

# Linear solutions for the frequency and amplitude modulation of ENSO by the annual cycle

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## ABSTRACT

We obtained linear solutions for the frequency and amplitude modulations of the El Niño–Southern Oscillation (ENSO) by the annual cycle using a modified harmonic oscillator equation. The frequency modulation by the annual cycle was capable of changing the ENSO phases and dominant frequency, but could not modify the ENSO amplitude. On the other hand, the amplitude modulation by the annual cycle intensifies the ENSO variability and also induces seasonal amplitude locking. The intensification rate of the ENSO amplitude with respect to the annual cycle becomes less sensitive in low-frequency regime of ENSO.

## 1. Introduction

One of unique features of the El Niño–Southern Oscillation (ENSO) is a seasonal locking of its variance, such that the lowest variability is in boreal spring and the highest variability is in boreal winter (Rasmusson and Carpenter, 1982; Galanti and Tziperman, 2000; An and Wang, 2001). This seasonal-locking phenomenon might be understood through the non-linear frequency locking of the ENSO to an annual period (Jin et al., 1994) or the seasonal change in the linear stability of ENSO (Tziperman et al., 1998). Conversely, the ENSO in turn modulates the strength of the seasonal sea surface temperatures (SST) and wind variations (Xie, 1995) such that the amplitude of the annual cycle in the eastern equatorial Pacific tends to be weaker during an El Niño episode and stronger during a La Niña episode (Gu and Philander, 1995; Xie, 1995). Theoretical studies have suggested that the two-way interaction between the annual cycle and the ENSO might enable the generation of deterministic chaos, which would explain the irregularity of the ENSO phenomenon (Chang et al., 1994, 1996; Jin et al., 1994; Tziperman et al., 1994, 1995; Jin, 1996; Wang and Fang, 1996; Wang et al., 1999; Timmermann et al., 2003). A somewhat different perspective concerning the annual cycle ENSO interaction is that of An et al. (2010), who pointed out that the long-term changes in the relationship between the ENSO and the annual cycle may be due to changes in the annual mean state. Furthermore, a two-way interaction between the ENSO and the annual mean state was

proposed (Jin et al., 2003; An and Jin, 2004; Choi et al., 2009), with researchers arguing that the annual mean state modifies the ENSO variability (Jin, 1996; Li and Hogan, 1999; Fedorov and Philander, 2000; An and Jin, 2001; Wang and An, 2001; An et al., 2004, 2006, 2008; Bejarano, 2006; Guilyardi, 2006) and that the ENSO influences the annual mean state through non-linear rectification (Jin et al., 2003; Rodgers et al., 2004; Schopf and Burgman, 2006; Sun and Zhang, 2006). Therefore, a comprehensive understanding of the interactions between the annual mean state, the annual cycle and the ENSO is necessary in order to determine the mechanisms behind tropical climate variability.

In this study, we confine the problem to only that of the effect of the annual cycle on the ENSO. This claim was investigated by solving the simplest possible linear dynamical system that represented the amplitude and frequency modulation of the ENSO according to the annual cycle. Solutions for the frequency and amplitude modulation cases are exhibited in Sections 2 and 3, respectively. In Section 4, our concluding remarks are given.

## 2. Frequency modulation

Since the climatic background state determines the stability of the ENSO, the annually varying background state modifies the frequency and amplitude of the ENSO on an annual time scale. Therefore, the effects of the annual cycle on the ENSO can be mathematically formulated by adding an annually varying frequency to the natural frequency in a harmonic oscillator equation, thus mimicking the ENSO. However, a shortcoming in this approach is that we do not know how the annual cycle modulates ENSO. Thus, rather than the actual process involved in the

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frequency modulation of ENSO by the annual cycle, we can only see the resultant frequency-modulated ENSO.

The equation for the modified harmonic oscillator with a frequency of  $\omega$  is

$$\begin{aligned} \frac{dT}{dt} &= \omega h, \\ \frac{dh}{dt} &= -\omega T, \end{aligned} \quad (1)$$

where  $\omega = \omega_0 + \omega_1(t)$  and  $\omega_1 = \delta \cos(\omega_A t)$  and, for our purposes,  $\omega_A$  is an annual frequency and  $\omega_0$  indicates the interannual frequency.  $T$  and  $h$  are considered to be the SST in the equatorial eastern Pacific and the thermocline depth averaged over the equatorial Pacific basin, respectively, which thus mimics the ENSO behaviour following the ‘Recharge paradigm of ENSO’ by Jin (1996). Actually, Burgers et al. (2005) modified Jin’s formula to a simpler version yet holding essential concept. Their formula is basically the same as eq. (4), and the eq. (1) is the same form except for no the damping term in Burgers’s formula. Therefore, our approach keeps the original idea of the recharge oscillator.

A solution for the aforementioned system would be

$$T = \hat{T}(t) \exp[i\omega_0 t], \quad h = \hat{h}(t) \exp[i\omega_0 t]. \quad (2)$$

After some manipulations, the solution reads as follows:

$$T = C \exp \left[ i \left( \omega_0 t + \frac{\delta}{\omega_A} \sin \omega_A t \right) \right], \quad (3)$$

where a real constant,  $C$ , was determined using the initial conditions. The details of the procedure are explained in Appendix A. This is a quasi-periodic solution with changing phase in time. However, the amplitude of the solution is  $|T|^2 = C^2$ ; thus, the amplitude of the annual cycle effect,  $\delta$ , does not alter the amplitude of  $T$  (i.e. the ENSO). Therefore, the annual cycle acts to set the phase of the interannual variation, but does not change the amplitude of the interannual variability.

To check the accuracy of the asymptotic solution given in eq. (3), we solved the system of (1) numerically using the fourth-order Runge-Kutta method. Here,  $\omega_0 = 0.5 \pi (\text{year}^{-1})$  and  $\omega_A(t) = 2\pi (\text{year}^{-1})$  were used, and the sensitivities of the system to a given value of  $\delta$  were obtained. As seen in Fig. 1, the amplitude of  $T$  does not depend on the value of  $\delta$ , indicating that the analytic solution is valid. In the weak modulation regime (Fig. 1a), the 4-year periodic oscillation is dominant. As the annual cycle influence was increased (i.e. increases in  $\delta$ ), variations were noticed in the dominant 4-year cycle (Fig. 1b). Further increases in  $\delta$  resulted in non-periodic oscillation due to strong modulation, as shown in Figs. 1c and d. Figure 1d shows that  $\delta = 10 \text{ year}^{-1}$ , which is beyond the asymptotic range; yet, it still provides an amplitude-constraint solution. However, a further, unrealistic increase in  $\delta$ , ( $>30 \text{ yr}^{-1}$ ), leads to a decrease in amplitude  $T$  (not shown here). As  $\delta$  increases, the fluctuation of  $T$  tends to be chaotic. Nevertheless, the phase relationship between  $T$  and  $h$  always maintains the pattern seen in the phase diagram of Fig. 1. It is important to note that the frequency-modulated model does not hold the seasonal locking of the ENSO amplitude

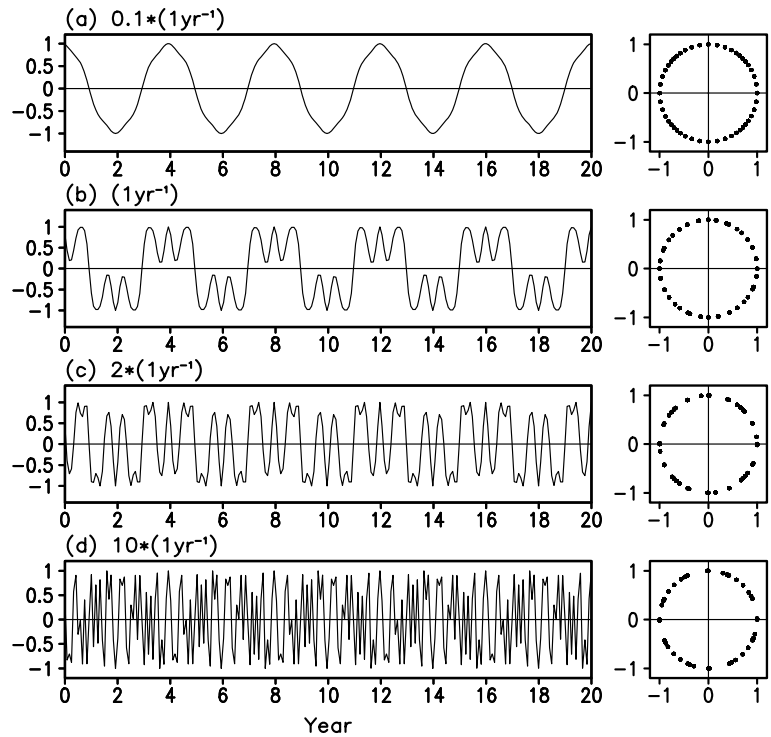


Fig. 1. Time series of  $T$  obtained from a frequency-modulation model for various values of modulation effects,  $\delta$ . (a)  $\delta = 0.1 (\text{yr}^{-1})$ , (b)  $\delta = 1.0 (\text{yr}^{-1})$ , (c)  $\delta = 2.0 (\text{yr}^{-1})$  and (d)  $\delta = 10.0 (\text{yr}^{-1})$ . The absolute values, which depend on the initial conditions, have no meaning. The right-hand panel shows a scatter diagram of  $T$  (horizontal axis) versus  $h$  (vertical axis).

(not shown here), therefore we can infer that the seasonal locking of the ENSO amplitude is not due to frequency-modulation.

### 3. Amplitude modulation

As previously mentioned, the annual variation of the climate state modulates the stability of the ENSO. This is because the stability conditions of ENSO are annually varying along with changes in the annually varying background states, which has been adopted to explain the amplitude (or phase) locking mechanism of ENSO to the annual cycle (Tziperman et al., 1998). Thus, we simply tested this annually varying stability effect on the ENSO. Annually varying stability was integrated into the linear oscillatory equation by controlling the damping term.

$$\begin{aligned}\frac{dT}{dt} &= -\lambda(t)T + \omega_0 h, \\ \frac{dh}{dt} &= -\omega_0 T,\end{aligned}\quad (4)$$

where  $\lambda(t) = \lambda_0 \cos(\omega_A t)$ ; thus, the time mean of  $\lambda(t)$  vanished. The damping effect has only the annual time scale. Above equation is almost identical to equation (8) of Burgers et al. (2005), which is a good approximation of the original recharge oscillator by Jin (1996).

The system of eq. (4) was solved using the perturbation method. The basic assumption in this method is a small value of  $\lambda_0$ . Thus, we choose  $\lambda_0$  as  $0 < \varepsilon \ll 1$ . The details of our procedures are described in Appendix B. The subsequent solution for

system (4) is

$$T \approx C \left[ \cos(\omega_0 t) + \frac{\varepsilon}{\omega_A^2 - 4\omega_0^2} \left\{ \omega_0 \cos(\omega_A t) \sin(\omega_0 t) - \frac{\omega_A^2 - 2\omega_0^2}{\omega_A} \sin(\omega_A t) \cos(\omega_0 t) \right\} \right], \quad (5)$$

where  $C$  is an arbitrary constant dependent on the initial conditions. This solution implies that the solution for the amplitude-modulated model is a combination of a regular oscillatory mode (first term) and the annually modulated mode (second term). The analytic solution of (5) indicates that the larger is  $\varepsilon$ , the larger will be the amplitude of  $T$ . Thus, the amplitude modulation of the ENSO due to the annual cycle results in the intensification of the ENSO. In addition, the larger is  $\omega_0$ , the larger will be the amplitude of  $T$ . Therefore, solution (5) implies that the annual cycle could amplify the ENSO through amplitude modulation, and that the amplification rate of the ENSO depends on the ENSO frequency.

As in the previous section, in order to check the accuracy of the asymptotic solution, a numerical integration of system (4) was performed and compared with the asymptotic solution. For the numerical solution,  $\omega_0 = 0.5 \pi$  (year<sup>-1</sup>) and  $\omega_A(t) = 2\pi$  (year<sup>-1</sup>) were used and the system sensitivities were obtained using a given value of  $\varepsilon$ . As seen in Fig. 2, the numerical solutions were well matched to the analytic solutions in an asymptotically appropriate range of  $\varepsilon$  (Figs. 2a–c). Even beyond the appropriate range (Fig. 2d), the qualitative characteristics were similar to each other. Thus, the analytic solution of (5) is valid. The

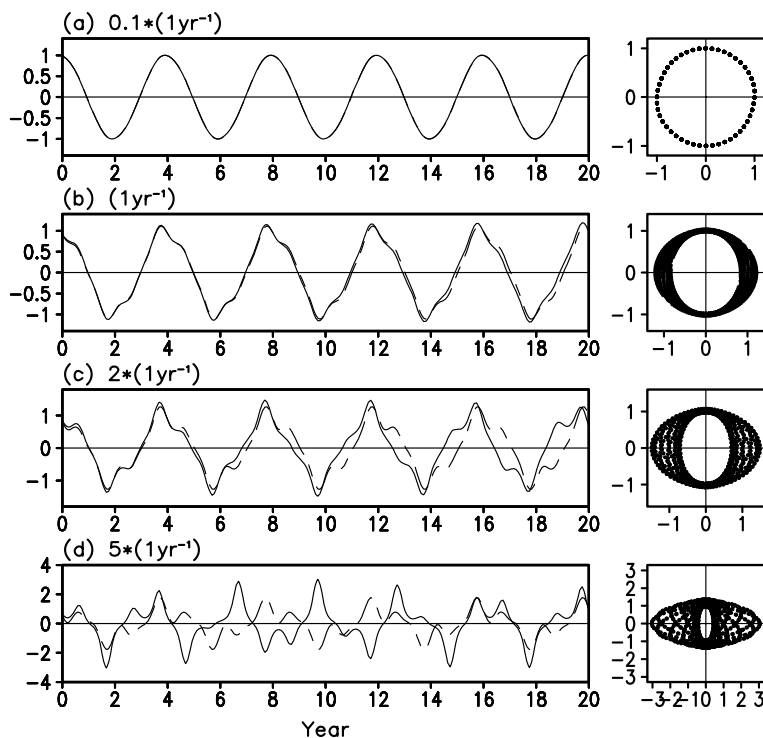


Fig. 2. As in Fig. 1, but for an amplitude-modulation model for various values of the modulation effect,  $\varepsilon$ . (a)  $\varepsilon = 0.1$  (yr<sup>-1</sup>), (b)  $\varepsilon = 1.0$  (yr<sup>-1</sup>), (c)  $\varepsilon = 2.0$  (yr<sup>-1</sup>) and (d)  $\varepsilon = 5.0$  (yr<sup>-1</sup>). All of the integrations started with the same initial conditions. The right-hand panels show a scatter diagram of  $T$  (horizontal axis) versus  $h$  (vertical axis). The solid and dashed lines indicate the numerical and analytical solutions, respectively.

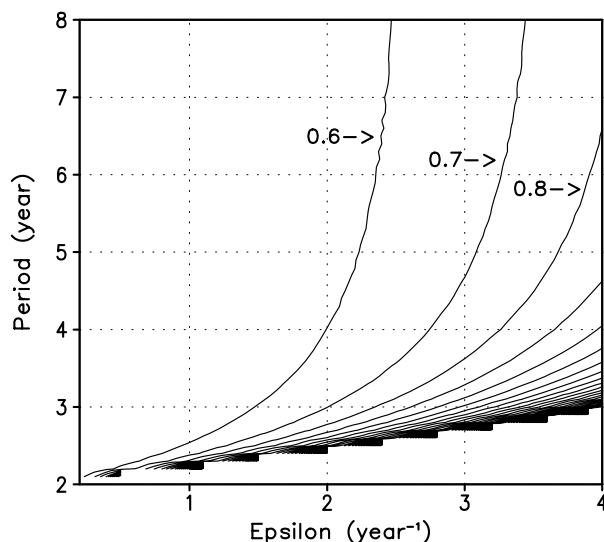


Fig. 3. Variance of  $T$  for a given  $\varepsilon$  ( $\text{yr}^{-1}$ ) and period ( $\text{yr}$ ;  $P = 2\pi/\omega_0$  in eq. (4)) obtained from the amplitude-modulation model.

amplitude of  $T$  depends on the value of  $\varepsilon$ . In a weak modulation regime (Fig. 2a), the 4-year periodic oscillation was dominant and its amplitude was unity. The variance of  $T$  was sensitive to both  $\varepsilon$  and  $\omega_0$ . The variance of  $T$  exponentially increased with respect to  $\varepsilon$  (Fig. 3), implying that the amplitude modulation of the annual cycle results in an exponential increase in the interannual variability. However, changes in the variance of  $T$  were not significant within a reasonable range of  $\varepsilon$  [i.e. near  $1$  ( $\text{yr}^{-1}$ )]. The variance of  $T$  linearly decreased with respect to the period, which implies that the longer is the ENSO period, the lesser will be the annual cycle influence. It is worthy of note that in this system a strong annual cycle does not mean to be associated with a strong amplitude modulation of the ENSO (i.e. a larger value of  $\varepsilon$ ); thus, our results do not contradict the general inverse relationship between ENSO amplitude and annual cycle intensity (Timmermann et al., 2007; An et al., 2010).

In order to verify the seasonal locking of the ENSO amplitude, the variance of  $T$  at each calendar month was calculated. Differing from the frequency-modulation model, the variance of  $T$  is seasonally locked and its maximum was observed in September (i.e. phase =  $3/2\pi$ ), due to the fact that the damping (or amplifying) coefficient of the system in eq. (4) is a cosine function (Fig. 4). Thus, there is a 3-month lag of ENSO amplitude with respect to the growth rate. This may support the hypothesis that the seasonal change in the linear stability of the ENSO causes the seasonal locking of the ENSO (e.g. Tziperman et al., 1998).

#### 4. Concluding remarks

In this study, we investigated the influences of the annual cycle on the ENSO using a simple linear harmonic oscillator that mimics the ENSO, particularly focusing on the frequency and amplitude

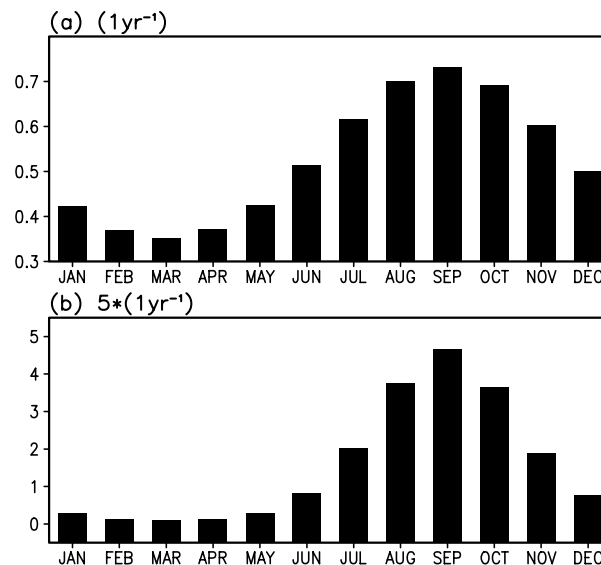


Fig. 4. Variance of the time series of  $T$  obtained from the amplitude-modulation model at each calendar month. (a)  $\varepsilon = 1.0$  ( $\text{yr}^{-1}$ ) and (b)  $\varepsilon = 5.0$  ( $\text{yr}^{-1}$ ).

modulation of ENSO by the annual cycle. By adding an annually varying frequency to the natural frequency of a linear oscillator system, we were able to examine the frequency-modulation effect. Both the analytical and numerical solutions showed that the phase of the ENSO was altered by the frequency-modulation effects, but the amplitude was not. The amplitude-modulation effect was examined by plugging the annually varying linear damping/growing term into a linear harmonic oscillator. Both the analytical and numerical solutions showed that the amplitude modulation resulted in the intensification of the ENSO and also caused a seasonal locking of the ENSO amplitude.

Our solution is valid in a linear system. So far, the linear theories such as ‘delay oscillator’ (Schopf and Suarez, 1988; Battisti and Hirst, 1989) or ‘recharge oscillator’ (Jin, 1996, 1997; An and Kang, 2000; An and Jin, 2001) have provided the mechanism that leads the oscillatory nature of ENSO and its interannual periodicity. Since the basic formulation in this study is almost identical to a simplified version of ‘recharge oscillator’, our study can be regarded as an extension of ‘recharge oscillator’. Therefore, solutions obtained here are applicable at least to the ENSOs in the stable regime or in the linear or weakly non-linear regime. For example, ENSOs during 1950s–1970s likely belong to the stable linear regime, while those during 1980s–2000s do to the unstable non-linear regime (An, 2004; An et al., 2005; An, 2009). In this regard, the findings here may be more applicable to the modulation of ENSO by the annual cycle during the early decades.

An irregular and quasi-periodic behaviours of ENSO may be attributed to the non-linear dynamics such as ‘frequency entrainment of the annual cycle’ or deterministic chaos (Jin et al.,

1994; Tziperman et al., 1994), which cannot be resolved in this study. On the other hand, a stochastic process (Chang et al., 1994; Penland and Sardeshmukh, 1995; Thompson and Battisti, 2000), which is also known to be a cause for the irregularity and quasi-periodicity of ENSO, may be explored by the similar way as used in this study. Therefore, a comprehensive analysis on the impact of the annual cycle on ENSO is necessary, not only in the linear stable regime that has been done here in a way, but also in the non-linear unstable regime.

## 5. Acknowledgments

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## 6. Appendix A: Solution for frequency modulation

Substituting eq. (2) into eq. (1) gives

$$\begin{aligned} \frac{d\hat{T}}{dt} + i\omega_0\hat{T} &= \omega_0\hat{h} + \omega_1\hat{h}, \\ \frac{d\hat{h}}{dt} + i\omega_0\hat{h} &= -\omega_0\hat{T} - \omega_1\hat{T}. \end{aligned} \quad (\text{A1})$$

The first-order balance in eq. (A1) is  $i\hat{T} \approx \hat{h}$  and  $i\hat{h} \approx -\hat{T}$ . Thus, the second-order balance becomes

$$\frac{d\hat{T}}{dt} = i\omega_1\hat{T}, \quad \frac{d\hat{h}}{dt} = i\omega_1\hat{h}. \quad (\text{A2})$$

The solution to (A2) is

$$\hat{T} = C \exp[i \int_t \omega_1(\tau) d\tau], \quad (\text{A3})$$

where  $C$  is a real constant. Thus, the solution to eq. (1) is

$$T = C \exp[i\omega_0 t + i \int_t \omega_1(\tau) d\tau]. \quad (\text{A4})$$

Let  $\omega_1$  change with the annual period  $\omega_A$ , that is,  $\omega_1 = \delta \cos(\omega_A t)$ .

By substituting this into (A4), the result is

$$T = C \exp \left[ i \left( \omega_0 t + \frac{\delta}{\omega_A} \sin \omega_A t \right) \right]. \quad (\text{A5})$$

## 7. Appendix B: Solution for amplitude modulation

In order to apply a perturbation method, the two equations in eq. (4) are combined into one.

$$\ddot{T} + \lambda \dot{T} + (\dot{\lambda} + \omega_0^2) T = 0, \quad (\text{B1})$$

where the superscript ‘dot’ indicates a time derivative and  $\lambda(t) = \lambda_0 \cos(\omega_A t)$ . In general, the amplitude modulation of the ENSO

according to the annual-mean state was larger than that of the annual cycle (An et al., 2010). Thus, the constant  $\lambda_0$  can be assumed to be  $\varepsilon$  ( $0 < \varepsilon \ll 1$ ). Therefore, eq. (B1) becomes

$$\ddot{T} + \varepsilon \cos(\omega_A t) \dot{T} + (\omega_0^2 - \varepsilon \omega_A \sin(\omega_A t)) T = 0. \quad (\text{B2})$$

From the definition of  $\varepsilon$ , eq. (B2) can be solved using the perturbation method (Glendinning, 1994). A possible solution can be expanded as

$$T = T_0 + \varepsilon T_1 + \varepsilon^2 T_2 + \dots \quad (\text{B3})$$

Subsequently substituting (B3) into (B2), from the first-order balance, we have

$$\ddot{T}_0 + \omega_0^2 T_0 = 0. \quad (\text{B4})$$

So, a solution of (B4) is  $T_0 = C \cos(\omega_0 t + \phi_0)$ , where  $\phi_0$  is determined by an initial condition. Alternately, we can assume  $\phi_0 = 0$  without losing the solution accuracy. The terms in  $\varepsilon^1$  give

$$\ddot{T}_1 + \omega_0^2 T_1 + \cos(\omega_A t) \dot{T}_0 - \omega_A \sin(\omega_A t) T_0 = 0. \quad (\text{B5})$$

After substituting  $T_0$  into (B5), we have

$$\begin{aligned} \ddot{T}_1 + \omega_0^2 T_1 - C \{ \omega_0 \cos(\omega_A t) \sin(\omega_0 t) \\ + \omega_A \sin(\omega_A t) \cos(\omega_0 t) \} = 0. \end{aligned} \quad (\text{B6})$$

A possible solution for (B6) is

$$\begin{aligned} T_1 = \chi \cos(\omega_A t) \sin(\omega_0 t) \\ + \xi \sin(\omega_A t) \cos(\omega_0 t). \end{aligned} \quad (\text{B7})$$

After some manipulation, we obtained two leading order terms of the expansion for (B1).

$$\begin{aligned} T \approx C \left[ \cos(\omega_0 t) + \frac{\varepsilon}{\omega_A^2 - 4\omega_0^2} \left\{ \omega_0 \cos(\omega_A t) \sin(\omega_0 t) \right. \right. \\ \left. \left. - \frac{\omega_A^2 - 2\omega_0^2}{\omega_A} \sin(\omega_A t) \cos(\omega_0 t) \right\} \right]. \end{aligned} \quad (\text{B8})$$

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