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Recent Advances in the Study of Convective Clouds and their Interaction with the Environment[1]

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Abstract

Recent developments in two main phases of cumulus cloud studies are discussed; first the relations between the clouds and their energy sources in the subcloud layer and second, the interaction between the clouds and their environment in the cloud layer. Under the first heading, the various originating impulses for cumulus formation are mentioned, and how the character of this impulse affects the spacing, scale and temperature structure of the clouds. The possible origins of trade cumuli are considered. The second phase is primarily concerned with the nature of the resistive forces operating against cloud growth, how these arise and how they affect the clouds' life cycle. The resistive forces are shown to be of importance in the transports of momentum, moisture and heat by the cumuli and to serve as controls in the feeding of latent energy into the high tropical troposphere and hence into the general circulation.

I. Introduction

For the past five years or more, the cumulus cloud problem has been developing very rapidly. A summary of the recent contributions, involving an attempt to relate them to each other, shows not only that our knowledge of convective elements and their detailed structure is today far advanced beyond five years ago, but that what has been learned about convective processes is vitally connected to the most important meteorological problems, such as the vertical transports of heat, moisture, and momentum, the structure of the trade inversion, and even to the general circulation of the atmosphere.

Many of the key questions and ideas upon which the new important developments have rested were originally advanced by European or American meteorologists as far back as the 1930's. The critical observations by which to test the hypotheses and from which the orders of magnitude of the quantities appearing in the equations could be compared, however, were largely lacking until the large-scale observational programs undertaken by American agencies in the decade since the war. Outstanding among these programs were the Woods Hole Oceanographic Institution Caribbean Expedition (led by Wyman and Woodcock) in 1946, the United States Weather Bureau Thunderstorm Project (field studies 1946 and 1947), the University of Chicago Project Tyrena (Pacific Trade observations, 1947), the Hawaiian Pineapple Research Institute Field Study in 1948, and General Electric Company's Project Cirrus (flight programs beginning 1946). The contributions

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[1] Contribution No. 595 from the Woods Hole Oceanographic Institution.

arriving as direct and indirect results of these field studies are still accumulating, and form the main material for this discussion. Nevertheless, the time is now approaching when many of the newly evolved questions raised by analysis of these data must await further measurements for their answers. One purpose of this summary will be to indicate the directions led by several of these questions and some possible observational approaches that they suggest.

For the purpose of critical evaluation, convection study may be broken down into three closely interwoven phases:

1. The detailed structure of thermals, cumulus, and thunderclouds; distribution in space and time of drafts, gusts, hydrometeors, entrainment, etc.
2. Thermals and convective clouds in relation to their energy sources.
3. Interaction of convective elements with their environment; their role in transport processes.

In the following discussion, results of the first phase will be introduced only as they contribute to the second and third phases of the problem. The physics of the condensation process, nuclei, state changes, and cloud electrification will be omitted since the present summary is almost entirely restricted to thermals and smaller cumulus in which icing does not occur. For these situations it is assumed that only the effects and not the details of the latent heat release need be considered.

II. Thermals and Convective Clouds in Relation to their Energy Sources

Because of the dominant role in the energy supply of cumulus played by condensation, and its regulation by the structure of the adjacent air, the major features of individual clouds may not differ widely, despite widely different initiating impulses. Nevertheless, the process which first raises parts of the air to saturation may significantly affect several features of the cloud layer as a whole, among them the spacing and scale of the clouds and often the sign and magnitude of the cloud-air temperature difference. This originating impulse may on some occasions be provided by organized thermal-convective circulations in originally unsaturated air, by unorganized thermal-convective bubbles, by occasional eddies in a well-mixed layer randomly hitting their condensation level, by wave motions within a layer or at an interface, or by updrafts caused by flow over barriers, convergence in the large-scale circulation patterns, sometimes due to fronts, seabreezes, or frictional differences, and possibly due to other effects not yet described. Many times, the effective impulse consists of a combination of several of these factors.

Lending itself most easily to theoretical analysis is the situation where the impulse for cloud formation lies in organized dry convection. So far, most of the analytical models attempting to describe the initiation of cumulus have had as their aim the prediction of the vertical motion field in unsaturated air. Two extreme physical models have been studied, namely, first, the motion field arising from uniform heating or cooling of an air layer at a horizontal boundary. This is the classical "convection cell" problem, first raised experimentally by **Bénard** (1900a; 1900b; 1901). The second model studies the convective motions resulting from localized differential heating. Despite the different physical situations, the underlying premises of all these approaches are similar. The basic hypothesis is that if convective updrafts are predicted in unsaturated, conditionally unstable air, and if sufficient moisture is present, cumulus clouds may be expected to form at the places of maximum lifting. The hydrodynamic equations of motion, continuity, state, and energy (either the heat conduction equation or the first law of thermodynamics) are set up and lead, usually by the method of perturbations, to a differential equation for the velocity and/or temperature field. In such a use of the hydrodynamic equations, an analysis strictly applicable only to laminar motion has been carried through and transformed by analogy to apply to the mean motions in a turbulent convective situation.

Bénard type convection cells and their relation to the cloud problem

The classical or Bénard-type convection cell theories have been well summarized in a review article by STOMMEL (1947a). The complexity of these perturbation equations is so great that only limited theoretical predictions are possible. The most famous is the criterion for the changeover from molecular conduction to laminar cellular convection first established by RAYLIEGH (1916). This criterion states that when the rate of temperature decrease with height (by analogy, potential temperature in the atmosphere) exceeds a critical value, steady polygonal cells should be observed. The critical temperature gradient depends directly upon the product of conductivity and viscosity and inversely upon the thermal expansion coefficient and fourth power of layer thickness. A preferred ratio between horizontal cell dimension and layer thickness is also derivable, giving cell diameters about 2.5—3.5 times their height. Controlled laboratory results (BÉNARD and AVSEC, 1938; SCHMIDT and SAUNDERS, 1938) appear to check these criteria in the case of laminar convection. They show steady, honeycomb-like, usually hexagonal cells which break down into rolls or strips as a horizontal flow is superposed. More recent experimental studies discussed by SEIDENTOPF (1941) and KUETTNER (1949) demonstrate that as temperature gradients are further increased to about 25 times the critical value (or, in practice, layer thickness increased by the fourth root of this factor, or about 2.2 times), a new change takes place in the flow and non-steady, growing and dying, convection cells occur. These cells have stronger and more localized updrafts, diameters approximately equal to their height, and a total lifetime given by the ratio of their height divided by the mean vertical velocity.

Intriguing though these results may appear, considerable caution must be exercised in applying them to the atmosphere and in seeking the origin of cumulus clouds in such cells. In the first place, uniform heating is assumed in the model and very rarely found in nature. Second, the models are based on constant conductivity and viscosity (laminar motion). Their large-scale analogies, the "eddy" viscosity and conductivity, even if they could be measured unambiguously, vary widely with space and time, and perhaps by orders of magnitude upon the onset of turbulent convective transport. This raises serious, though not necessarily insurmountable, difficulties before an observational attempt to determine whether the critical condition for the onset of Bénard-type convection cells is in real cases reached or exceeded. Therefore, in order to see if such models have any usefulness in application to the real atmosphere, it first must be ascertained whether regular polygonal or strip-like convective patterns are found. Some qualitative evidence has been presented (MAL, 1930; DURST, 1932; 1933; WALKER, 1933; BÉNARD and AVSEC, 1938; BRUNT, 1938). In the case of cumulus formation, Bénard-type cells should be sought only in the subcloud layer over the open sea, where the heating from below can be expected to be the most uniform, and where also the lapse rate is superadiabatic. Under these conditions, there exists some suggestive but not conclusive evidence. Among the most recent and pertinent is a brilliant observational paper by WOODCOCK (1940) deducing columnar and strip-like convective patterns over the autumnal Atlantic from the soaring routines of herring gulls. This study was followed by photographic investigations of the behavior of smoke trails over the sea off Massachusetts, Florida, and San Juan, Puerto Rico; the deflections in the smoke plumes were consistent with the existence of polygonal cells in the lowest hundred or so meters above the sea (WYMAN and WOODCOCK, 1947). The hope that similar polygonal patterns could be found in trade cumulus and that their roots lay in Bénard-type convection cells led to the now famous Wyman-Woodcock (Woods Hole Oceanographic Institution) expedition to the San Juan area in 1946. Disappointingly enough, not only were no polygonal or detectably regular cloud patterns found (see Fig. 1), but these observations showed that the existence of Bénard-type cells throughout any thick portion of the subcloud layer were almost certainly precluded, since the mean lapse rate in this area was slightly subadiabatic, becoming more stable as the cloud base level was approached (see BUNKER, HAURWITZ, MALKUS, STOMMEL, 1949) thereby shielding the cloud layer from such convective motions as might exist below. Thus the question of the origin of trade cumulus was left open and is still far from settled. Due to the great importance of the trade cumulus, and their role in the budgets of the air over the tropical oceans, this question

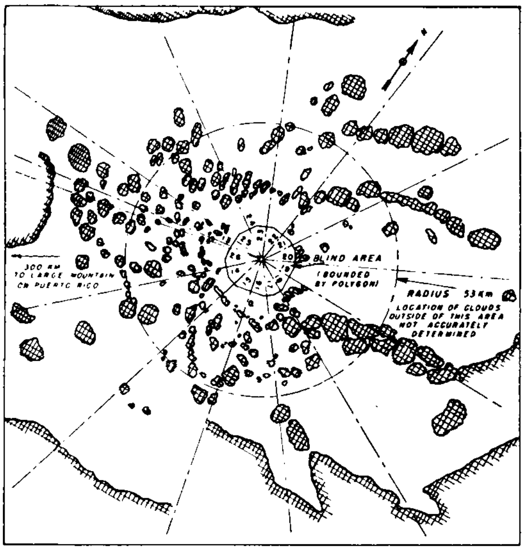

Fig. 1. Map showing distribution of trade cumuli calculated from high altitude photographs Nos. 13—26 made by the Wyman-Woodcock Caribbean expedition on the afternoon of April 25, 1946. Scale: 1 cm = 10 km. Location of center 20°00′ N., 64°25′ W. The wind was east-southeast, decreasing slightly with elevation. Note the rather irregular spacing of cloudy and clear areas. Only the medium and large clouds within the 53-km circle have been accurately located. The shaded areas are distant cloud banks. See Figure 4 for one of the original photographs. After J. WYMAN et AL. (1946).

will be reconsidered in the concluding paragraphs of the present section. Meanwhile, attention will be turned to a model of cumulus origin in organized dry convection which has had rather definite observational verification.

A model of cumulus origin based on differential heating; cloud streets

Following the results of the Wyman-Woodcock Caribbean expedition, in which the problem of the origin of trade cumulus appeared rather obscure and complicated, the attention of the Woods Hole group became directed toward the broader question of the origin and roots of cumuli in general. It was then sought to determine whether any clear-cut cases occurred in which convective clouds could be related to a specific energy source and what role they played in the transports of heat and vapor from such a source to a passing air stream. The hope was to find a sample situation which could be studied both theoretically and observationally, and that the results of such a study in which only relatively few factors were operating on the airflow might eventually provide a starting point for the examination of more complex cases.

A rather striking example of patterned convective clouds with an identifiable "source" are the cloud "streets" formed by islands on sunny summer days. These periodically-spaced rows of small cumuli (see Figures 2 and 3) are frequently seen stretching far downwind of small, flat oceanic islands when slightly stable air flows across them.

The basic hypothesis underlying this study was that the heating by the island excites one or more of the "free periods" of the unsaturated stable airstream, creating a wave-like pattern in the stream lines downstream of the "obstacle" and that condensation may sometimes occur at the wave crests. If the air is also conditionally unstable, these may be preferential spots for the outbreak of cumuli, and the wavelength of the oscillations may then be inferred from the cloud spacing. A theoretical analysis (MALKUS and STERN, 1951) was undertaken, similar in principle to those on convection cells, but in which following LANGWELL (1951a) and others, the first law of thermodynamics was the energy equation. Thus a distribution of heating as a function of space could be assumed, so that the air would be heated over the island and not elsewhere. From this, it is predicted that the production or non-production of lee waves (visible as cloud streets if sufficient moisture is present) does not depend, within wide limits, upon the character or space distribution of the heat source, but upon the properties of the air stream before it reaches the island. Specifically, no significant downstream oscillations are expected unless the atmosphere possesses some non-uniformity in the vertical, such as a change in wind speed or a very strong inversion. Second, the wavelengths of the lee waves, when they do occur, depend only upon the undisturbed stability, windspeed, and height of the discontinuity and are independent of the exact form of the heating. For islands about 10 km wide under normal insolation, the amplitude of the vertical motions in unsaturated air depends directly on the heating, inversely on the square of the windspeed and may exceed $^1/_2$ m/sec.

An observational program carried on by **Malkus** and **Bunker** (1952) showed that in cases where the island cloud streets were observed, a wind change at some upper level was found. The spacing checked the predicted wavelength in every instance (see Fig. 2 and 3). In addition to verifying the major premises of the simple theory, these observations suggested several improvements and new lines of inquiry, leading to the removal of the assumption of arbitrarily distributed heat sources and the inclusion of the effects of turbulent eddying and a well-mixed ground layer. Even without the refinements, however, this investigation has demonstrated that in some instances convective clouds do have their origin in unsaturated thermal-convective motions, and that perturbation methods predicting these motions have been successful in predicting the occurrence and scale of the clouds.

Fig. 2. Typical cloud street formed over and to the lee of Nantucket Island, Aug. 14, 1950. The low-level wind blows from right to left. The first three clouds from the right were over the island; those remaining were over the downwind waters. The spacing of the clouds was measured as 1 km, within experimental error of the predicted wavelength of lee waves. After **Malkus** and **Bunker** (1952).

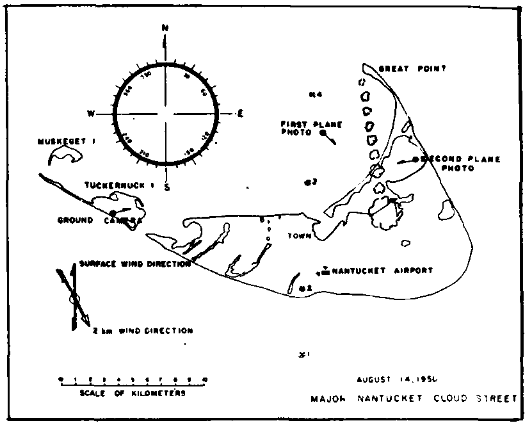

Fig. 3. Scale map of cloud street shown in Fig. 2. The numbered crosses indicate the positions at which airplane soundings were made. After **Malkus** and **Bunker** (1952).

Other cases of cloud streets have been observed which may have a similar origin, such as those formed by the flatter portions of the Hawaiian Islands.[1] In the lake country of the far north, especially on the Scandinavian peninsula, a common sight is 50—100 km long rolls of cumuli extending parallel to the wind. These have been described by **Kuettner** (1949). Since they appear mainly on sunny days following cold fronts, it is quite likely that their origin also lies in differential heating, although wind shear and the earth's rotation must play an important role in their maintenance.

Certain common features, however, may be sought in all cumuli whose individual origin thus lies in any organized dry-convective updrafts and these may provide a way to recognize this class of clouds from preliminary observations. First, they will usually form in recognizable patterns more or less simultaneously; second, such clouds will possess "roots" or detectable updrafts of cloud scale extending well below their bases and through a large fraction of the subcloud layer; and third, like several other categories of cumuli, they may at times be colder than their surroundings (discussed by **Bleeker** and **Andre**, 1950) if the initial updrafts are sufficiently strong.

Cumuli formed in lines or areas of convergence

Frequently over heated peninsulas or islands, of cumulus clouds appear which can definitely be associated with a "seabreeze front". These have been studied in Hawaii by **Leopold** (1949) and over Cape Cod, a peninsula on the eastern

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[1] Described to the writer by Mr. Wendell A. Mordy of the Hawaiian Pineapple Research Institute.

United States seaboard, by **Malkus, Bunker,** and **McCasland** (1951). These seabreeze clouds owe their existence only indirectly to heating, which may not be strong enough to release the conditional instability except upon the addition of a sharp line of convergence at the zone where the seabreeze meets the gradient wind or the opposite seabreeze. Whether the updrafts caused by the clash of these two wind systems serve merely as a trigger to release self-sustaining convective clouds with warmer temperatures than their surroundings, or whether the initial upward velocity is sufficient to maintain clouds which consume energy and may thereby be colder than their surroundings probably varies from instance to instance. In any case, the individual clouds would not be expected to show deep roots. This expectation was supported by the Cape Cod study, in which the seabreeze cumuli were noticeably warmer than their environment at all levels.

Another way in which convergence extended over wider areas may serve to trigger the outbreak of cumuli is by vertical stretching of the so-called "mixed" or ground layer. If the top of the mixed layer over a wide peninsula, for example, lay somewhat below the condensation level and then a seabreeze circulation or convergent flow of other origin were superposed, so that in places the top of the mixed layer approached or reached the condensation level of the air parcels within it, random eddying would, under suitable conditions of instability, cause clusters of cumuli to break out in these places. Such spreadout, gradual convergence probably plays a significant role in the onset of convective cloudiness over the Cape Cod and Florida peninsulas (see **Byers** and **Rodebush**, 1948; **Byers** and **Braham**, 1949) even when no sharp "seabreeze front" is present, and convergent flow (perhaps of other origin) probably remains as the most plausible explanation of the cluster-like formations of trade cumuli, as shall be suggested presently. Clouds triggered in this manner clearly will not show roots in the sense this term has been defined in the preceding discussion, although warm, moist pockets may be detectable in the subcloud layer prior to their appearance.

Probably the most important effect exerted by convergence on convective clouds, however, lies not just in triggering their outbreak, but in determining to a large extent how far they will develop once condensation has occurred. The role of convergence in the interaction of clouds and environment is discussed in the concluding paragraphs of this paper.

On the possible origins of trade cumuli

The trade cumuli are one of the most dramatic and vital features of the subtropical atmosphere (see **Riehl**, 1950). Day and night they grow and decay over a hundred million square kilometers of warm oceans, their bases at the small stable stratum topping the mixed surface layer (~ 600 m), their tops pushing against, and some of them penetrating, the trade inversion (base averages 2 000 m). Their degree of development, ranging from scattered cumulus humilis to large cumulonimbus, is controlled by the height of this inversion, which in turn is regulated by the divergence field of the passing synoptic systems. In relatively undisturbed weather, they have the characteristic appearance shown in Fig. 4, and form in clusters separated by about 30-km wide clear areas, as shown in Fig. 1. As these cloud columns rise, eventually mixing partially with the inversion air aloft, they carry with them vast amounts of energy in the form of latent heat, raised initially from evaporation at the sea surface. It was shown by **Riehl, Yeh, Malkus,** and **LaSeur** (1951) that the net heat and vapor transported upward by these cumuli is easily sufficient to account for the wellknown downstream rise in height of the trade inversion, which goes on in the face of overall subsidence and divergence. Therefore, the relation between these clouds and their moisture and energy sources in the subcloud region warrants study.

Evidence on these points is still extremely scarce, and at first glance, perhaps even contradictory. However, for one particular locality, namely the Caribbean near San Juan, in one relatively undisturbed April period, certain deductions may be safely drawn. First, these individual clouds did not have roots. This is not surprising in view of the preceding discussion (page 75) which showed that even if thermals or convection cells were found at low levels above the sea (at ship's deck level, for example), the stabilization of the air as

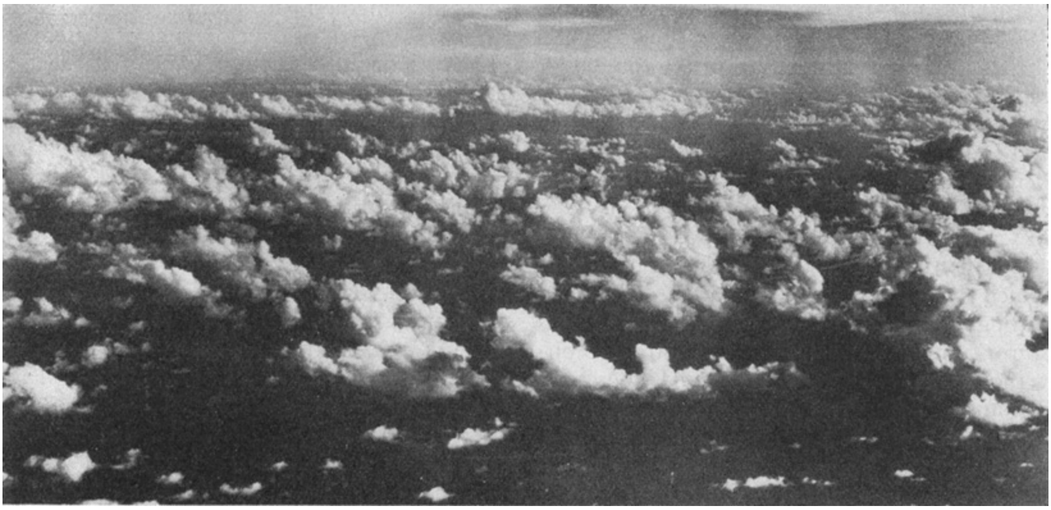

Fig. 4. High altitude photograph No. 15 of trade cumulus made by the Wyman-Woodcock expedition on April 25, 1946. One of the series of photographs used to construct Fig. 1. The camera was pointed outward from the center along line 15 in Fig. 1, so that the photograph gives a cross-section exactly in the plane of the east-southeast wind. The wind therefore blows from left to right across the photograph and decreases with height. After WYMAN et AL. (1946).

cloud base is approached would damp them out long before saturation. Furthermore, BUNKER et AL. (1949) showed that the major features of the subcloud layer, with maximum turbulence at 300 m, decreasing to a minimum at cloud base, were incompatible with the deep penetration of convective updrafts. Supporting evidence was found in horizontal accelerometer runs flown below cloud base, in which no increased turbulence appeared under the clouds.

A second significant feature of the trade cumulus studied by the Wyman-Woodcock expedition was their buoyancy. The in-cloud temperatures were, on the average, warmer than those of the environment, wavering between a + 1° C excess and a — 2° C deficiency near the cloud tops. The buoyancy forces were calculated in two cases by STOMMEL (1951) and in each case were positive through the lower portion of the cloud, becoming negative in the top few hundred meters, indicating overshooting. It seems likely that during a major fraction of their life cycles, these clouds were self-driving and not running on the energy provided by an external impulse.

For these two reasons, it is not reasonable to seek the origin of these clouds either in dry thermals of cloud size or in very sharp convergence lines in the flow, but most plausibly in gradual variations in the depth of the mixed surface layer. On the average, the top of the mixed layer was found to be at about 85 % the height of the lifting condensation level of the lowest air. Variations of several hundred meters from day to day and even within a few hours on the same day were noted. On some occasions its top reached or exceeded the condensation level, and it was, in the mean, noticeably higher in cloudy than in clear areas. One or more mechanisms, then, causing spatial oscillations in the thickness of the mixed layer are needed. These are probably of the dimensions of the cloud clusters, random eddying in these regions accounting for the outbreak of individual clouds. It is readily shown that fluctuations in the thickness of the mixed layer are only very weakly coupled with variations in the height of the trade inversion and the depth of the moist layer as a whole. Attention must therefore be directed to independent, weakly convergent flows set up within the subcloud layer itself. LANGWELL (1951b) has proposed unstable gravitational waves at the interface between the nearly adiabatic ground layer and the isothermal stratum at its top. One should also inquire about more random causes such as slight inhomogeneities in sea surface roughness, radiative properties, or temperatures on a 10—30 km scale.

The deductions just made should not be generalized without more observations to apply to the origin of trade cumuli in all cases. In several examples of Pacific clouds, Barrett and Riehl (1948) have presented evidence which may indicicate negative buoyancy throughout a cloud. To be sure, their examples were cumulonimbus, indicating strong convergence in the large-scale flow. Under such circumstances it is conceivable that an initial updraft might provide enough kinetic energy to build a cloud against both friction and slight stability. The orders of magnitude cannot be estimated accurately until more is known of the resistive forces on clouds. In conclusion, the evidence gathered so far by no means precludes the possibility that the origin of trade cumuli may be quite different over different parts of the subtropical oceans or even over the same region at different times. This suggestion is supported in the next section when resistive forces on clouds are discussed.

III. The Interaction of Convective Elements with their Environment

The preceding section has dealt with the relation between the clouds and the subcloud layer. Various mechanisms were discussed for raising heat and moisture through this layer so that they are available for cumulus formation. How these properties are further transported within and through the cloud layer depends upon the dynamics of the individual clouds, and particularly upon the interaction of the cloud with its environment.

This approach to the cloud problem as part of many larger ones was relatively new with the Wyman-Woodcock expedition and was phrased clearly in the first written report of this group (Wyman and Collaborators, 1946). In the classical development of meterology, cumulus clouds were regarded almost entirely as "weather phenomena", to be predicted as part of the synoptic situation. The main interest was, therefore, in "stability criteria" and the early work on convection consisted primarily of the evolution of thermodynamic methods to predict the onset and probable degree of development of the clouds. Their internal structure was regarded as interesting mainly as it led to precipitation or affected aviation. The interaction of the ascending parcel or column with its environment was ignored.

Dissatisfaction with these results soon appeared, however, when it was frequently found that cumuli failed to grow tall even when the parcel method stated that a vast amount of energy was available to them. Papers by several prewar writers pointed to the inhibitory effects exerted by the environment, in particular a brilliant paper by Namias (1939), who described the cutting off of tall cumulus towers by mixing. Christians (1935) and other European authors set up a theoretical framework for discussion of the several resistance forces operating against the growth of cumulus clouds, but at that time there were not adequate data even to estimate correctly their orders of magnitude, and hence their possible importance in a cloud's life cycle. Observations leading to the quantitative calculation of one of these resistance effects were for the first time provided in 1946 by the Wyman-Woodcock expedition to the trades.

This expedition found that temperature lapse rates inside clouds were, contrary to classical expectations, not moist adiabatic but considerably steeper, and that the in-cloud temperatures fluctuated with height, as if some sort of turbulent mixing process were taking place. At this point, Stommel (1947b) introduced, by analogy with the simple fluid jet, the idea that environment air was continuously "entrained" and mixed with the rising cloud air. From comparison of temperatures and mixing ratios inside and outside the cloud, he devised a method to determine the fractional amount of the entrainment, level by level. In the tradewind clouds studied, it frequently corresponded to a doubling of the mass flux of the rising air in 1 km ascent.

While the purpose of the entrainment paper by Stommel was specifically to explain the departure from a moist adiabatic lapse rate inside trade cumulus, it provided the cornerstone for many contributions to follow. From consideration of the effect exerted by the surrounding air upon a cloud and its logical converse, the effect exerted by the cloud upon the surrounding air, a framework can be set up to investigate many previously baffling problems. For example, it can now be understood why observations sometimes showed cloud tops which ceased growing while warmer than the environment, and why "pulsating" cumulus towers are rare. A suggestion can be made concerning the different features observed in trade cumuli contrasted with their middle-latitude counterparts. The effects of windshear upon convection can be discussed analytically, leading to at least preliminary models of convective clouds as transport elements. In effect, the role of cumuli, especially of trade cumuli, as an integral part of the layer in which they grow, rather than as incidental photogenic objects sometimes acting up to produce aircraft hazards and rainfall, is now a fruitful topic of investigation. Clearly, the question of predicting the onset and growth of these clouds from initial air conditions becomes considerably more complex as a result of entrainment and other resistive effects.

Stability criteria and their modifications by resistive forces

The model of the air parcel which rises without interaction with its environment, while now regarded as a drastically oversimplified one to describe real cases of convection, still serves an important function as a clearcut limiting case, just as the undamped harmonic oscillator is a clearcut limiting case in studying the behavior of real, frictionally damped oscillators. The original purpose in setting up stability criteria was to establish the conditions of thermal stratification in the environment under which an air parcel when displaced slightly from its equilibrium position would acquire an initial acceleration away from its origin. Since a rising, saturated, non-interacting air parcel would cool at the moist adiabatic lapse rate, the well-known result was obtained that if the environment cooled more rapidly with height than this rate, the ascending parcel would become less dense than its surroundings and would accelerate upward.

Strictly speaking, these criteria regarding initial accelerations stand unmodified, since frictional and resistive forces come into action only after an initial velocity has been acquired. The difficulty arises in attempting to apply them to finite convective motions. The first quantitative recognition that buoyancy forces were cut down by the parcel's environment came with the slice method of J. BJERKNES (1938). He recognized that the compensatory sinking (usually dry adiabatic) of the outside air would cause its warming, thus cutting down the relative temperature excess of the rising parcel and thereby its buoyancy. This effect was shown to be greater the more a region was filled with updrafts because of the requirement thereby of more limited, hence stronger downdrafts and consequent intensified warming of the sinking air.

The entrainment of drier environment air, however, ordinarily reduces buoyancy forces far more than does compensatory sinking. Simple calculations show that even under exaggerated conditions of 3/10 of the area occupied by active updrafts, the effects of compensatory sinking reduce the buoyant energy predicted by the parcel method only by about 40 %. Upon entrainment of a normally observed fraction of outside air with relative humidity about 70 %, the buoyant energy may be reduced by over 90 % from that predicted by parcel considerations.

AUSTIN (1948) has discussed in detail the reduction in buoyant energy under various assumed rates of entrainment and environment humidities. He showed what the lapse rates inside clouds would become, and hence what environment lapse rates would be necessary for cloud growth, thus illuminating the inhibitory effects of a dry environment upon the development of large cumuli. Such methods, however, can only be expected to give statistical probabilities for cloud growth, rather than clearcut predictions from an initial stratification. The reason is that one or more assumptions, usually including that of an average entrainment rate, is required. Actually, the entrainment rate which will prevail at a given time, under given conditions is itself a complex, adjusting function of the vertical motions, cloud size and other factors, and may be highly variable. Hence it is probably no longer fruitful to discuss convection from the point of view of onset criteria, parcels, and initial perturbations, except for reference, as is the undamped harmonic oscillator in physics. The more difficult problem of a cloud's "life cycle" and the interacting adjustments between buoyancy and resistance after the updrafts are of finite size must sooner or later be tackled.

Present knowledge of the resistive effects on cumulus clouds

It has been shown that the buoyancy forces producing cloud growth are reduced by two causes: first, by the compensatory sinking of the environment; and second and more drastically by the entrainment of drier outside air. Clearly, if the buoyancy forces on its air are upward, the cloud grows, and hence for stability criteria, the net buoyancy is all that need be determined. Nevertheless, these two effects, compensatory sinking and entrainment, further reduce the kinetic energy in an existing updraft via ordinary friction or reduction of the draft's upward momentum. Therefore, these too must be considered in a "life cycle" approach, since anything reducing the vertical motions in turn acts back to reduce the resistive effects, which control the buoyant energy release, and so forth. The nature of these "drag" forces has been set forth in an excellent treatise by SCHMIDT (1947) who shows that they are exactly the same forces which would oppose the motion of a non-buoyant jet of water created within a resting fluid. The first he calls "form" or "profile drag", and the second "skin friction", similar to terminology in aerodynamics. The form drag is due to the pushing aside of the surrounding medium as the jet grows and accelerates it back as a "countercurrent". The skin friction arises from viscous forces at the moving boundary, which in the fluid jet give rise to entrainment. In contrast to aerodynamics, however, in which solid bodies are moved through fluids, the form or profile drag may prove to be neglectable in many aspects of the cloud problem. Clearly, it will be most important as the cloud tower grows rapidly upward and the countercurrent is being established, and less so when the cloud later approaches a steady state. Entrainment and its consequent reduction of upward velocity, however, is clearly operative throughout the entire life of the updraft. This conclusion concerning relative drag magnitudes may be invalid, though, in the later considerations of the horizontal motion of the updraft through the air, since a horizontal countercurrent must then continually be produced similar to the case of a moving solid, thus creating a pressure difference in the horizontal across the draft.

For preliminary models of a cloud's growth and life cycle, it would indeed be fortunate were both resistive effects due to countercurrents neglectable compared to those due to entrainment. In view of the above discussion it seems plausible to proceed upon this tentative assumption, especially since both buoyancy reduction and momentum reduction by entrainment can be readily dealt with analytically. The buoyancy reduction by countercurrents was shown to be relatively smaller, and their frictional effects seem prohibitively complex at present. Intensive observational tests of such models will then be called for, to determine whether this assumption requires later modification.

Preparatory to any analytic treatment of the interaction of entrainment and buoyancy in cloud dynamics, however, a further discussion, of the entrainment mechanism is required. Two points of view prevail at present concerning this topic. It is this writer's belief that these points of view are by no means incompatible, but rather arise from two different operational definitions of entrainment. STOMMEL (1947b) calculated the proportion of environment air of known properties which must have mixed with the rising air to produce the observed in-cloud distribution of temperature and moisture. By his analogy with the spreading jet in fluid mechanics he implicitly assumed that all this mixed air was dragged along with the updraft, permitting its cross section to expand with height. Other writers (AUSTIN, 1948; HOUGHTON and CRAMER, 1951) have assumed explicitly that the draft cross section remains nearly constant with height, in more apparent agreement with existing sparse observations. The latter writers contend that the major entrainment is that which is required to preserve continuity in a constant cross-sectional cloud whose ascent rate increases upward. The observed entrainment rates called upon by them are based upon direct measurement of inflow winds around clouds, such as the calculations described by BYERS and BRAHAM (1949) and are much smaller than those found by Stommel. Clearly, there is no conflict between these two viewpoints if the possibility is admitted that all the air which is in or becomes mixed with that ascending is not retained indefinitely within the updraft, but that some is "detrained" during the ascent.

Stommel's calculations might be said to give the "gross entrainment" or the total amount of environment air mixed with cloud air, while the Austin, Houghton-Cramer method considers "net" or as they call it "dynamic" entrainment, the difference between the two amounts in a given cloud being detrained. An observational test of this hypothesis would be provided if a vertical velocity profile of the cloud were measured simultaneously with in-cloud and environment soundings. The amount of net influx required to preserve continuity could then be calculated and compared to the total amount of outside air mixed with cloud air. Although such measurements are still wanting, some less direct evidence supporting this hypothesis appears in the following sections.

Recent approaches to cloud dynamics

HOUGHTON and CRAMER (1951) have constructed the first completely analytic model of the interaction of buoyancy, vertical motions, and entrainment in a cloud. Although their work, because of the assumptions necessitated by present data, cannot yet be regarded as a "life cycle" model, it nevertheless represents a significant step in this direction.

Under the basic assumption that dynamic entrainment be the only resistive effect, these authors obtained and solved differential equations for the height distribution of vertical motions, net mass influx, and liquid water. These equations were derived from the laws of mass, momentum and energy conservation under the additional restriction to a steady-state cloud of constant cross section. Their work has the great advantage over previous attempts in that the entrainment rate need not be assumed or inferred but appears as one of the dependent variables in the system of equations and hence can be predicted. The vertical velocities so calculated are of the observed order of magnitude, but probably somewhat too great. Their derived influx rates are comparable to but larger than those found by the Thunderstorm Project (BYERS and BRAHAM, 1949) and smaller than most of those found by STOMMEL (1947b). They found that although the liquid water content of their clouds was considerably less than that predicted by the parcel method, dynamic entrainment of even very dry outside air does not suffice totally to "dry out" the cloud. This appears to be in conflict with many observations (MALKUS, 1952). Clearly, then, the greatest departure from realism of such a model is that it fails to provide a way in which updrafts may die out with height, other than by increasing stabilization of the environment. Dynamic entrainment ceases to operate as a resistance when the ascent rate has stopped increasing upward, since no more influx is then required to maintain continuity. The authors, however, themselves suggested that additional resistances, such as form or profile drag, weight of liquid water, or further mixing with the environment probably should be introduced.

A second entrainment paper by STOMMEL (1951) appearing simultaneously with that of Houghton and Cramer provides interesting comparison with their work. He computed, again using the Wyman-Woodcock soundings, the vertical velocity distribution within these trade cumuli. Similar to Houghton and Cramer, he assumed a steady state and that the only resistive effect was the entrainment inferred from observed soundings. The vertical velocities obtained were comparable to those of Houghton and Cramer, although the environment air of the tradewind clouds was far moister. Some more striking differences also appeared. Although Stommel's rate of increase of updraft with height was nearly the same as that predicted by Houghton and Cramer, the measured "gross entrainment" was far greater than that needed to satisfy continuity and continued at a high rate above that level where the vertical motions began to diminish upward.

If further observational work corroborates these indications, a mechanism to cause such additional entrainment or mixing must be sought. A factor not included in the Houghton-Cramer model which operates in real situations is shear in the external wind. Observations of clouds in the Woods Hole (eastern U. S.) area seem to indicate that those clouds which cease to grow despite warmer temperatures at their tops occur on days of pronounced wind shear. The Thunderstorm Project (BYERS and BRA-

HAM, 1949) also found that strong shear exerted an inhibitory effect upon thunderstorm development.

Some effects of windshear upon convection

The crux of these investigations is that momentum is a vector quantity. While the preceding discussion has shown that the vertical momentum relations in a cumulus cloud are extremely complex, due to the interaction of buoyancy and friction, the horizontal component is altered by friction alone. If a parcel or updraft (the presence of liquid water is, for the moment, unimportant) were to rise without interaction through an environment in which the external wind changes with height, it would acquire a velocity relative to the air, since its forward momentum would remain that of its original level. If the shear vector between any two levels is defined as the upper minus the lower wind, the parcel's relative velocity would be in the sense opposite to this.

It has been shown by MALKUS (1949a; 1952) that frictional forces act to reduce but do not eliminate this relative velocity. A striking verification of this prediction was found near Woods Hole, when on several days of marked wind turning with height, small cumulus were actually observed moving at an angle of 45° to the wind at their midlevel. This case has been discussed quantitatively by MALKUS, BUNKER and MCCASLAND (1949).

The entrainment of outside air is one factor operating to alter the momentum of the updraft toward that of its surroundings. The total amount of outside air mixed with the updraft must be included, hence the gross entrainment rates found by STOMMEL (1947b) were used in these considerations. When this is assumed to be the only resistive force, relative horizontal velocities of about 2 m/sec should be common about 1 km above draft base, for normal wind shears. If form or pressure drag creates additional resistance of equal magnitude, the relative velocities are decreased about 40 %. Direct verification of relative cloud-air velocities was found in the Thunderstorm Project's tracking of clouds by radar (see BYERS and BRAHAM, 1949). At roughly 1 km above cloud base, they found average relative speeds of 2 m/sec, increasing to an average of 5 m/sec at 6 km (fairly large average windshears).

The relative velocity between draft and environment has been shown by MALKUS (1949a) to produce important asymmetries between the location of updrafts and that of the visible, liquid cloud. How these may arise is easy to see. If the horizontal speed of the draft through the air is of the order of 1 m/sec, it will be greater than the normal rate of inflow of entrained air, which for a stationary draft might be fairly symmetrical around the periphery. Hence by vector addition, the inflow rate on the relative upwind side is greatly increased and actual outflow or detrainment of cloudy air from the updraft may take place on the opposite side. If verifiable observationally, a mechanism has clearly been presented whereby the total influx or "gross entrainment" is caused to be greater than the dynamic or "net entrainment" required to preserve continuity.

Several types of observation verify this prediction. The most striking are time-lapse motion pictures (motion pictures exposed at long intervals, say one every five seconds and projected at normal speed of 16 per second). A number of sequences made in the Woods Hole area showed cloud fragments streaming out from one side of the moving cumuli, becoming caught in the adjacent downdraft and dissipating, while the cloud grew vigorously and put out new towers on the opposite side. In each case, the cloud dissipated on the downwind side of the shear vector and grew on the upwind side (see discussion by MALKUS, 1949b; 1952). Nineteen sequences of films made by the Thunderstorm Project in Florida were also checked and the prediction verified in every case that the windshear for the region could be determined reliably (17 cases). In Fig. 4, showing trade cumulus, the wind blows from left to right across the photograph and decreases upward so that the shear vector points toward the left. Signs of active growth, new turrets, and turbulence are apparent on the right edges of the clouds, while they appear fuzzy and weak on the left edges.

An additional verification of these asymmetries in trade cumulus was found from the accelerometer records made in the horizontal airplane traverses by the Wyman-Woodcock expedition. Fig. 5 presents an average of eight such horizontal traverses through trade cumuli, in each of which the ambient wind increased upward. If it can be assumed that active updrafts are associated with cloud-scale turbulence the data are easily interpretable as follows: In the region δ, where cloud air has been blown off the updraft, the water droplets may be expected to evaporate and the cloud die. In the region D, where air is being brought into the updraft, new cloud may be expected to form. Similar observations were reproduced by MALKUS, BUNKER and MCCASLAND (1949) in the Woods Hole area. Thus it is seen that a cumulus is not simply an aggregate of droplets, drifting passively with the wind, but rather is a transient, dynamic balance between rapid growth and equally rapid destruction. The air is seen to have a component of motion *through* the updraft. An extreme case of air motion through a draft is found in clouds which appear to remain standing near mountain tops for hours despite strong winds.

The asymmetrical distribution of upward accelerations within thunderclouds and its relation to windshear has been discussed analytically by LANGWELL (1951c). She showed that a large cumulus is most likely to begin transition to a mature stage on its edge toward which the shear vector points (δ region), because there the updrafts are weakest or absent and yet liquid particles plentiful enough to cause a significant downward drag. Her predictions concerning the side of convective clouds that first produces rain were well verified from data taken by the Thunderstorm Project. The downward transport of momentum by the downdraft obeys principles similar to those just presented and is discussed by BYERS and BRAHAM (1949). The role of this downward transport of usually very high westerly momentum has been shown to be important in the cold air outflow from thunderstorms, the initiation of new thunderstorm cells, and the formation and movement of squall lines.

The slopes of cumuli; trade cumuli vs. those of middle latitudes

The slope of an updraft at any level depends only upon the ratio of the vertical speed to the departure of the horizontal speed at that level from that at its base. Therefore, it is very easy to calculate the slopes of the clouds discussed in Stommel's second entrainment paper (1951). The horizontal speed of the clouds can be calculated from the entrainment rate and observed wind shear using the technique evolved by MALKUS (1949a). Stommel already calculated the vertical speeds. The result of this calculation is reproduced in Fig. 6. This should be compared with the photograph presented earlier, in Fig. 4, which was made in the same area at the same time as the sounding used by Stommel. Due to negative shear (easterlies decreasing upward), the predicted slope of the cloud is upwind, as is the observed slope. Most of the clouds in the photograph appear to slope more strongly backward than does the theoretical cloud. This cannot be due to the 18 000 ft altitude from which the photograph was taken, since even the nearest clouds were about 70 000 ft from the camera. It may be due to additional resistive forces such as form drag, not included in the theory, or it may be that most of the clouds pictured here were in a later stage of their life cycles than Stommel's, thus having

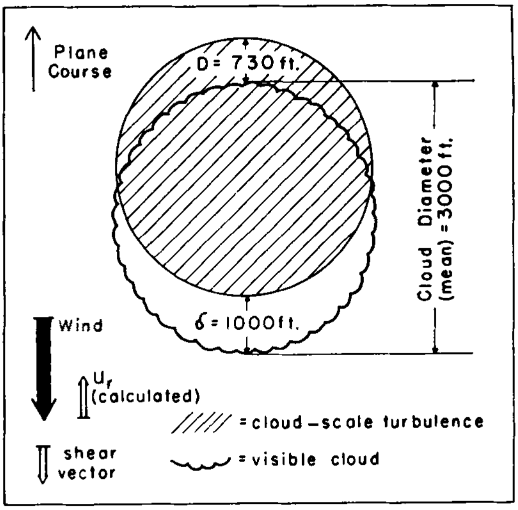

Fig. 5. Average of eight horizontal airplane traverses with accelerometer through middle levels of trade cumuli. Traverses were made by the Wyman-Woodcock Caribbean expedition, April 1946. In each of these cases, the wind increased with height about 1 mps/km. The cloud-scale turbulence was at least an order of magnitude in excess of clear-scale turbulence. The time of entrance and exit into visible cloud was recorded directly on the accelerometer record. The arrow labelled U_r is the cloud motion seen by an observer moving with the wind; an observer riding on the cloud would feel a wind in the opposite direction. After MALKUS (1949a).

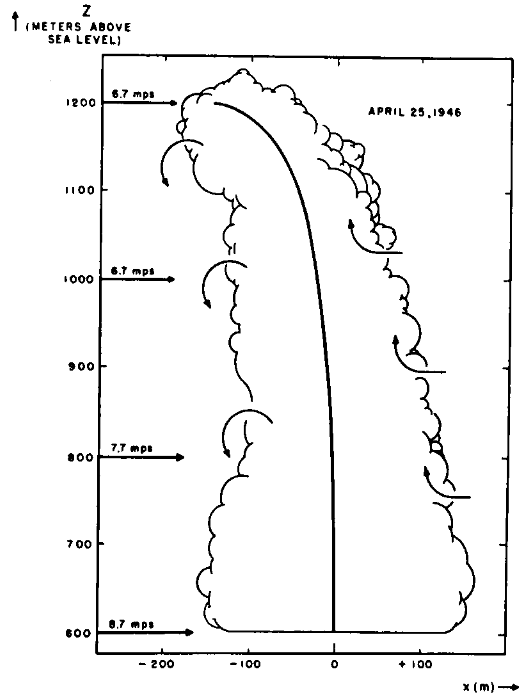

Fig. 6. Theoretical slope of the trade cumulus on April 25, 1946 whose vertical velocity profile was calculated by Stommel (1951). The dark arrows to the left are the observed external wind. The light curved arrows represent schematically the predicted air motions into and out of the cloud, according to Malkus (1949b). The cloud should be building on the right and decaying on the left side as it moves from left to right more rapidly than the external wind. Compare with Fig. 4.

slower ascent speeds. A more detailed study of cloud slopes (Malkus, 1952) shows that greater horizontal friction forces or weaker updrafts produce more slant, as does stronger external shear, other conditions remaining the same.

The inhibitory effect of a strong slant upon cloud growth has been discussed by Byers and Braham (1949). They demonstrated that a cloud grows by the production of successive turrets at or near its top. If wind shear is weak the new turrets grow in air already moistened by dissipated predecessors. If external shear is strong, the old turrets become strongly slanting as their updrafts decay. The new turrets with strong drafts grow nearly vertically and hence are penetrating drier air. That small cumuli as well as thunderclouds grow in this manner has been verified by time-lapse movies in the Woods Hole area where individual towers have been observed to have life spans lasting from six to sixteen minutes.

An additional inhibitory effect of wind shear upon cloud growth has been suggested in the foregoing discussions. The relative cloud-air horizontal velocity increases linearly with wind shear. If it can be shown, as proposed here, that additional entrainment results from this motion of the cloud through the air, then the resistive effects and buoyancy reduction are substantially augmented by wind shear. A suggestion concerning the difference between trade cumulus and their extra-tropical counterparts may now be feasible.

The constancy of the trade wind through the subtropical cloud layer is well known. The average shear vector through a corresponding layer in the Woods Hole area, for example, was 2—3 times greater (6 mps/km) in the summer months than near San Juan during the Wyman-Woodcock expedition (2—3 mps/km). From the data of Riehl et al. (1951), the shear in the Pacific Trade appears to be even smaller (~ 1 mps/km). Trade cumuli may therefore be operating against considerably smaller resistive forces than middle latitude cumuli. This would mean that the clouds are more easily triggered by weaker impulses (see concluding paragraphs of Section II) and that smaller buoyancy forces are required to produce comparable vertical speeds. Turbulence within trade cumuli could also be expected to be less than that in middle latitude clouds for the same ascent rates. This would happen because the in-cloud lapse rates need not be so much steeper than moist adiabatic. Austin (1948) has pointed out that turbulence within a cloud should be greater the greater the departure of the in-cloud lapse rate from moist adiabatic, because an unstable state is thereby presented for the small-scale, moist adiabatic overturning near cloud center.

It was mentioned that the observed clouds in Fig. 4 may have slanted more than predicted by the Stommel-Malkus theory because of the omission of horizontal form drag from the model. Observations from the Thunderstorm Project also indicate that form drag may be important in reducing the relative horizontal cloud-air velocity. Some entirely different evidence presented by Riehl et al. (1951) regarding the transports in the subtropical cloud layer further support this possibility.

Transports by convective elements

Most of the work on cumulus clouds discussed so far emphasizes the effects of the environment upon the cloud. The data gathered by the University of Chicago Project Tyrena (reported on by RIEHL, YEH, MALKUS and LA SEUR, 1951) permitted for the first time framing the problem in reverse. Clearly, if the environment exerts a frictional force on the cloud, the cloud exerts an equal and opposite force on the environment. Also, according to the picture drawn above, the environment not only supplies drier air to the cloud, but by "detrainment" the cloud supplies moister air to the environment.

From the Tyrena observations it was possible to calculate effective large-scale momentum and moisture transport coefficients layer by layer for a section of the Pacific Trade. In the cloud layer the momentum "Austausch" was found to be significantly larger than the moisture "Austausch". Since the clouds themselves are the sole contributors to these coefficients within this layer (BUNKER et AL. 1949, have shown the turbulence in between clouds is at least an order of magnitude smaller than cloud scale) these data suggest that for equal gradients, trade cumulus clouds are far more effective transporters of momentum than of water vapor. If form drag is of roughly equal importance as entrainment in the horizontal momentum relations of the clouds, this observation is explainable qualitatively. If by means of pushing the air around itself, the cloud thereby exerts a force on the environment, it imparts some of its horizontal momentum to the surrounding air. It also exchanges horizontal momentum with the surrounding air by means of mixing or entrainment. It exchanges water vapor with the environment only by the latter process. Hence one might expect intuitively that the measured "eddy" viscosity for a convective layer might be considerably in excess of the "eddy" diffusivity. It can be shown analytically, even by a rather crude model, that if form drag and entrainment contribute friction forces (horizontal) of roughly equal magnitude on the elements of a convective layer, that this ratio comes out considerably greater than one. A required assumption is that the individual elements somehow overshoot their equilibrium positions and sink partially back down.

STOMMEL (1951) has demonstrated the overshooting of trade cumuli in the Caribbean, and RIEHL et AL. (1951) have further discussed how penetration of even a small fraction of the clouds into the trade inversion and later dissipation of their tops by mixing can provide sufficient moisture to account for the observed downstream rise in height and weakening of the inversion.

These latter calculations emphasize the tremendous amounts of energy continually pumped into the troposphere by the trade cumuli. The magnitude may perhaps be grasped by the following: If a one square kilometer area is 50 % covered by trade cumulus, only a small fraction of which need be actively rising at any one time, they can easily provide a net upward transport of latent energy in water vapor equal to that released by a 500 kg bomb of trinitrotoluene (3 000 cal/gm) exploding every minute. If this is summed up over all the tropical oceanic area normally covered by trade cumuli, a net upward transport of latent energy is given which is greater by a factor of roughly four or five, than the rate of kinetic energy dissipation by all the wind systems of the globe.

It has been strikingly pointed out by RIEHL (1950) however, that tropical convection does not uniformly feed energy into the atmospheric heat engine, but mainly in the easterly waves and polar troughs which, in passing, superpose strong convergence upon the flow. This large-scale convergence results in a greatly deepened "moist layer" and a tremendous diminution in the resistive forces operating against the growth of the cumuli. In the first place, the stable lid usually provided by the trade inversion is raised or removed. Second, net ascending motion in the region diminishes the required compensatory sinking and hence the reduction in buoyancy predicted by the slice method (CRESSMAN, 1946, has discussed this effect in detail) and finally, the growing clouds are surrounded by a deep layer of moist environment up to great heights so that entrainment is less effective in reducing their buoyancy.

How this energy, once made available in these disturbed regions to the upper subtropical atmosphere, is then injected into middle latitudes is being studied by Riehl and other investigators of the role of the tropics in the general circulation. It is now apparent, however, that cumulus clouds are important driving elements, and the forces restraining them are important controls, in the basic machinery of the atmospheric heat engine.

IV. Conclusion

In concluding, it can be seen that many more questions concerning cumulus cloud dynamics have been raised by this paper than have been answered by it. Many of these are framed in terms of specific sets of measurements which must be carried out. Consequently, the Woods Hole Oceanographic Institution has planned a new program of meteorological observations, to commence in the Spring of 1952. The principal observing tool is to be a large, slow-flying aircraft equipped with psychrograph, accelerometer, variometer, liquid water collector, drift sight, and cameras. By numerous flights through and near both trade and middle latitude cumuli, it is hoped to answer some of the existing questions and upon these answers to base new inquiries which are better evolved and more searching.

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