On the Scale of Auroral Model Experiments

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Abstract

The possibilities to scale down the physical properties of gaseous discharges with altered linear dimensions are considered with special regard to model experiments on aurorae. It is shown that the drift orbits can be transformed correctly, although the spiralling motion around the magnetic field lines cannot. This spiralling motion constitutes a very small ripple on the drift orbits in nature but a greater ripple in the experiments. However, this ripple seems to be of minor importance, and reasons are given for the view that the recent auroral experiments represent nature in a satisfactory way.

Introduction

When making model experiments of large scale phenomena in nature, it is not always possible to scale down appropriately every important detail. Model experiments of cosmic ray orbits in the earth's magnetic field or in other types of fields can be scaled down properly in every respect. It is only necessary to relate the particle energies and the field strength in nature and in the experiments in the appropriate way, and the experimental results are then immediately applicable to nature.

On the other hand it is impossible to alter the dimensions and time scale of gaseous discharges and at the same time transform their physical properties correctly in *every* respect (see ENGEL-STEENBECK II; 1934, pp. 95—102). If the ordinary similarity laws are used, it is theoretically possible to transform the orbits of the particles, the motion by diffusion and in electric and magnetic fields, the density and the pressure, if the temperature can be kept constant. Some atomic processes, however, such as recombination and cumulative ionization cannot even in theory be subjected to a scale transformation, but ionization in one step is allowed if the similarity laws are fulfilled.

In addition to these strictly theoretical considerations, practical difficulties may prevent a proper scaling in cases, where the scale factor is very large or very small. This is the case in the auroral model experiments, performed by BIRKELAND (1913), MALMFORS (1946) and BLOCK (1955). The latter paper will be referred to as paper I.

In these experiments lengths in nature have been reduced by approximately $1:10^8$, which means that according to the similarity laws the experimental magnetic field should be 10^8 times the earth's field, i.e., $6 \cdot 10^7$ gauss at the poles. It is thus techniqually impossible to fulfill this condition. We want to represent nature by a scale model of natural phenomena, and must therefore be sure that the significant details of the phenomena in nature are scaled down correctly. We must therefore consider two things.

a) What is important in nature?

b) What is scaled down correctly and what is not?

Particle orbits

The particle orbits are governed by the equation

$$m\frac{d\mathbf{v}}{dt} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \qquad (1)$$

In this equation we will introduce a characteristic length λ, time τ, voltage φ and magnetic field β. Then we can write

$$\frac{m}{e}\frac{\lambda}{\tau^2} = \frac{\varphi}{\lambda} + \frac{\lambda}{\tau}\cdot\beta \tag{2}$$

Suppose now a model where the length is changed by a factor k_λ, the time by k_τ, the voltage by k_φ, and the magnetic field by k_β. Then (2) changes into

$$\frac{m}{e}\frac{k_\lambda \lambda}{(k_\tau \tau)^2} = \frac{k_\varphi \varphi}{k_\lambda \lambda} + \frac{k_\lambda \lambda}{k_\tau \tau}\cdot k_\beta \beta \tag{3}$$

Assuming the particle orbits to be correctly transformed, the following conditions must be fulfilled

$$\frac{k_\lambda}{(k_\tau)^2} = \frac{k_\varphi}{k_\lambda} = \frac{k_\lambda k_\beta}{k_\tau} \tag{4}$$

There are four unknowns but only two equations, which permits us to choose our scale factors arbitrarily to some extent. Eliminating k_τ we get

$$\frac{(k_\lambda \cdot k_\beta)^2}{k_\varphi} = 1 \tag{5}$$

In the special case that $k_\lambda = 1$ we see that $k_\varphi = (k_\beta)^2$ which is equivalent to the equations (2) and (11) in paper I. The similarity laws mentioned in the beginning of this paper imply that $k_\lambda k_\beta = 1$ and $k_\varphi = 1$. This is due to the assumption that the same energies should be involved in the collisions. In equation (1) collisions are neglected.

In our experiments we have k_λ of the order of 10^{-8}. The magnetic field may be as large as 6,000 gauss at the poles of the terrella or 10^4 times the earth's field. Thus $k_\beta = 10^4$. This gives $k_\varphi = 10^{-8}$. As a characteristic voltage in nature we may choose the voltage over a distance of the order of the earth's radius. This may be 10^4 volts. In the experiment the potential over a distance equal to the terrella radius should then be 10^{-4} volts, which of course is impossible in a laboratory experiment. This is also pointed out in connection with the equation (34) of paper I (p. 84).

There is, however, another possibility which has been used. There are two kinds of motion, one drift motion and one spiralling motion around the magnetic field lines. It is shown in paper I, that the drift motion is transformed properly (see pp 80—81). The drift motion in the equatorial plane as calculated by ALFVÉN (1950) is shown in Fig. 1.

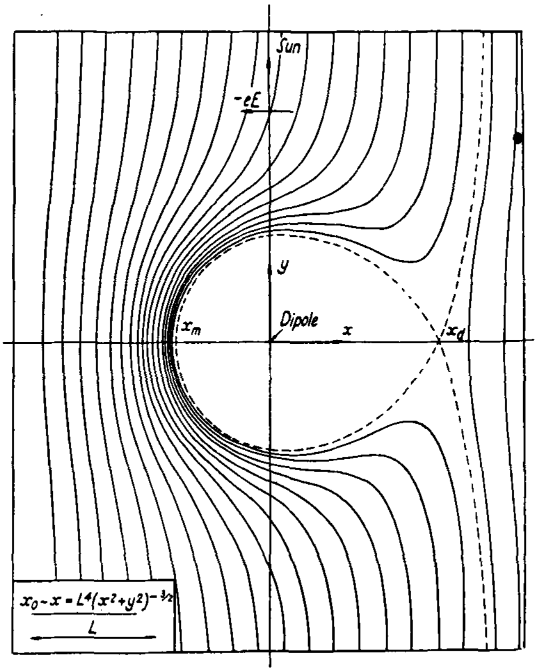

Fig. 1. Motion of the electrons in the equatorial plane of a magnetic dipole field on which is superimposed a homogenous magnetic field, under the action of a homogenous electric field.

The equations of the drift motion have been calculated by ALFVÉN (1950, p. 23).

$$\mathbf{u}_\perp = -\frac{1}{eB^2}[\mathbf{B} \times (\mathbf{f} + \mathbf{f}^m + \mathbf{f}^i)] \tag{6}$$

where in our case

$$\mathbf{f} = e\mathbf{E} \tag{7}$$

$$\mathbf{f}_m = -\mu \nabla B \tag{8}$$

$$\mathbf{f}^i = -m\frac{d}{dt}\mathbf{u}_\perp \tag{9}$$

$$\mu = \frac{W_\perp}{B} \tag{10}$$

$\mathbf{u}_\perp$ = drift velocity perpendicular to the magnetic field
$W_\perp$ = kinetic energy perpendicular to the magnetic field

It is shown by ALFVÉN (1955), that $\mathbf{f}^i$ is negligible at distances less than approximately 30 earth radii from the earth. Since the experiments cannot simulate phenomena so far from the earth, we will neglect $\mathbf{f}^i$ in the remainder of this paper. Therefore, if we transform the drift orbits, their geometry will be unchanged if $\mathbf{f}$ and $\mathbf{f}^m$ are multiplied by the same factor.

$$e\mathbf{E} \sim \frac{W_\perp}{B} \nabla B \qquad (11)$$

or

$$e\frac{k_\varphi \varphi}{k_\lambda \lambda} \sim \frac{W_\perp}{k_\lambda \lambda} \qquad (12)$$

so that

$$W_\perp \sim e k_\varphi \varphi \qquad (13)$$

i.e., the particle energies are proportional to the applied voltage. This makes the characteristic length L of the forbidden zone proportional to λ because

$$L^4 = c \cdot \frac{\mu a}{eE} \qquad (14)$$

c = a constant dependent on the unit system. a = magnetic dipole moment of the terrella or the earth.

Thus

$$L^4 \sim \frac{W_\perp}{B} \cdot \frac{B\lambda^3}{\varphi/\lambda} = \frac{W_\perp}{\varphi} \cdot \lambda^4 \qquad (15)$$

However, (13) is not always valid. If collisions can be neglected, electrons starting in crossed electric and magnetic fields get a mean energy

$$W_\perp \sim \left(\frac{E}{B}\right)^2 \qquad (16)$$

A combination of (12) and (16) gives

$$\frac{e k_\varphi \varphi}{k_\lambda \lambda} \sim \frac{(k_\varphi \varphi)^2}{(k_\beta \beta k_\lambda \lambda)^2} \cdot \frac{1}{k_\lambda \lambda}$$

or

$$\frac{k_\varphi}{(k_\beta k_\lambda)^2} = \text{constant} \qquad (17)$$

It can easily be seen that this is again equivalent to the condition that $L \sim \lambda$ at the transformation, if $W_\perp$ is subject to (16). It may be noted that (5) is a special case of (17).

We can summarize all this by saying that the drift orbits are correctly scaled down if the following conditions are fulfilled.

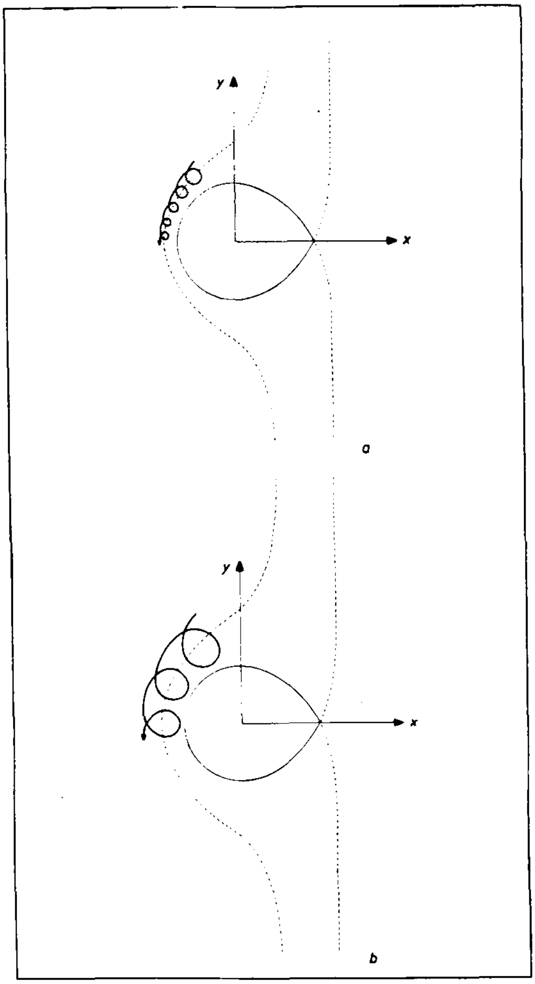

Fig. 2. Comparison of spiralling motions superimposed on the drift motion in nature (a) and in experiment (b). The spiralling radius of curvature is for clarity exaggerated in both cases.

a) The magnetic field must be so strong that the radius of curvature ϱ is small enough for the equation (6) to be at all applicable. Expressed in mathematical terms this condition is

$$|(\varrho \ \mathrm{grad})B| \ll |B|$$

which in our case can be expressed as, say $\varrho \ll R$ = terrella radius

b) The characteristic length L of the forbidden zone must be correctly scaled down.

These conditions are shown to be fulfilled in the experiments described in paper I, pp. 80—81.

The only difference from nature is then that the spiralling radius is relatively much greater than in nature. The ratio of the spiralling-radius and the size of the forbidden zone is about 0.0001 in nature and 0.05 in the experiments (see Fig. 2). If the magnetic field of the terrella is increased this ratio will be diminished but the latitudes of the auroral zones and the whole appearance of the discharge is substantially unchanged. This is very important, because it makes it plausible that the incorrect scaling down of the spiralling motion is of minor importance.

Particle density and plasma oscillations

It is important to make clear the role of collisions in the discharge. The total cross section for collisions is never larger than about 10^{-15} cm^2. In the experiment the density is 10^{12} or 10^{13} cm^{-3}. Thus the mean free path is a few metres, and this is larger than the dimensions of the vacuum chamber. Even if we allow for the spiralling of the particles, they can never experience more than a few collisions during their mean life in the discharge.

In nature the density may be of the order of 1 per cm^3. Then the mean free path will be at least 10^{15} cm or 10^5 times the size L of the forbidden zone. An estimate of the total length of an electron orbit near the forbidden zone shows that it may be of the order 10^3 or 10^4 times L and therefore less than the mean free path. Thus both in nature and experiment collisions are of minor importance and the collision probabilities are about the same in both cases. This would not be true if the magnetic field of the terrella could be increased so much that also the spiralling would be correctly transformed because then the effective orbital length in the experiment would be much longer than the mean free path. It would then be necessary to decrease the pressure to about 10^{-8} mm Hg in the vacuum chamber.

In ordinary gases it is the collisions that maintain statistical equilibrium and isotropic velocity distribution. As suggested in paper I it is probable that in the auroral discharges – in nature or in a model – the isotropic velocity distribution is maintained by what is usually called plasma oscillations. It is shown in the experiments that a kind of "noise" is present when the magnetic field is so strong that auroral zones appear on the terrella and this noise is substantially constant up to the strongest magnetic fields (see paper I, p. 70). This effect is similar to what happens in a magnetron, where the current should be cut off suddenly at a certain magnetic field, but noise sets in and the cut-off is made more smooth (COLLINS, 1948, and REVERDIN, 1951). It is still more similar to investigations made by ÅSTRÖM (1948). He used a tube with crossed electric and magnetic fields, where electrons emitted from a cathode move in throchoidal orbits. If the magnetic field and the current were not too strong, the electron beam was well defined, proceeding in the vicinity of an equipotential surface with the same potential as the cathode. If the magnetic field or the current were increased above a certain limit, the beam became blurred, noise appeared, and electrodes several hundred volts negative with respect to the cathode were reached by a substantial part of the electrons emitted from the cathode. In some cases as much as 90 % of the electrons could reach negative electrodes.

The measurements on plasma oscillations in magnetic fields hitherto made (see e.g. BOHM, BURHOP, MASSEY and WILLIAMS, 1949) seem to indicate that "oscillations" are set up whenever the gradient of the particle density becomes considerable somewhere in the discharge tube. The oscillations act so as to decrease the density gradients. We may call this an entropy increasing effect where the electron gas is transferred from a more "ordered" state to a less ordered one. It is clear that the auroral experiments confirm this view because noise is found whenever a forbidden zone is present, and at the boundary of the forbidden zone the density gradient is certainly large. If this view is correct, the presence of noise is thus subject to the same conditions as the proper scaling down of the drift orbits.

Discussion

It is obvious that model experiments of this kind can never *prove* anything; they can only make one or another hypothesis about the aurorae more or less probable. In the author's opinion it seems very probable, however, that the experiments described in paper I constitute a good model of all the most significant details of the auroral mechanism. The point where noise sets in constitutes a sudden transition from a device, which has nothing to do with aurorae, to a good representation of the auroral mechanism. The character of the discharge suddenly changes at this point and at higher magnetic fields no substantial change of the discharge takes place. The only important difference between discharges at strong and at weak fields above the "transition" point is the latitudes of the auroral zones. It has been possible in some cases to increase the magnetic field by a factor ten above the "transition" point.

Clearly the mechanism described by the electric field theory of aurorae can be realized at least in the laboratory, i.e., in the model experiments made by Malmfors and Block. In particular the charges, which are set up in the equatorial plane, discharge along the field lines to the auroral zones, rather than destroy the theoretical picture of the orbits in the equatorial plane. The drift orbits are shown to agree well with the theoretical orbits. The more similarities that can be found between the laboratory experiment and nature, the greater will be the probability, that the electric field theory describes what happens, not only in the model experiments, but also in nature.

The weakest details of the experiments are the orbits of the positive ions. Their radius of curvature is still of the same order as the terrella radius, and not even their drift orbits are therefore correctly scaled down. As far as we know now, however, it is always the electrons that determine the main character of a gaseous discharge.

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