Shower Formation in Large Cumulus

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Abstract

A distinction is drawn between the appearance of evanescent fibrous streaks in decaying cumulus towers (*fibrillation*), and the *glaciation* of cumulus. A technique of measuring cumulus is described, and used to show that fibrillation, which marks the onset of shower formation, occurs when cumulus towers reach a *fibrillation zone* or a *fibrillation level;* towers which rise appreciably above this zone glaciate if supercooled. The fibrillation level varies from occasion to occasion, apparently according to the strength of the cloud updraughts, and in a manner which indicates the coalescence process to be the general shower-*forming* agency in clouds observed in Sweden, although the temperatures at the summit-level of incipient showers may be as low as — 20° C.

1. Introduction

It is well known that the size which cumulus clouds attain before producing showers varies from place to place and occasion to occasion. Sometimes, especially over or near the ocean, showers fall from cumulus only about two kilometres deep (see, for example, ALPERT, 1955), while at other times, especially in the interior of continents, no showers form in clouds having a depth of several kilometres. Attempts have been made to relate shower formation to the temperature at the level of the cloud summits, but this, just as the critical cloud depth, seems to vary between wide limits. For example, in New Mexico BRAHAM, REYNOLDS and HARRELL (1951) detected radar echoes from only one fifth of summer cumulus whose summit-level temperatures lay between - 12 and - 24° C; PEPPLER (1940) concluded from observations made on routine aircraft soundings in Germany that showers usually occur when the summit-level temperatures fall below - 12° C; SCHWERDTFEGER (1948) found - 3° C to be the most frequent value of this temperature in over 250 aircraft observations of shower clouds near 60°N 13°W, while it is now generally recognised that in the tropics showers commonly form in clouds whose summit-level temperatures are several degrees above the melting-point. Such observations cannot be reconciled with the idea that showers form only when the cloud tops become cold enough for ice nuclei to promote the appearance of significant concentrations of ice crystals amongst supercooled water droplets, as envisaged in the Bergeron-Findeisen theory of shower formation, and it is now accepted that processes of droplet coalescence are important, not only in the small shower clouds of tropical regions, but probably also in those clouds large enough to have strongly supercooled tops which become "glaciated" in the course of shower production. LUDLAM (1951, 1952) emphasised that processes of cloud-particle growth require a certain period to accomplish the production of particles of precipitation-element size, and regarded the

[1] This work was done while the writers were on attachment to the International Institute of Meteorology, Stockholm.

period available in a particular cumulus as determined by the time taken for the cloud updraughts to lift the small cloud particles from near the base, where they are formed (or some higher level in the case of ice crystals), to the level of the summits, where they are rapidly evaporated. Thus the vertical extent, and hence also the summit-level temperature, of those cumulus which become shower clouds should depend very much on the strength of the updraughts in their interiors, a parameter not previously considered. The observations discussed in the following paragraphs were made to examine more closely the size of cumulus in which showers form, and the relation to the speed of the cloud updraughts.

2. The significance of the fibrillation and glaciation of cumulus tops

In an ordinary population of cumulus the clouds reach a variety of heights before subsiding and shrinking or dissolving. Usually the number of summits attaining a particular level falls rapidly as this level is raised, and this is true also of the individual towers, or bubbles (LUDLAM and SCORER, 1953), of which the larger clouds are composed.

During the diurnal development of cumulus overland, showers do not form unless the largest clouds reach a certain size, or summit-level. At first the showers are slight and widely scattered; it is uncommon for the development to cease just at this stage: usually they very soon become more intense and numerous as the clouds become still larger and greater numbers exceed the critical size. In temperate latitudes the shower clouds are nearly always more than 2 km deep, and so the temperature in their tops is usually at least a few degrees below the melting-point. The formation of a shower in one of the larger clouds is characteristically accompanied or preceded (often several minutes before precipitation is seen below the cloud base) by a transformation in the appearance of its upper parts. At first the cumulus tops are well-defined, with sharp outlines and a wealth of details provided by many minor domes or nodules and their shadowed interstices; if the cloud tower continues to rise its upper cap retains a sharp and bulging outline, but soon on its sides and lower parts the details dissolve into a silky sheen, and the edges become more diffuse (as in Figs. 5—7). Eventually the whole of the tower acquires the lustre and the fibrous texture which is typical of ice clouds, and the largest clouds often develop persistent fibrous anvils which from optical phenomena and flight observations are known to be composed largely, if not wholly, of ice particles. Consequently this striking transformation is called *glaciation*.

Sometimes, in a cumulus tower which has reached the peak of its ascent, traces only of this transformation can be seen (as in Figs. 1—3). It may be difficult or even impossible to detect until the evaporation of the tower begins; usually this is accompanied and hastened by a subsidence, and within a minute or two the tower becomes emaciated and ragged; amongst the last shreds of cloud, which have well-defined edges, diffuse or distinctly fibrous upright streaks become visible. If the whole cloud is subsiding, or if a wind shear has displaced the dissolving tower to one side of it, then the streaks are left in clear air, and may themselves survive only another minute or two before evaporating. No shower develops, although a brief, faint radar echo may have been detected: evidently the streaks consist of particles much larger than the cloud droplets, and for this reason sink rather quickly and are noticeably more reluctant to evaporate. It is remarkable that cloud towers do not usually shrink steadily as they dissolve; rather, after they have become emaciated, a stage is suddenly reached at which only a few ragged fragments exist here and there throughout the volume previously occupied by the tower. Sometimes these frayed residues may have a rather fibrous arrangement, but in a clear atmosphere an experienced observer rarely has difficulty in distinguishing this appearance from the more diffuse and persistent streakiness which is the only trace of the transformation called glaciation.

The fibrous texture characteristic of ice clouds is attributed to the large size of their particles, which fall in trails from localised condensation-regions. The crystals can be regarded as *precipitation-elements*, which can fall hundreds of metres or even some kilometres from their birth-place before evaporating (LUDLAM, 1948; 1956). Their large size (linear dimension about 200 microns) is attained because of their very small initial concentration (about 1/1000 of the concentration of droplets characteristic of liquid clouds), and because, forming only at approximate saturation with respect to liquid water, their fall commonly takes them through a layer a kilometre deep or more, which although cloudless is supersaturated with respect to ice, so that an abundant store of vapour is available for condensation. Consequently the particles of small ice clouds can by condensation alone reach the size of precipitation elements, whereas droplets can reach this size only by aggregation processes inside rather thick clouds. Nevertheless, the trails of raindrops which fall from the localised regions of copious condensation inside cumulus also have a fibrous structure, although their nearness to a ground observer and poor illumination may make this coarser than seen in the high ice clouds, and it is clear that the fibrous structure does not signal the *nature* of the particles, but only shows that they have the *size* of precipitation elements (dimension 200 microns or more). The transformation of cumulus tops into fibrous trails often occurs[1] in those tropical shower clouds which are wholly below the level of the 0° C isotherm, and is therefore not appropriately called a "glaciation". It seems equally inadvisable to use this term to describe the traces of streak formation which occur in cumulus tops which are supercooled, but apparently insufficiently for the development of an undoubted glaciation. We therefore propose that this name should be reserved for the transformation of cumulus summits into a large and persistent ice cloud, while the more evanescent streakiness which occurs in decaying towers should be called a "fibrillation".

One of the most common and striking features of glaciation in a large cloud is the cessation of sinking motions on the sides of the cloud summits. The nodules which constantly emerge on the upper cap of a growing cumulus tower effect the rapid mixing of the cloud air with the clear environment; this mixing is accompanied by a chilling as the cloud particles evaporate, and thus by a loss of buoyancy. The nodules can be seen to sink aside from the cap of the cloud, which grows upwards by the protrusion of its interior. The sinking motions on the cloud sides are particularly conspicuous on time-lapse films. At temperatures considerably below 0° C the humidity in the surrounding clear air may be well below 100 % with respect to liquid water, but may correspond to near-saturation with respect to ice. Evaporation during mixing processes at the boundaries of a glaciated cloud then becomes less intense, the cloud edges become less sharply defined, and the chilling becomes insufficient to cause noticeable sinking motions and to promote evaporation. Instead, the residues of successive glaciating towers slowly accumulate and produce the characteristic anvil cloud, so that the typical shape of cumulus, which taper upward, gives place to the typical shape of mature cumulonimbus, which taper towards the middle levels. On the other hand, the streaks which are observed in fibrillated cumulus summits are *essentially* evanescent; they are observed to sink rapidly, dissolving into clear air, disappearing behind fresh cloud towers or settling into the bulk of the cloud.

Evidently fibrillation marks a critical stage of shower formation, occurring in clouds which have been barely capable of producing particles of precipitation-element size. This transformation can be related to the maximum height, or dimensions, attained by the parent cloud tower. In contrast, it is hardly possible to define the exact stage at which the glaciation of a large shower cloud occurs, because this is a transformation which occurs more slowly, and which spreads sideways to later towers, and downwards, so that eventually the whole cloud, and the precipitation from it, may acquire the fibrous texture. In the observations to be described, particular attention was paid to determining the stage of cumulus development in which fibrillation occurred.

3. Observational techniques

The observations were made during the summer months of 1954 and 1955 near Östersund, in the province of Jämtland, central Sweden (Fig. 8). The field-station is a few kilometres north of Östersund, on a hill near the east shore of the lake Storsjön. On

[1] As seen in films of J. S. MALKUS and V. J. SCHAEFER. See also, for example, SCHAEFER, 1949, p. 7.

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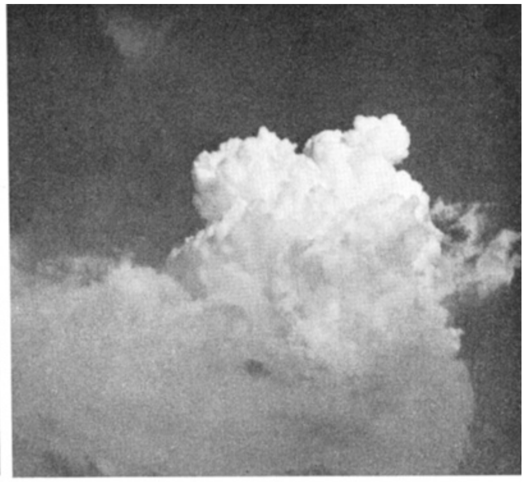

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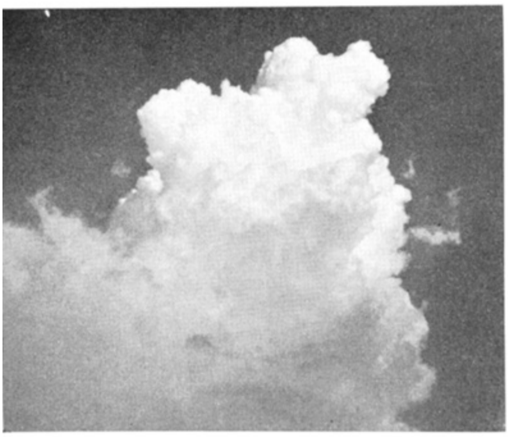

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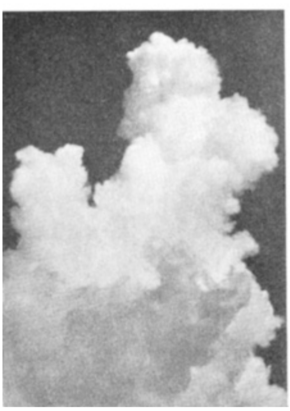

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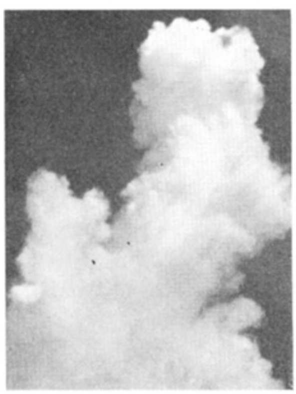

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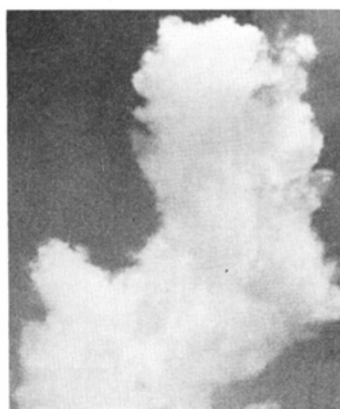

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Figs. 1–7. Fibrillation and glaciation in cumulus towers. On the left and extreme right of Fig. 1 are disintegrating towers. That on the left has *fibrillated* and has left diffuse streaks near its peak level, above shreds of droplet cloud which evaporate more quickly. On figs. 2 and 3 traces of diffuse streaks can be seen beneath the subsiding remnants of the right-hand tower ('marginal' fibrillation). The central cloud towers build rapidly to much greater heights, and in figs. 5 to 7 acquire the soft texture associated with *glaciation*. The interval between successive pictures is about 2 min.

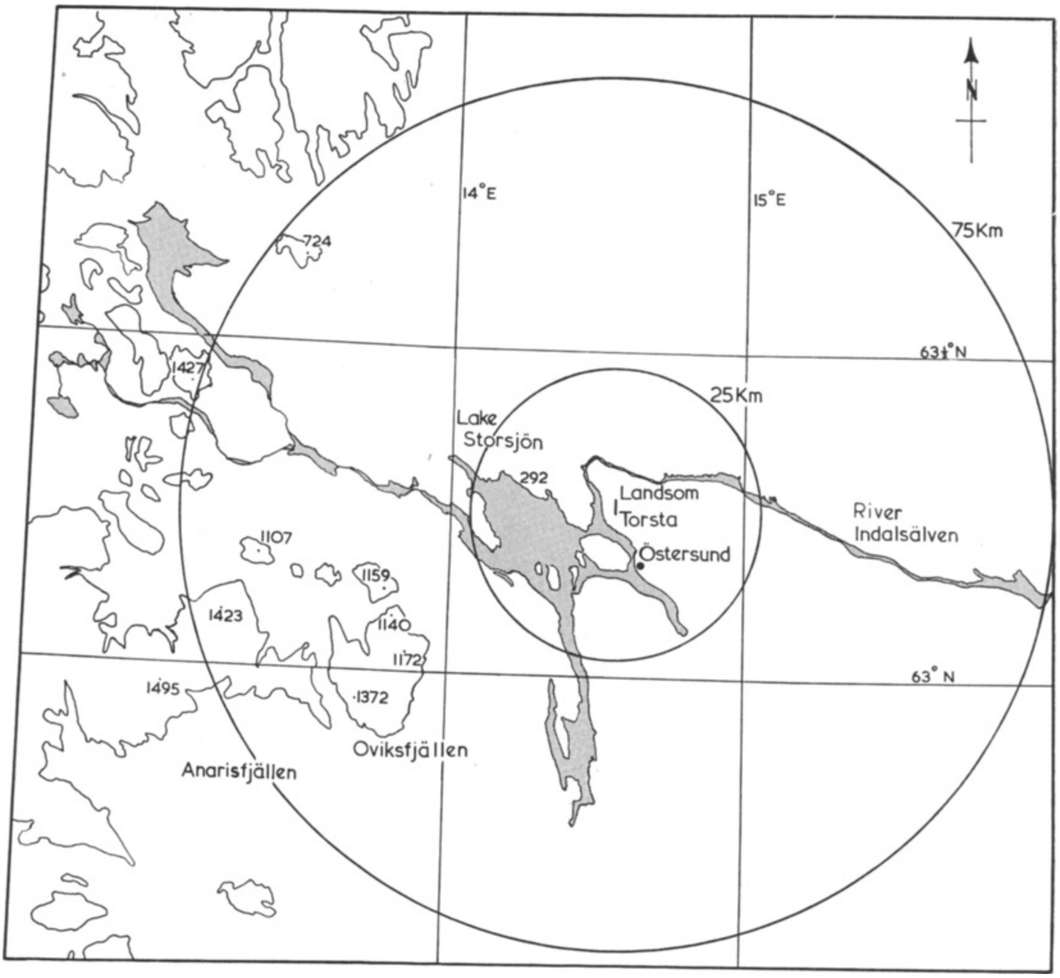

Fig. 8. Location and surroundings of observing station at Ås, Sweden. The theodolite base-line, just over 2 km long, extends between Torsta and Landsom. The treeline (about 600 m above sea level) is marked, and peak heights are given in metres.

days of scattered cumulus the air over the cool waters of this lake is usually cloud-free, so that we have unobstructed views of the clouds which develop over the mountainous areas which lie farther west. The nearest of these are the Ovik and Anaris, which have a number of peaks above 1,000 m. Over the whole eastern hemisphere the ground undulates between 300 and 400 m above sea-level; it is mostly covered by forest, with numerous small rivers and lakes.

The air is very clean, and on bright days it is possible from the ground to survey and measure cumulus clouds at distances of 100 km or more. The measurements have been made by observers equipped with pilot-balloon theodolites and in telephonic communication, at either end of a N–S base-line just over 2 km long. The technique of observing has already been described in a preliminary report (LUDLAM, 1955). The observations made from the ground are supplemented by aircraft soundings, under the direction of ground observers, and by routine and special radio-sonde ascents from the nearby station in Östersund. Occasional use has also been made of 3 cm radar, but simply for the detection of showers when the clouds are scattered the eye is superior. In very cloudy weather, and when the visibility is comparatively poor, this is of course no longer true. On one or two occasions when the visibility decreased to about 50 km the observational work was hampered.

4. Accuracy of the observations

After some practice the observers are able to identify and train their theodolites upon a particular detail of a cloud tower some 20 to 30 seconds after one of them has chosen it for measurement. This detail is usually the top of one of the nodules which continually emerge from the cap of the tower, or some other detail of size less than 10 m, which at a distance of about 50 km subtends an angle of less than 0.01°. Its azimuth and elevation are measured simultaneously with each theodolite on a signal from one observer. The tower is kept under observation; the measurement may be repeated, the rate of rise of the tower may be measured, and the peak height reached by the tower before its dissolution begins is recorded. Each theodolite[1] is orientated by sighting the other, and can be read to 0.01°; after each few observations the theodolites are levelled and freshly orientated: the bearing of the other instrument is rarely found to have wandered by more than 0.01° (usually in the same sense at each theodolite).

The observations are given to a computer who uses a 20-inch slide rule to evaluate the distance (d) and the height (h) of the cloud detail with respect to the control station, from the equations

$$d = c \sin \lambda / \sin (\phi - \lambda) \qquad (1)$$

where c is the length of the base-line (2,154 m), and ϕ and λ are the azimuths from the control and second station, respectively; and

$$h = d \tan \Theta + 6.8 \times 10^{-8} d^2 \qquad (2)$$

where Θ is the elevation from the control station; h and d are expressed in metres. The last term in equation (2) is a correction-term to take into account the curvature of the earth and the refraction of light; it was used after it had been found to give correspondence within a few metres with the paths of rays calculated to have an average refraction.

The principal errors which arise in the calculated distance of a cumulus detail are due to errors in the angular measurements of the azimuths. If these amount to $\delta\phi$, $\delta\lambda$, it may readily be shown that the error in the distance d is given by

$$\delta d = -d\,(\delta\phi - \delta\lambda) / \sin(\phi - \lambda) \qquad (3)$$

The algebraic difference of the azimuth errors is not likely to exceed 0.02°; using this value in equation (3), the percentage error in the calculated distance of the cloud can be derived as a function of the true distance and azimuth, and is shown in Fig. 9. Two-theodolite measurements are not usually attempted on clouds lying within 20° of the direction of the base-line. Fig. 9 also shows the resultant error in measuring the height of a cloud detail 5,000 m above the control station, assuming an additive error of 0.02° in observing its angular elevation. It can be seen from the diagram that, since most of the observations are made on clouds at ranges between 25 and 75 km, the errors in the height determinations are not likely to exceed 100 m.

It is not always possible for both observers to identify a detail in the cloud bases, and their height is therefore estimated by noting the elevation of the cloud base each time a tower is measured, and assuming it to have the same range. The heights deduced in this way are probably correct within 50 m, and agree well with measurements made by aircraft. There is usually a marked diurnal variation in the height of cloud base, and if this is taken into account it appears that on most days the cloud base at a particular time is uniform within about 100 m, although a consistent difference exceeding this may occur over certain sectors of the sky, or even in the same locality when the wind is light or when there are showers. When therefore the general level of the cloud base is known, a useful estimate of the height of a cloud top can be made with one theodolite simply by measuring the angular elevations of the top and base of the cloud. The error of such a determination is not likely to exceed about 10 %, i.e., 500 m at 5 km. In this way, and more accurately by using radar to determine the range of clouds containing precipitation, or aircraft observa-

[1] The theodolites used were British Meteorological Office balloon theodolites, made by E. R. Watts and Sons.

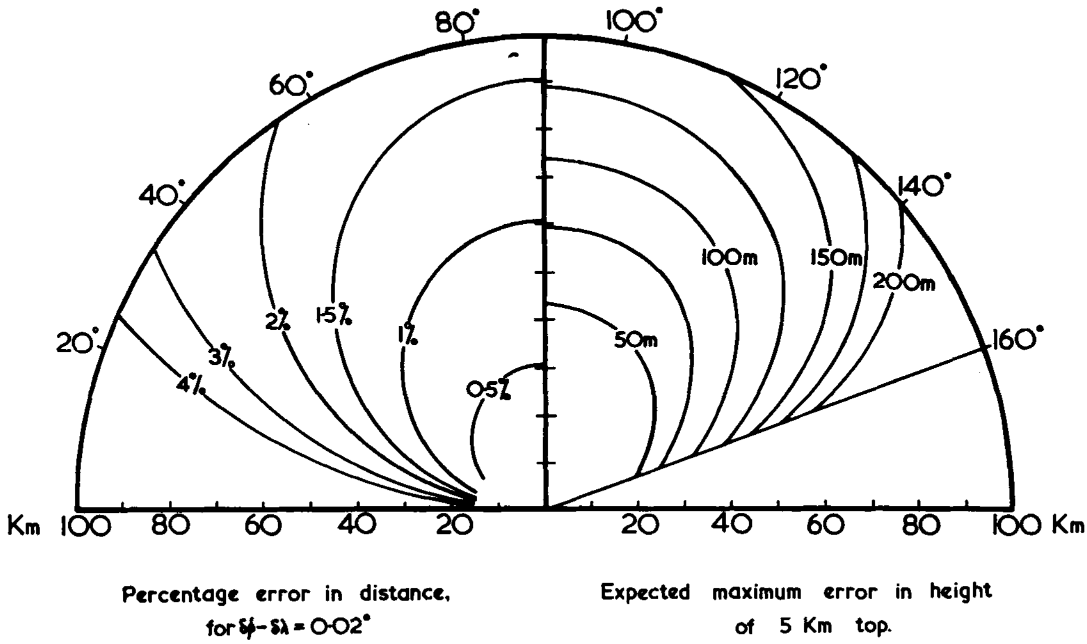

Fig. 9. Maximum likely errors in distance and height of cloud tops derived from theodolite observations, as a function of distance and angular bearing away from the direction of the base-line. An error of 0.02° is assumed in the difference of the azimuths observed from the ends of the base-line, and an additive equal error in the angular elevation measured at the control station.

tions, measurements may be made on clouds obscured from one observer and on clouds lying in directions close to that of the base-line.

The theodolite observations are supplemented by 16 mm time-lapse colour films and sequences of photographs on 6 × 9 cm films, which form an invaluable record. Experience of making measurements from these pictures leads us to suppose that by using a large negative size and taking a number of precautions, the analysis of photographs taken from either end of a similar base-line could provide height determinations with errors not more than three times as great as those arising in the 2-theodolite measurements. Although this photographic method has the advantage of providing a permanent record, and possibly more information at less personal trouble, its reduction has to be postponed until the film has been processed and is more tedious: the practised theodolite observers and their computer obtain a more accurate range and height within a minute of selecting a cloud. When this rapidity has been achieved, between 50 and 100 clouds can be measured in a day, and the number of observations is more often limited by the number of visible, interesting clouds than by the stamina of the observers. Their measurements, supported by the film records, are probably more satisfactory, and certainly cheaper, than those obtained primarily by the photographic method.

5. The fibrillation level

Observations on a number of days of scattered cumulus are illustrated in Figs. 10 to 14. These diagrams show the peak heights reached by individual cumulus towers, and whether or not fibrillation or glaciation of the towers occurred. When only traces of fibrillation were observed the occurrence was noted (before the height of the tower was computed) as a "marginal" transformation. The diagrams include estimates of the height of the cloud bases, from which the considerable diurnal variation is apparent.

These days are typical of those on which the depth of the cumulus exceeds about 2,500 m: it is then possible to recognise a layer within

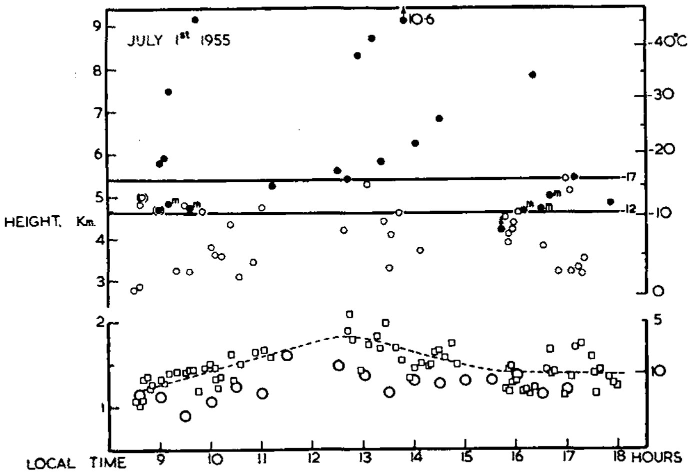

Fig. 10. Observations of cumulus bases and tops on several days in 1955. a) *peak heights reached by ascending towers*: an open circle indicates that no fibrillation was observed; a black dot indicates that fibrillation or glaciation occurred. If only traces of fibrillation were detected the letter '*m*' is added; if a fibrillated tower was observed only after it had subsided an unknown but small distance from its peak height, an upward-pointing arrow is added. b) *cloud bases*: a large open circle indicates the convective condensation level of air about 2 m above the ground. Shower symbols are placed beside points representing bases measured near showers, which are often below the general base-level. Distinction is sometimes made between the general level of the cumulus bases in the mountainous (SW) sector and in the flat terrain near and east of the field station (NE).

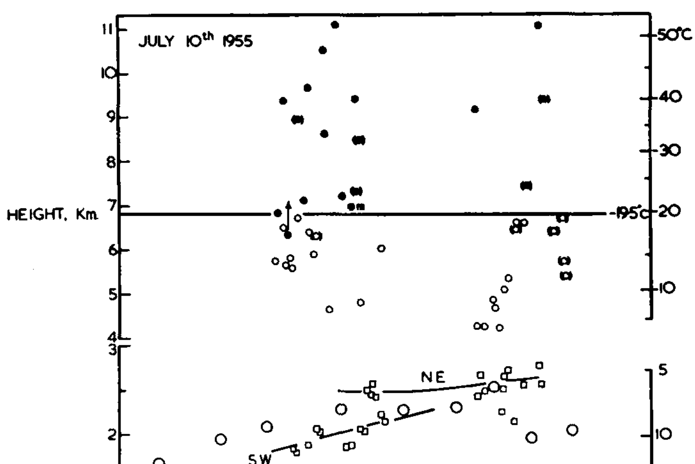

Fig. 11. See Fig. 10.

which some of the cumulus tops become fibrillated; usually this layer, the *fibrillation zone*, is not more than a few hundred metres thick, and contains most of the observations of marginal transformations, and all clouds which rise above the upper limit of the layer show marked fibrillation or glaciation. Quite commonly the zone is apparently only one or two hundred metres deep, and it is permissible to speak of the *fibrillation level.* On most days the cumulus have not become large enough for their tops to become fibrillated until about noon, after which time the variation in the height of cloud bases is rather small, so that, although we shall relate the fibrillation to the height of the towers above the bases, we have not usually tried to find a diurnal trend in the fibrillation level. Sometimes a variation has seemed to be present and it has been desirable to distinguish between conditions in the morning and afternoon, but mostly the observations are hardly sufficient to justify this.

That it should be possible to recognise a fibrillation level, and that it should be found at practically the same height on successive similar days (for example, 10 to 12 July, 1955), gives us confidence in the accuracy of our observing methods. It is still more encouraging to discover such evidence of orderly behaviour in populations of cumulus whose individuals superficially have a very varied character, growing over a variety of terrain and attaining a range of sizes and durations.

The height and temperature of the fibrillation level, and its height above the cloud bases, vary considerably according to the properties of the air mass, as shown in Table 1, which includes all days during two summer periods when a fibrillation level could be determined. The height of the fibrillation level varies over a range from 4,000 to 6,900 m, and its tempera-

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ture from −9 to −23° C, while its height above the cloud base varies from 2,500 to 4,600 m.

On days when the depth of cumulus is restricted to about 2,500 m or less it is not usually possible to observe fibrillation in the cloud summits. Observation is frequently hindered because the individual towers do not rise far out of the general cloud mass, having a smaller size and upward speed than is usual in larger clouds. Often they have entered a very stable layer or inversion which is suppressing the convection, have "overshot" their equilibrium level and acquired a negative buoyancy, which causes them to subside and disappear again from sight before disintegrating, unlike towers below the extreme summit-levels on days of much larger clouds, which are brought to rest by erosive mixing after emerging rapidly from the parent clouds. Frequently, too, the towers of the small clouds spread out at the base of the stable layer, producing expanding shelves of stratocumulus cumulogenitus which obscure succeeding towers.

When, therefore, showers develop in such small clouds, it is not possible to relate their formation to a fibrillation level, but only to refer to the level which the summits attain. Usually this is only a few hundred metres above the peak heights reached by the towers of still smaller clouds which fail to produce showers. Table 2 then, which summarises our observations on days of small shower clouds, can be regarded as an extension of Table 1, and shows that showers have occurred in clouds having a depth as little as 900 m, and summit-level

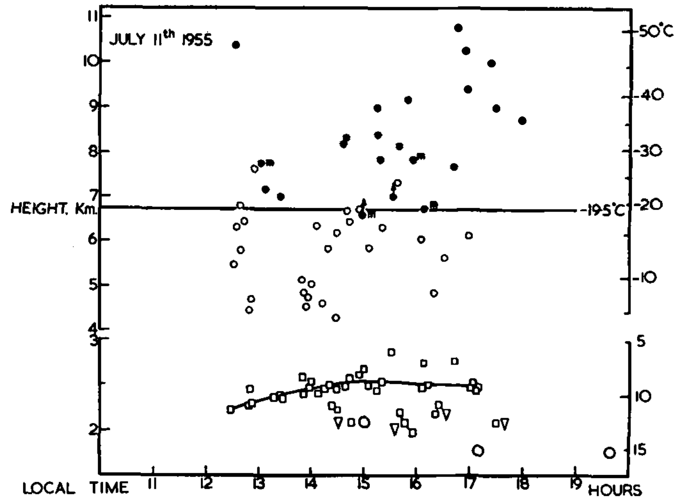


Fig. 12. See Fig. 10.

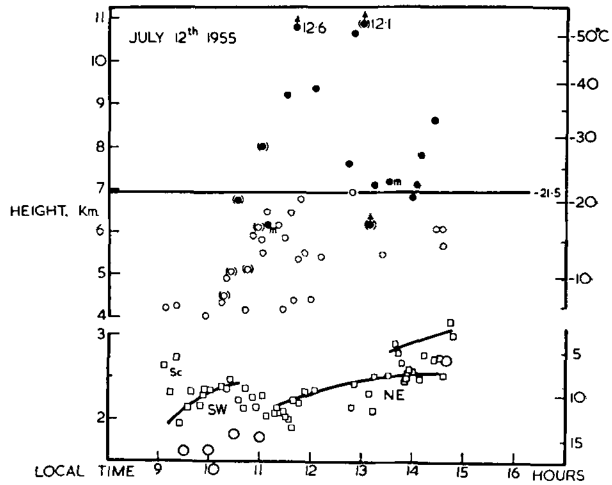


Fig. 13. See Fig. 10.

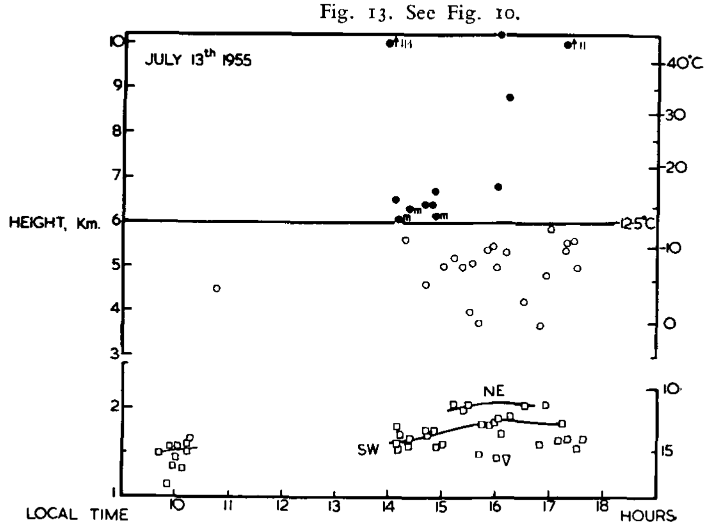


Fig. 14. See Fig. 10.

Table 1. Observed values of height and clear-air temperature of the fibrillation level, or of the average height of the fibrillation zone, and its height above the cloud base.

Date	Height above S.L. m	Temperature °C	Height above cloud base m
1954			
9 Aug........	4,800	−13	3,500
13 Aug........	4,000	−9	2,500
18 Aug........	4,000	−11	2,500
13 Sep.........	5,200	−23	4,000
1955			
28 Jun (3)[1]....	5,000	−19	2,600
29 Jun (4).....	5,300	−16	3,100
1 Jul (5)......	5,000	−13	3,500
10 Jul (6)......	6,800	−19	4,300
11 Jul (7)......	6,800	−19	4,300
12 Jul (8)......	6,900	−21	4,400
13 Jul (9)......	6,000	−13	4,000
14 Jul (10).....	6,600	−19	4,600
8 Aug (11)....	4,300	−9	3,300
10 Aug (12)....	4,800	−10	3,300
15 Aug (13)....	5,600	−15	3,300
16 Aug (14)....	5,400	−12	3,300

[1] The numbers in brackets identify the occasion of the curves of Fig. 17 and the points of Fig. 19. Two of these occasions are listed in Table 2.

Table 2. Observed values of height and clear-air temperature at the tops of cumulus which became small shower clouds, and their height above cloud base.

Date	Height above S.L. m	Temperature °C	Height above cloud base m
1954			
19 Aug a.m. (1).	3,900	−10	2,400
p.m. (2).	3,800	−9	2,000
20 Aug a.m.....	3,200	−4	1,700
p.m....	3,300	−4	1,400
31 Aug........	2,300	−3	900
8 Sep.........	2,400	−2	1,500
14 Sep.........	2,900	−7	1,600
1955			
26 Jun.........	3,900	−9	2,300
2 Aug........	2,700	−4	1,000
28 Aug........	2,300	+3	1,200

(On some of these occasions the cumulus tops penetrated a few hundred m above an inversion and the temperatures within the tops may conceivably have been 3 or 4° C lower than the clear-air temperature at the summit level)

temperatures as high as +3° C. Together, these tables illustrate the great variation in the sizes and summit temperatures at which showers form, even in one place during the same season, and doubtless the range could be increased further by including observations from other regions.

In 1954 it was noticed, as previously reported (LUDLAM, 1955), that the height of the fibrillation level above the cloud base seemed to be related to the vigour of the convection, being great when the cloud towers ascended rapidly, and small in the little clouds of shallow polar air masses, which probably contain only weak updraughts. This circumstance, and the remarkably high temperatures at the lower of the fibrillation levels, suggest that it is the coalescence process, rather than the growth of ice particles, which produces the precipitation elements in fibrillating cloud tops, and hence which is the predominant shower-*forming* agency. It is therefore necessary to discuss the course of the coalescence process in cumulus clouds.

6. The coalescence process in cumulus

The coalescence of cloud droplets in cumulus has been considered in a previous paper (LUDLAM, 1951). Here the discussion was based upon two premises:

1. In the lowest few hundred metres of the cloud there are some droplets of radius 20 to 40 μ, in concentrations of order 100/m^3, amongst a droplet population having uniform radii of several microns.
2. The droplets are lifted through the cloud by convective bubbles, in which the concentration of condensed water is that corresponding to adiabatic rise from the cloud base, until the bubbles arrive at the cloud summits and emerge as individual towers. There the mixing with clear air causes a rapid evaporation of all small droplets.

It was shown that if, during the rise of a bubble to the summit-level, the large droplets have attained radii of about 150 μ, then their further growth into raindrops and the formation of a shower is practically inevitable. This is because droplets of such a size have a fall speed exceeding 1 m/sec, so that they tend to have settled some hundreds of metres through the bubble and remain poised in the general cloud tops with a good chance of escaping the evaporation to which the towers are exposed; because even if involved in the evaporation they are able to settle through a few hundred metres of unsaturated air to re-enter the cloud mass with only slight shrinkage, and because in a moderate concentration of cloud water further growth by coalescence beyond this radius to raindrop size is very rapid, occupying only a few minutes.

Equally it may be considered that the presence of droplets of about this size and concentration is necessary for the occurence of fibrillation in a dissolving cloud tower, so that the summit-levels calculated in the previous paper (as a function of cloud base temperature, which governs liquid water content within the cloud, and of rate of rise of bubbles) to be necessary for the formation of showers, may be regarded as theoretical fibrillation levels.

However, these premises need some reconsideration, and are discussed in the following paragraphs.

7. The size spectrum of droplets in cumulus

In the previous paper it was shown that droplets of the size assumed to occur in rather small concentrations near the cloud base and efficiently initiate the coalescence process, are readily introduced into the clouds of maritime air masses by the giant hygroscopic nuclei, which there have concentrations of order $10^3/m^3$. These nuclei may be absent, or present in much smaller concentrations, in continental air masses, and even in maritime air masses if the winds are too light to produce sea-spray. It was therefore suggested that in these circumstances the coalescence process may be hindered.

Recently observations have been made of the size spectra of droplets in the lower parts of cumulus in Russia (ZAYTSEV, 1950), in New Jersey and Florida (WEICKMANN and AUFM KAMPE, 1953), and in England (MURGATROYD, 1955). These writers apparently agree that small cumulus always contain abundant concentrations of droplets having radii of about 30 μ.

According to Zaytsev droplets of radii up to 25 μ are encountered 200 m above the base of small clouds, and "some droplets of the order of 100 μ diameter are found, on the average, from 350 m above the cloud base". The diagrams of Weickmann and aufm Kampe show that most of their observations in cumulus humilis were made at 500 or 900 m above the cloud bases, and their summary of the droplet size spectrum typical of these clouds shows the presence of droplets of radius about 30 μ in concentrations of order $10^{-1}/cm^3$. Finally, Murgatroyd states that well inside cumulus at heights of more than 300 m above the level of the bases the mean droplet radius is about 10 μ, and that there are a few large droplets of radius 40—50 μ above 300 m, and of radius exceeding 100 μ above 1,000 m.,

Remarkably enough, none of these authors specifies the volumes of the samples of cloud air from which their droplet spectra were obtained, but since all of the measurements were made by microphotography of small slides exposed during flight, it can be assumed that their spectra refer to volumes of up to 100 cm^3. Naturally if much smaller volumes had been sampled, there would have been no mention of the large droplets, and equally it may be speculated that if much larger volumes are examined, smaller concentrations of even bigger droplets will be discovered. Clearly it is important to relate a given spectrum to the volume which it represents.

However, on the basis of the measurements quoted, it appears that at heights about 300 m above the bases of most small cumulus, droplets of radius about 30 μ exist in concentrations between 10^{-2} and $10^{-1}/cm^3$. It is most unlikely that these large droplets could arise by condensation alone on giant hygroscopic nuclei. According to WOODCOCK (1953), the typical concentrations of giant nuclei near cloud base in air over the oceans with fresh surface winds (Beaufort force 4—5) are those shown in the second column of Table 3. The last columns show the radii which are attained by these

Table 3. Growth of giant sea-salt nuclei by condensation.

Mass of salt in nucleus g	Typical concentration (Woodcock) cm^{-3}	Radius at cloud base (Keith and Arons) μ	Radius in cloud after 300 sec μ	Radius in cloud after 1 hr μ
10^{-10}	$2 \cdot 10^{-2}$	10	14	22
10^{-9}	10^{-3}	18	23	35
10^{-8}	10^{-5}	35	40	56

nuclei at the level of the cloud base if they are lifted from low levels at speeds of about 1 m/sec, according to KEITH and ARONS (1954). The fourth column shows the radii attained at a height of 300 m above the cloud base if rise continues at this rate, calculated as described previously (LUDLAM, 1951, p. 404), but taking into account the heat economy of the droplets.

A large cloud may commonly have existed for about an hour before reaching a size at which it is likely to produce a shower, and it is possible that at the end of such a period a bubble rising through the lowest parts of the cloud becomes infected with droplets which have survived the whole of this time. The last column of the table therefore shows the sizes of the giant nuclei after they have been suspended an hour inside the cloud. It can be seen that even if they infected fresh bubbles in undiminished concentration, and even if this concentration is that found in oceanic air, then the large droplets grown upon the nuclei would still be about 10 times fewer than those actually encountered.

It is therefore evident that the observed large droplets are grown predominantly by the coalescence process, which is presumably active in small clouds even though it does not succeed in producing precipitation. Recent developments in the bubble theory of convection (SCORER and LUDLAM, unpublished) recognise that during the rise of a buoyant bubble there is an intense mixing with the environment through the cap of the bubble, while there are strong stirring motions within the bubble itself. It must therefore be supposed that a bubble which rises through a cloud which has been in existence for some time incorporates numbers of the particles of preceding bubbles, in ever-decreasing concentration the greater their age. In this way a small proportion of the cloud particles are able by prolonged condensation and coalescence to attain an abnormal size, and the size spectra of the droplets become very broad: it is impossible otherwise to account for the breadth of the observed spectra, for the rise of air unmixed from the level of the cloud base produces very narrow spectra containing droplets of practically uniform size (see, for example, HOWELL, 1949). According to this view, even in small clouds the size spectra may include radii much greater than 30 μ if the sampling volume is sufficiently increased.

In this connection we recall previous observations in England (BROWNE, DAY and LUDLAM, 1955), in which droplets of radius about 150 μ were detected in concentrations of order 1/m^3 inside cumulus of depth 1,500 m: they were found on three successive days at heights between about 600 m and 1,200 m above the cloudbases, in clouds which did not produce showers. We believe that droplets of this radius and concentration commonly exist in cumulus having this size, and that this accounts for the readiness with which such clouds produce echoes from suitable radar (see, e.g., PLANK, ATLAS and PAULSEN, 1955). The size spectra which have been detailed in recent literature must be regarded as extending continuously to radii of at least 150 μ in small cumulus, and of course to values appropriate to raindrops in larger clouds which produce showers.

8. The concentration of liquid water in cumulus

It has previously been emphasised (LUDLAM and SCORER, 1953) that only within actively rising bubbles, which occupy only a fraction of the total volume of a large cloud, can it be expected that the concentration of condensed water approximates to the value corresponding to adiabatic rise from the cloud base. The rate of dilution of a rising bubble is very considerable, and the departure from the saturated adiabatic process within one bubble must greatly depend upon how soon it follows in the path of another, that is, upon the rate of bubble production. Only if this rate is increased beyond a certain value can a whole cloud *grow*, with successive bubbles retaining a greater buoyancy during their rise and hence reaching greater heights. Probably only during the building phase of a large cloud do the conditions within rising bubbles closely approach values appropriate to the rise of air unmixed from the cloud base. If the rate of bubble production falls, then the cloud ceases to grow or even shrinks, and although bubbles may still emerge as towers in the summits, there must be a stronger departure from adiabatic conditions at all levels in the cloud.

Perhaps for this reason, and because of the difficulty of selecting for examination clouds in the building phase, and then making traverses through the internal bubbles, only rarely have flight measurements with improved instruments indicated liquid water concentration close to the adiabatic values (WARNER, 1955, MURGATROYD, 1955).

9. Calculation of the course of the coalescense process in cumulus

In view of the conclusions of the two previous sections, it must be questioned whether the considerations mentioned in section 6 form a satisfactory basis on which to estimate the course of the coalescence process. Every cloud which approaches the size associated with shower formation has been in existence for some time and probably already contains very small concentrations of drops of nearly the size of raindrops, and almost certainly larger than the radius of 150 μ previously specified as a critical one for shower formation.

Nevertheless, it is observed that when the cloud is large (depth about 2,500 m or more), during a building phase successive bubbles form separate towers in which the production of fibrillation, of a detectable radar echo, or even of a shower, occurs rather suddenly, suggesting that within a particular bubble the coalescence process has progressed beyond a well-defined threshold. Evidently at this stage the concentrations of drops of radius greater than about 150 μ have suddenly increased from insignificant values of $1/m^3$ or less to values exceeding about $100/m^3$, and it will be shown below that this is readily interpretable as due to the growth to this size, within the individual bubble, of those droplets of radius about 30 μ with which it became infected at the beginning of its ascent through the cloud mass. In such a large cloud, in a building phase which brings it to the point of shower formation, it may still be reasonable to expect the liquid water content within rising bubbles to approximate to the adiabatic values, and altogether the previous simple model still seems a reasonable basis for a calculation of the course of the coalescence process.

On the other hand our observations suggest that in small clouds, of depth less than about 2,500 m, the showers develop more slowly, for it is often difficult to define a stage at which a shower has formed. Usually no fibrillation can be seen in the summits, and often the first visible signs of precipitation are faint fallstreaks beneath spreading shelves of stratocumulus cumulogenitus. Gradually, over a period of perhaps half an hour or more, these become more extensive until most of the cloud acquires a glaciated structure, so that it appears as a miniature anvil cloud. The showers which fall from these clouds are very slight, and they may even fail to reach the ground. In such clouds the evaporation near the summits is very slow and the growth of the cloud particles into precipitation elements proceeds over a long period within a succession of convective bubbles. The simple model in which the cloud droplets within a bubble are rapidly evaporated when it reaches the summit-level of the cloud, and in which the period of droplet growth is regarded as limited to the period taken for the bubble to traverse the cloud from base to summits, is therefore quite inappropriate, for it considers only the history of the bubble and not the history of the entire cloud.

Only during the building phase of a large, vigorous cloud is the model likely to be at all satisfactory, and even then it is necessary to circumvent consideration of the *cloud* history (i.e., of the early stages of the coalescense process), by appealing to the flight observations which show the existence of significant concentrations of 30 μ-radius droplets near the bases of practically all cumulus, and which therefore can be supposed to be present at the beginning of a *bubble* history.

With this reservation in mind, and remembering also that the assumption of adiabatic concentrations of liquid water is not then unreasonable but is still not supported by direct measurements, we can estimate the course of the coalescence process as a bubble rises from the cloud base towards the summits.

As before (LUDLAM, 1951) we restrict attention to the growth of an individual droplet of radius R and fall speed V, settling through a cloud of smaller droplets of uniform radius r and fall speed v, as described by the equations

$$4\varrho dR = E_1 E_2 (V - v)\, w\, dt \qquad (4)$$

where ϱ is the density of water, E_1 is the collision efficiency, E_2 the coalescence efficiency, and w is the concentration of water in the form of the smaller droplets; and

$$dz = (U^1 - V)\ dt \qquad (5)$$

where U^1 is the updraught speed and z is height (conveniently measured above cloud base).

Certain simplifications of these equations are reasonable. In the absence of any information to the contrary, the coalescence efficiency E_2 is assumed equal to 1. As before, the collection efficiency E_1 is taken as the value E_2 calculated by Langmuir, adjusted to take some account of the finite size of the collected droplets. Under average conditions of air density and temperature, and assuming the collected droplets to have a uniform radius of $10\,\mu$, the value of E_1 varies with R as shown in Table 4; these values cannot be regarded as very secure, particularly at the lower radii. Reported attempts at experimental verification give efficiencies both higher and lower than Langmuir's values (see, for example, KINZER and COBB, 1956).

Table 4. Fall-speed V and efficiency of catch $E_1 = (E_L^{1/2} + r/R)^2$, where E_L is Langmuir's efficiency of catch, of a droplet of radius R falling through a cloud of droplets of radius $r\,(= 10\,\mu)$. Air pressure 700 mb, temperature $-7°$ C.

R μ	V cm/sec	E_L	E_1
21	5.5	0.31	1.08
30	11.2	0.44	0.98
50	26.7	0.55	0.89
100	77	0.67	0.85
150	127	0.72	0.85

The liquid water concentration w is assumed to be that corresponding to adiabatic ascent from the level of the cloud base, m. In equation (5), which has been used to determine the height of the growing droplet and hence the liquid water concentration in its environment, it is preferable to replace $(U^1 - V)$, where U^1 is the speed of the updraught in the cloud, by U, the speed of ascent of the bubble containing the droplet. This is permissible in view of the circulatory and stirring motions within a bubble, and because we are not concerned with values of V greater than about 1 m/sec, which may allow the growing droplet to fall out of a bubble.

The equations can then be written

$$4\varrho dR = E_1\ (V - v)\ m\ dt \qquad (6)$$

$$\text{and} \qquad dz = U\,dt \qquad (7)$$

$$\text{whence } 4dR/E_1\ (V - v) = m dz/U\varrho \qquad (8)$$

If now E_1 and V are regarded solely as functions of R, and as m and U are functions of z, we obtain the following relation for the growth of a droplet from an initial radius R_0, at the cloud base, to a final radius R_1 at a height z_1 above the cloud base:

$$\int_{R_0}^{R_1} 4\,dR/E_1\,(V - v) = \int_0^{z_1} (m/U\varrho)\,dz \qquad (9)$$

The value of the integral on the left can be computed for various values of R_0 and R_1, while that on the right can be computed if the height and temperature of cloud base, and the variation of U with z, are given. The hypothesis that fibrillation is due to the growth of droplets from radii of about $30\,\mu$ to radii of about $150\,\mu$ can be tested by inserting these limits to the integral on the left and comparing its value with that on the right when the height above the cloud base of the fibrillation level is taken as z_1. To make this comparison it is necessary to determine the variation of U with height.

10. The speed of cloud updraughts

Probably the best method of observing the speed of cumulus updraughts is to measure the rising speed of a glider soaring inside the clouds, since the pilot is able to manoeuvre his aircraft into the updraught cores. Estimates of vertical air speed can also be made during horizontal traverses of clouds by powered aircraft, but are difficult to interpret, for the aircraft may not have passed through the core of an updraught, which occupies only a portion of the cloud. On the other hand, the existence of a circulatory motion within a cloud bubble results in the vertical air speed in its interior exceeding, by a considerable fraction, the speed of rise of the bubble (as defined by the motion of its cap), which is the appropriate value to consider when examining the growth of droplets within it. In view of these practical difficulties in exploring cumulus with aircraft we have tried to assess the rising speeds of bubbles by observing from the ground the rate of ascent of the caps of individual cloud towers.

The towers become visible when ascending bubbles emerge from the bulk of the cloud. In our experience, provided they are not glaciated, they are nearly always within one minute observed to be decelerating, and come to rest after rising a distance about equal to their diameter at the time of emergence (about 400 m). This is attributed to the strong effect of mixing with clear air, as opposed to the saturated air through which the bubble rises within the cloud. The evaporation which occurs during mixing with clear air is accompanied by a buoyancy-destroying chilling, and the volume of air which can continue to rise quickly diminishes, so that the bubble is said to suffer a rapid "erosion" (as discussed by Malkus and Scorer, 1955). According to the bubble theory, a bubble cap rises at a speed approximating to a limiting speed U given by a relation of the form

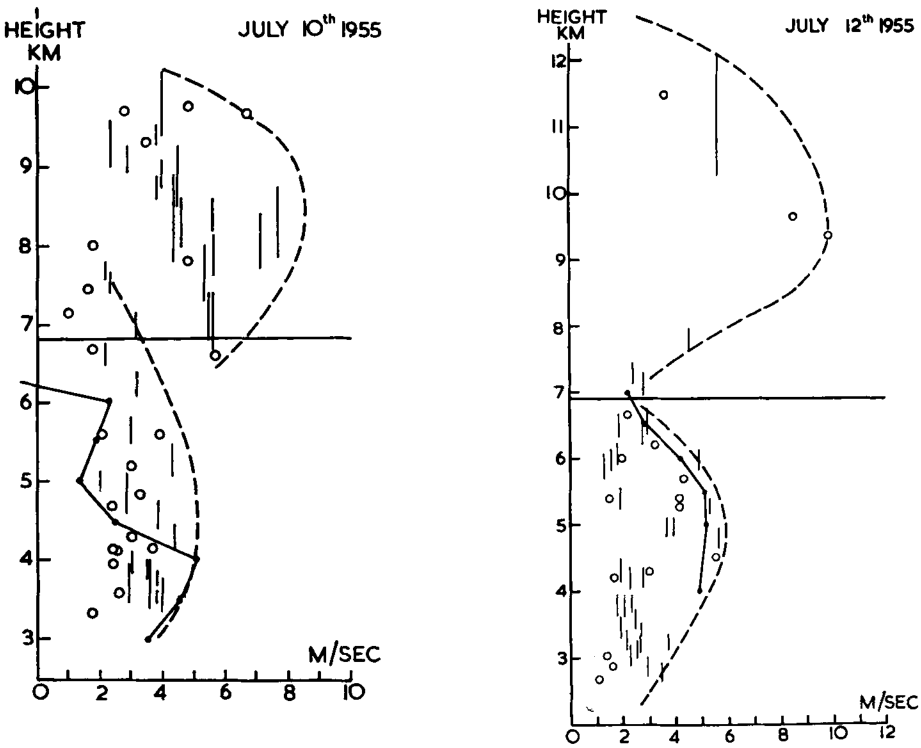
$$U^2 = kRgB \qquad (10)$$

Here R is approximately the half-width of the bubble, B is the buoyancy (usually expressed as $\triangle T/T$, where T is the virtual temperature and $\triangle T$ the excess of virtual temperature within the bubble, adjusted in the case of cloudy air to include the effect of the condensed water upon the air density), and k is a constant. The deceleration of cloud towers in clear air is associated with a decrease in R, and perhaps also in the buoyancy B, during mixing and erosion.

Evidently only the rate at which a cloud tower emerges from a cumulus is at all likely to be representative of the rate of rise of bubbles within the cloud mass. Our attention was therefore concentrated on observing the rate of rise of cloud towers as soon as possible after they became identifiable. Measurements were made directly by theodolite and subsequently also from time-lapse film records. Both kinds of observations provided an angular rate of rise, which was converted into a true rate of rise by assessing the distance and height of the towers, and, equally necessary, their horizontal motion. Usually the distance and height were known from the double-theodolite measurements; on other occasions they were deduced from the angular height of the cloud base (whose true height throughout the day was observed) or obtained from the range of associated radar echoes. The horizontal motion of the cloud towers was assumed to be that of the wind at their level, which was known from observations of the radio-sonde and special pilot balloons, and from theodolite observations of the displacement of measured clouds. On occasions of considerable wind shear the horizontal motion of a cloud tower may differ from the wind at its level by several m/sec (Scorer and Ludlam, 1953); however, the wind shear within a layer occupied by vigorous convective clouds is usually rather small, and the effect of horizontal motion was kept as little as possible by observing clouds mainly on azimuths at right angles to the wind direction. Consequently we think that the errors made in deducing the true rates of rise of the cloud towers are not likely to exceed 10 %.

When the rates of rise of emergent towers are plotted as a function of height, as in Fig. 15—16, a cloud of points is obtained. We have drawn an envelope to this cloud to indicate the maximum speeds observed, and assume that this curve represents the maximum rates of rise of bubbles within building clouds of all sizes, as a function of height. Where the observed values are smaller we suppose this to imply that the observations were made after some deceleration had already occurred, or that the bubbles had a size or buoyancy less than the maximum appropriate to their level. Strictly, it would have been better to take into account the varying size of the observed towers. However, in this exploratory study it was supposed that the smaller bubbles had already been partially eroded or had originated at levels well above the cloud base; in either circumstance they were likely to contain significantly less condensed water and therefore be less likely to produce precipitation elements than the bubbles having the maximum upward speeds. From a few observations, also, it seemed that the introduction of a tower width was not certain to reduce greatly the scatter of the points on the height-speed diagrams.

Fig. 17 shows the maximum rate of rise of cloud towers, as a function of height, from near the cloud base to the fibrillation



Figs. 15, 16. Rates of rise of emergent cumulus towers, as a function of height, on two days in 1955. Vertical lines show mean upward speeds between levels corresponding to the ends of the lines, as measured from time-lapse films. Circles represent emergent speeds over intervals of about 200 m, as measured by theodolite. The full lines show speeds deduced from soundings, on the basis of bubble theory. The pecked curves, drawn as an envelope to the points representing individual towers, are assumed to represent the maximum rates of rise of cloud bubbles inside building clouds of all sizes, as functions of height.

level, observed on one day in 1954 and on 12 days in 1955. We assume these curves define the rates of rise of bubbles inside building clouds, and use them to evaluate the integral on the right of equation (9).

11. The coalescence process and the production of a fibrillation level

Two of these curves are reproduced on figure 18, on which the ordinates now represent height above cloud base. One curve represents conditions on a day of vigorous convection (12 July, 1955) when rising speeds were as great as 6 m/sec; the other is for the afternoon of 19 August 1954, when the speeds barely exceeded 2 m/sec. We have not measured smaller rising speeds, these being characteristic of days of small clouds on which the difficulties of observation are greater.

The diagram also shows the value of the expression $S = \int_0^z (m/U\varrho)\, dz$, as a function of the height z above the cloud base, on these two occasions. In so far as adiabatic values of liquid water concentration are actually attained within rising bubbles, and in so far as the integral on the left of equation (9) has been correctly obtained, the value of the integral S at any level corresponds to a particular radius of a growing droplet lifted from near the base of the cloud by the bubbles. The value of this radius depends upon the initial radius, and scales of radii appropriate to initial radii of 20 μ and of 30μ are entered along the upper margin of the diagram. Thus at the level where the value of the expression S reaches 19.6×10^{-4} sec, a droplet of initial radius 20 μ will have attained a radius of 150 μ. It can be seen that in the lowest few

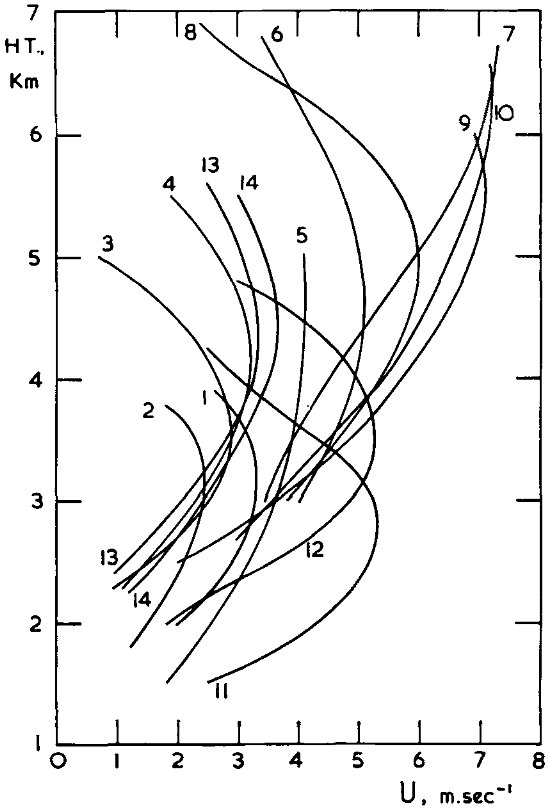


Fig. 17. Maximum rates of rise of cloud towers on the 14 occasions numbered in Table 1, as a function of height above sea level, from near cloud base to the observed fibrillation level.

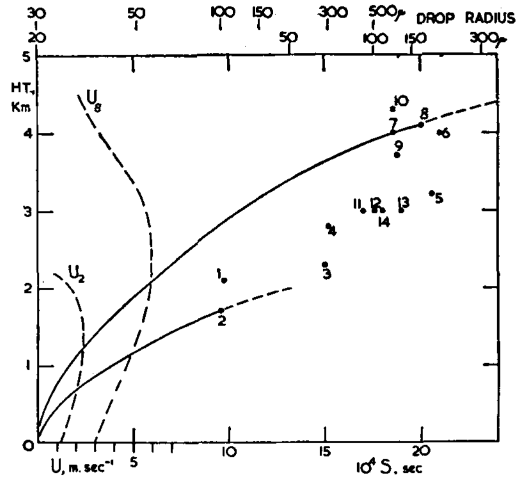


Fig. 18. Maximum rates of rise of cloud towers as a function of height above cloud base on 19 August 1954 (U_2) and on 12 July 1955 (U_8), and the variation with height of the values of the integral $S = \int (m/U\varrho)dz$ on these two days. The values at levels 300 m below the fibrillation levels are marked by dots, and the curves are extended as pecked lines to the fibrillation levels. The other dots show corresponding values on other observation occasions. The scales on the upper margin show the sizes of a droplet growing from an initial radius of 20 μ or 30 μ, which correspond to the values of the integral S.

hundred metres of the clouds, where the liquid water content is small, the contribution to the value of the integral S is small, particularly when the updraughts are strong. Thus the exact level near the base at which the growing droplets are introduced into a rising bubble hardly affects the level at which they attain radii of 150 μ, or any larger size.

We assume that inside a rising bubble at any level the drop-size corresponding to concentrations significant for shower production is determined by the size attained by the droplets of radius about 30 μ which were present in these concentrations within a few hundred metres of the cloud base and have been lifted in the bubble. This does not preclude the existence already in the cloud at this level of smaller concentrations of drops of similar size, which have had a longer history. A very important feature is the rapid droplet growth which occurs in the bubble as it approaches the cloud summits. Supposing, for example, initial radii of only 20 μ, then on 12 July 1955 the significant droplet radius at a level 3,400 m above the base is only 50 μ; at 3,900 m it is 100 μ and less than 200 m above it attains the critical value of 150 μ, while in a further 400 m of ascent the droplets have achieved the size of raindrops (radius 500 μ). This implies that in a cloud with summits about 3,900 m above the base there are no droplets in concentrations sufficient for shower production of greater radius than 100 μ, which is too small to allow them to escape evaporation in the summit towers. In a cloud only 200 m taller these droplets attain radii of 150 μ, probably cause fibrillation, and are likely to settle into the cloud bulk during the dissolution of the towers and grow into raindrops at lower levels. If the cloud tops rise a further 400 m the towers themselves become charged with raindrops. It is therefore understandable that there should be a well-defined fibrillation level, and that quite intense showers should form rather suddenly in clouds which grow only slightly beyond a critical size.

In Fig. 18 we have marked the curves which show the trend of the integral S with a dot at a level 300 m below the fibrillation level, and extended the curve upwards as a pecked line. This is because during the last three hundred metres or so of the rise of a tower its erosion is very pronounced, so that in all probability its water content diminishes rapidly and there can be practically no growth of the large droplets in this stage. As a tower dissolves the fibrillation is usually discerned about this distance below the peak height reached by the tower, and so we regard the value of the integral S at a level of 300 m below the fibrillation level as the value appropriate to the growth of drops of 150 μ radius.

The remaining 12 dots show the values attained at this distance below the fibrillation level on other days for which the calculation could be made. It will be observed that on the days when the fibrillation level is well above the cloud base the integral S has attained values of about 20×10^{-4} sec, corresponding to the growth by coalescence of droplets of initial radius about 20 μ. There is a noticeable tendency for the value of the integral corresponding to fibrillation to decrease as the height of the fibrillation level above the cloud base falls, and in fact for the afternoon of 19 August 1954, on which the integral has its lowest value, the fibrillation level implied is actually the minimum height of the peaks of clouds which produced showers, fibrillation not being observed.

In view of the impossibility of defining an initial droplet radius, and the uncertainty in the evaluation of the integrals in equation (9), the diagram can only indicate that there is a relation between the size of a cloud which is necessary for shower formation and the speed of its updraughts, and that the relation is consistent with what is known of the coalescence process. It is evident that the process is more efficient in small clouds having weak updraughts, probably because the individual clouds have long durations and are often protected from rapid evaporation at the summits by the formation of stratocumulus cumulogenitus shelves beneath stable layers limiting the cloud growth. No satisfactory models for estimating the course of the coalescence process in these small clouds and of its initial stages in large clouds have yet been evolved.

12. Summary. The role of the coalescence process in shower formation

We have shown that on a number of days when cumulus clouds were carefully observed, showers formed in clouds whose towers for the first time rose above a rather well-defined level, the fibrillation level. During the decay of towers not far surpassing this level, an *evanescent fibrous structure*, due to the presence of precipitation elements, could be seen. The height of the fibrillation level varied from occasion to occasion.

Previous flight observations of the spectra of droplet size in cumulus cloud indicate the activity of the coalescence process of droplet aggregation, even in small clouds. It appears that with some reservations the course of the coalescence process towards shower formation can be represented by a simple model of cumulus structure, in which droplets are lifted in convective bubbles from near the cloud base to the summits, where they are subject to rapid evaporation unless they have attained the size of precipitation elements (minimum radius 150 μ). The model is applicable to large clouds containing vigorous updraughts, and not to small, weak shower clouds, in which the history of the entire cloud appears more important.

The speed of bubbles rising inside building clouds can be estimated on particular days as a function of height, by observing the speeds at which bubbles rise out of clouds to produce towers. Assuming also that inside such clouds processes are not far from adiabatic, the course of the coalescence process within an ascending bubble can be estimated, and can be shown to have progressed to the stage of producing precipitation elements by the time the bubble reaches the observed fibrillation level.

It is concluded that the coalescence process has been responsible for the formation of the observed showers, even though the cloud tops sometimes have been strongly supercooled.

Other observations, to be examined in a second paper, show that the ice phase may develop in the tops of clouds when they are supercooled, even only slightly, but apparently not usually before the coalescence process causes the formation of a shower. It seems that the coalescence, having produced *large* drops of which some freeze, introduces crystal multiplication processes which after a time lead to a glaciation, or more extensive development of the ice phase than can occur only by the action of freezing nuclei in *small* cloud droplets. Glaciation in large clouds frequently causes what CRADDOCK (1949) has aptly called a "second phase" in the growth of cumulonimbus, in which new towers rise beyond the fibrillation level with high speeds and produce impressive anvils. These upper appendages to cumulonimbus may, however, have little significance for shower formation. Although in general there is a close relation between glaciation and shower formation, it appears to us that these are not necessarily cause and effect, but are often both consequences of an advanced coalescence process.

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