Experiments on Non-Laminar Flow of Mercury in Presence of a Magnetic Field

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Abstract

Two examples are given of non-laminar motion in an electrically conducting liquid in the presence of an external magnetic field.

Introduction

In many cosmical applications, e. g. in the interior of the sun, in the interstellar space, in the interior of the stars, in the ionosphere and when cosmic radiation is concerned, the presence of a magnetic field gives rise to forces, which are comparable with other forces. The purpose of the following discussions is to give some special examples of these effects, acting upon a liquid which is in a state of non-laminar motion.

The behaviour of an incompressible liquid conductor is strongly influenced by the presence of an external magnetic field. This is due to the fact that every motion perpendicular to the magnetic field will induce electric currents which interact with the magnetic field, resulting in forces on the elements of the liquid. In this way it can be shown that one layer of the liquid is coupled to the neighbouring layers, not only by viscous forces but also by electro-magnetic forces.

The laminar state of motion in such a liquid has been treated theoretically and experimentally by HARTMANN (1937), ALFVÉN (1942), WALÉN (1944), LUNDQUIST (1949) and many other authors. On the other hand, until now, little attention has been paid to the treatment of a non-laminar state. HARTMANN (1937) gives some aspects on this subject and the spectrum of magneto-hydrodynamic turbulence is discussed by BATCHELOR (1950) and CHANDRASEKHAR (1951).

While studying the flow of mercury in channels placed in a strong magnetic field HARTMANN (1937) found that the transition between laminar and non-laminar motion as well as the non-laminar state itself were influenced by the field. The measurements with rotating cylinders, described in Sec. 1, are in agreement with Hartmann's results as regards the non-laminar state and the explanation seems to be the same as pointed out by Hartmann, viz. a suppression of the whirls by the magnetic field. A more detailed discussion is given in a previous paper (LEHNERT, 1951).

Sec. 2 gives an example of a vortex-configuration which in some respects may be similar to KÁRMÁN's vortex-street (KÁRMÁN, 1911).

1. Measurement of the Apparent Viscosity with Rotating Cylinders

The measurement was carried out with the well-known method of balancing the torque between two concentric, non-conductive cylinders, one of which is at rest while the other

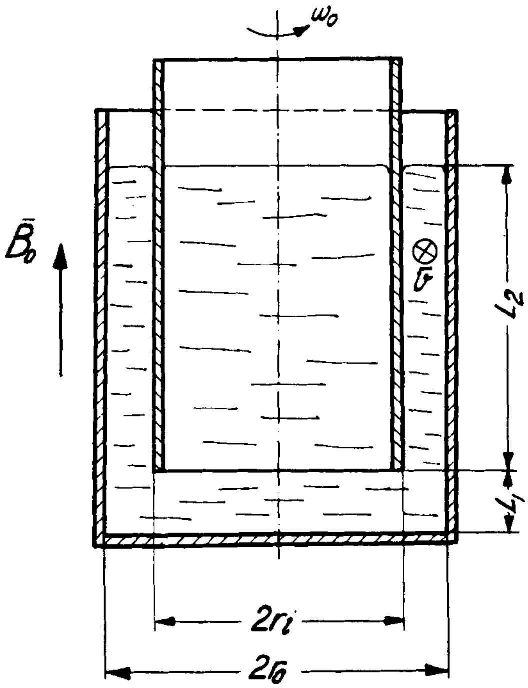


Fig. 1. Co-axial cylinders rotating with relative angular velocity ω_0. Magnetic field, B_0, in axial direction, and particle-velocity, $\bar{v}$, in a plane perpendicular to the axis.

is rotating with constant angular velocity ω_0. The space between the cylinders was filled with mercury and the external magnetic field, $\bar{B}_0$, parallel to the axis (fig. 1). The torque acting on the cylinder at rest was indicated by an elastic wire and a mirror, reflecting a light-ray on a scale. fig. 2 shows the result with the outer cylinder rotating and fig. 3 with the inner cylinder rotating.

The curves for the purely mechanical case differ distinctly from each other. According to Taylor (1922) the instability arises earlier when the inner cylinder is rotating than in the opposite case. It may be convenient to distinguish between two domains of the curves:

(i) Small velocities. The presence of a magnetic field gives rise to electric polarization. The velocity-distribution is practically undisturbed in the upper regions of the cylinders, where the velocity $\bar{v}$ varies insignificantly with the coordinate in the direction of the field. In the region between the lower edge of the inner cylinder and the bottom, however, the distribution of the mechanical case is strongly modified by the field resulting in a great increase in torque, as long as the motion is laminar. But if this "edge-effect" alters the velocity-distribution a non-laminar, thin boundary-layer may arise at the walls in presence of the field, even if the mechanical case is laminar. This may explain the fact that the increase in torque in reality becomes rather moderate.

(ii) Large velocities. The edge-effect will increase the torque both in a laminar and in a non-laminar case. But if the motion is non-laminar the field may have a second effect in respect to the torque, acting in the same or in the opposite direction as the edge-effect. If it acts in the same direction the field will always increase the

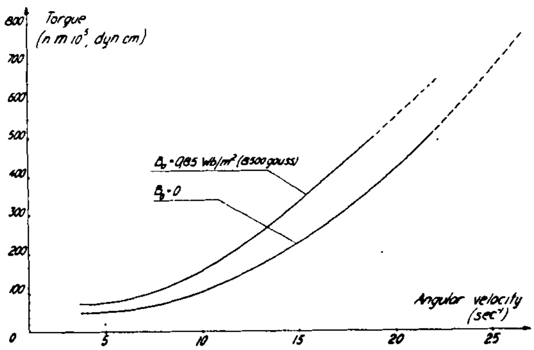


Fig. 2. Torque in newtonmeters · 10^5 (= dyn.cm) as a function of angular velocity. Outer cylinder rotating. $r_i = 37.0$ mm; $r_0 = 50.5$ mm; $l_1 = 9$ mm; $l_2 = 78$ mm.

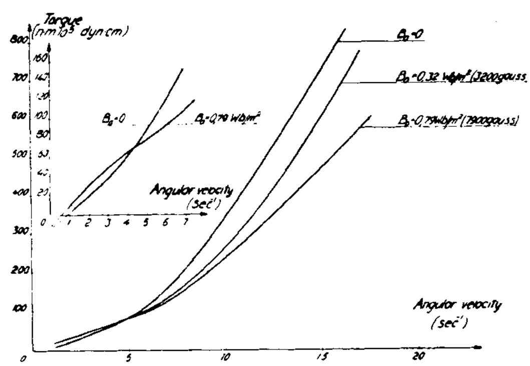


Fig. 3. Inner cylinder rotating.
$r_i = 37.0$ mm; $r_0 = 50.5$ mm; $l_1 = 11$ mm; $l_2 = 95$ mm.

torque. If it acts in the opposite direction, however, the net effect will be an increase or decrease due to the dominating one of the effects above. fig. 3 shows that the net effect is a decrease, which makes the assumption very plausible that a *non-laminar motion of a conducting liquid is suppressed by an external magnetic field, at least in certain cases.*

2. A Magneto-Hydrodynamic Vortex-Configuration

If a motion is started in a liquid with infinite conductivity and no mechanical viscosity, the presence of an external magnetic field will force the motion to be propagated along the field-lines leaving the regions outside undisturbed. In a liquid with finite, but large, conductivity and small mechanical viscosity the motion will spread outside the fluxtubes which contain the region where the motion is started, but if the fluxtubes of the liquid considered are not too long the behaviour is roughly the same as in the former case.

A special form of motion is studied with

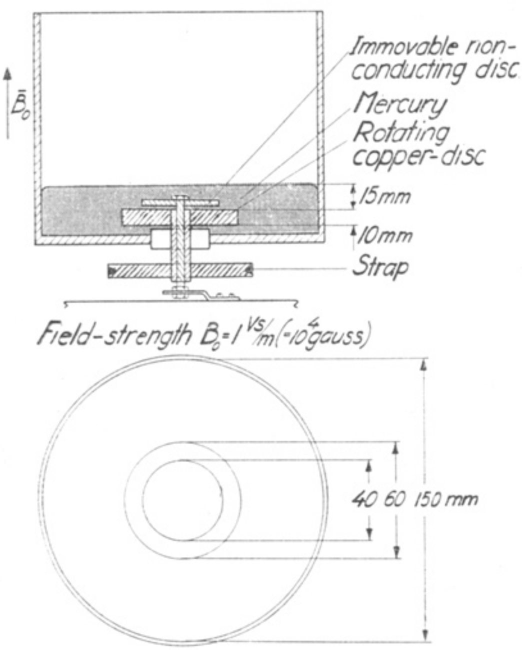


Fig. 4. Arrangement for producing the configurations of Figs. 5 and 6. The strap is driven by a motor. Field-strength $\overline{B}_0 \approx 1$ Vs/m² (= 10^4 gauss).

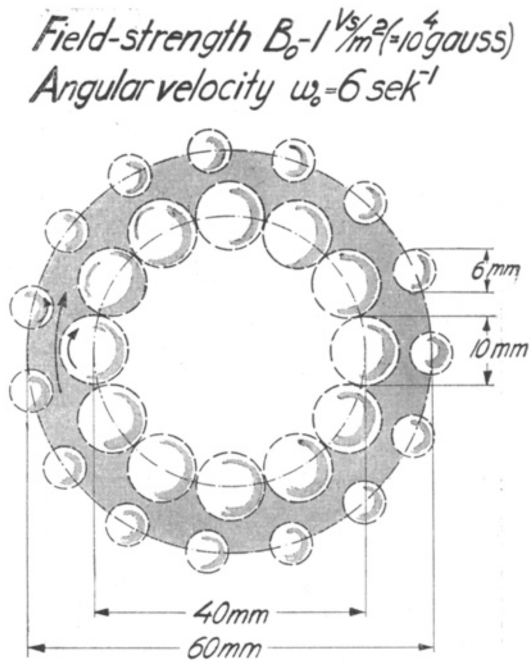


Fig. 5 a. Outline of the surface as observed when looking from the top of the vessel in fig. 4. The dark region follows the motion of the copper-disc. The small arrows indicate the direction of rotation of the whirls.

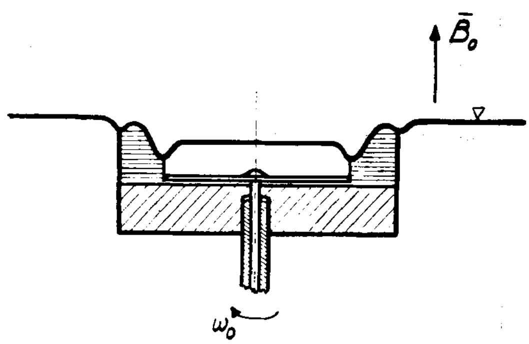
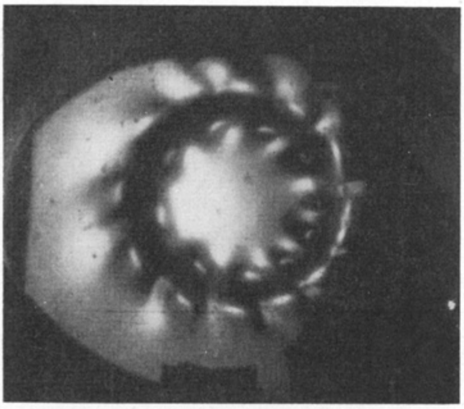
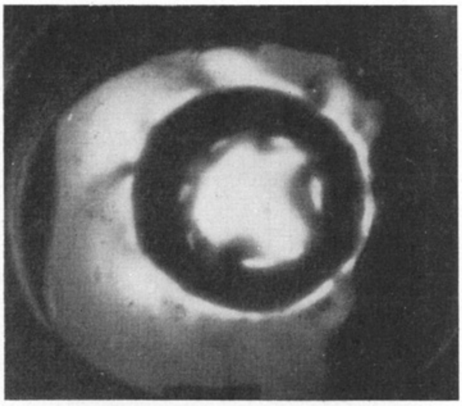


Fig. 5 b. Cross-section of the surface. The dark region corresponds to the layers following the motion of the copper-disc. The surface is horizontal in the regions at rest and the whirls are indicated by the cavities at the boundaries.

the apparatus of fig. 4. At the bottom of the vessel a rotating copper-disc, driven by a motor, is situated. The surface of mercury is about 15 mm above the copper-disc. Immediately above the latter a thin, immovable disc of non-conducting material is situated,



a



b

c

Fig. 6 a. Picture of the surface when the disc is being accelerated.
b. Stationary motion, ω_0 about 10 sec.$^{-1}$.
c. Disc suddenly stopped. In this case five large whirls moved towards the centre.

which screens off the inner part of the copper-disc. The magnetic field is normal to the discs.

If the magnetic field is absent no motion is observed at the surface when the copper-disc is set into rotation. With a magnetic field-strength of 1 Vs/m^2 (= 10^4 gauss) a motion is observed on the surface as shown by fig. 5:

(i) The region outside the radius of the copper-disc as well as the region which is screened by the immovable disc are at rest, whereas all layers situated between the radii of the two discs form a cylindrical sheet, moving with the same angular velocity as the copper-disc. The dark ring in fig. 6 b corresponds to this region.

(ii) If the linear dimensions are suitably chosen two vortex-rows are superposed on the motion of the cylindrical sheet and the centres of the whirls are situated above the edges of the discs. The rows move slowly along the sheet and the rotation is directed in such a way that they seem to roll like a ball-bearing between the moving and immovable regions.

Fig. 6 a shows the surface when the disc is being accelerated and the whirls appear distinctly. In fig. 6 b the motion is stationary with a rather high angular velocity (about 10 sec.$^{-1}$) and fig. 6 c shows the motion of some large whirls towards the centre as the disc is suddenly stopped. Stationary rotation at small angular velocities gives a picture similar to fig. 6 a. At a first glance the configuration seems to be similar to Kármán's vortex-street (Kármán 1911) in pure hydrodynamics. Kármán has shown that the only stable configuration of vortex-rows in a purely hydrodynamic case consists of two rows, the centres of the whirls in one row being situated in front of the spacing between the whirls in the other. There are, however, at least two differences between Kármán's case and that of fig. 6:

(i) The positions of the whirls in one row seems to be independent of the others as shown by fig. 6 a.

(ii) A stable system consisting of only one vortex-row exists in the magneto-hydrodynamic case; if the immovable disc is taken away the outer row still exists, which is distinctly shown by using a copper-disc of about the double radius of that in the experiment described.

Both cases above are clear examples of the importance of a magnetic field for the non-laminar motion in media with as good electrical conductivity as mercury. In cosmical applications, where the linear dimensions are much larger, the conditions for magneto-hydrodynamic phenomena are much more favourable than can ever be realized in the laboratory, and the effects will have a still greater influence on the motion.

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