The effect of mechanical stirring on horizontal convection

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ABSTRACT

An important experimental result, as yet poorly understood, is that mechanical stirring can significantly enhance the strength of horizontal convection. A contentious issue is whether this necessarily implies that the mechanical stirring replaces the buoyancy forcing as the main source of energy driving the observed overturning circulation, as has been suggested for the Atlantic meridional overturning circulation (AMOC). In this paper, rigorous energetics considerations and idealized numerical experiments reveal that the rate at which the surface buoyancy forcing supplies energy to the fluid, as measured by the production rate of available potential energy G(APE), does not solely depend upon the buoyancy forcing, as is often implicitly assumed, but also upon the vertical stratification, such that the deeper the thermocline depth, the larger G(APE). This suggests that mechanical stirring enhances horizontal convection because it causes more energy to be extracted from the buoyancy forcing. It does so by enhancing turbulent mixing, which allows surface heating to reach greater depths, which increases the thermocline depth and hence G(APE). This paper therefore proposes a new hypothesis, namely that mechanically stirred horizontal convection and the AMOC are best described as mechanically controlled heat engines.

1. Introduction

Characterizing and quantifying the relative importance of the mechanical and thermodynamic forcing in driving and stirring the oceans has been a much debated issue in oceanography for over a century now. Historically, oceanographers have tended to assume that the large-scale ocean circulation could be regarded as the sum of an apparently independent wind-driven and buoyancy-driven component despite the highly non-linear character of the system. Thus, the wind-driven component is usually identified with the horizontal vertically averaged circulation (as described by Ekman and Sverdrup theories, for instance), while the buoyancy-driven component is usually identified with the zonally averaged circulation taking place in the latitudinal/meridional vertical plane. Thermodynamics somehow legitimates this separation, since the entropy and buoyancy of the fluid parcels can only be significantly affected by diabatic effects due to the surface buoyancy forcing and internal molecular diffusive processes.

In the classical view, for example, Colin de Verdière (1993), the two key physical processes controlling the buoyancy-driven circulation, often called the Thermohaline Circulation (THC),

are high-latitude cooling and vertical (diapycnal) mixing. In this view, cooling provides the destabilizing mechanism setting up the Atlantic meridional overturning circulation (AMOC) into motion, while the turbulent diapycnal mixing serves to transfer the surface equatorial heating down to the abyss to avoid the oceans filling up with cold and dense waters. The key role played by turbulent diapycnal mixing, however, raises the question of whether it makes physical sense to regard the AMOC as being entirely decoupled from the mechanical forcing. Indeed, such a view seems to require that all of the turbulent diapycnal mixing be sustained only by the mechanical stirring due to the buoyancy forcing. In the oceans, however, dissociating the effects of the mechanical forcing (wind, tides) and thermodynamic forcing (heat and freshwater fluxes) in sustaining the observed turbulent diapycnal mixing rates seems impossible. On this basis, it would appear that the AMOC must be at least partly controlled mechanically, calling for a overhaul of the classical view to take the role of mechanical forcing into account. The above observations, first made by Munk and Wunsch (1998, MW98 thereafter), prompted a renewal of interest in ocean energetics, and generated many new research questions not previously considered, such as:

(1) In absence of mechanical forcing, could the buoyancy forcing support an overturning circulation and turbulent diapycnal mixing of the observed strength on its own?

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(2) Is the power input of mechanical energy due to the wind, tides, and buoyancy forcing sufficient to sustain the observed rate of stirring and diapycnal mixing in the oceans?

Subsequently to MW98, the main idea that developed is that the buoyancy forcing is very inefficient at producing mechanical energy when applied at the top of a stratified fluid, as first suggested by Sandström (1908) [recently translated from the German by Kuhlbrodt (2008)], so that it could sustain only a very weak AMOC and low turbulent diapycnal mixing rates on its own. As a result, it has become widely agreed that it is the mechanical forcing, such as the winds and tides, rather than the surface buoyancy fluxes, that are the main supplier of the energy required to drive and sustain the AMOC against dissipation, in sharp contrast with the classical held view, for example, see the review by Kuhlbrodt et al. (2007). Interestingly, these ideas appear to be confirmed by the recent laboratory experiments of mechanically stirred horizontal convection by Whitehead and Wang (2008), which show that horizontal convection can increase dramatically when acted upon by the lateral motions of a stirrer.

Although the importance of the mechanical forcing as a control mechanism of the AMOC appears to be now well established, it still remains difficult to understand how the energy supplied by the winds and tides in the oceans, or by the stirrer in Whitehead and Wang (2008)'s experiments, can eventually end up as the highly organized form of large-scale kinetic energy characterizing the AMOC. Indeed, it seems clear that in order to sustain turbulent diapycnal mixing, the mechanical stirring must be a source of turbulent kinetic energy and available potential energy at 'small scales'. Supporting the AMOC, on the other hand, clearly requires producing kinetic energy at the 'basin scale'. Therefore, if one is to accept the idea that it is the mechanical forcing, rather than the buoyancy forcing, that is the direct supplier of the large-scale kinetic energy of the AMOC, then clearly a physical mechanism is required to convert the highly disorganized form of small-scale turbulent kinetic energy into highly organized large-scale kinetic energy. In other words, some form of inverse energy cascade appears to be required. So far, however, inverse energy cascades have only been discussed in the context of two-dimensional or QG stratified turbulence, mostly in relation with horizontal motion. We are not aware that the possibility of an inverse energy cascade in the vertical plane has ever been discussed before.

With little, if any, convincing evidence for an inverse energy cascade in the vertical plane, can we account for the existence of a mechanical control of the AMOC without the mechanical forcing having to be the main supplier of the AMOC large-scale kinetic energy? The purpose of this paper is to explore an alternative idea that has not received any attention so far, namely that the buoyancy forcing be the main driver of the AMOC, as in the classical view, but with the new twist that the mechanical forcing controls the mechanical energy production due to the buoyancy forcing. The advantage of this idea is that it resolves the above scale problem, since the spatial scales associated with the buoyancy forcing match those assumed to pertain to the AMOC. In the proposed new view, the results of Whitehead and Wang (2008) would then be explained by the lateral action of the stirrer increasing the power input due to the buoyancy forcing. In seeking support for such an idea, the main difficulty is probably to convince the reader that the current widespread idea that surface buoyancy fluxes cannot produce a significant amount of mechanical energy when applied at the top of a stratified fluid is wrong and without theoretical basis. Indeed, accepting the possibility that the work done by the surface buoyancy fluxes can be large in the oceans or in laboratory experiments of horizontal convection is crucial to accepting the ideas developed in this paper. The purpose of this paper is to examine the theoretical basis for each of the two ideas, and demonstrate that only the latter appears to be consistent with first principles. To that end, Section 2 offers a preliminary review and discussion of the relevant issues. Section 3 reviews the theoretical basis in more details, based on the recent papers by Tailleux (2009a,b). Section 4 offers numerical illustrations of the previous theory, in the context of mechanically stirred horizontal convection. Finally, Section 5 offers a summary and discussion of the results.

2. Review and discussion of relevant issues

2.1. MW98 constraint on the sources of stirring

The first important issue relates to MW98 derivation of a constraint on the power input of mechanical energy required to stir and mix the oceans at the observed rates. In summary, MW98's argument is that turbulent mixing must replenish the background gravitational potential energy (GPE_r) at the same rate ($W_{r,\text{forcing}}$) at which high-latitude cooling depletes it. According to classical turbulence theory (for a Boussinesq fluid with a linear equation of state), the rate of GPE_r increase due to turbulent mixing is equal to the rate of available mechanical stirring energy $G(KE)$ times the 'mixing efficiency' parameter γ_{mixing},[1] e.g., Osborn (1980). Thus, equating $\gamma_{\text{mixing}} G(KE)$ with $W_{r,\text{forcing}}$ yields the following constraint on the rate of

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[1] Oakey (1982) shows that γ_{mixing} can be expressed as $\gamma_{\text{mixing}} = \varepsilon_P/\varepsilon_K$, that is, as the ratio of the local rate of available potential energy dissipation by turbulent molecular diffusion over the local rate of turbulent kinetic energy dissipation by molecular viscosity. This definition makes it clear that the 'mixing efficiency' γ_{mixing}, despite what its name suggests, is not necessarily bound to be lower than unity. On the other hand, the parameter $R_f = \gamma_{\text{mixing}}/(1 + \gamma_{\text{mixing}})$ is always comprised between 0 and 1, so that many authors favour reserving the term 'mixing efficiency' to R_f rather than γ_{mixing}, for example, Peltier and Caulfield (2003). R_f is the so-called flux Richardson number.

mechanical energy input $G(KE)$

$$G(KE) = \frac{W_{r,\text{forcing}}}{\gamma_{\text{mixing}}}. \tag{1}$$

Estimating $W_{r,\text{forcing}} = O(0.4\,\text{TW})$ from an observational estimate of the rate of deep water formation, and using the canonical value $\gamma_{\text{mixing}} \approx 0.2$ of Osborn (1980), MW98 concluded that $G(KE) \approx 2\,\text{TW}$ were needed to sustain turbulent diapycnal mixing in the oceans. Since the wind is known to contribute for about $O(1\,\text{TW})$, about $O(1\,\text{TW})$ needs to be attributed to other sources of stirring . The most natural candidate is the buoyancy forcing, which Oort et al. (1994) estimated to contribute for $G(APE) = 1.2 \pm 0.7\,\text{TW}$, where $G(APE)$ is the production rate of available potential energy by surface buoyancy fluxes. MW98 argued, however, that Sandström (1908)'s 'theorem' precludes $G(APE)$ to be large, and that only the work rate done by the tides can realistically close the energy budget. According to MW98, the buoyancy forcing alone could not produce an overturning circulation and turbulent diapycnal mixing rates such as those observed in the oceans on its own because of Sandström (1908)'s 'theorem'.

MW98's rather controversial and somewhat provocative stance succeeded in generating a considerable renewed interest in the issues of ocean energetics, tidal mixing, and driving mechanisms of the AMOC, which had until then been regarded as rather well understood. The interested reader may consult Huang (1999), Wunsch and Ferrari (2004), Nycander et al. (2007) and Kuhlbrodt et al. (2007) for a survey of the most recent developments. The main questions raised by Munk and Wunsch (1998)'s study, which seem to have attracted the most attention so far, can be summarized as follows:

(i) Does Sandström (1908)'s 'theorem' really imply that $G(APE)$ should be negligibly small in the oceans? Alternatively, is it true that Sandström (1908)'s 'theorem' implies that the buoyancy fluxes can produce only negligible work rate when applied to the top of a stratified fluid?

(ii) How accurate is the value $W_{r,\text{forcing}} = 0(0.4\,\text{TW})$ estimated by Munk and Wunsch (1998)? Does it overestimate or underestimate $W_{r,\text{forcing}}$?

(iii) What is the nature of the so-called mixing efficiency γ_{mixing} parameter? What is its physical origin? Should we accept the canonical value $\gamma_{\text{mixing}} \approx 0.2$ widely assumed by oceanographers?

(iv) How do the non-linearities of the equation of state affect Munk and Wunsch (1998)'s conclusions regarding the energy sources for mechanical stirring?

2.2. The work rate done by surface buoyancy fluxes

One of MW98's most controversial assertion is perhaps that the power input of mechanical energy due to the surface buoyancy forcing, denoted by W_{buoyancy} henceforth, must be negligible because of Sandström (1908)'s 'theorem'. So far, it seems fair

to say that this issue is still the subject of considerable debate among oceanographers. To a large extent, the current controversy owes its origin to the current lack of agreement on how W_{buoyancy} should be defined and quantified.[2] The situation is somewhat peculiar, though, because until recently, MW98 and a majority of earlier studies all appeared to agree that W_{buoyancy} should be measured by the APE production rate $G(APE)$, as is commonly done in the atmospheric case following the pioneering work of Lorenz (1955). Given that there exist known expressions for $G(APE)$, it is in principle rather straightforward using observations to assess its magnitude, as done by Oort et al. (1994), even if large error bars arise in doing so. Why Sandström (1908)'s 'theorem', which is merely an attempt at empirically rationalizing a number of laboratory experiments that others have failed to reproduce, for example, Coman et al. (2006), and for which a sound theoretical basis is lacking, should imply that $G(APE)$ cannot be large in the oceans is rather mysterious.

To add to the confusion, a dramatic paradigm shift was surreptitiously introduced over the past decade by a number of authors who started to advocate that W_{buoyancy} should rather be measured by the following quantity:

$$B = \int_V \kappa g z \nabla^2 \rho \, \mathrm{dV} = \kappa g \left[\langle \rho \rangle_{\text{bottom}} - \langle \rho \rangle_{\text{top}} \right] \tag{2}$$

for example, Paparella and Young (2002) and Wang and Huang (2005), which physically represents the work of expansion/contraction due to the molecular diffusive heating/cooling in the Boussinesq approximation, where $\langle \rho \rangle_{\text{bottom}}$ and $\langle \rho \rangle_{\text{top}}$ denote the bottom and surface integrated density respectively, with g the acceleration of gravity, z the vertical coordinate, and κ the molecular diffusion. From the viewpoint of APE theory, it can be shown that the above quantity B is related to $G(APE)$ by the formula:

$$B = G(APE) - D(APE), \tag{3}$$

for example, Tailleux (2009a), where $D(APE)$ denotes the APE dissipation rate due to molecular diffusion. For typical oceanic values, Wang and Huang (2005) show that $B = O(15\,\text{GW})$, which is smaller by several orders of magnitude than $G(APE) = O(1.2 \pm 0.7\,\text{TW})$ estimated by Oort et al. (1994). Obviously, regarding B rather than $G(APE)$ as the definition of W_{buoyancy} represents a dramatic paradigm shift, which does not appear to have encountered much resistance so far, so much that it is B that is implicitly used as the definition of W_{buoyancy} in the reviews by Wunsch and Ferrari (2004) and Kuhlbrodt et al. (2007), which do not even cite Oort et al. (1994)'s estimate for $G(APE)$. Apparently, regarding B as the definition of W_{buoyancy} owes much of its success to being conveniently small and vanishing in the limit of vanishing molecular diffusion, in apparent agreement with a widespread interpretation of Sandström (1908)'s 'theorem'.

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[2] We note that Sandström (1908) does not provide any indication on how to compute W_{buoyancy}.

Physically, B represents the work done by *all* buoyancy fluxes. The latter not only include the surface buoyancy fluxes, but the interior buoyancy fluxes due to turbulent molecular diffusion as well. Therefore, identifying B with W_{buoyancy} is no more justified than identifying the work rate done by the velocity against the stress tensor $\mathbf{S}$, namely,

$$\mathbf{v} \cdot \nabla \cdot \mathbf{S} = \underbrace{\int_S \mathbf{u}_s \cdot \tau \, \mathrm{d}S}_{G(KE)} - \underbrace{\int_V \rho \varepsilon_K \, \mathrm{d}V}_{D(KE)}, \tag{4}$$

as the work rate done by the wind W_{wind}, where ε_K is the viscous dissipation rate. As for B, the quantity $\mathbf{v} \cdot \nabla \cdot \mathbf{S}$ includes the work done by the surface wind stress as well as the work done against interior turbulent molecular viscous forces. The fact that both B and $\mathbf{v} \cdot \nabla \mathbf{S}$ incorporate internal irreversible processes opposing the power input due to the buoyancy or wind forcing clearly speaks against using the latter as a definition of W_{buoyancy} and W_{wind} respectively. Moreover, we note that in the same way that W_{wind} explicitly depends upon the wind forcing, so should W_{buoyancy} explicitly depend upon the surface buoyancy forcing. The latter property is satisfied by $G(APE)$, but obviously not by B. Making the distinction between $G(APE)$ and B is crucial, because Paparella and Young (2002)'s 'anti-turbulence theorem' shows that while the difference between $G(APE)$ and $D(APE)$ must always be very small for Boussinesq horizontal convection (see eqs 2 and 3), there is no such constraint precluding $G(APE)$ from being potentially large, as further discussed below.

What is then the appropriate value of $G(APE)$ in the oceans? In support of MW98, it seems fair to say that a known shortcoming of Oort et al. (1994)'s methodology is its reliance on the so-called Lorenz approximation, which usually overestimates APE [and plausibly $G(APE)$ as well], as discussed by Huang (1998). But can Lorenz's approximation be so bad as to completely invalidate Oort et al. (1994)'s $G(APE)$ estimate? To resolve this issue, a deeper look at exact expressions of $G(APE)$ is needed. This was done by Tailleux (2009a), who used both simple scaling arguments and an indirect method to suggest that $G(APE) = O(0.4 - 0.5 \, \text{TW})$ is a plausible estimate for typical oceanic values, which coincides with Oort et al. (1994)'s lower bound for $G(APE)$. The indirect method relies on the possibility to establish from first principles the following result:

$$G(APE) \approx W_{r,\text{forcing}}, \tag{5}$$

which physically states that the production rate of APE is approximately equal to the rate at which GPE_r decreases as the result of high-latitude cooling (with a strict equality for a Boussinesq fluid with a linear equation of state), see Tailleux (2009a). This result, which has been overlooked so far, is important, because it shows that for consistency, $W_{r,\text{forcing}}$ and $G(APE)$ must be assumed to be similar numbers. In contrast, MW98 assumed $G(APE) \approx 0$ but $W_{r,\text{forcing}} \approx 0.4 \, \text{TW}$. Of course, as for any observational estimate, especially in the oceans, the accuracy of MW98's estimate for $W_{r,\text{forcing}}$ could itself be debated, as done

by some authors, see Hughes and Griffiths (2006) and the review by Kuhlbrodt et al. (2007), but this is beyond the scope of this paper. The important point, for the present purposes, is that $G(APE)$ and $W_{r,\text{forcing}}$ should always be similar numbers, plausibly $O(0.4 - 0.5 \, \text{TW})$ in the oceans, even though it would be worthwhile to re-assess these numbers more carefully from the most recent observations, using exact expressions for $G(APE)$ rather than Lorenz's approximation. See also Hughes et al. (2009) on related views.

2.3. Alternative views on MW98's energy constraint

An important consequence of eq. (5) is to allow for a rewriting of MW98's energy constraint eq. (1) as follows:

$$\frac{G(APE)}{G(KE)} \approx \gamma_{\text{mixing}}. \tag{6}$$

This result is important for understanding how W_{buoyancy} should enter MW98's constraint. Indeed, the issue has remained unclear so far owing to MW98 discarding W_{buoyancy} and $G(APE)$ on the basis of Sandström's 'theorem'. Based on the existing literature, it would seem that the most natural approach to extend MW98's constraint to account for a non-zero value of $W_{\text{buoyancy}} = G(APE)$ would be by simply adding $G(APE)$ to $G(KE)$ in eq. (1), which would lead to

$$[G(APE) + G(KE)] = \gamma_{\text{mixing}} W_{r,\text{forcing}}. \tag{7}$$

However, eq. (6) above shows that this would be incorrect. In fact, it turns out that MW98's neglect of $G(APE)$ led to the correct result, not so much because $G(APE)$ is actually negligible, as discussed above, but rather because adding $G(APE)$ to $G(KE)$ would somehow amount to the double counting of the effects of cooling. This conclusion is surprising and far from intuitive, as realizing that eq. (7) is erroneous requires recognizing that the same physics sets the values of both $W_{r,\text{forcing}}$ and $G(APE)$. Specifically, the cooling, while lowering the centre of gravity of the background stratification at the rate $W_{r,\text{forcing}}$, also liberates a similar amount of energy at the rate $G(APE)$ that is available for reversible conversion into kinetic energy and hence available for stirring as well.

The fact that it is MW98's eq. (1) that is correct, rather than the more intuitive eq. (7), seems to conflict with the classical interpretation of eq. (1) as a constraint on the power input of mechanical energy given the observed rate of deep water formation. Before addressing the latter issue, it is useful to discuss how eq. (1) should be generalized to account for a non-linear equation of state. Indeed, note that eq. (1) implicitly assumes a Boussinesq ocean with a linear equation of state. Tailleux (2009a) addressed this question by introducing the parameter ξ defined by

$$\xi = \frac{W_{r,\text{turbulent}}}{D(APE)}, \tag{8}$$

where $W_{r,\text{turbulent}}$ is the rate of change of the background GPE_r and $D(APE)$ is the APE dissipation rate due to turbulent molecular diffusion, integrated over the domain considered. As shown previously by Winters et al. (1995), $\xi = 1$ for a Boussinesq fluid with a linear equation of state. For water or seawater, however, one may show that $-\infty < \xi < 1$, as discussed in Tailleux (2009a,b). The precise value of ξ depends on the characteristics of the stratification, and cannot be determined without a full analysis of the problem at hand. In particular, estimating the relevant value of ξ for the oceanic case remains to be done. Hints as to what parameters control ξ are discussed in Tailleux (2009b). Assuming that ξ has been determined in some way, Tailleux (2009a) shows that it modifies eq. (1) as follows:

$$G(KE) = \frac{1 + (1 - \xi)\gamma_{\text{mixing}}}{\xi\gamma_{\text{mixing}}} G(APE)$$
$$= \frac{1 - \xi R_f}{\xi R_f} G(APE), \qquad (9)$$

where $R_f = \gamma_{\text{mixing}}/(1 + \gamma_{\text{mixing}})$ is a globally defined flux Richardson number. MW98's eq. (1) is recovered in the limiting case $\xi = 1$, and by using the above result that $G(APE) \approx W_{r,\text{forcing}}$. Note that a non-linear equation of state increases the requirement on $G(KE)$ for a given value of $G(APE)$ and γ_{mixing}, which is consistent with the findings of Gnanadesikan et al. (2005).

An interesting consequence of the above extension and generalization of MW98's results is to question whether eq. (9) (and hence eq. 1) can be legitimately interpreted as a constraint on the amount of $G(KE)$ required to sustain diapycnal mixing in the oceans, as proposed by MW98. Indeed, MW98's interpretation seems to assume implicitly that both $G(APE)$ and γ_{mixing} are fixed in some sense. But eq. (9) can be rewritten equivalently either as

$$G(APE) = \frac{\xi\gamma_{\text{mixing}}}{1 + (1 - \xi)\gamma_{\text{mixing}}} G(KE), \qquad (10)$$

or as

$$\xi R_f = \frac{G(APE)}{G(KE) + G(APE)}, \qquad (11)$$

which one may want to regard either as a constraint on $G(APE)$ or on the parameter ξR_f. For instance, in the latter case, one could estimate the right-hand side of eq. (11) from the plausible observational values $G(KE) \approx 1\,\text{TW}$ and $G(APE) \approx 0.5\,\text{TW}$, yielding

$$\xi R_f \approx \frac{0.5}{1.5} \approx 0.3,$$

and ask whether this value for ξR_f, which physically represents the relative fraction of the total mechanical energy input due to the buoyancy forcing, can be ruled out from what is known empirically about ξ and R_f (or γ_{mixing}). Regarding γ_{mixing}, although a majority of studies tends to assume $\gamma_{\text{mixing}} \approx 0.2$, it is important to note that the value of γ_{mixing} is actually highly dependent upon the mixing process considered. Thus, while γ_{mixing} appears to be

indeed close to $\gamma_{\text{mixing}} \approx 0.2$ for mechanically driven turbulent mixing (as in shear flow instability for instance), buoyancy-driven turbulent mixing is generally found to be more efficient, with values as high as $R_f = 0.5$ ($\gamma_{\text{mixing}} = 1$) being reported by Dalziel et al. (2008) in the context of Rayleigh–Taylor instability, see Tailleux (2009a) for a discussion about this point.

2.4. On the nature of mixing efficiency

A few comments on γ_{mixing} are in order, owing to the importance of this parameter for the whole issue of ocean energetics. The way such a parameter appears in Tailleux (2009a)'s theory is as the ratio

$$\gamma_{\text{mixing}} = \frac{D(APE)}{D(KE)}, \qquad (12)$$

where $D(APE)$ and $D(KE)$ are the global APE and KE dissipation rates,

$$D(APE) = \int_V \rho\varepsilon_P\,dV, \qquad D(KE) = \int_V \rho\varepsilon_K\,dV \qquad (13)$$

with ϵ_P and ϵ_K being the local dissipation rates of APE and KE due to molecular diffusive and viscous processes, respectively. In other words, the parameter γ_{mixing} entering all above formula is a globally defined bulk value. This value is therefore possibly different from the quantity actually measured in field campaigns or in laboratory experiments, which by contrast is locally defined over a considerably smaller geographical region by $\gamma_{\text{mixing}}^{\text{local}} = \varepsilon_P/\varepsilon_K$, for example, Oakey (1982). Note here that the concept of local APE dissipation requires the concept of APE density, see Holliday and McIntyre (1981) or Roullet and Klein (2009) for a clarification about this idea. The link between the bulk value of γ_{mixing} and its geographically variable local counterpart $\gamma_{\text{mixing}}^{\text{local}}(\mathbf{x})$ can be clarified if one writes

$$\gamma_{\text{mixing}} = \frac{D(APE)}{D(KE)} = \frac{\int_V \rho\varepsilon_P\,dV}{\int_V \rho\varepsilon_K\,dV} = \frac{\int_V \gamma_{\text{mixing}}^{\text{local}}\rho\varepsilon_K\,dV}{\int_V \rho\varepsilon_K\,dV}$$
$$= \frac{\sum_i \gamma_{\text{mixing}}^{(i)} \int_{V_i} \rho\varepsilon_K\,dV}{\sum_i \int_{V_i} \rho\varepsilon_K\,dV} = \sum_i W_i\,\gamma_{\text{mixing}}^i, \qquad (14)$$

where the latter equation was obtained by decomposing the total ocean into subdomains V_i over which $\gamma_{\text{mixing}}^{\text{local}}(\mathbf{x})$ is approximately constant, the weights W_i being defined by

$$W_i = \frac{\int_{V_i} \rho\varepsilon_K\,dV}{\int_V \rho\varepsilon_K\,dV} = \frac{\int_{V_i} \rho\varepsilon_K\,dV}{D(KE)}. \qquad (15)$$

This result, therefore, shows that the bulk value γ_{mixing} used in the above formula (and in MW98's paper) actually represents a weighted average of the geographically variable $\gamma_{\text{mixing}}^{\text{local}}$, the subdomains V_i with the largest turbulent kinetic energy dissipation rates having the most weight. Therefore, even if $\gamma_{\text{mixing}}^{\text{local}} = 0.2$ over 90% of the oceans say (with a weight $W = 0.9$), having $\gamma_{\text{mixing}}^{\text{local}} = 2$ over the remaining 10% (with a weight $W = 0.1$) could be potentially sufficient to yield $\gamma_{\text{mixing}} \approx 0.38$. Note

that because such a value of γ_{mixing} is nearly twice as large as the commonly used one, it would nearly halve the requirement on the energy constraint derived by MW98, thereby eliminating the apparent shortfall in mechanical energy that prompted so much interest in tidal mixing over the past decade. This point is important, because it establishes that the bulk value of γ_{mixing} relevant to the oceans cannot be known accurately without sufficient knowledge of the places where $\gamma_{\text{mixing}}^{\text{loacl}} \gg 0.2$. An important outcome of this paper is to provide numerical empirical evidence suggesting that $\gamma_{\text{mixing}}^{\text{local}} > 1$ are possible in convectively unstable regions.

2.5. Structure of the wind and buoyancy forcings

As said above, interpreting eq. (9) as an energy constraint on the mechanical sources of stirring is debatable. To shed light on this issue, some important points about the nature and structure of $G(KE)$ and $G(APE)$ that have been so far overlooked need to be raised. First, recall that the work rate done by the surface wind stress τ is usually expressed as follows:

$$G(KE) = \int_S \tau \cdot \mathbf{u}_s \mathrm{d}S, \qquad (16)$$

where $\mathbf{u}_s$ is the ocean surface velocity. Note that $G(KE)$ appears as a correlation between the external forcing (the wind stress) and a parameter depending upon the ocean circulation (the surface velocity); in other words, a key property of $G(KE)$ is that it is not a function of the forcing alone, but of the solution of the problem as well. This point is important, because it makes it possible for the buoyancy forcing to exert some control on the wind-driven circulation through its effect on the surface velocity. To assess the importance of such an effect, one could for instance compute $G(KE)$ for a purely wind-driven homogeneous ocean model, and compare it with that obtained for a wind- and buoyancy-driven ocean model, which is beyond the scope of this paper. For the present purposes, we shall take as our working assumption that $G(KE)$ is primarily determined by the wind forcing, which a majority of oceanographers would probably agree is a reasonable assumption.

The work rate done by surface buoyancy fluxes, on the other hand, was shown by Tailleux (2009a) to be given by the following expression (for a compressible thermally stratified fluid):

$$G(APE) = \int_S \frac{T - T_r}{T} Q_{\text{surf}} \mathrm{d}S, \qquad (17)$$

where T is the surface temperature of the fluid parcels, and T_r the temperature the surface parcels would have if displaced adiabatically to their level in Lorenz (1955)'s reference state, while Q_{surf} is the diabatic rate of heating cooling/heating due to the surface heat fluxes. A useful approximation to eq. (17) can be obtained by expanding T as a Taylor series around the surface pressure P_a, that is, $T \approx T_r + \Gamma_r(P_a - P_r) + O[(P_a - P_r)^2]$,

where $\Gamma_r = \alpha_r T_r/(\rho_r C_{pr})$ is the adiabatic lapse rate, leading to

$$\begin{aligned} G(APE) &\approx -\int_S \frac{\alpha_r(P_r - P_a)}{\rho_r C_{pr}} Q_{\text{surf}} \mathrm{d}S \\ &\approx \int_S \frac{\alpha_r g z_r}{C_{pr}} Q_{\text{surf}} \mathrm{d}S, \end{aligned} \qquad (18)$$

by using the approximation $P_r - P_a \approx -\rho_0 g z_r$. The last expression in eq. (18) is actually that derived by Winters et al. (1995) in the context of the Boussinesq approximation, with α_r and C_{pr} constant [the suffix r indicates that the variables have to be estimated in Lorenz (1955)'s reference state]. Like $G(KE)$, $G(APE)$ is a correlation between a term linked to the external forcing (the surface heating/cooling rate) and a parameter depending on the particular state of the system, namely $(T - T_r)/T \approx \alpha_r(P_a - P_r)/(\rho_r C_{pr}) \approx \alpha_r g z_r/C_{pr}$. Equations (17) and (18) make it possible, therefore, for the wind (and tides) to control the buoyancy-driven circulation through their effects on the stratification. Equation (18) can be written as follows:

$$G(APE) \approx \frac{\alpha g}{C_p}\left(h_r^{\text{cool}} - h_r^{\text{heat}}\right) Q_{\text{heating}} \qquad (19)$$

where h_r^{cool} and h_r^{heat} are appropriate values for the reference depths associated with the net cooling Q_{cooling} and net heating Q_{heating} respectively, with $Q_{\text{cooling}} = Q_{\text{heating}}$ in steady state conditions. Specifically, h_r^{heat} and Q_{heating} are defined by

$$h_r^{\text{heat}} = \frac{\int_{S^+} |z_r| Q_{\text{surf}} \mathrm{d}S}{\int_{S^+} Q_{\text{surf}} \mathrm{d}S}, \qquad (20)$$

$$Q_{\text{heating}} = \int_{S^+} Q_{\text{surf}} \mathrm{d}S, \qquad (21)$$

assuming α and C_p constant, where S^+ represents the fraction of the surface restricted to positive values of Q_{surf}, with a symmetric definition for h_r^{cool} and Q_{cooling}. Physically, h_r^{heat} is likely small, as heating in the oceans occurs primarily over the warmest (and hence lightest) temperatures, while h_r^{cool} is a direct measure of the thermocline depth. Plausible oceanic values $\alpha = 10^{-4}\ K^{-1}$, $C_p = 4 \times 10^3$ J.K^{-1} kg^{-1}, $g = 10$ m s^{-2}, $Q_{\text{heating}} = 2$ PW $= 2 \times 10^{15}$ W, $h_r^{\text{cool}} - h_r^{\text{heat}} = 10^3$ m yields $G(APE) = 0.5$ TW, suggesting that $G(APE)$ is comparable with the work rate done by the wind, consistent with the findings of Oort et al. (1994). Equations (18) and (19) show that the wind and tides can control the amount of energy extracted from the buoyancy forcing by controlling the thermocline depth. Interestingly, this behaviour appears to be consistent with the recent laboratory experiments by Whitehead and Wang (2008) showing the strength of horizontal convection to be significantly enhanced by the lateral action of a stirring rod (see Hughes and Griffiths, 2008 for a review on horizontal convection). Indeed, the authors find that the enhanced overturning strength corresponds to a deepening of the stratification, which suggests that the stirring rod causes an increase in $G(APE)$, thereby allowing more energy to be extracted from the buoyancy forcing, resulting in a stronger buoyancy-driven circulation.

3. Theoretical results about the effects of mechanical stirring on the work rate done by surface buoyancy fluxes

3.1. Summary of Tailleux (2009)'s theory

The following seeks to summarise and review the key elements of the theoretical framework describing the energetics of thermodynamically and mechanically forced turbulent stratified fluids recently developed by Tailleux (2009a), building upon the previous works by Winters et al. (1995) and Lorenz (1955). This framework is based on the decomposition of the total energy into five distinct energy reservoirs, namely: (1) volume-integrated kinetic energy (KE); (2) volume integrated available potential energy (APE); (3) volume integrated background gravitational potential energy (GPE_r); (4) volume integrated exergy IE_{exergy} (a subcomponent of internal energy IE associated with the reference vertical temperature gradient) and (5) volume integrated dead internal energy IE_0 (a subcomponent of internal energy IE associated with the equivalent thermodynamic equilibrium temperature of the system), for which the evolution equations are expressed by

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = -C(KE, APE) + G(KE) - D(KE), \tag{22}$$

$$\frac{\mathrm{d}APE}{\mathrm{d}t} = C(KE, APE) + G(APE) - D(APE), \tag{23}$$

$$\frac{\mathrm{d}GPE_r}{\mathrm{d}t} = \underbrace{W_{r,\text{turbulent}} + W_{r,\text{laminar}}}_{W_{r,\text{mixing}}} - W_{r,\text{forcing}}, \tag{24}$$

$$\frac{\mathrm{d}IE_0}{\mathrm{d}t} = (1 - \Upsilon_0)\dot{Q}_{\text{net}} + D(APE) + D(KE) - G(APE), \tag{25}$$

$$\frac{\mathrm{d}IE_{\text{exergy}}}{\mathrm{d}t} = \Upsilon_0 \dot{Q}_{\text{net}} - \underbrace{[W_{r,\text{turbulent}} + W_{\text{laminar}}]}_{W_{r,\text{mixing}}}, \tag{26}$$

such that

$$TE = KE + APE + GPE_r + IE_0 + IE_{\text{exergy}}$$

represents the total energy of the fluid. The precise definitions of all energy reservoirs and energy conversion terms are given below. Note that the above evolution equations are exact, but for the neglect of very small terms, see Tailleux (2009a) for justification. Moreover, it is also important to note that the above evolution equations do not rely on any form of Reynolds decomposition and associated closure assumption, unlike most current treatment of ocean energetics.

The total kinetic energy is defined by

$$KE = \int_V \rho \frac{\mathbf{v}^2}{2} \, \mathrm{d}V;$$

the available potential energy is defined by

$$APE = \underbrace{\int_V \rho \left[gz + I(\Sigma, \upsilon) \right] \mathrm{d}V}_{PE} - \underbrace{\int_V \rho \left[gz_r + I(\Sigma, \upsilon_r) \right] \mathrm{d}V}_{PE_r}$$

defined as in Lorenz (1955) as the difference between the potential energy PE (the sum of the gravitational potential energy GPE and internal energy IE) of the actual state minus the background potential energy PE_r of the reference state, for example, the PE of the state of minimum potential energy achievable in an adiabatic rearrangement of the fluid parcels (subscript r), where g is gravity, ρ is density, z is the vertical coordinate pointing upward and I the specific internal energy, viewed as a function of specific entropy Σ and specific volume υ; the gravitational potential energy of the background reference state is defined by

$$GPE_r = \int_V \rho g z_r \, \mathrm{d}V,$$

where $z_r = z_r(\mathbf{x}, t)$ represents the vertical position of the fluid parcels in their reference state; the dead part of internal energy is defined by

$$IE_0 = \int_V \rho_0 I(T_0, P_0) \mathrm{d}V$$

defined as the internal energy of the isothermal state of temperature T_0 having the same potential energy as the background reference state, that is, such that

$$PE_0 = PE_r,$$

where the specific internal energy I is best regarded here as a function of temperature and pressure; the exergy part of internal energy is defined by

$$IE_{\text{exergy}} = IE_r - IE_0,$$

where IE_r is the internal energy of Lorenz (1955)'s reference state. Physically, the separation of internal energy into its dead and exergy parts is motivated by the need to distinguish between processes primarily affecting the equivalent thermodynamic temperature T_0 rather than the vertical variations of the reference temperature profile $T_r'(z, t) = T_r(z, t) - T_0$. For instance, Tailleux (2009a) argues that the diffusive dissipation of APE at the rate $D(APE)$ primarily affects T_0, whereas the GPE_r variations due to turbulent molecular diffusion occurring at the rate $W_{r,\text{mixing}}$ primarily affects $T_r'(z, t)$.

The total energy is $TE = KE + GPE + IE$, where $GPE = AGPE + GPE_r$, and $IE = AIE + IE_r = AIE + IE_{\text{exergy}} + IE_0$. The total energy satisfies the evolution equation

$$\frac{\mathrm{d}TE}{\mathrm{d}t} = G(KE) + \dot{Q}_{\text{net}} \tag{27}$$

obtained by summing (22)–(26). In a steady state, we see that $\overline{Q}_{\text{net}} = -\overline{G(KE)}$, that is, there must be some net cooling in order to balance the energy budget.

In eqs (22) and (23), the term $C(KE, APE)$ represents the reversible conversion between KE and APE. Its explicit expression

is given by

$$C(KE, APE) = \int_V \rho \left(gw - P \frac{\mathrm{D}\upsilon}{\mathrm{D}t} \right) \mathrm{d}V \tag{28}$$

where P is the pressure, w is the vertical velocity, $\upsilon = 1/\rho$ is the specific volume. This expression reduces, in the Boussinesq approximation, to the more familiar expression

$$C(KE, APE) = \int_V \rho gw \, \mathrm{d}V \tag{29}$$

which is often referred to as the 'buoyancy flux'.

The terms $G(KE)$ in eq. (22), $G(APE)$ in eq. (23), $\dot{Q}_{\mathrm{net}}$ in eqs (25) and (26) and $W_{r,\mathrm{forcing}}$ in eq. (24) represent forcing terms, in the sense that these terms involve the external forcing (i.e. the surface stress τ or surface heat flux Q_{surf}) explicitly. The explicit expressions for these terms is given by

$$G(KE) = \int_S \mathbf{u}_s \cdot \tau \, \mathrm{d}S, \tag{30}$$

$$G(APE) = \int_S \left(\frac{T - T_r}{T} \right) Q_{\mathrm{surf}} \mathrm{d}S, \tag{31}$$

$$\dot{Q}_{\mathrm{net}} = \int_S Q_{\mathrm{surf}} \, \mathrm{d}S, \tag{32}$$

$$W_{r,\mathrm{forcing}} = -\int_S \frac{\alpha_r P'_r}{\rho_r C_{pr}} Q_{\mathrm{surf}} \mathrm{d}S. \tag{33}$$

Note that in the case of the APE eq. (23), $G(APE)$ can be regarded both as a forcing term as well as a conversion between dead internal energy IE_0 and APE. Here appears the connection with the classical thermodynamic theory of heat engines, in the sense that $G(APE)$ appears as a surface integral of the product of a thermodynamic efficiency like coefficient (i.e. $(T - T_r)/T$) times the surface heat flux Q_{surf}. From that viewpoint, the production of available potential energy explicitly appears as the conversion of internal energy into mechanical energy, which is precisely what heat engines do. As discussed in Section 2, a Taylor series expansion of the surface temperature T in eq. (3) shows that at leading order $(T - T_r)/T \approx -\alpha_r P'_r/(\rho_r C_{pr})$, and hence that $G(APE) \approx W_{r,\mathrm{forcing}}$. Nevertheless, the fact that in reality $G(APE)$ and $W_{r,\mathrm{forcing}}$ are never exactly equal motivates the introduction of the parameter $\xi_2 = W_{r,\mathrm{forcing}}/G(APE) = O(1)$ to capture the difference. As discussed in the introduction, the differences between $G(APE)$ and $W_{r,\mathrm{forcing}}$ vanish in the Boussinesq approximation, in which case

$$G(APE) = W_{r,\mathrm{forcing}} = \int_S \frac{\alpha g z_r}{C_p} Q_{\mathrm{surf}} \mathrm{d}S. \tag{34}$$

The above equations suggest that a thermodynamic efficiency appropriate for the ocean should be based on the parameter

$$\Upsilon = \frac{\alpha P}{\rho C_p} \tag{35}$$

for suitably chosen values of of α, P, ρ and C_p. Note that among these four quantities, only P can vary significantly. The parameter Υ_0 is defined by

$$\Upsilon_0 = \frac{\int_{V_0} \alpha_0 P'_0 \mathrm{d}V_0}{\int_{V_0} \rho_0 C_{p0} \mathrm{d}V_0}, \tag{36}$$

where all the quantities with subscripts 0 refer to quantities defined for the equivalent isothermal thermodynamic equilibrium state of temperature T_0 defined above.

The quantities whose magnitude increase due to to turbulence are $D(KE)$, $W_{r,\mathrm{mixing}}$ and $D(APE)$, whose explicit expressions are given by

$$D(KE) = \int_V \rho \varepsilon_K \mathrm{d}V, \tag{37}$$

$$W_{r,\mathrm{mixing}} = -\int_V \kappa \rho C_p \nabla T \cdot \nabla \left(\frac{\alpha_r P'_r}{\rho_r C_{pr}} \right) \mathrm{d}V, \tag{38}$$

$$D(APE) = \int_V \kappa \rho C_p \nabla T \cdot \nabla \left(\frac{T - T_r}{T} \right) \mathrm{d}V, \tag{39}$$

where $P'_r = P_r - P_a$, with P_a the atmospheric pressure. As is well known, $D(KE)$ represents the dissipation rate of kinetic energy into internal energy. The nature of $D(APE)$ and $W_{r,\mathrm{mixing}}$, however, was only clarified by Tailleux (2009a), who showed that $W_{r,\mathrm{mixing}}$ represented the conversion rate between IE_{exergy} and GPE_r, while $D(APE)$ represent the dissipation of APE into IE_0. The intensification of $W_{r,\mathrm{mixing}}$ is best evidenced by looking at its form in the case of a Boussinesq fluid with a linear equation of state for which one has

$$\begin{aligned} W_{r,\mathrm{mixing}} &= \int_V \kappa \|\nabla z_r\|^2 \alpha g \frac{\partial T_r}{\partial z_r} \mathrm{d}V \\ &= \int_V \kappa \|\nabla z_r\|^2 \rho_0 N_r^2 \mathrm{d}V = \int_V (K_T + \kappa) \rho_0 N_r^2 \mathrm{d}V, \end{aligned} \tag{40}$$

where K_T is the turbulent effective diffusivity defined by Winters et al. (1995). It is customary to write $W_{r,\mathrm{mixing}} = W_{r,\mathrm{laminar}} + W_{r,\mathrm{turbulent}}$ as the sum of a laminar and turbulent (or non-laminar) contributions,

$$W_{r,\mathrm{laminar}} = \int_V \kappa \alpha g \frac{\partial T_r}{\partial z_r} \mathrm{d}V = \kappa g \left[\langle \rho_r \rangle_{\mathrm{bottom}} - \langle \rho_r \rangle_{\mathrm{top}} \right] \tag{41}$$

$$W_{r,\mathrm{turbulent}} = \int_V \kappa \left[\|\nabla z_r\|^2 - 1 \right] \alpha g \frac{\partial T_r}{\partial z_r} \mathrm{d}V, \tag{42}$$

where $\langle . \rangle$ denotes a horizontal integral, with top and bottom referring to the surface integral estimated at the top and bottom of the domain respectively. For simplicity, only horizontal surfaces are considered, that is, topographic effects are not considered here. Equation (42) shows that $W_{r,\mathrm{turbulent}}$ will differ from zero as soon as the isopycnal surfaces depart from their reference position, and hence even in quasi-laminar situations, such as the particular cases considered in this paper.
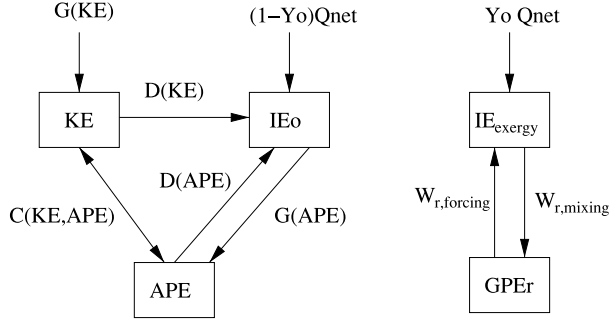
Fig. 1. Schematic diagram of the energy conversions taking place in a mechanically and thermodynamically forced turbulent thermally stratified fluid corresponding to eqs (22)–(26) previously derived by Tailleux (2009).

The two terms $D(APE)$ and $W_{r,\text{turbulent}}$ are generally found to be strongly correlated. This correlation was discussed empirically in Tailleux (2009b), who characterized the correlation in terms of the following ratio:

$$\xi = \frac{W_{r,\text{turbulent}}}{D(APE)} \tag{43}$$

which is generally such that $-\infty < \xi < 1$ for water or seawater. It is shown that the case of a Boussinesq fluid corresponds to the idealized limit $\xi \longrightarrow 1$. The fact that $W_{r,\text{turbulent}}$ and $D(APE)$ coincide for a Boussinesq fluid with a linear equation of state was first established by Winters et al. (1995). Figure 1 schematically illustrates the energetics of a mechanically and thermodynamically forced stratified fluids associated with the above equations. For all practical purposes, $G(APE)$ and $G(KE)$ represent the work rate done by the buoyancy and mechanical forcing respectively, whereas $D(APE)$ and $D(KE)$ represent the two terms dissipating the 'available' mechanical energy $ME = APE + KE$.

3.2. *Coupling between $G(APE)$ and $G(KE)$*

A central point of this paper is to emphasize the fact that $G(APE)$ depends sensitively upon the oceanic stratification, making it possible for the mechanical forcing to control the value of $G(APE)$ by modulating the thermocline depth via its control on turbulent diapycnal mixing. This dependence of $G(APE)$ on the mechanical forcing is reflected by the possibility to link $G(APE)$ to the mechanical power input $G(KE)$ due to the wind and tides, as previously seen with the rewriting of MW98's energy constraint as eq. (6). The following shows how to derive this result from first principles, using Tailleux (2009a)'s framework summarized above. To that end, we first construct the mechanical energy balance by summing the steady-state version of the KE and APE equations, which yields

$$G(APE) + G(KE) = D(APE) + D(KE). \tag{44}$$

Now, by combining this equation with the definition of the globally defined flux Richardson number

$$R_f = \frac{\gamma_{\text{mixing}}}{1 + \gamma_{\text{mixing}}} = \frac{D(APE)}{D(APE) + D(KE)}, \tag{45}$$

one may write

$$D(APE) = R_f[G(APE) + G(KE)]. \tag{46}$$

Next, we turn to the steady-state GPE_r balance, namely,

$$W_{r,\text{turbulent}} + W_{r,\text{laminar}} = W_{r,\text{forcing}}. \tag{47}$$

As mentioned above, $W_{r,\text{forcing}} \approx G(APE)$ to a very good approximation. To be mathematically rigorous, it is possible to introduce a parameter $\xi_2 = W_{r,\text{forcing}}/G(APE) \approx 1$. Using the definition of $\xi = W_{r,\text{turbulent}}/D(APE)$ above, allows one to rewrite the GPE_r budget as follows:

$$\xi D(APE) + W_{r,\text{laminar}} = \xi_2 G(APE). \tag{48}$$

Combining this result with eq. (46) yields

$$G(APE) = \frac{\xi R_f}{\xi_2 - \xi R_f} G(KE) + \frac{W_{r,\text{laminar}}}{\xi_2 - \xi R_f}. \tag{49}$$

Equation (49) is one of the most important results of this paper, as it represents one way to demonstrate the interconnection between the mechanical and thermodynamic power input in the most general settings. Note that eq. (9) is an exact result, valid whenever steady-state conditions can be assumed. A useful limit is the case of the widely used Boussinesq model for which $\xi = \xi_2 = 1$, in which case eq. (49) simplifies to

$$\begin{aligned} G(APE) &= \frac{R_f}{1 - R_f} G(KE) + \frac{W_{r,\text{laminar}}}{1 - R_f} \\ &= \gamma_{\text{mixing}} G(KE) + (1 + \gamma_{\text{mixing}}) W_{r,\text{laminar}}, \end{aligned} \tag{50}$$

Equation (49) shows that in absence of mechanical forcing, the work rate done by the surface buoyancy fluxes is given by

$$G(APE) = \frac{W_{r,\text{laminar}}}{\xi_2 - \xi R_f} \tag{51}$$

or, for a Boussinesq fluid

$$G(APE) = \frac{W_{r,\text{laminar}}}{1 - R_f} = (1 + \gamma_{\text{mixing}}) W_{r,\text{laminar}}. \tag{52}$$

The latter two results are interesting, because they do not rule out the possibility for $G(APE)$ to be large, provided that the parameters $\xi_2 - \xi R_f$ or $1 - R_f$ can become small enough. The problem, however, is that the latter conditions can be met only if R_f can become close to unity, or equivalently if $\gamma_{\text{mixing}} \gg 1$ (for a Boussinesq fluid), which is much larger than the widely and commonly used canonical value $\gamma_{\text{mixing}} \approx 0.2$. On the other hand, we are not aware that γ_{mixing} has ever been measured in the context of horizontal convection, leaving the question open. To settle the issue, therefore, one needs to establish whether or not values of $\gamma_{\text{mixing}} \gg 1$ are possible in this context. Proving that γ_{mixing} can never be much larger than unity would be sufficient

to definitively establish that the buoyancy power input can never be large for horizontal convection, which would support Sandström (1908)'s view. On the other hand, showing that γ_{mixing} can indeed be much larger than unity would establish, in contrast, that Sandström (1908)'s 'theorem' is invalid, and hence that buoyancy forcing in the context of horizontal convection can support both a strong overturning circulation and turbulent diapycnal mixing rates. The idealized numerical experiments presented below seek to get insight into this issue, by showing for the first time evidence of values for $\gamma_{\text{mixing}} \gg 1$.

Perhaps the most important feature of eq. (49) is to reveal that horizontal convection becomes mechanically controlled when the mechanical power input is such that $\xi R_f G(KE) \gg W_{r,\text{laminar}}$, which is possible even for relatively low values of γ_{mixing}—a very important point. This theoretical result suggests therefore that $G(APE)$ can be dramatically enhanced by the presence of mechanical stirring, provided that the latter provides positive work to the system (i.e. such that $G(KE) > 0$). Negative work, on the other hand, should reduce the strength of the buoyancy-driven circulation. The numerical examples studied next seek to illustrate these different ideas.

4. Numerical experiments

The physical implications of eqs (49) and (50) are explored in the following by means of numerical experiments of horizontal convection affected or not by the effects of mechanical stirring.

4.1. Model description

The idealized configuration considered here is similar to that used in many previous studies, for example, Beardsley and Festa (1972) and Paparella and Young (2002). The effect of mechanical stirring is modelled here as a forcing term in the vorticity equation. This is different from Beardsley and Festa (1972) who considered the effects of a stress applied to the external boundaries. A stress condition is not used in this study, in order to control the spatial extent over which the mechanical stirring is acting. This is found necessary to avoid the buoyancy-driven cell to be completely overcome by the mechanically driven cell, as is the case in Beardsley and Festa (1972). The numerical implementation of such a model was initially that described by Marchal (2007), which is based on a grid with collated temperature and streamfunction points. Despite the use of Arakawa Jacobians, however, it was found that the method did not conserve the mean temperature, resulting in a spurious net heat flux even in steady-state conditions. For this reason, the code was slightly modified to use a staggered grid instead, with temperature points being at the centre of the grid boxes. Such an approach was found to ensure the conservation of the mean temperature and its variance.

The equations solved by the numerical model are the Boussinesq equations for a fluid with a linear equation of state

$$\frac{\partial \omega}{\partial t} = J(\Psi, \omega) - g\alpha \frac{\partial T}{\partial x} + \nu \nabla^2 \omega + F(x, z, t) \qquad (53)$$

$$\frac{\partial T}{\partial t} = J(\Psi, T) + \kappa \nabla^2 T, \qquad (54)$$

where $\omega = \nabla^2 \Psi$ is the vorticity, T is the temperature, $J(a, b) = \partial a/\partial x \, \partial b/\partial z - \partial a/\partial z \, \partial b/\partial x$ is the Jacobian operator, g is the acceleration due to gravity, α is the thermal expansion coefficient assumed to be constant, ν is the kinematic viscosity, κ is the thermal diffusivity and F is a term aimed at modelling the effect of mechanical stirring that is further described later.

Following Marchal (2007), the above system is made dimensionless as follows: $t = (L^2/\kappa)t^*$, $(x, z) = L(x_*, z_*)$, $\omega = (\kappa/L^2)\omega_*$, $T = \Delta T T_*$ and $\Psi = \kappa \Psi_*$, where the starred quantities are the dimensionless ones. The dimensionless forms of the above equations become, after dropping the stars for clarity

$$\frac{\partial \omega}{\partial t} = J(\Psi, \omega) + P_r \left(-R_a \frac{\partial T}{\partial x} + \nabla^2 \omega \right) + F^*, \qquad (55)$$

$$\frac{\partial T}{\partial t} = J(\Psi, T) + \nabla^2 T, \qquad (56)$$

where $R_a = g\alpha\Delta T L^3/(\nu\kappa)$ is the Rayleigh number, and $P_r = \nu/\kappa$ is the Prandtl number. In the numerical experiments described here, we used $P_r = 10$ and $R_a = 10^5$. For comparison, note that typical oceanic values are $R_a = O(10^{20})$, for example, Paparella and Young (2002).

As in Paparella and Young (2002), the fluid is forced by a time-independent surface temperature boundary condition varying linearly in x. All experiments were performed at 4 different numerical resolutions in a 2-D square geometry, 50×50, 100×100, 150×150, and 200×200 in order to verify the robustness of our results to the model resolution. At equilibrium, the forcing term in the vorticity equation is associated with the work rate

$$G(KE) = \int \int_V F\Psi \, \mathrm{d}x \mathrm{d}z \qquad (57)$$

which can be in principle either positive or negative. In this paper, the mechanical forcing takes the following analytical form

$$F(x, z, t) = I \sin^2 [(3z + 2)\pi] \sin^2 (2\pi x), \qquad (58)$$

with I taking on different values detailed in the next paragraph. The shape of the forcing is illustrated in Fig. 2.

4.2. Experiments

Four idealized experiments were considered:

(i) Purely buoyancy-driven (i.e. $I = 0$, no mechanical forcing).

(ii) 'Direct' Mechanical forcing ($I = 30$).

(iii) Weak 'indirect' mechanical forcing ($I = -30$).
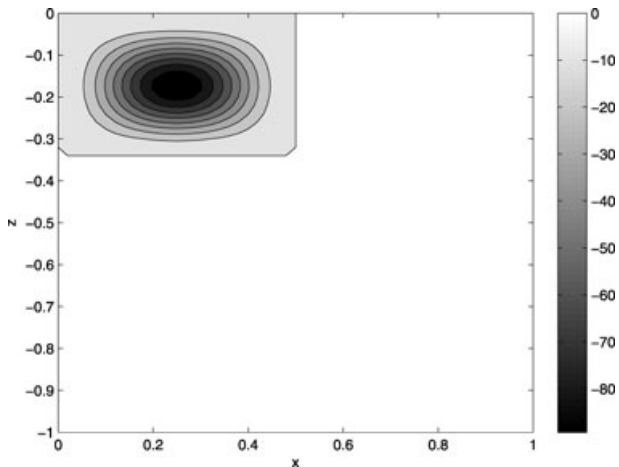
(iv) Strong 'indirect' mechanical forcing ($I = -90$).

Fig. 2. Shape of the forcing applied in case (d).

crease $G(APE)$ and hence the strength of the buoyancy-driven cell, with a corresponding increase in KE, APE, Ψ and $D(APE)$ over the purely buoyancy-driven case.

In contrast, experiments (iii) and (iv) seek to explore the effect of 'indirect' mechanical forcing, that is, which corresponds to the case where the mechanical forcing tends to oppose the thermal forcing. The case (iii) is designed so that the mechanical forcing is too weak to reverse the sense of the buoyancy-driven cell, in which case it always opposes the buoyancy-driven cell, and hence results in $G(KE) < 0$. According to eq. (50), a decrease in all quantities KE, APE, Ψ, $D(APE)$ and $G(APE)$ is expected in that case. In case (iv), the mechanical forcing becomes strong enough to create an 'indirect' cell flowing in the sense opposite to the buoyancy-driven cell, for which $G(KE) > 0$. According to eq. (50), this should result in the increase of all the above quantities.

4.3. Results

The results are synthesized in Figs. 3 and 4, which illustrate the streamfunction and temperature associated with the 4 different experiments (resolution 150×150). Fig. 5 illustrates the forcing and dissipation quantities $G(KE)$, $G(APE)$, $D(APE)$ and $D(KE)$ for the four experiments and model resolutions used. Tables 1–5, respectively list the results for Ψ_{max}, the maximum strength of the buoyancy-driven overturning cell, KE, APE, γ_{mixing} and

By definition, the purely buoyancy driven case develops a thermally direct circulation. This is the reference 'control' case to be compared with the mechanically stirred ones. The purpose of Experiment (ii) seeks to investigate the effect of 'direct' mechanical forcing on horizontal convection, i.e., which corresponds to the case where the mechanical forcing tends to enhance the buoyancy-driven cell. As the mechanical and thermodynamic forcings act in the same way, this case is always associated with $G(KE) > 0$. Accordingly, eq. (50) suggests that this should in-
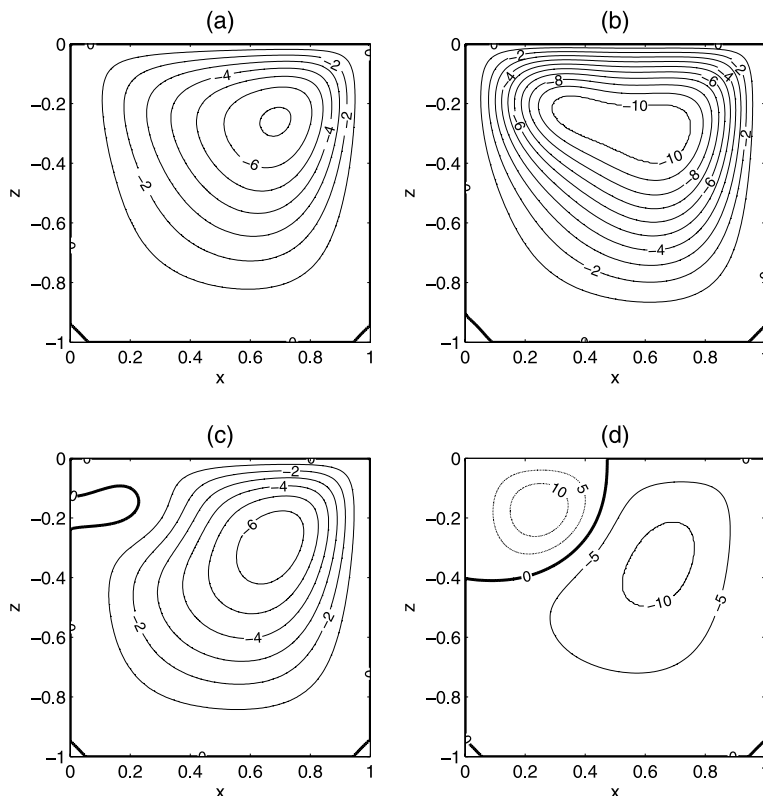

Fig. 3. Streamfunction for the different experiments: (a) no mechanical forcing; (b) 'direct' mechanical forcing; (c) weak 'indirect' mechanical forcing and (d) strong 'indirect' mechanical forcing.
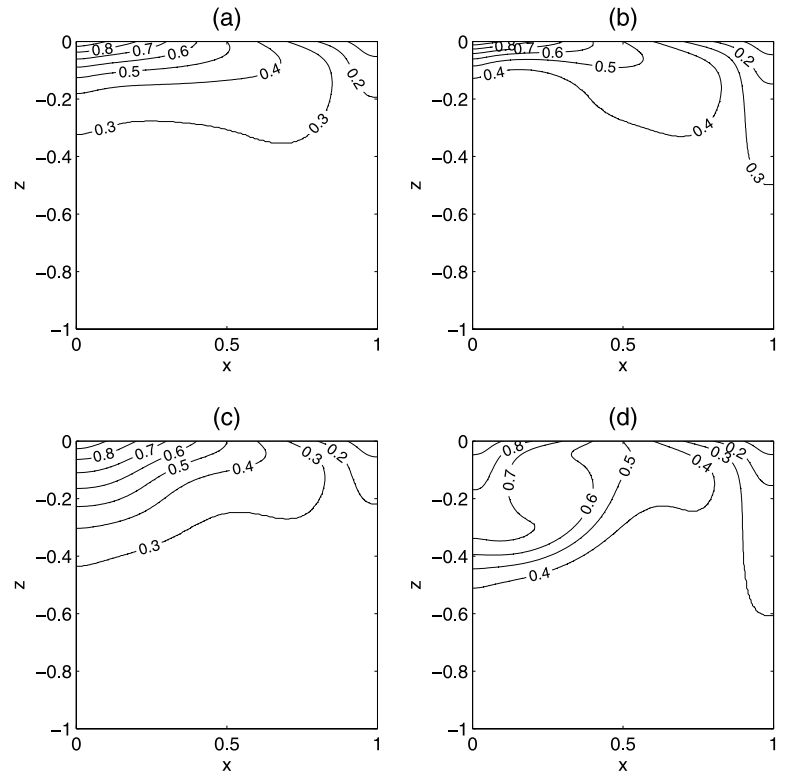
Fig. 4. Temperature field for the different experiments: (a) no mechanical forcing; (b) 'direct' mechanical forcing; (c) weak 'indirect' mechanical forcing and (d) strong 'indirect' mechanical forcing.
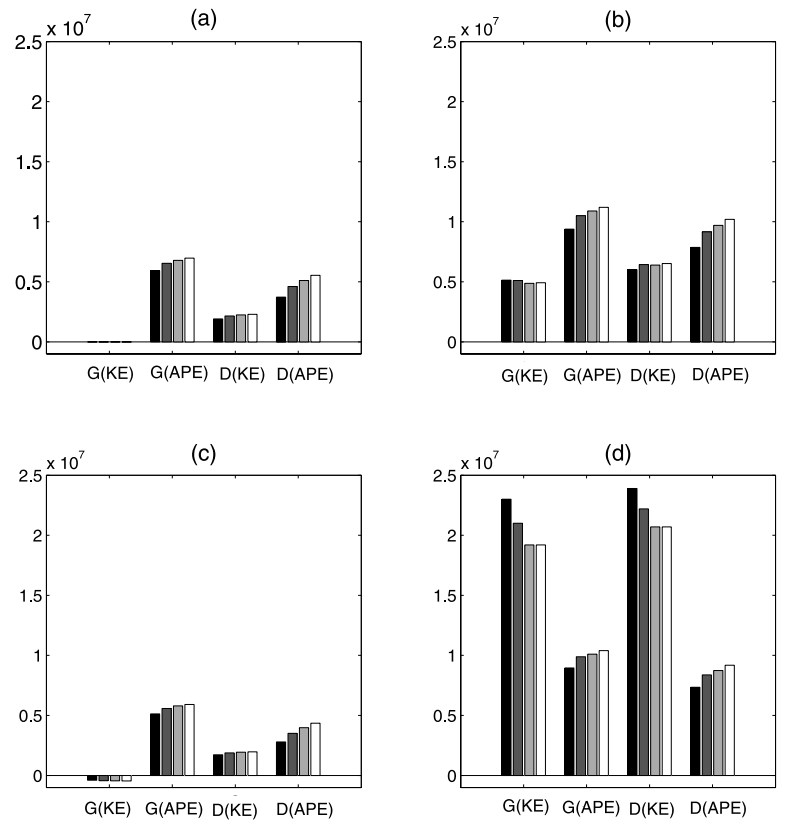

Fig. 5. Values of G(KE), G(APE), D(KE) and D(APE) for the different experiments, at resolution 50 × 50, 100 × 100, 150 × 150 and 200 × 200.

Table 1. Values of $\Psi_{\max}$, maximum of stream function for the thermally direct cell

	50×50	100×100	150×150	200×200
(a)	6.95	7.09	7.14	7.16
(b)	11.11	11.02	10.80	10.89
(c)	6.51	6.61	6.67	6.68
(d)	11.10	11.30	11.20	11.30

Table 2. Values of KE, kinetic energy

	50×50	100×100	150×150	200×200
(a)	120.8	129.8	132.9	134.4
(b)	397.5	410.3	396.7	403.1
(c)	106.7	112.6	115.2	115.8
(d)	955.2	903.8	836.5	838.8

Table 3. Values of APE, available potential energy

	50×50	100×100	150×150	200×200
(a)	3.83×10^3	3.96×10^3	4.00×10^3	4.02×10^3
(b)	5.36×10^3	5.54×10^3	5.51×10^3	5.55×10^3
(c)	5.03×10^3	5.13×10^3	5.12×10^3	5.14×10^3
(d)	1.07×10^4	1.10×10^4	1.08×10^4	1.08×10^4

Table 4. Values of γ_{mixing}, mixing efficiency

	50×50	100×100	150×150	200×200
(a)	1.95	2.14	2.28	2.42
(b)	1.31	1.42	1.51	1.57
(c)	1.62	1.87	2.05	2.20
(d)	0.31	0.38	0.42	0.44

Table 5. Values of $H_{\max}$, maximum of heat transport

	50×50	100×100	150×150	200×200
(a)	1.031	1.092	1.117	1.129
(b)	1.678	1.645	1.678	1.699
(c)	0.810	0.858	0.877	0.879
(d)	1.318	1.394	1.412	1.422

$H_{\max}$, the maximum value of the heat transport, for the four experiments and model resolutions used. Most of the quantities are rather straightforward to diagnose from the model outputs. The only real difficulty lies in the estimation of *APE*, *D(APE)* and *G(APE)*. To that end, one first needs to compute the reference state, which is easily achieved by sorting the temperature profile at each time step. The difference in *GPE* between the actual and sorted states yields the APE. The above sorting method yields a permutation, which can then be used to sort the diabatic heating term due to molecular diffusion. Separating the contributions due to the surface fluxes from that due to the internal heating yields the terms *G(APE)* and *D(APE)*. The procedure used is close to that described in Winters et al. (1995) and several other recent studies.

As expected, all plots in Fig. 3 show a well-marked thermally direct cell in all cases. Fig. 3(b) shows that the buoyancy-driven overturning cell is greatly strengthened by the addition of direct mechanical forcing. This can be explained by both the observed increase in *G(APE)* (Fig. 5, panel b), as well as by the direct effect of the mechanical forcing as a source of vorticity of the same sign as the thermal forcing. In such a case, therefore, the mechanical forcing is able to act both directly as a source of vorticity of the right sign, and indirectly by increasing diapycnal mixing and hence *G(APE)*. Figure 4(b) shows that the increase in

G(APE) is associated with the dense plumes penetrating deeper in that case.

Figure 3c shows that the weak indirect mechanical forcing results in a decrease of the buoyancy-driven overturning cell, which is expected from the theory when $G(KE) < 0$, and which is shown by the weaker values of $\Psi_{\max}$, KE and APE given in Tables 1–3. The weakening of the overturning cell results from the decrease in *G(APE)*, despite an apparent increase in the thermocline depth seen in Fig. 4. Note, however, that the increase in thermocline depth is expected to increase *G(APE)* only in absence of changes in the surface heat fluxes. Whether the latter occur can be determined from the value of the maximum heat transport $H_{\max}$ listed in Table 5. Here, we see that the increase in thermocline depth is simultaneously associated with a significant reduction in the heat transport over the purely buoyancy-driven case, as seen in Table 5.

Figure 3(d) shows that when the indirect mechanical forcing becomes strong enough to generate an indirect cell, so that $G(KE) > 0$, the buoyancy-driven overturning cell is also increased. In that case, there is no ambiguity that the thermally direct cells increase as the result of the increase in *G(APE)*, since the mechanical forcing is a source of opposite vorticity to that induced by the thermal forcing.

The resolution is found to have a varying impact depending on the quantity and type of experiment considered. For instance, Table 1 shows that $\Psi_{\max}$ increases slightly with increasing resolution in the purely buoyancy-driven case (a) and weak indirect mechanical forcing case (c), whereas the opposite occurs for the direct mechanical forcing (c). No systematic effect occurs in the strong indirect mechanical forcing case (d). In general, however, it is difficult to detect any systematic effect of the resolution on the results, which overall appear to be similar independently of the resolution. The main exception is the dissipations ratio or mixing efficiency $\gamma_{\mathrm{mixing}} = D(APE)/D(KE)$, which appears

to be systematically increasing with the model resolution regardless of the types of experiment, as seen in Table 4. Apart from this, γ_{mixing} varies significantly across the different experiments. The most important result is the finding of values as high as $\gamma_{\text{mixing}} = 2.42$ in the purely buoyancy-driven case, achieved at the highest model resolution 200×200 considered. Importantly, the mixing efficiency appears to be always reduced over its value in the purely buoyancy-driven case. The strongest reduction is found in the strong indirect mechanical forcing (d), for which the mixing efficiency reduces to $\gamma_{\text{mixing}} \approx 0.4$, which is only twice as large as the canonical value $\gamma_{\text{mixing}} = 0.2$. The main point here is to show that the value $\gamma_{\text{mixing}} = 1$ obtained in the context of buoyancy-driven turbulent mixing resulting from Rayleigh–Taylor instability by Dalziel et al. (2008) is not the maximum possible value for γ_{mixing}.

Another interesting issue concerns the effect of mechanical stirring on the maximum heat transport, which was also raised in MW98's study. Table 5 shows that the maximum heat transport is enhanced in the two cases where the mechanical forcing acts as a net energy supply to the fluid, that is, (b and d), whenever $G(KE) > 0$, irrespective of whether the mechanical forcing is direct or indirect. The strongest effect, however, appears to occur for the direct mechanical forcing case (b). The weak indirect mechanical forcing case (c), on the other hand, yields a maximum heat transport that is weaker than in the purely buoyancy-driven case. In the actual oceans, the wind forcing drives surface Ekman cells that tend either to reinforce or oppose the large-scale thermally direct cell in the Atlantic ocean responsible for the Atlantic heat transport. Overall, however, they are expected to act as a net energy supply to the oceans. The above results suggest, therefore, that the wind may contribute in the oceans, possibly significantly, to enhance the poleward heat transport over the purely buoyancy-driven case. From the above results, we can see that it is difficult to determine whether the wind stress in the Atlantic Ocean mainly forces the AMOC directly by acting as a source of vorticity in the meridional/vertical plane of the same sign as the buoyancy forcing, as in (b), or whether it is acting indirectly by controlling the turbulent diapycnal mixing and the thermocline depth, as in (d). The importance of the case (c) for the oceanic case is unclear, but may be important regionally whenever the surface wind stress opposes surface ocean currents.

5. Discussion and conclusions

Over the past decade, a widespread recurrent argument has been that buoyancy fluxes, because they are primarily acting at the ocean surface, are too inefficient to contribute significantly to the mechanical energy production in the oceans. The only theoretical basis supporting this idea, however, appears to rely either on the controversial Sandström (1908)'s 'theorem', or more recently, on Wang and Huang (2005)'s definition of W_{buoyancy}, which takes its roots in the 'anti-turbulence theorem' of Paparella

and Young (2002). These two approaches, however, appear to be incompatible with that proposed by Lorenz (1955) APE theory, which established more than 50 yr ago that W_{buoyancy} should be measured by the APE production rate $G(APE)$, for which explicit and rigorous definitions are available both for Boussinesq and non-Boussinesq fluids, regardless of whether the equation of state is linear or non-linear. Importantly, $G(APE)$ provides a definition of W_{buoyancy} in which the buoyancy forcing appears explicitly, which is not the case of Wang and Huang (2005)'s quantity; as to Sandström (1908)'s paper, it does not offer any explicit formula for W_{buoyancy}. Assessing the importance of the mechanical energy production due to surface buoyancy fluxes, therefore, simply amounts to estimating plausible values for $G(APE)$, by all available theoretical and observational means. If one accepts that $G(APE)$ is the relevant measure of W_{buoyancy}, then one is inevitably led to the conclusion that surface buoyancy fluxes must significantly contribute to the mechanical energy production in the oceans, similarly as the winds and tides. The main new subtlety here, however, is to point out that $G(APE)$ is not determined by the surface buoyancy fluxes alone, but is also a strong function of the ocean stratification. This is crucial, because this allows for $G(APE)$ to be strongly controlled by the winds and tides. Such an approach establishes, among other things, that Sandström (1908)'s 'theorem' cannot be used to make any meaningful predictions about $G(APE)$. If anything, it is APE theory that should serve to evaluate Sandström (1908)'s 'theorem', not the reverse.

Physically, the theoretical mechanism discussed in this paper is as follows. The work done by the stirrer causes an increase in turbulent kinetic energy and available potential energy, which in turn causes an increase in turbulent molecular diffusion, thereby increasing the downward rate of transfer of buoyancy from the surface. This causes a deepening of the thermocline, as is clearly evident in Whitehead and Wang (2008)'s results, which in turn allows the dense plumes to reach deeper, thereby increasing $G(APE)$. This results in more power being extracted from surface buoyancy fluxes, resulting in a stronger buoyancy-driven overturning circulation. An important advantage of the present mechanism is that it does not require an inverse energy cascade, which seems the only way by which the energy supplied by the stirrer could directly end up enhancing the strength of the large-scale overturning circulation. Moreover, the present mechanism is also attractive, for it offers a natural way to account for two important empirical results, namely the sensitivity of the numerically simulated AMOC to the turbulent diffusivity K_v, for example, Colin de Verdière (1993) and southern winds at the latitude of Drake passage (the so-called Drake passage effect) first pointed out by Toggweiler and Samuels (1995). In the latter case, this is because the southern winds drive a northward Ekman transport that constantly brings dense waters over lighter waters, causing static instability and intense stirring. This idea finds support in the numerical experiments by Vallis (2000), in which the simulated ocean stratification becomes much shallower if the

ACC is suppressed. This is only possible if the presence of the ACC results in considerably more mixing of ocean waters.

The interdependency of the power input by the mechanical and thermodynamical forcings is further illustrated by the formula $G(APE) \approx \gamma_{\text{mixing}} G(KE)$ relating $G(APE)$ and $G(KE)$ via the bulk mixing efficiency γ_{mixing}. This formula, which is simply a rewriting of MW98 energy constraint (1) accounting for the result $W_{r,\text{forcing}} \approx G(APE)$, unambiguously demonstrates the control exerted by the mechanical forcing and turbulent diapycnal mixing on $G(APE)$. Whether such a formula can be interpreted as a constraint on $G(KE)$, as suggested by MW98, is however not entirely clear because this implicitly assumes that both $G(APE)$ and γ_{mixing} can be regarded as known and fixed parameters. But this fails to be supported by the present results, which by contrast suggest that both $G(APE)$ and γ_{mixing} are strongly sensitive to the particular settings considered. In fact, our impression is that of all three quantities $G(KE)$, $G(APE)$, and γ_{mixing}, it is $G(APE)$ that is likely to be the less fixed and most variable in the oceans, a postulate that can in principle be investigated more systematically by means of numerical experiments with coupled climate models. As a result, the correct interpretation of the above formula, whose generalizations include eqs (49) and (50) seems to be that there is a two-way control of the mechanical forcing on the buoyancy-driven circulation and vice versa. In this paper, horizontal convection is found to be systematically stronger whenever the mechanical forcing acts as a net energy source for the fluid, that is, when $G(KE) > 0$. Interestingly, this behaviour seems to be also present in numerical ocean general circulation models. For instance, De Boer et al. (2008) found no overturning circulation in a coarse resolution OGCM when the wind forcing is suppressed.

The present theoretical framework is also useful to shed light on the much debated issue of whether surface buoyancy forcing alone can support a strong overturning circulation and high rates of turbulent diapycnal mixing. In this regard, the empirical evidence is not really conclusive, for example, see Hughes and Griffiths (2008). From a theoretical viewpoint, the problem amounts to establishing whether horizontal convection can give rize to large value of $G(APE)$. From the present results, eqs (50) and (47) show that in absence of mechanical forcing, the APE production and dissipation rates $G(APE)$ and $D(APE)$ are given by

$$G(APE) = (1 + \gamma_{\text{mixing}})W_{r,\text{laminar}}$$
$$= (1 + \gamma_{\text{mixing}})\kappa g \left(\overline{\rho}_{\text{bottom}} - \overline{\rho}_{\text{top}} \right), \qquad (59)$$

$$D(APE) = \gamma_{\text{mixing}}W_{r,\text{laminar}}. \qquad (60)$$

Because $W_{r,\text{laminar}}$ is by construction a 'laminar' rate, 'small' by definition, the above formula shows that $G(APE)$ and $D(APE)$ can only be large if the bulk mixing efficiency γ_{mixing} can reach values far exceeding unity. At first sight, this appears problematic, because as far as we are aware, the highest value of

mixing efficiency ever reported is $\gamma_{\text{mixing}} = 1$ in the context of buoyancy-driven mixing associated with Rayleigh–Taylor instability by Dalziel et al. (2008). Tailleux (2009a) suggested, however, that there is fundamentally no reason why γ_{mixing} could not be larger than unity, provided that APE dissipation can be made larger than KE dissipation. (Recall here that according to Oakey (1982), γ_{mixing} is simply the ratio of the APE dissipation rate over the KE dissipation rate.) In this paper, we show that this is precisely what happens for horizontal convection, with the particular numerical experiments considered here illustrating that values $2 < \gamma_{\text{mixing}} < 3$ are possible. In fact, the preliminary results of an ongoing investigation suggest that the mixing efficiency can further increase as the Rayleigh number is increased. On the basis of these results, we speculate that γ_{mixing} can possibly become very large in regions of deep water formation for instance. This point is important with respect to deriving an upper bound for the bulk mixing efficiency γ_{mixing} in the oceans. Note that having $\gamma_{\text{mixing}} = 0.2$ in 98% of the oceans, with $\gamma_{\text{mixing}} = 10$ in only 2% of the oceans, would be sufficient to significantly alter the bulk value

$$\gamma_{\text{mixing}} = 0.98 \times 0.2 + 0.02 \times 10 = 0.396$$

which is nearly twice as large as the canonical value that is currently widely used. This study demonstrates, therefore, that in order to fully understand ocean energetics, it is crucial to better understand how large γ_{mixing} can be, and whether such large values can occur over significant areas in the oceans.

We believe that the present results are exciting, because they suggest a way to reconcile the different theories for the driving mechanisms of the AMOC. So far, there has been a tendency to offer a dichotomy between apparently mutually exclusive theories based on either purely mechanically driven theories, with deep water formation acting as a control mechanism, or as purely buoyancy-driven. Purely mechanically driven theories are exemplified by the papers by Gnanadesikan (1999) or Kuhlbrodt et al. (2007). On the other hand, the theory of multiple equilibria of the thermohaline circulation tends to rely on the buoyancy-driven view of the THC. How to incorporate the mechanical control on the multiple equilibria theory was recently discussed by Johnson et al. (2007) and Nof et al. (2007). Multiple equilibria have usually been investigated in models with fixed turbulent eddy diffusivity. This study, however, suggests that γ_{mixing} may be possibly quite different for each equilibria, which could affect these results, an issue for future research.

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