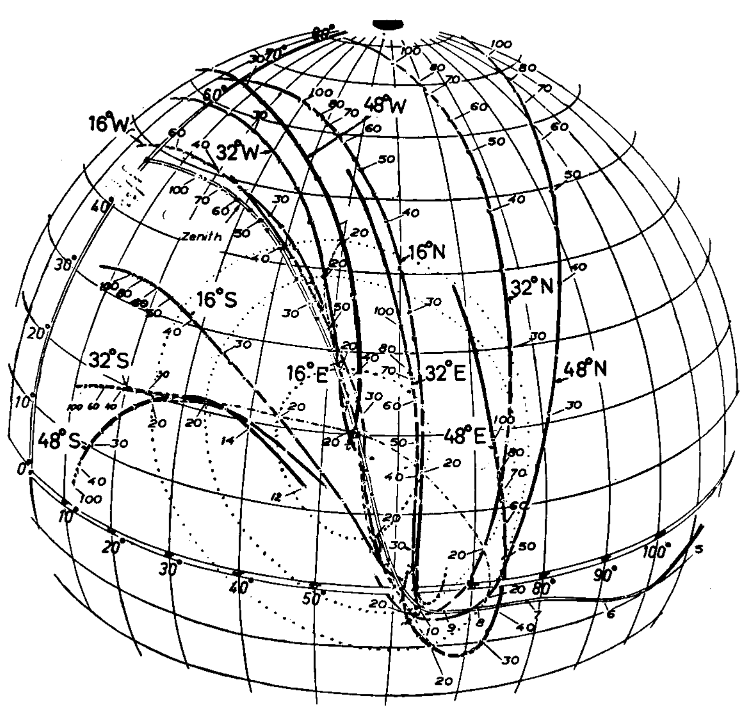

Fig. 13. Friedrichshafen.

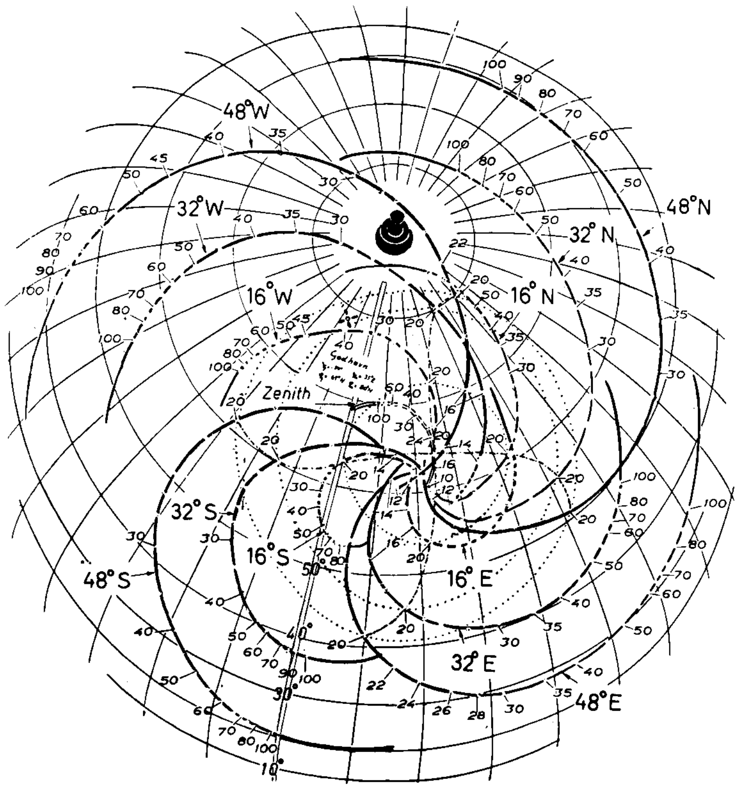

Fig. 14. Godhavn.

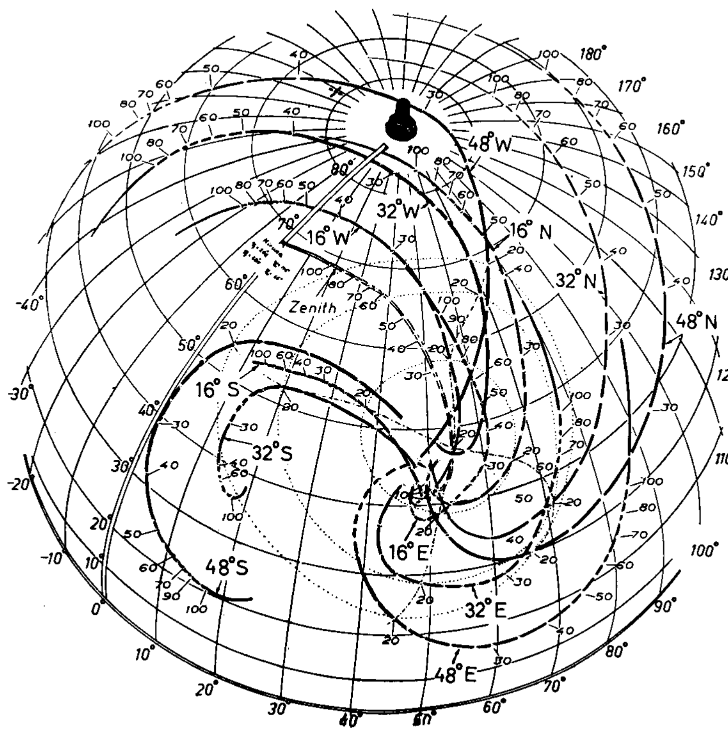

Fig. 15. Kiruna.

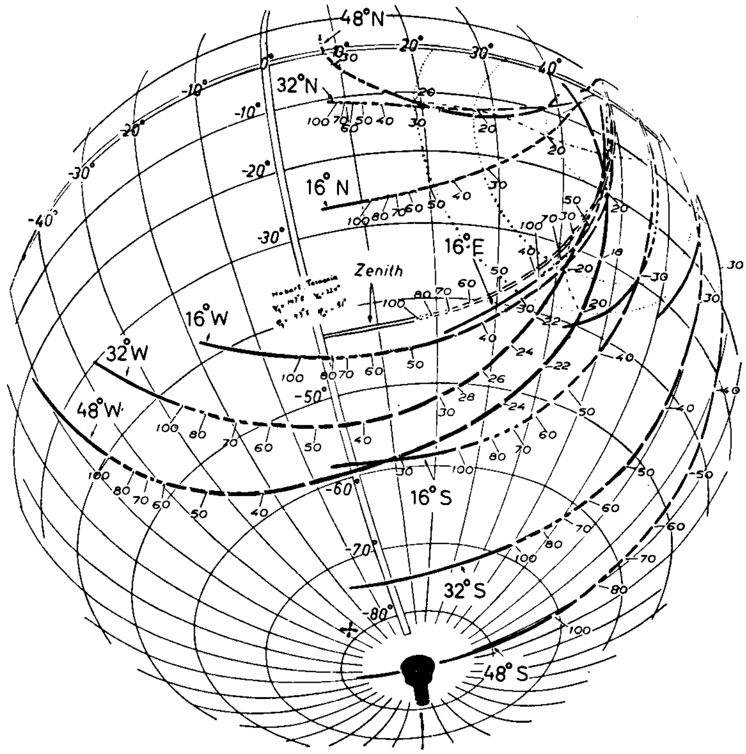

Fig. 16. Hobart.

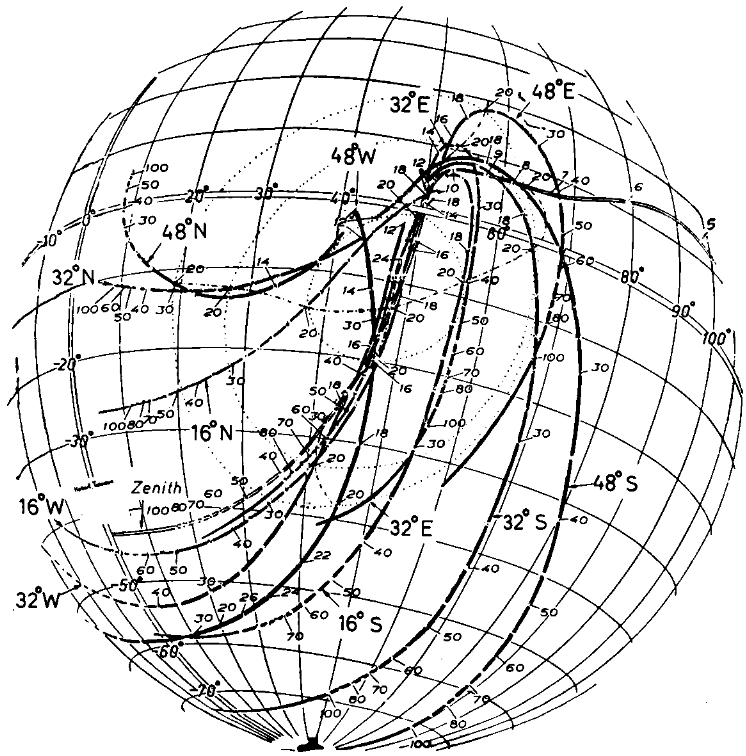

Fig. 17. Hobart.

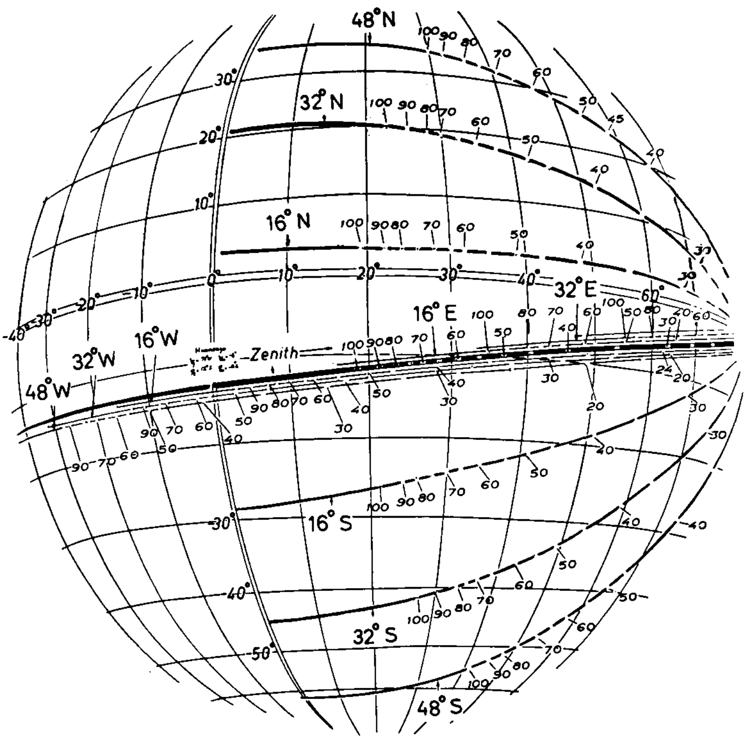

Fig. 18. Huancayo.

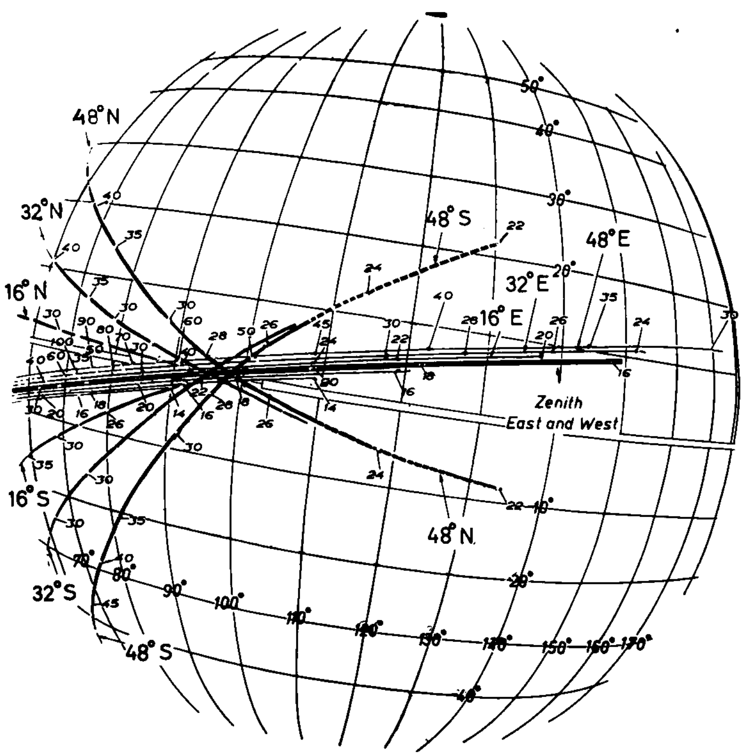

Fig. 19. Huancayo.

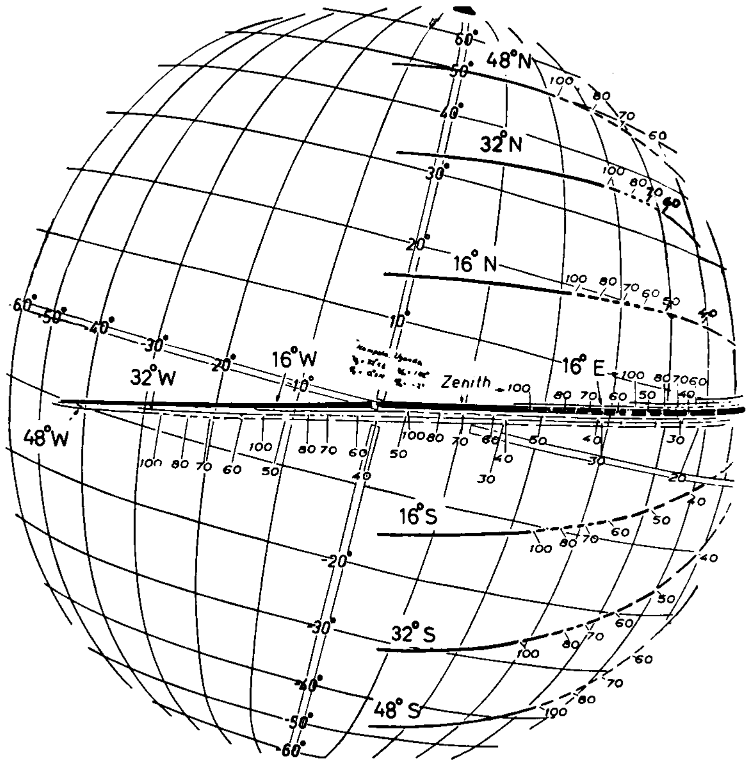

Fig. 20. Kampala, Uganda.

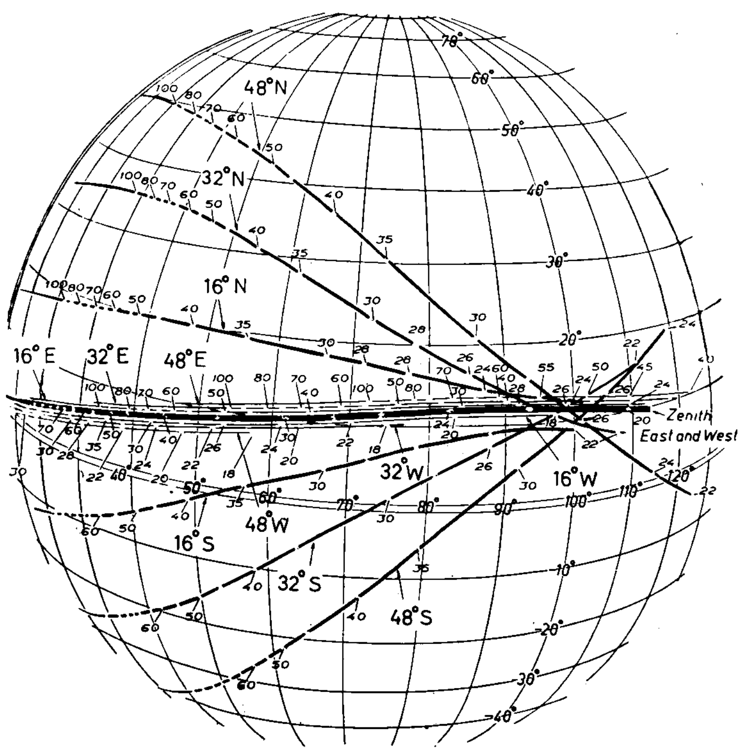

Fig. 21. Kampala, Uganda.

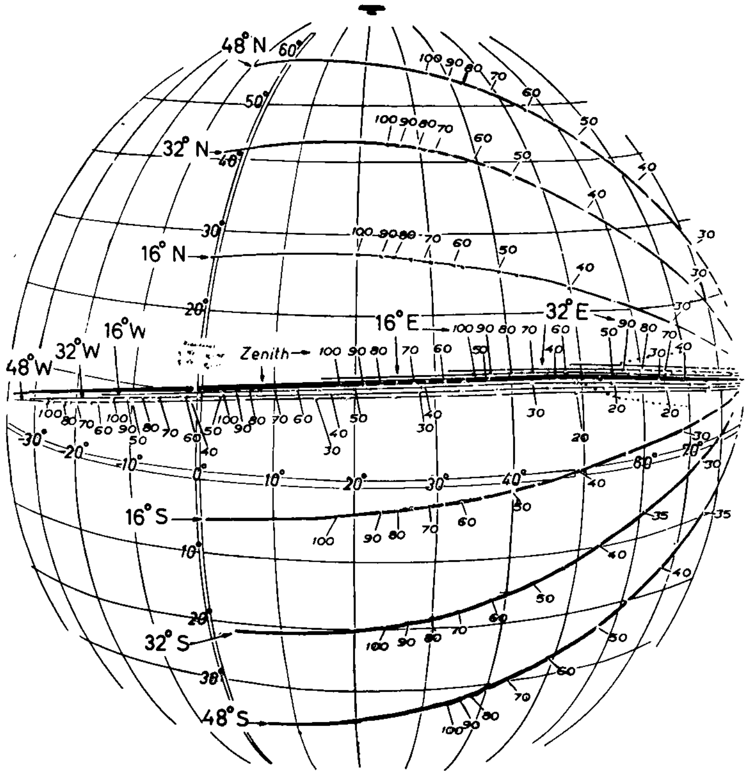

Fig. 22. Kodaikanal.

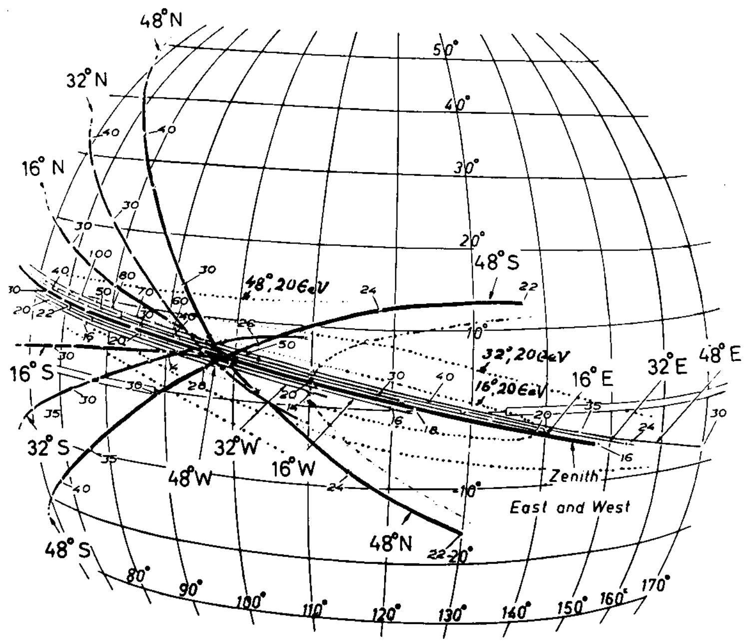

Fig. 23. Kodaikanal.

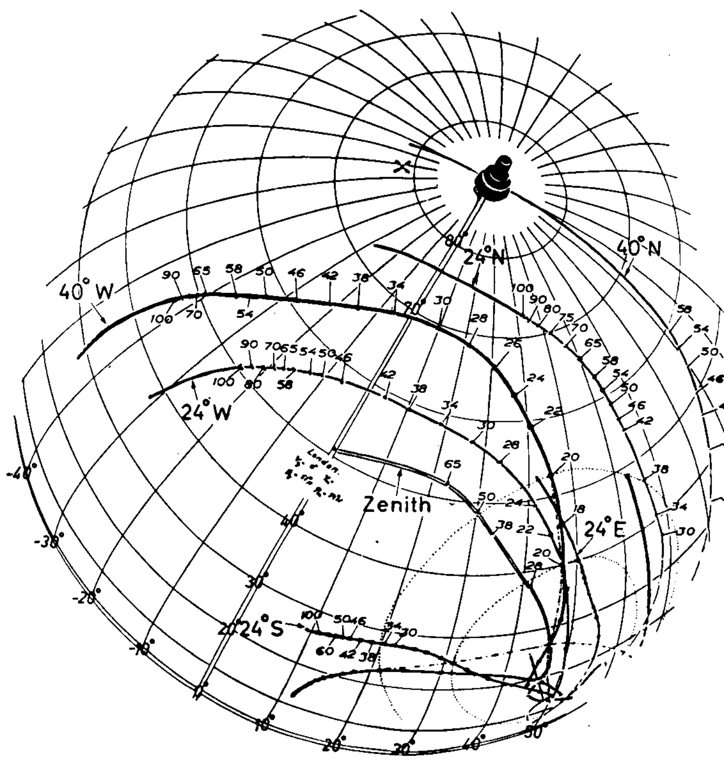

Fig. 24. London

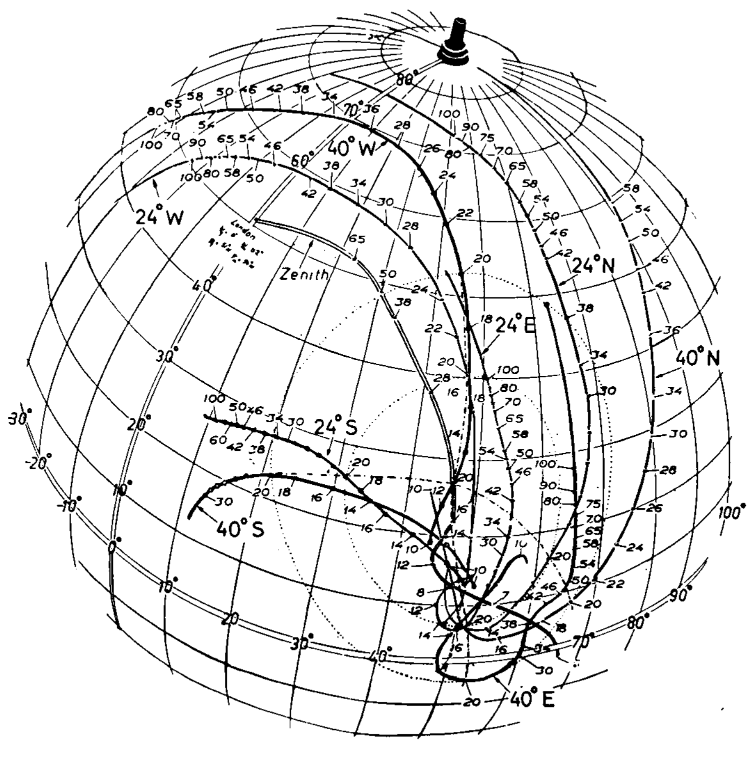

Fig. 25. London

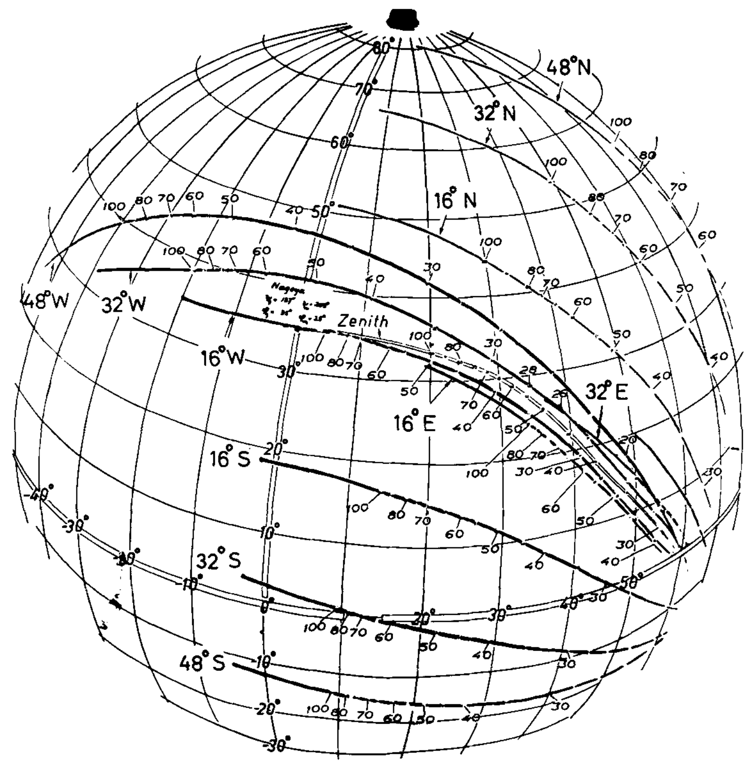

Fig. 26. Nagoya.

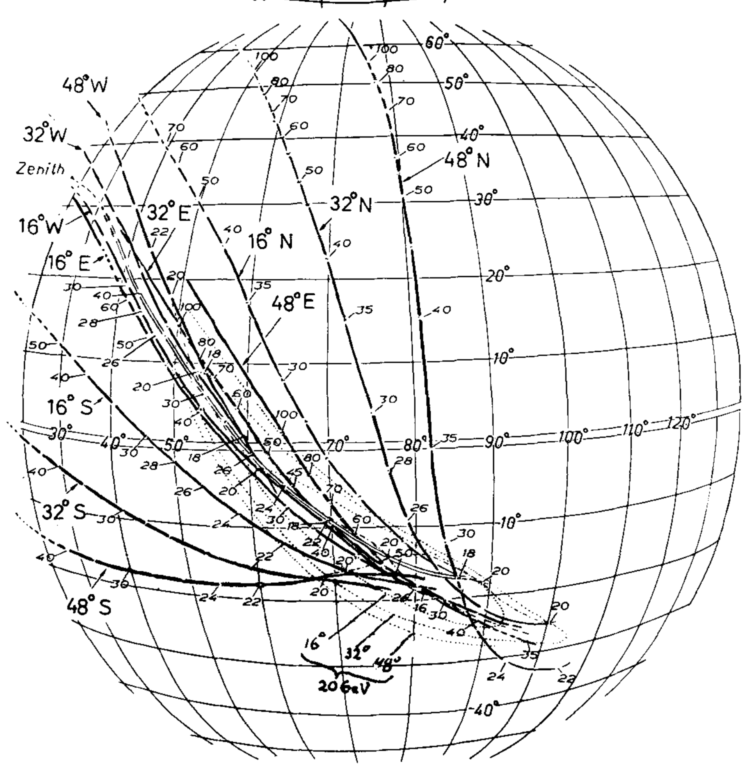

Fig. 27. Nagoya.

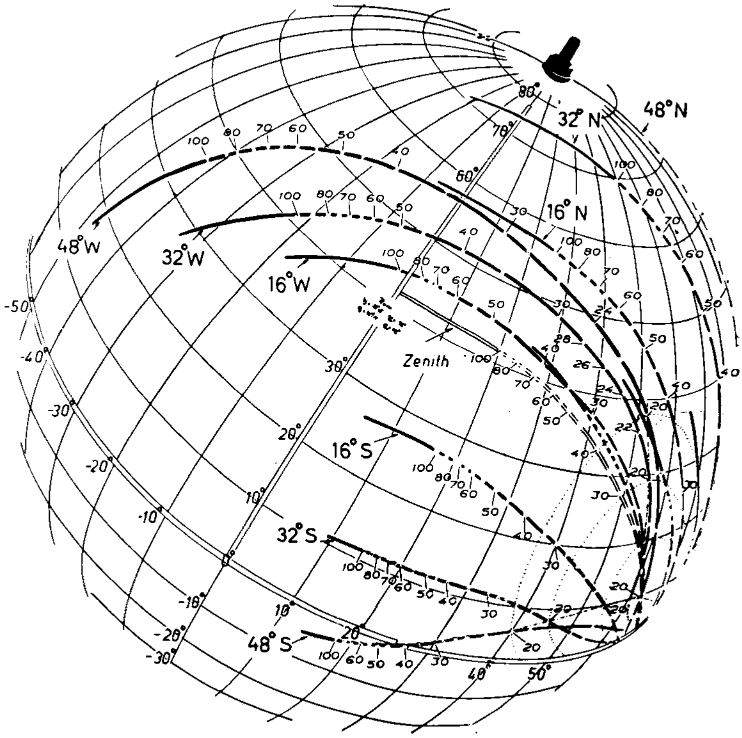

Fig. 28. Rom.

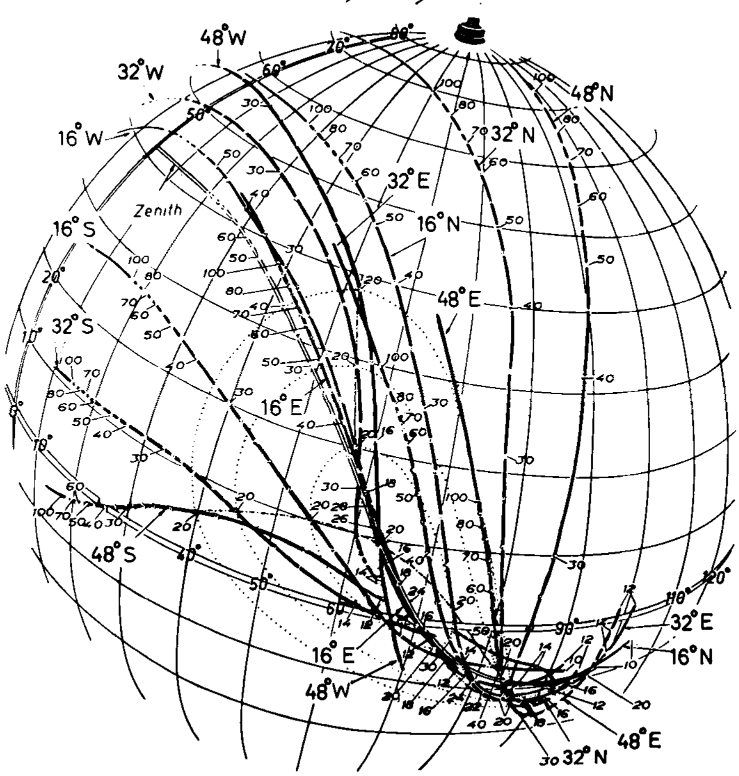

Fig. 29. Rom.

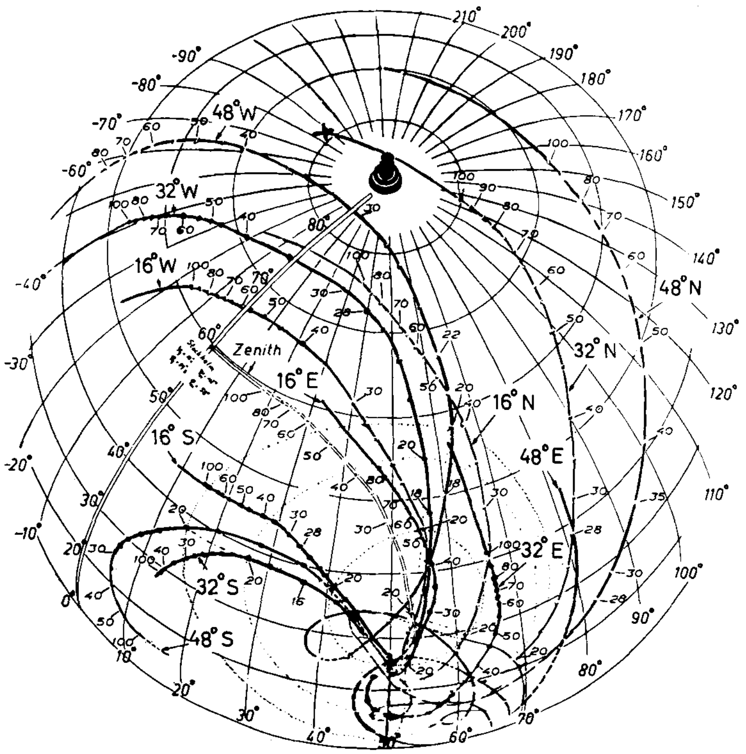

Fig. 30. Stockholm.

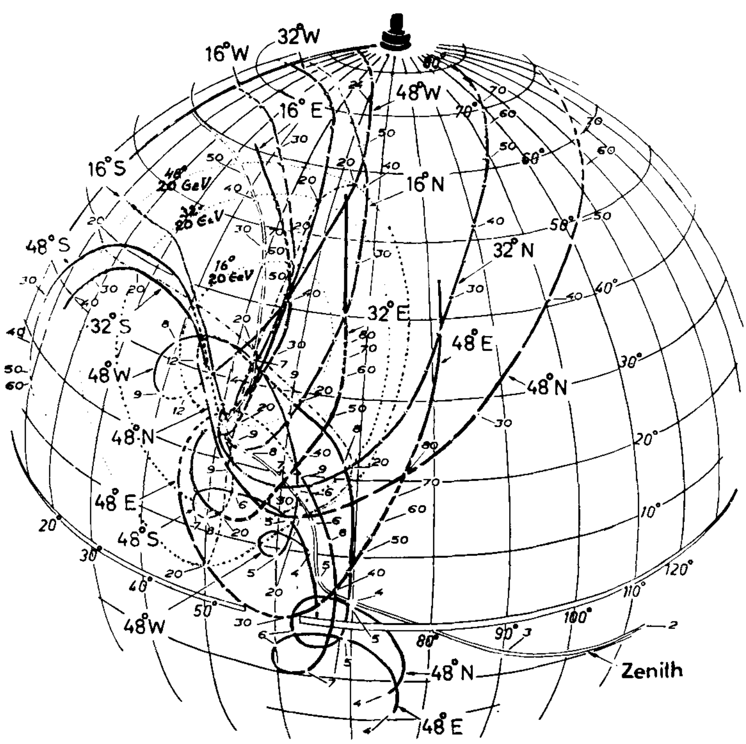

Fig. 31. Stockholm.

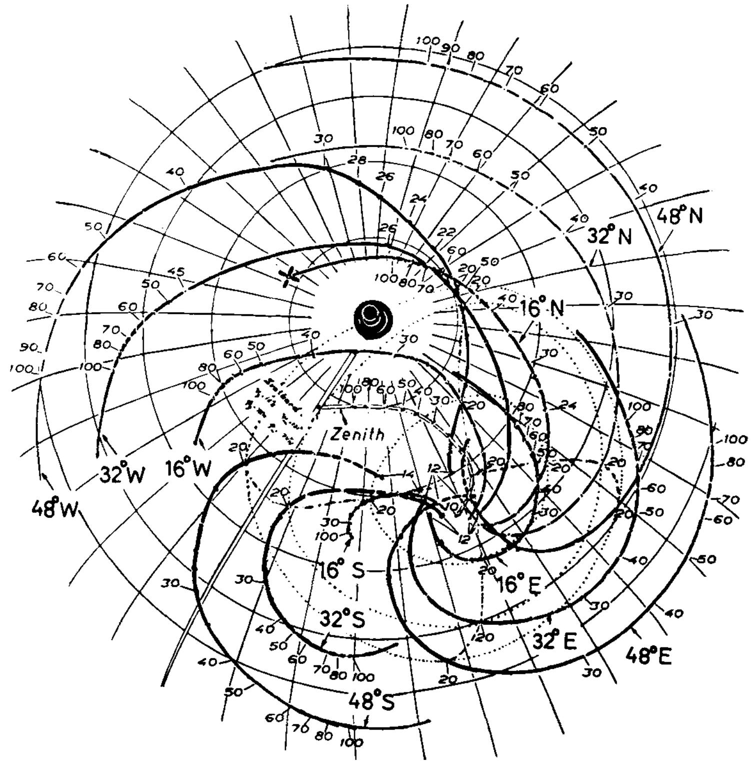

Fig. 32. Svalbard.