On the Electric Field Theory of Magnetic Storms and Aurorae

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Abstract

The electric field theory is discussed and further developed. The basic assumption of the theory is that the sun emits beams of rarified ionized gas which are electrically polarized. The electric field of the beam produces a discharge of Malmfors-Block's type around the earth. At the same time the electric field produces a variation in the Cosmic Ray intensity.

The theoretical treatment starts by an analysis of the motion of charged particles in the equatorial plane of the earth under the action of the electric field. This phenomenon is supposed to be of basic importance. The motion produces space charge in certain regions. This charge is supposed to leave the equatorial plane along the magnetic lines of force. These two basic assumptions, which are supported by the model experiments, lead to a theory which gives a fairly good picture of magnetic storms and aurorae.

In the earlier presentation of the theory the inertia of the beam was neglected. The theory gave a description of the main phase of a storm. The motion in the equatorial plane produces a clockwise ring current at about 7 earth-radii from which a discharge to the auroral zones takes place.

In the present paper also the inertia of the beam is taken into account. The result is that a second ring current is produced, which flows anti-clockwise at about 30 earth-radii. This current which seems to account for the *initial* phase of a storm, should be associated with an "*inner auroral zone*" with a polar distance of 5°—10°. The importance of observing aurorae and magnetic disturbances in this region is stressed.

Introduction

Like Chapman-Ferraro's theory the electric field theory of magnetic storms and aurorae is founded on Schuster's idea that the cause of these phenomena is a beam of ionized gas emitted from the sun, but this is almost the only point of agreement. The assumptions about the most important properties of the beam are different, and so are the descriptions of the phenomena which occur when the beam reaches the earth.

Concerning the beam Chapman-Ferraro's theory assumes that the *magnetic field inside the beam* is zero. This means that before the beam has reached the neighbourhood of the earth, the electric field in the beam is zero (apart from fields produced by diffusion and similar effects). Contrary to this the electric field theory assumes that the beam possesses a magnetic field H which is "frozen in" into the highly conducting matter, and that due to the motion v of the beam this magnetic field produces an electric field $E = -\frac{1}{c} v \times H$. Although it is possible that H and E have a complicated structure, only the simple case when E is homogeneous and tangential to the earth's orbit has been discussed.

The assumption that the magnetic field inside the beam is zero was natural at the time when Chapman and Ferraro made it. Since then, however, it has become evident that magneto-hydrodynamic phenomena are much more important in cosmical physics than assumed earlier, and today it seems very artificial to ignore the magnetic field. Indeed, it implies that the beam is produced in a region of the solar atmosphere where the field is zero.

The idea of an electric field associated with the beam has recently got much support from cosmic ray results (BRUNBERG and DATTNER 1954). In fact, it seems possible to explain the changes in Cosmic Ray intensity associated with magnetic storms as produced by the electric field of the beam, whereas this effect seems to be difficult to explain according to Chapman-Ferraro's theory.

When the beam approaches the earth it is braked by the earth's magnetic field and a system of currents is produced. In Chapman-Ferraro's theory the currents are produced by the inertia of the beam, and the current system which results is identified with the currents during the initial phase. No clear picture of the main phase is given. In some papers it is claimed that the main phase is due to a ring current of Birkeland's type. In other papers the whole current system is supposed to be localized in the upper atmosphere.

In the electric field theory the electric field of the beam is supposed to be its most important property and a magnetic storm is considered as the result of the application of a field to the environment of the earth. In the earlier development of the electric field theory the main phase of a storm was described but no explanation of the initial phase was given. According to the theory the current system during the main phase consists of an *eccentric* ring current in the equatorial plane, produced by particles moving in trochoidal orbits. This current is closed by currents along the lines of force and in the auroral zones.

The purpose of the present paper is to reconsider the electric field theory. The inertia of the beam which was neglected in the earlier presentations of the theory is included in the present paper. The result of this comes out to be that the theory describes not only the main phase but also the *initial phase*. Another result of the theory is the prediction of an "*inner auroral zone*" inside the normal auroral zone.

§ 2. *Basic assumptions*

The simplest assumption we could make is that *a magnetic storm is produced by a homogeneous electric field* in the environment of the earth.

We represent the earth's field by the field from a dipole a pointing in the $-z$ direction. The beam carries a magnetic field H_0 with it, and we must make an assumption about the direction of this field. We assume that it is parallel to the z-axis, and that it is homogeneous in the relatively small region around the earth, which we are considering. Hence the total field in the equatorial plane is

$$H = H_0 + aR^{-3} \tag{1}$$

Suppose that the velocity vector v_0 of the beam points in the $-y$ direction. This means that the sun is far away in the $+y$ direction. The electric field E of the beam is

$$\vec{E} = -\frac{1}{c}\vec{v}_0 \times \vec{H}_0 \tag{2}$$

This means that the electric field of the beam points in the x-direction. We assume that E is constant over the whole region around the earth.

The assumptions about H_0 and E which we have made are the same as made in the theory of storm variations of the Cosmic Radiation (ALFVÉN, 1949, 1950, 1954).

We shall later (in § 6) make a detailed discussion of the motion of charged particles of the beam in the magnetic field from (1) under the influence of the electric field (2), but first we shall discuss the problem qualitatively.

As in most discharges electrons are the most important carriers of electric current it is reasonable to start by discussing the motion of the electrons in the beam. As the energy is much below Cosmic Ray energies, the radius of curvature of their paths is small compared to the distance to the earth. This means that they will move in trochoidal orbits and their motion can be treated by the methods outlined in Cosmical Electrodynamics Chapter II (ALFVÉN, 1950). Due to the electric field E their motion in the equatorial plane will be a drift in the $-y$ direction. Near the dipole this drift is superimposed on an anti-clockwise drift due to the inhomogeneity of the magnetic field. The result is that the electrons in the beam will drift in orbits as shown in Fig. 1. Superimposed on this motion there may be oscillations through the equatorial plane.

§ 3. *The ring current*

It is immediately seen that this motion is equivalent to a ring current in the equatorial plane. As the electrons move anti-clockwise, the current goes clockwise. This current gives a decrease in the magnetic field in the equatorial plane near the dipole. The electrons go closer to the dipole in the $-x$ direction (which corresponds to 18^h in the afternoon if the y-axis points towards the sun). Hence the decrease in magnetic field should be a maximum

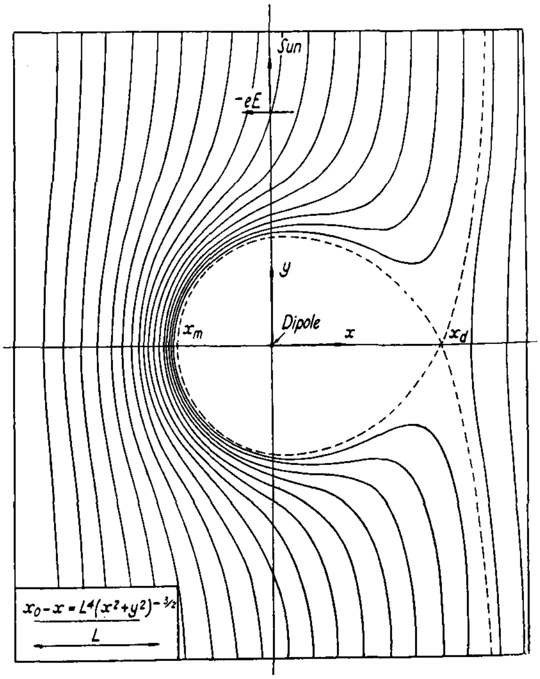

Fig. 1. Motion of the electrons in the equatorial plane of a magnetic dipole field on which is superimposed a homogeneous magnetic field, under the action of a homogeneous electric field.

at that time. This agrees very well with the equatorial disturbance during the main phase of a magnetic storm. (This can be seen by adding D_s and S_D. The general decrease in D_s during the main phase is superimposed by a negative value of S_D at 18^h and by a positive value at 6^h.)

It is important to note that in this case the ring current consists of electrons moving in trochoidal orbits. It is well known that these orbits are perfectly stable. It is of interest to compare this current with the ring current in Chapman-Ferraro's theory. In order to explain the decrease of the equatorial field during the main phase, Birkeland and Störmer supposed that there is a ring current in the equatorial plane at a few times the earth's radius. This conception has been included in Chapman-Ferraro's theory. Without any detailed arguments for it they assume that when the ionized cloud is stopped at some distance from the earth, charged particles are emitted in such a way that they move in circles around the earth, thus producing a ring current.

It has never been proved that a ring current of this type is stable. From the betatron theory we know under what conditions a particle moving in a circle with radius R in a magnetic field H has a stable orbit: If

$$H = cR^n$$

where c is a constant, the condition for stability is

$$0 > n > -1$$

As in the dipole field of the earth we have

$$n = -3$$

a circular orbit is certainly not stable. This holds for a single particle. It is very likely that it also holds for a multitude of particles. Until the contrary is proved it is reasonable to assume that a ring current of this type cannot exist.

§ 4. *The auroral zone*

Returning to the electric field theory, it is of interest that there is a certain "forbidden region" around the dipole which the electrons cannot reach. The electron density is a maximum near the border of the forbidden region. At the same time as the electrons drift in orbits of Fig. 1 they may oscillate perpendicular to the equatorial plane. When doing so they move along the magnetic lines of force (see Cosmical Electrodynamics Chapter II). If the amplitude of these oscillations is large enough they may hit the surface of the earth. As the electron density is a maximum near the border of the forbidden region it is important to see where electrons starting from this line hit the earth. This is found by projecting the borderline along the lines of force upon the earth's surface. The line which is obtained in this way has the shape shown in Fig. 2. This line is identified with the *auroral zone*. It goes closest to the pole at 6^h and its polar distance is a maximum at 18^h. The equation of the curve is given in Cosmical Electrodynamics, p. 182.

The polar distance of this curve varies with the time angle and it is important to note that the variation is of the same type as the observed time variation of the polar distance of the magnetic polar disturbance and of the aurora.

§ 5. *Terrella experiments*

We have seen that already the simple case we have discussed gives certain features, which are characteristic to a magnetic storm: viz. an equatorial ring current and a region of disturbances which may be identified with the auroral zone. The question is whether these

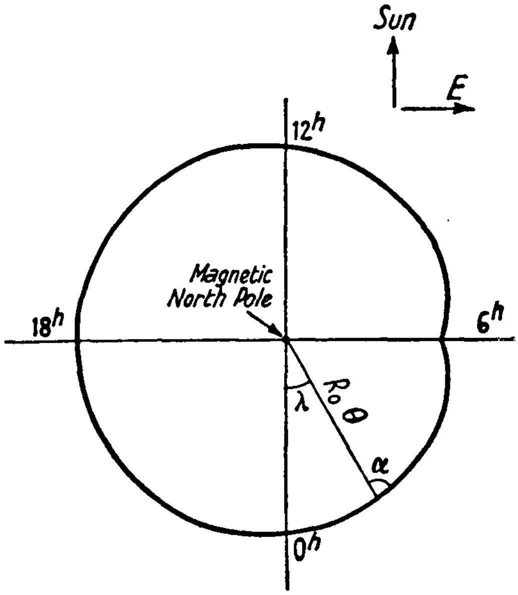

Fig. 2. Auroral curve (*I*-curve).

results are furtuitous or not. The answer depends upon how much the electric field is disturbed by the space charge, which is produced by the electrons and the ions when they enter the earth's magnetic field. Is it reasonable or not to approximate the resultant electric field as a homogeneous electric field?

It is very difficult to answer this question by a theoretical analysis. However, some light is thrown on the problem by an experiment by MALMFORS (1946) which recently has been repeated by BLOCK (1955). In some respects Malmfors-Block's experiment is similar to the famous experiment, which Birkeland made many years ago. Malmfors and Block placed a magnetized sphere ("terrella") in a chamber at low pressure and applied an electric field so as to reproduce the conditions assumed in the electric field theory. A beam of ionized gas was shot towards the terrella. It was found that eccentric luminous rings were produced around the poles as predicted by the theory. The rings are similar to the auroral zones. See Fig. 3.

It was found that even if no ionized gas was shot towards the terrella a similar phenomenon is obtained. In this case the ionization is produced by an electric discharge by the electric field. The discharge ionizes the gas mainly in the neighbourhood of the singular point x_d (in Fig. 1) and the electrons drift around the terrella following essentially a line which is similar to the border of the forbidden region. At the same time they oscillate along the lines of force and hit the terrella in the auroral zones.

In the terrella experiment the density of electrons and ions is certainly so large that space charge effects might be of importance. In spite of that the electrons move essentially according to the pattern, which has been derived without taking into account space charge effects. This indicates that space charge effects do not change the motion in the equatorial plane in a decisive way.

The reason for this may be that if space charge is accumulated in some region, it is discharged along the lines of force of the terrella. The electric field which is needed to send particles away along the lines of force is small compared to the field which is necessary in order to change the drift considerably.

Hence the terrella experiment indicates that we probably can treat the problem in the following way:

a. We calculate the motion in the equatorial plane under the influence of the given field E, taking no account of fields from space charge.

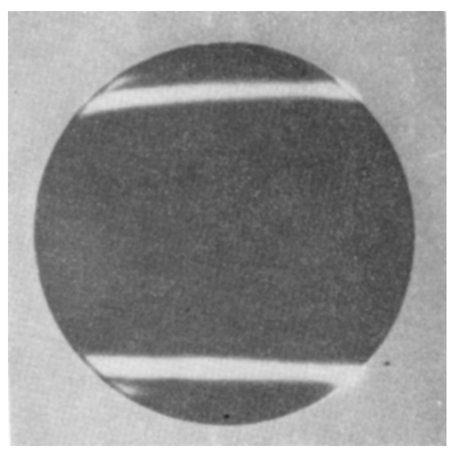

Fig. 3. *Luminous rings on the terrella, seen from the 'nightside'*, BLOCK, (loc. cit.).

b. The space charge, which is accumulated in certain regions, is assumed discharged along the lines of force to the earth (terrella).

To a second approximation we should also consider the electric fields from the space charge and the changes in the given magnetic field due to the currents.

§ 6. Motion of charged particles in a dipole field under the influence of an electric field

We study the motion of electrons and ions under the assumption that their motion is confined to the equatorial plane. The kinetic energy of the particles is supposed to be so small that the motion can be treated by the perturbation method. In the case of particles in the terrestrial field this means only that the energy must be below Cosmic Ray energies.

The motion of the particles consists of a circular motion which is superimposed upon a drift motion with the velocity

$$\vec{u} = -\frac{c}{eH^2}\vec{H}\times(\vec{f}+\vec{f^m}+\vec{f^i}) \tag{1}$$

with

$$\vec{f} = e\vec{E} \tag{2}$$

$$\vec{f^m} = -\mu \operatorname{grad} H \tag{3}$$

$$\vec{f^i} = -m\frac{d\vec{u}}{dt} \tag{4}$$

$$\mu = \frac{W_\perp}{H} \tag{5}$$

where $W_\perp$ is the kinetic energy of the particle. During the motion μ remains constant. (See Cosmical Electrodynamics p. 13.)

In the earlier development of the theory (ALFVÉN 1939, 1940, 1950) the importance of the term f^i was not realized and this term was neglected. The inclusion of this term means that we take account of the inertia of the beam from the sun. We shall find that this gives rise to the initial phase of a storm.

Because of (4), equation (1) is a differential equation. In the cases of interest to us it is conveniently solved if we calculate a first approximation to u from (1) neglecting f^i, and later introduce the value of u into (4) and find a second approximation.

The electric field gives according to (1) and (2) a drift

$$u_x^E = 0 \tag{6}$$

$$u_y^E = -\frac{cE}{H} \tag{7}$$

The inhomogeneity of the magnetic field gives the drift

$$u_x^m = \frac{3ca\mu}{e}\cdot\frac{y}{HR^5} \tag{8}$$

$$u_y^m = -\frac{3ca\mu}{e}\cdot\frac{x}{HR^5} \tag{9}$$

Introducing as unit length

$$L = \left(\frac{a\mu}{|e|E}\right)^{1/4} \tag{10}$$

we obtain

$$u_x^m = \frac{3cE}{H}\left(\frac{L}{R}\right)^4\frac{y}{R}\cdot\frac{e}{|e|} \tag{11}$$

$$u_y^m = -\frac{3cE}{H}\left(\frac{L}{R}\right)^4\frac{x}{R}\frac{e}{|e|} \tag{12}$$

As we shall see in the following the inertia term (4) is of importance only in the region so far from the dipole that (7) is much larger than (8) and (9). Hence we may compute the inertia drift from the time derivative of (7):

$$\frac{du_y^E}{dt} = \frac{cE}{H^2}\cdot\frac{dH}{dy}\cdot u_y^E = \left(\frac{cE}{H}\right)^2\cdot\frac{3ay}{HR^5} \tag{13}$$

which gives

$$u_x^i = -\left(\frac{cE}{H}\right)^2\cdot\frac{3cma}{eH^2R^4}\cdot\frac{y}{R} \tag{14}$$

$$u_y^i = 0 \tag{15}$$

The total drift velocity u is composed of u^E, u^m and u^i

$$u_x = u_x^m + u_x^i \tag{16}$$

$$u_y = u_y^E + u_y^m \tag{17}$$

We introduce the distance

$$\lambda = (a/H_0)^{1/3} \tag{18}$$

where the dipole field equals the homogeneous field H_0 and put

$$\eta = \frac{R}{\lambda} \tag{19}$$

This gives

$$H = H_0 (1 + \eta^{-3}) \tag{20}$$

and

$$u_x^i = -\frac{3mv_0^3}{eE\lambda} \cdot \frac{y}{R} \cdot \frac{\eta^8}{(1+\eta^3)^4} \tag{21}$$

with

$$v_0 = \frac{cE}{H_0} \tag{22}$$

which is the drift velocity in the beam far away from the dipole.

The value of u_x^i is small unless $\eta \approx 1$. When the particles drifting with the velocity (7) pass the region $\eta \approx 1$ they are displaced in the x-direction a distance x_λ which is:

$$x_\lambda = \int u_x^i dt = \int_\infty^0 \frac{u_x^i}{u_y^E} dy \tag{23}$$

or from (21) and (7)

$$x_\lambda = \frac{mv_0^2}{2eE} \int_\infty^0 \frac{6\eta^5}{(1+\eta^3)^3} \cdot \frac{y}{R} \frac{dy}{\lambda} \tag{24}$$

If x_λ is small so that x can be considered as approximately constant during the motion we have $y dy / R\lambda = d\eta$. The value of the integral is -1. This gives

$$x_\lambda = -\frac{mv_0^2}{2eE} \tag{25}$$

The displacement in the x-direction is so large that the increase eEx_λ in electrostatic energy exactly compensates the loss in kinetic energy $\frac{1}{2} mv_0^2$. As u_x^i decreases very rapidly on both sides of a maximum near $\eta \approx 1$ we could approximately describe what happens by saying that when the particles reach the distance λ from the dipole they are suddenly stopped and displaced x_λ.

§ 7. *Numerical values*

In order to estimate the numerical values of the important quantities, we tentatively put

$$v_0 = 2 \cdot 10^8 \text{ cm/sec} \tag{1}$$

a value which corresponds to a motion from the sun to the earth in $0.75 \cdot 10^5$ sec (≈ 1 day). Further we put

$$E = 1.4 \cdot 10^{-5} \text{ volt/cm} = 0.5 \cdot 10^{-7} \text{ e.s.u.} \tag{2}$$

a value which has been derived from the Cosmic Ray storm effect during a moderate storm (ALFVÉN, 1949, 1950, 1954). This gives from 6 (22)

$$H_0 = \frac{cE}{v_0} = 0.7 \cdot 10^{-5} \text{ gauss} \tag{3}$$

From 6 (18) we find with $a = 8 \cdot 10^{25}$ gauss cm^3:

$$\lambda = 2 \cdot 10^{10} \text{ cm} \tag{4}$$

The value of L could be derived from the polar distance Θ of the auroral zone. The polar distance of the auroral curve (compare Cosmical Electrodynamics, p. 182) varies between arcsin $0.872 \sqrt{R_0/L}$ and arcsin $1.164 \sqrt{R_0/L}$, so for a rough average Θ we have

$$L = R_0 \sin^{-2}\Theta \tag{5}$$

where R_0 is the earth's radius. With $\Theta \approx 22°$ this gives

$$L = 5 \cdot 10^9 \text{ cm} \tag{6}$$

Equations 6 (10), 6 (5), and 6 (18) give

$$\left(\frac{L}{\lambda}\right)^4 = \frac{W_\perp^0}{eE\lambda} \tag{7}$$

where $W_\perp^0$ is the kinetic energy of the circular motion of the electrons in the field H_0. This is a measure of the electron temperature in the beam. From (2) and (4) we find

$$eE\lambda = 2.8 \cdot 10^5 \text{ e volt} \tag{8}$$

The values (4) and (6) give $W_\perp^0 \approx 10^3$ e volt, corresponding to an electron temperature of 10^7 degrees. The ion temperature is probably much smaller.

The translational energy of the protons ($m = 1.6 \cdot 10^{-24}$g) in the beam is

$$\frac{m_H v_0^2}{2} \approx 2 \cdot 10^4 \text{ e volt} \tag{9}$$

and of the electrons

$$\frac{m_e v_0^2}{2} \approx 10 \text{ e volt} \tag{10}$$

From (2), (8), (9) and 6 (25) we find that for protons x_λ is of the order 10^9 cm, whereas for electrons x_λ is only 10^6 cm. Hence we can neglect the inertia drift of the electrons. We can also neglect quadratic terms of u^i/u^E for ions.

§ 8. *Currents produced by the motion*

The currents produced by the trochoidal motion of charged particles consist of the currents due to the spiralling motion and the currents due to the drifts. The magnetic field of the spiralling motion is the same as the field from the "equivalent dipoles". This corresponds to a diamagnetic effect of the gas and is usually not so important as the field produced by the drift currents so we shall primarily discuss the latter effect.

Suppose that per unit volume there are n^+ ions with charge e^+ and n^- electrons with charge e^-. The drift of the ions is u^+ and the drift of the electrons u^-. Then the current density is

$$j = n^+e^+u^+ + n^-e^-u^- \tag{1}$$

As according to § 6 u^+ and u^- have three components each it is convenient to split up the current into these three components:

$$j = j^E + j^i + j^m \tag{2}$$

As we shall see j^E and j^i are responsible for the initial phase and j^m for the main phase of a storm.

Putting $e = e^+ = -e^-$, we have from (1) and 6 (6)—6 (15)

$$\begin{cases} j_x^E = 0 & (3) \\ j_y^E = e(n^- - n^+)c\dfrac{E}{H} & (4) \end{cases}$$

$$\begin{cases} j_x^i = -(n^+m^+ + n^-m^-)\dfrac{3v_0^3}{E\lambda}\dfrac{y}{R}\cdot\dfrac{\eta^8}{(1+\eta^3)^4} & (5) \\ j_y^i = 0 & (6) \end{cases}$$

$$\begin{cases} j_x^m = (n^+\mu^+ + n^-\mu^-)\,3ca\dfrac{y}{HR^5} & (7) \\ j_y^m = -(n^+\mu^+ + n^-\mu^-)\,3ca\dfrac{x}{HR^5} & (8) \end{cases}$$

Here m^+ and m^-, μ^+ and μ^- refer to the ions and to the electrons.

Before the beam reaches the earth's magnetic field we must in average have $n^+ \approx n^- = n_0$ in order to avoid very large space charge.

During the motion in the earth's magnetic field the density will change. We can compute the density from the equation

$$\frac{\partial(nu_x)}{\partial x} + \frac{\partial(nu_y)}{\partial y} = 0 \tag{9}$$

As the inertia drift of the electrons is negligible, the electrons in the λ-region have only a velocity in the $-y$ direction. This gives

$$n^-u_y = n^-c\frac{E}{H} = \text{const.} \tag{10}$$

or

$$n^- = n_0\frac{H}{H_0} = n_0\frac{1+\eta^3}{\eta^3} \tag{11}$$

As the magnetic drift j^m goes in circles around the dipole, (11) is generally valid (except of course in the forbidden region).

For the ions the drift u_x^i gives the current (5) which is the x-component of a circular current the y-component of which is (4). This gives

$$j_x^i = -\frac{y}{x}j_y^E \tag{12}$$

From (4) and (5) we obtain after simple calculations the ion density

$$n^+ = \frac{n}{1 + x_\lambda\dfrac{x}{R}\cdot P^i} \tag{13}$$

where

$$P^i = -\frac{1}{\lambda}\cdot\frac{6\eta^5}{(1+\eta^3)^3}$$

As $x_\lambda\, P^i \ll 1$ we get approximately

$$n^+ = n^-\left(1 - x_\lambda\frac{x}{R}P^i\right) = n_0\frac{1+\eta^3}{\eta^3}\left(1 - x_\lambda\frac{x}{R}P^i\right) \tag{14}$$

This satisfies (9) (neglecting quadratic terms).

According to (7) and (8), the magnetic drift gives also currents in circles around the dipole.

§ 9. *The current system*

Hence we obtain a system of *circular* currents:

$$i = j^i + j^m \tag{1}$$

Introducing 8 (11) we find from 8 (5), 6 (22) and 6 (20)

$$j^i = K^i P^i \tag{2}$$

with

$$K^i = \frac{c}{H_0} \cdot \frac{n_0 m_0 v_0^2}{2} \tag{3}$$

and

$$P^i = -\frac{1}{\lambda} \frac{6\eta^5}{(1+\eta^3)^3} \tag{4}$$

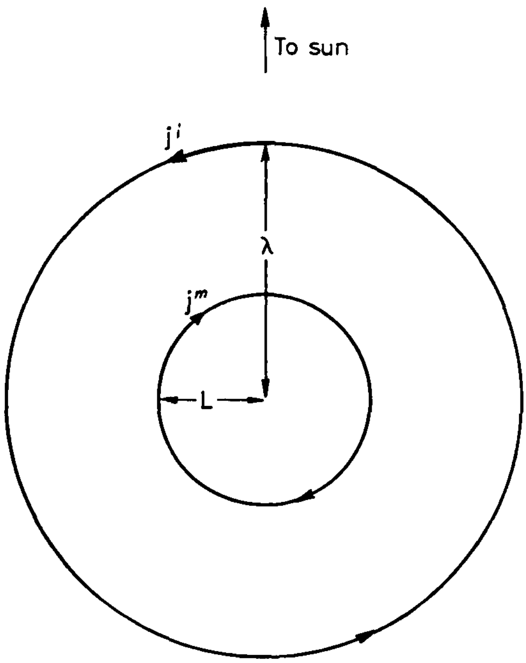

Fig. 4. Current system in the equatorial plane.

where $m_0 = m^+ + m^-$ is the mass of an ionized atom in the primary beam. Further we have from 8 (7), 8 (8), 8 (11), 6 (20) and 6 (22)

$$j^m = K^m P^m \tag{5}$$

with

$$K^m = e n_0 v_0 L_0 \tag{6}$$

and

$$P^m = \frac{3}{L_0} \left(\frac{L_0}{R}\right)^4 \tag{7}$$

where

$$L_0^4 = L_+^4 + L_-^4 = \frac{a}{eE} (\mu^+ + \mu^-) \tag{8}$$

As remarked earlier $\mu^+ \ll \mu^-$, which means that

$$L_0 \approx L_- = L$$

In a disk in the equatorial plane with the thickness 1 cm the total inertia current equals K^i and the magnetic drift current between $R = L_0$ and infinity equals K^m. The ratio between the two currents is

$$\vartheta = \frac{K^i}{K^m} = \frac{m_0 v_0^2}{2} \cdot \frac{1}{eEL} \tag{9}$$

The length L is related to the auroral zone and according to 7 (6) a reasonable value is $L = 5 \cdot 10^7$ cm. With 7 (2) and 7 (9) this gives

$$\vartheta = 0.3 \tag{10}$$

With this value given and $\lambda/L = 4$ it is possible to compute the current density as a function of the distance R from the dipole:

$$j = K^m (P^m + 0.3 P^i) \tag{11}$$

The result is plotted in Fig. 5. It is seen that outside $R \approx 0.75\ \lambda$ there is a current in the anti-clockwise direction tending to produce an increase in the magnetic field in the central parts of the equatorial plane. Inside $R \approx 0.75\ \lambda$ the current flows in the clockwise direction and gives a decrease in the magnetic field near the dipole. (Compare Fig. 4.)

According to (7) the current density goes towards infinity when we approach the dipole. However, the particles of the beam are deflected by the magnetic gradient drift so that there is a "forbidden region" around the dipole. The shape of the forbidden region for the electrons is shown by Fig. 1 which also shows the path of the electrons. The border of the forbidden region intersects the x-axis at the points $x = +\sqrt[4]{3}\,L = 1.32\,L$ and $x = -0.74\,L$ and the y-axis at the points $y = \pm 0.83\ L$. Inside the forbidden region the positive ions produce a current

$$\vec{j} = e n^+ (\vec{u}_E^+ + \vec{u}_m^+) \tag{12}$$

For the positive ions there is a similar forbidden region which is the minor image of the forbidden region for the electrons, but enlarged or diminished in the proportion L^+/L^- where L^+ and L^- are given by 6 (10).

The development of a magnetic storm

§ 10. Deformation of the front surface.

We shall now discuss what happens when a beam approaches the dipole. We assume that the beam has a plane front surface so that at a certain time t_0 there are no charged particles for $y<y_0'$ where y_0' is supposed to be much larger than λ. For $y>y_0'$ the particle density is constant $n_0^+=n_0^-=n_0$.

When $x=\infty$ the position y_1 of the front at the time t is

$$y_1=y_0-v_0(t-t_0') \tag{3}$$

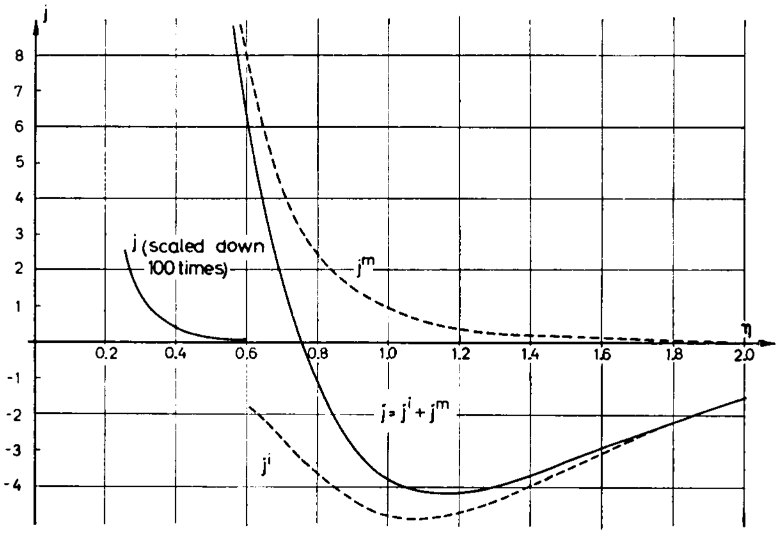

Fig. 5. The current density $j=j^i+j^m$ as a function of the distance $\eta=R/\lambda$ from the dipole.

We still neglect the electric fields produced by space charge. This means that our treatment holds only for the case when n_0 is infinitely small.

In a region where the magnetic drift is negligible (i.e. $R\gg L$) the front of the beam moves with the velocity

$$\frac{dy}{dt}=v^E=-\frac{cE}{H_0+aR^{-3}} \tag{1}$$

Putting $\lambda=(a/H_0)^{1/3}$, $R=(x^2+y^2)^{1/2}$ and $v_0=cE/H_0$ we obtain after integration

$$y_0'-y+\frac{\lambda^3}{x^2}\left(1-\frac{y}{R}\right)=v_0(t-t_0') \tag{2}$$

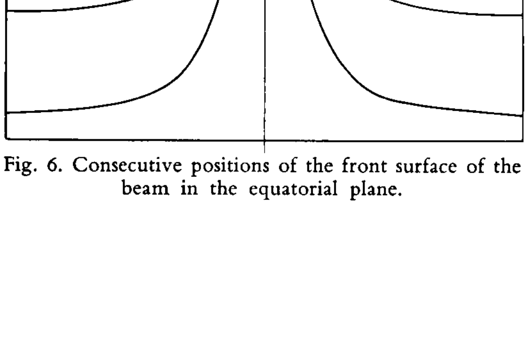

Fig. 6. Consecutive positions of the front surface of the beam in the equatorial plane.

The dipole field retards the front so that it lags behind a distance Δy. If

$$y=y_1+\Delta y \tag{4}$$

we have

$$\Delta y=\frac{\lambda^3}{x^2}\left(1-\frac{y}{R}\right)$$

or

$$\Delta y=\frac{\lambda^3}{R^2+Ry} \tag{5}$$

The shape of the front is given by (4) and (5). Fig. 6 shows its consecutive positions.

At the y-axis the position of the front is given by

$$y=y_0'-v_0(t-t_0')+\frac{\lambda^3}{2y^2} \tag{6}$$

or

$$\frac{\lambda^3}{2y^2}-y=v_0(t-t_0) \tag{7}$$

where t_0 is the time at which the point reaches $y=\lambda/\sqrt[3]{2}$.

Thus the front forms a "hollow". Its geometry has some similarity with the hollow in Chapman-Ferraro's theory, but its electromagnetic properties are quite different. When the front has reached the distance L the magnetic drift becomes important. Electrons moving anti-clockwise around the dipole will close the hollow and transform it into the forbidden region of Fig. 1.

§ 11. Current across the y—z plane

When the gas cloud approaches the dipole the currents near $R=\lambda$ will be developed first. This gives anti-clockwise currents, producing an increase in the magnetic field near the dipole. This phenomenon should be identified with the *initial phase*. When later the cloud has reached the vicinity of L clockwise currents are produced giving the decrease in the terrestrial magnetic field which is characteristic for the *main phase*.

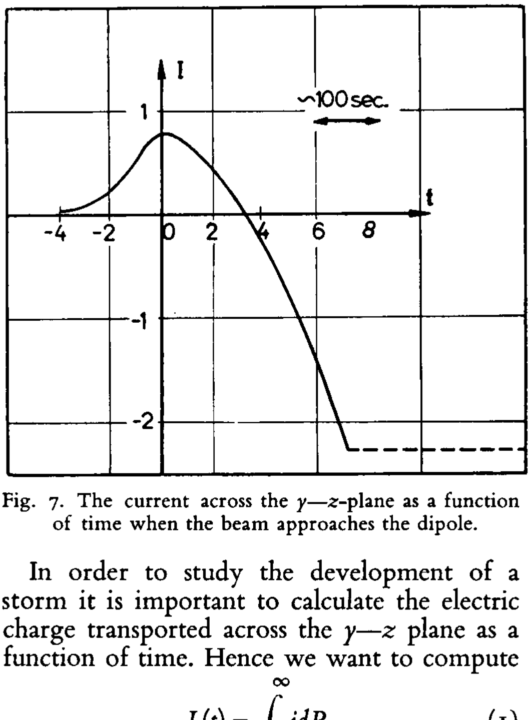

Fig. 7. The current across the y—z-plane as a function of time when the beam approaches the dipole.

In order to study the development of a storm it is important to calculate the electric charge transported across the y—z plane as a function of time. Hence we want to compute

$$I(t) = \int_y^\infty j dR \tag{1}$$

where j is given by 9 (11) and y is a function of time according to 10(7).

As

$$\int_y^\infty P^i dy = \int_\eta^\infty \frac{6\eta^5 d\eta}{(1+\eta^3)^3} = \frac{1+2\eta^3}{(1+\eta^3)^2} \tag{2}$$

and

$$\int_y^\infty P^m dy = \frac{3\lambda}{L}\int_\eta^\infty \left(\frac{L}{\lambda\eta}\right)^4 d\eta = \left(\frac{L}{\lambda}\right)^3 \frac{1}{\eta^3} \tag{3}$$

we find from (1)

$$I = K^m\left[\left(\frac{L}{\lambda}\right)^3 \frac{1}{\eta^3} - \vartheta \frac{1+2\eta^3}{(1+\eta^3)^2}\right] \tag{4}$$

which together with

$$\frac{1}{2\eta^2} - \eta = \frac{v_0}{\lambda}(t-t_0) \tag{5}$$

defines the transport of charge across the y—z-plane as a function of time.

Fig. 7 shows I as a function of time. The current crossing the y-axis can be considered as consisting of a current at a distance λ, flowing in the $-x$ direction and a current at the distance L flowing in the x direction. The λ current starts at first and increases towards an asymptotic value. The L current starts some time later and soon causes the total current in the $-x$ direction to decrease to zero and then to a negative value. When the cloud has advanced to a distance of approximately L from the dipole the magnetic gradient drift deflects the particles. The conditions become more complicated. The main result is that the current does not increase any more.

When particles reach the negative y-axis behind the forbidden region an L current with the reverse sign begins to build up. Later also a λ current at $y=-\lambda$ starts. When the whole field (except the forbidden region) is filled up the total current across the y—z plane is zero and stationary conditions are reached. The λ current flows anti-clockwise along the whole circle $R=\lambda$ and produces an increase in H near the dipole which is more than compensated by the stronger clockwise current at the distance L.

The charge transported across the y—z plane by the λ current builds up a positive space charge at the circle $R=\lambda$ on the negative side of the y—z plane and a negative charge on the positive side. The λ current near $(x=-\lambda, y=0)$ and $(x=\lambda, y=0)$ is given by 10 (4), and the difference $n^+ - n^-$ is a result of the space charge. In the stationary state there is a space charge on the circle $R=\lambda$, which is constant in time. Its density is proportional to $-x$.

The space charge produces an electric field E' which inside the circle λ amplifies the primary field E. As we are still discussing the case when the density of the charged particles is so low that electric fields due to space charge are negligible we have $E' \ll E$. Displacement currents due to the increase of E' close the current circuit until the stationary state is reached.

§ 12. *Discussion*

The theory which has been presented so far is a straight-forward calculation of what should happen when electrons and ions drift in the equatorial plane of a magnetic field 6 (1) under the influence of an electric field E. Up to now it has been assumed that the density is so low that electric fields from space charge and magnetic fields from currents are small compared to the given fields H and E.

In order to estimate the order of magnitude of the important quantities λ, L, ϑ, and the time-scale, it was necessary to introduce the values of E and H_0 of the beam. In order to avoid all *ad hoc* assumptions these values were taken from the theory of Cosmic Ray storm effects. The only value which was derived from magnetic storm observations and hence may by considered as an *ad hoc* assumption was the value of L which was derived from the polar distance of the auroral zone. (Also the assumption $\mu^+ \ll \mu^-$ or $L^+ \ll L^-$ has a similar character, but this assumption is not essential for the present discussion.)

The theory, which by neglecting space charge effects can at best be a first approximation, gives a current which at first flows anticlockwise and later is superimposed by a much stronger clockwise current. An identification with the initial phase and the main phase is natural.

The duration of the initial phase is according to Fig. 7 of the order of 10 minutes. This is less than one tenth of the observational value which is a few hours.

§ 13. *Production of space charge*

The current system which we have derived gives rise to space charge in some regions:

a. At the circle λ and around it positive charge is accumulated mainly on the $-x$ side of the y-axis and negative charge mainly on the positive side.
b. During the initial phase space charge is accumulated at or near the advancing front surface. In the λ region the space charge distribution of the stationary state is reached immediately when the front has passed. In the L region, however, a transient phenomenon occurs. When the L current across the y—z plane begins to flow the advancing front has the shape as shown in Fig. 6. The current flows along circles and produces a positive space charge at the $+x$ side of the advancing stream and a negative charge at the $-x$ side.
c. As the L system flows along circles but the forbidden region is limited by a non-circular curve, space charge is accumulated at the border of the forbidden region.

§ 14. *Currents along the lines of force*

The motion of charged particles which we have studied produces space charge in certain regions. When we go from the infinitely small densities which we have considered in § 6—12 to the densities which we have to deal with during a real storm, we find that the electric fields from the space charge become very large compared to E. There are two different possibilities:

a. The space charge effects disturb the whole type of motion so considerably that the accumulation of space charge is diminished very much.
b. The space charge produces currents along the lines of force and is discharged in this way without changing the motion in the equatorial plane.

The real case is probably somewhere between these extreme cases.

A detailed theoretical analysis of the problem is extremely difficult because it is likely that the discharge along the lines of force is associated with plasma oscillations, and at present our knowledge of these complicated phenomena is rudimentary. The assumption that plasma oscillations are important has been strengthened by Block's terrella experiment, in which the discharge easily produces "noise", showing the presence of plasma oscillations.

There is no doubt that the space charge will modify the motion of the particles to some extent. On the other hand the terrella experiment indicates that even when space charge is produced the change in the type of motion is not very large. This implies that most of the space charge is carried away along the lines of force.

Hence the picture we make of a magnetic storm is the following:

a. The motion of charged particles in the equatorial plane in the case of infinitely small density gives the basic pattern.
b. The space charge which is accumulated in some regions is discharged along the magnetic lines of force.
c. Some effects due to electric field from the space charge modify the motion but do not change it fundamentally.

If b) is applied to the currents in the L region we obtain the current system of the main phase. The circular currents in the equatorial plane close as currents along the lines of force to the auroral zones and as currents in the upper atmosphere in the auroral zones. This theory has been worked out long ago (ALFVÉN, 1939, 1940, 1950) and the magnetic field from this current system represents the real disturbances rather well over the whole earth. A possible exception is found inside the auroral zones where the real field possibly is somewhat stronger than the theoretical field (KIRKPATRICK, 1952). It is not quite clear whether this discrepancy is significant or due to some secondary effect which have not been considered. A possible explanation is that the discharge produced by the λ current gives an additional field.

If we apply the same principle to the λ current we find that the positive space charge accumulated at the negative side of the y-axis and the negative charge at the positive side should discharge along the lines of force to the polar caps. They hit the ionosphere in a region which is the projection along the magnetic lines of force of the circle on the surface of the earth.

A current system of this type seems to give a magnetic disturbance which is reconcilable with the field measured during the initial phase.

§ 15. Space charge accumulated in the equatorial plane

According to our assumptions the space charge produced in the equatorial plane should be discharged along the lines of force to the polar caps. Hence the current flowing to one of the polar caps from the surface element $d\sigma$ of a layer of thickness $\triangle$ near the equatorial plane is

$$di = -\frac{\triangle}{2}\,\mathrm{div}\,j d\sigma \qquad (1)$$

The current density is

$$j_z = \frac{di}{d\sigma} = -\frac{\triangle}{2}\,\mathrm{div}\,j \qquad (2)$$

Here j means the total current density

$$j = j^E + j^i + j^m \qquad (3)$$

As j^E is the same for electrons as for ions, it does not produce any space charge. Further j^m goes in circles around the dipole and does not either produce space charge (outside the forbidden region). Hence for the currents in the λ region we obtain from 8 (5), 8 (6) and 9 (2)

$$j_z = -\frac{\triangle}{2}\mathrm{div}\,j^i = -\frac{\triangle}{2}\cdot\frac{\partial j^i_x}{\partial x} = -\frac{\triangle}{2}K^i\cdot y\frac{\partial}{\partial x}\left(\frac{P^i}{R}\right) \qquad (4)$$

or as

$$\frac{\partial}{\partial x} = \frac{x}{\lambda^2\eta}\frac{\partial}{d\eta} \qquad (5)$$

we find

$$i_z = -\frac{\triangle}{\lambda^2}\cdot 3K^i \sin\lambda' \cos\lambda'\eta\frac{d}{d\eta}\left[\frac{\eta^4}{(1+\eta^3)^3}\right] \qquad (6)$$

or

$$j_z = \frac{\triangle}{\lambda^2}\cdot 3K^i \sin\lambda' \cos\lambda'\,\phi \qquad (6\text{ a})$$

with

$$\phi = \frac{5\eta^3 - 4}{(1+\eta^3)^4}\eta^4 \qquad (7)$$

Here λ' means the longitude angle counted positive eastwards from the midnight direction:

$$x = \lambda\eta\sin\lambda';\quad y = -\lambda\eta\cos\lambda' \qquad (8)$$

The function $\phi(\eta)$ is plotted in Fig. 8. It is evident, that in the first and third quadrants a large positive space charge is accumulated outside $\eta = \sqrt[3]{0.8}$, whereas there is a small negative charge inside this limit. In the second and fourth quadrants the signs are reversed.

The physically important phenomenon is that the circular λ current of § 9

$$j^i = K^iP^i \qquad (9)$$

consists of one component

$$j^i_1 = K^iP^i\cos^2\lambda' \qquad (10)$$

which is produced by the relative motion of the ions and the electrons and another component

$$j_2^i = K^i P^i \sin^2 \lambda' \quad (11)$$

which is produced by the displacement of a net space charge. As at the densities we have to consider in reality the latter effect becomes negligible, only the first component is of importance. This current is closed by currents along the lines of force with the current density

$$j_z' = \frac{\triangle}{2} K^i P^i \frac{d \cos^2 \lambda'}{\lambda \eta d\lambda'} \quad (12)$$

$$j_z' = \frac{\triangle}{\lambda^2} K^i \sin \lambda' \cos \lambda' \frac{6\eta^4}{(1 + \eta^3)^3} \quad (13)$$

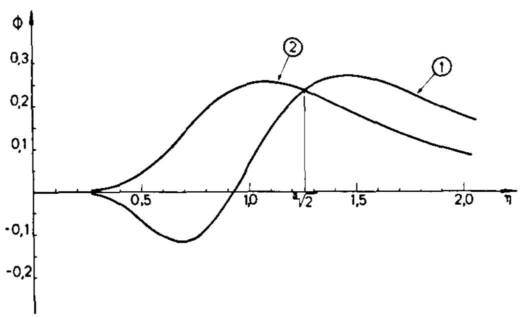

Fig. 8. Space charge accumulated in the equatorial plane.
Curve 1 calculated from eq. 15 (7)
Curve 2 " " " 15 (13)

Superimposed on this effect is a relatively small effect due to the radial component of j_2^i, which carries charge outwards. This effect changes (13) into (6). It is not so sure that this effect is more important than other secondary effects which we have neglected. For example, if the emission of space charge from the equatorial plane is delayed somewhat, the effect may vanish and (13) may be closer to reality than (6). Equation (13) is also represented in Fig. 8.

§ 16. *Projection of the λ region upon the earth's surface*

In order to find how the λ current closes over the polar cap we must calculate the polar distance Θ where the earth's surface is reached by a line of force which passes the equatorial plane at the distance $\eta\lambda$.

The flux ϕ_1 which penetrates the earth's surface between the parallel circle Θ and the equator is

$$\phi_1 = \left(\frac{2a}{R_0^3} - H_0\right)(\pi R_0^2 - \pi R_0^2 \sin^2 \Theta) \quad (1)$$

The flux ϕ_2 through the equatorial plane between the circle $\eta\lambda$ and the circle R_0 is

$$\phi_2 = \int_{R_0}^{\eta\lambda} \left(H_0 + \frac{a}{R^3}\right) 2\pi R dR = \frac{2\pi a}{R_0} - \frac{2\pi a}{\eta\lambda} + \\ + \pi(\eta^2\lambda^2 - R_0^2) H_0 \quad (2)$$

The condition $\phi_1 = \phi_2$ gives

$$\sin^2 \Theta = \frac{\eta_0}{\eta} \cdot \frac{2 - \eta^3}{2 - \eta_0^3} \quad (3)$$

with

$$\eta_0 = \frac{R_0}{\lambda} \quad (4)$$

$$\lambda = \left(\frac{a}{H_0}\right)^{1/3} \quad (5)$$

As $\eta_0 \ll 1$ we have approximately

$$\sin^2 \Theta = \frac{\eta_0}{\eta}\left(1 - \frac{\eta^3}{2}\right) \quad (6)$$

This shows that in the equatorial plane the circle

$$\eta = \sqrt[3]{2} \quad (7)$$

is the border between those lines of force which go to the earth and those going to infinity. Only space charge produced inside (7) can be discharged over the polar cap. The lines of force through this circle go to the pole.

Differentiating (6) we obtain

$$\frac{d\eta}{d\Theta} = -\frac{\eta^2 \sin 2\Theta}{\eta_0(1 + \eta^3)} \quad (8)$$

A surface element $d\sigma$ in the equatorial plane is

$$d\sigma = \lambda^2 \eta d\eta d\lambda' \quad (9)$$

where λ' is the longitude angle. A surface element of the polar cap is

$$dS = R_0^2 \sin \Theta d\Theta d\lambda' \quad (10)$$

where R_0 is the earth's radius. If a current di along the lines of force has the current density

$$j_z = \frac{di}{d\sigma} \quad (11)$$

when it leaves the equatorial plane, its density is J when it reaches the polar cap:

$$J = \frac{di}{dS} \tag{12}$$

The relation between j and J is

$$JR_0^2 \sin \Theta d\Theta = j_z \lambda^2 \eta d\eta \tag{13}$$

or

$$J = \frac{2\eta^3 \cos \Theta}{\eta_0^3 (1 + \eta^3)} j_z \tag{14}$$

§ 17. Currents to the polar caps

Introducing 15 (6) and 15 (13) into 16 (14) we obtain

$$J = \frac{6K^i \cos \Theta}{\eta_0^3} \sin \lambda' \cos \lambda' \cdot \Psi \tag{1}$$

with

$$\Psi = \frac{(5\eta^3 - 4)\eta^7}{(1 + \eta^3)^5} \tag{2}$$

if we derive it from 15 (6)
or

$$\Psi = \frac{2\eta^7}{(1 + \eta^3)^4} \tag{3}$$

if derived from 15 (13). These equations should be combined with 16 (3) in order to give the current as a function of the polar distance Θ. The formula (2) is derived if we apply our assumptions in a straight-forward way. However, it means that along the lines of force from the λ-region there should flow currents with opposite signs very close together. This is perhaps not very likely. The charges will probably get mixed so that what reaches the polar cap is more like (3) than (2). Both functions are plotted in Fig. 9, assuming

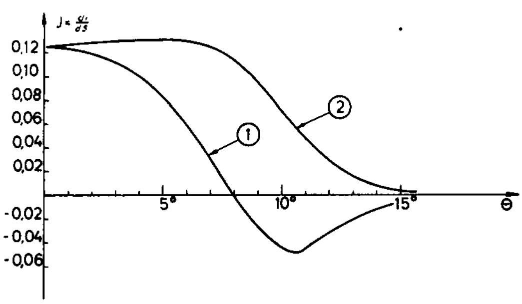

Fig. 9. Current density to the polar cap.
Curve 1 calculated from eq. 17 (2)
Curve 2 " " " 17 (3)

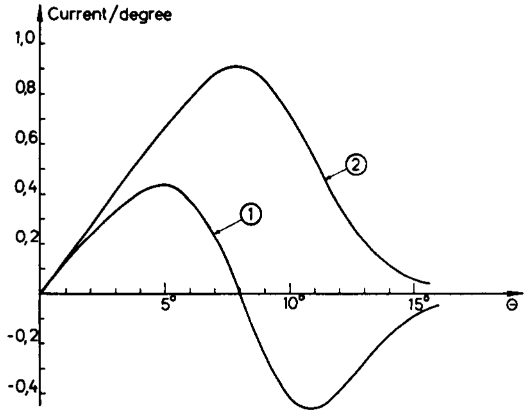

Fig. 10. Current/latitude degree to the polar cap.
Curve 1 calculated from eq. 17 (2) multiplied by $\sin \Theta \cos \Theta$
Curve 2 calculated from eq. 17 (3) multiplied by $\sin \Theta \cos \Theta$

$$\eta_0 = \frac{R_0}{\lambda} = 0.03$$

Fig. 10 shows current per latitude degree to the polar cap

$$\frac{di}{d\Theta} \sim \Psi \sin \Theta \cos \Theta$$

It should be observed that only the charge to the left of $\eta = \sqrt[3]{2} = 1.26$ in Fig. 8 is discharged over the polar cap. The rest should move away along the lines of force of the interplanetary field. Also in this respect a certain mixing may take the place, so that eventually even some of the charge outside $\eta = \sqrt[3]{2}$ is discharged over the polar caps.

The currents have maxima not very far from $\eta = 1$ which projected on the polar cap by means of 16 (3) gives

$$\sin^2 \Theta \approx \frac{\eta_0}{2} \tag{4}$$

or from 16 (4) and 16 (5)

$$\Theta \approx \left(\frac{1}{2} R_0\right)^{1/2} \left(\frac{H_0}{a}\right)^{1/6} \tag{5}$$

Introducing the earth's magnetic field at the equator

$$H_e = \frac{a}{R_0^3} \tag{6}$$

we find

$$\Theta = 0.7 \left(\frac{H_0}{H_e}\right)^{1/6} \quad (7)$$

Hence the polar distance is determined by the strength of the magnetic field H_0 in space outside the earth.

Introducing the value of H_0 from 7 (3) and putting $H_e = 0.3$ gauss we find

$$\Theta = 0.7 \cdot \left(\frac{7 \cdot 10^{-6}}{0.3}\right)^{1/6} = 0.12 = 7^\circ \quad (8)$$

§ 18. *On the existence of an "inner auroral zone"*

According to the electric field theory the main phase of a magnetic storm is mainly due to a ring current in the equatorial plane at the distance $L \approx 5 \cdot 10^9$ cm which is closed by currents along the lines of force to the auroral zones at $\Theta \approx 20^\circ$. As shown earlier (ALFVÉN, 1939, 1940, 1950) the magnetic field of this current system gives a good agreement with the magnetic disturbances which are actually observed and it also accounts for some important features of the aurorae.

In the present paper we have found that besides the L current system we should expect another system at a distance of about $\lambda \approx 2 \cdot 10^{10}$ cm. This phenomenon is a direct effect of the inertia of the storm-producing beam. The currents in the equatorial plane are likely to close in a way similar to the L current. If this occurs we should expect *aurora at a polar distance of 5°—10°*, corresponding to the projection of λ on the earth's surface.

Hence besides the ordinary auroral zone at a polar distance of $\Theta \approx 20^\circ$ we should expect an *inner auroral zone* with a polar distance which is less than half of the ordinary auroral zone. According to 17 (7) its polar distance is determined by the interplanetary field H_0.

Aurorae in the inner auroral zone should start simultaneously with the initial phase of a storm, whereas aurorae in the ordinary auroral zone should essentially be associated with the main phase. Even during the main phase there should be aurorae in the inner zone. The auroral activity should be accompanied by geomagnetic disturbances, which should have a secondary maximum in the inner auroral zone. As according to § 9 the total current in the λ-region is a few tenths of the current in the L-region, we should expect the auroral activity and the magnetic activity to be a few tenths of those of the ordinary auroral zone.

These theoretical predictions should be checked by observations. Present observational data seem to give some support of the theory but this is far from conclusive. Certainly aurorae are frequently observed inside the ordinary auroral zone, but it is not known whether the frequency decreases in a monotonous way towards the pole or—as predicted by the theory—has a secondary maximum at 5°—10°. SUCKSDORFF (1947) has analyzed the geomagnetic results from some polar stations and concludes that there are very strong currents near the geomagnetic pole. VESTINE (1947) has reported one case when there is a strong magnetic disturbance at Thule and nothing at other stations. This gives additional support for the existence of an inner auroral zone. Of course much more material is necessary before it is legitimate to draw any certain conclusions.

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