Tellus (2006), 58A, 385–391

TELLUS

Thermobaric effect on slantwise convection in cold seawater

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(Manuscript received 23 May 2005; in final form 24 November 2005)

ABSTRACT

A theoretical investigation shows that the pressure dependence of the thermal expansion coefficient (the thermobaric effect) acts to destabilize stratified geostrophic flows when both temperature and salinity contribute positively to the vertical stability. For vertical stability where the salinity contributes negatively, the thermobaric effect may act to stabilize the flow. The considered disturbances are small-amplitude two-dimensional rolls with axes aligned along the basic geostrophic current, and the growth mechanism is symmetric baroclinic instability. The boundaries of the marginally stable convection cells are essentially parallel to the slanting isopycnals of the basic state; therefore, the term slantwise convection is used to describe this phenomenon. Furthermore, the thermobaric effect induces a shift of the centres of the convection cells towards the lower part of the fluid layer, as previously demonstrated for buoyancy-driven convection. The width of the marginally stable cells is small and determined by turbulent diffusion processes in the fluid.

1. Introduction

In frontal areas, in the upper ocean or in oceanic bottom boundary layers, we find regions of slanting isopycnals. Here the associated geostrophically balanced current may be unstable to symmetric perturbations in the form of rolls aligned along the flow direction, for example, Allen and Newberger (1996, 1998) for the near-bottom case. Similar situations arise in the atmosphere, and the phenomenon is referred to as symmetric instability (Ooyama, 1966; Stone, 1966; Hoskins, 1974). Sometimes the word slantwise convection is used (Emanuel, 1994), since the associated cell boundaries are parallel to the sloping isopycnals. We will adopt the term slantwise convection in the present paper. In the non-diffusive case, infinitely thin cells (zero wavelength in the cross-flow direction) constitute the most unstable mode. When diffusive processes are taken into account, the value of the critical baroclinicity parameter increases, and so does the associated critical wavelength (Walton, 1975; Emanuel, 1979; Weber, 1980).

As pointed out by Gill (1973) in his analysis of the water masses in the Antarctic, the thermal expansion coefficient for cold seawater varies considerably with the pressure, or equivalently, with depth. Since this affects the buoyancy of a water mass, it will have important implications for the mixing and convection in polar waters (Killworth, 1977, 1979; McDougall, 1984, 1987a,b; Garwood et al., 1994; Løyning and Weber, 1997; Akitomo, 1999a,b). The term thermobaric is commonly used to denote the effect of pressure on the thermal expansion coefficient. Since the geostrophic current induced by a horizontal density gradient in a rotating ocean does depend on the thermal expansion coefficient, it will be influenced by the thermobaric effect, hereafter TE for short. The equations that govern the perturbations of this basic state will be modified accordingly. The aim of this paper is to investigate how TE alters the conditions for the onset of slantwise convection in stratified geostrophic flows. This paper is organized as follows: In Section 2, we define our model and write down the basic state of the system. Section 3 contains the equations that govern the linear stability of the basic state, subject to two-dimensional perturbations with axes aligned along the basic flow. In Section 4, we solve the perturbation problem in terms of a power series expansion, assuming that the thermobaric number is a small parameter, and in Section 5, the perturbation equations are solved numerically for arbitrary thermobaric numbers to obtain the marginal stability curves and the marginally stable perturbation stream function. Finally, Section 6 contains some concluding remarks.

2. Model and basic flow

The ocean model considered here is unlimited in the lateral directions and bounded vertically by two parallel horizontal planes at a distance H. A Cartesian coordinate system (x,y,z) is defined

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DOI: 10.1111/j.1600-0870.2006.00179.x

such that the z-axis is vertical and positive upwards. The lower plane is situated at $z = 0$. The basic temperature and the basic salinity distributions are taken to vary linearly in the y- and z-direction, that is,

$$\begin{aligned} T &= T_r + \frac{\Delta T_h}{L} y + \frac{\Delta T_v}{H} z, \\ S &= S_r - \frac{\Delta S_h}{L} y - \frac{\Delta S_v}{H} z. \end{aligned} \tag{1}$$

Here, T_r, S_r are constant reference values, and $\Delta T_h/L$, $\Delta T_v/H$ are the horizontal and vertical temperature gradients, respectively, while $\Delta S_h/L$, $\Delta S_v/H$ are the corresponding salinity gradients. The density ρ is taken to be a linear function of the temperature and salinity, but with a thermal expansion coefficient α that varies with the pressure (TE). Converting from pressure dependence to depth, we can write

$$\rho = \rho_r\{1 - \alpha(z)(T - T_r) + \beta(S - S_r)\}, \tag{2}$$

where

$$\alpha(z) = \alpha_0 + \alpha_1(H - z). \tag{3}$$

β is the constant expansion coefficient for salt. Typical values of the parameters in (2) for cold seawater will be $\alpha_1 = 2.6 \times 10^{-8}\ \mathrm{K}^{-1}\ \mathrm{m}^{-1}$ (Killworth, 1977; Gill, 1982), $\alpha_0 = \alpha(z = H) = 2.5 \times 10^{-5}\ \mathrm{K}^{-1}$ and $\beta = 8 \times 10^{-4}\ \mathrm{psu}^{-1}$. For more details, we refer to Løyning and Weber (1997). We assume that the ocean is incompressible, and stably stratified in the vertical, that is, $\partial\rho/\partial z < 0$.

The basic flow is taken to be geostrophically balanced in the y-direction and hydrostatic in the vertical, yielding a current U along the x-axis. Utilizing the Boussinesq approximation, we then obtain for the vertical current shear

$$\frac{\partial U}{\partial z} = -\frac{g\alpha_0 \Delta T_h}{Lf}\left(1 + \frac{\alpha_1}{\alpha_0}(H - z)\right) - \frac{g\beta\Delta S_h}{Lf}. \tag{4}$$

3. Perturbation analysis

We investigate the stability of the basic state by introducing small perturbations into the solutions for the velocity, temperature and salinity. The considered disturbances are independent of the x-coordinate. Such disturbances are frequently referred to as symmetric disturbances. The flow field is taken to be non-divergent. We introduce non-dimensional variables by taking H, f^{-1}, Hf, $H^2 f^2$, $Hf^2/(\alpha_0 g)$ and $Hf^2/(\beta g)$ as units of length, time, velocity, pressure per unit density, temperature and salinity, respectively. In the case of symmetric disturbances, we may define a stream function ψ for non-divergent flow such that the perturbation velocity components may be written $(u,\ \psi_z,\ -\psi_y)$, where the subscripts denote partial differentiation. We write the horizontal perturbation velocity u, stream function ψ, pressure p, temperature θ and salinity s as complex Fourier components, that is,

$$u, \psi, p, \theta, s = \{u(z), \psi(z), p(z), \theta(z), s(z)\} \exp(ily + \omega t), \tag{5}$$

where l is a real wavenumber in the y-direction, and ω is the complex growth rate.

As mentioned in the introduction, the instability mechanism in this problem is essentially independent of the diffusive processes. However, since the most unstable mode in the inviscid case has zero wavelength, we realize that internal dissipation actually sets the horizontal scale of motion. Accordingly, we expect the most unstable horizontal wavelength to be short, but finite, and the walls of the corresponding convection cells to be closely parallel to the isopycnals. This means that in modelling the internal dissipation processes qualitatively, we may assume for the vertical derivative in the dissipation terms that $\partial^2/\partial z^2 = \kappa^{-2}\ \partial^2/\partial y^2$, where $\kappa = \tan\varphi$, and φ is the angle between the cell walls and the horizontal axis. Furthermore, we assume that the turbulent eddy coefficients for momentum, heat and salt are effectively equal. Then double-diffusive processes will not occur in our problem. The common turbulent diffusion coefficient will be denoted by A.

Utilizing the Boussinesq approximation, and eliminating the pressure from the momentum equation, the linearized perturbation equations for the remaining four non-dimensional variables can be written as

$$\omega u = -ilB_T\{1 + \varepsilon(1 - z)\}\psi - ilB_S\psi + D\psi - El^2 u, \tag{6}$$

$$\begin{aligned} \omega(D^2 - l^2)\psi = &-Du - il\{1 + \varepsilon(1 - z)\}\theta + ils \\ &-El^2(D^2 - l^2)\psi, \end{aligned} \tag{7}$$

$$\omega\theta = -B_T D\psi + ilG_T\psi - El^2\theta, \tag{8}$$

$$\omega s = B_S D\psi - ilG_S\psi - El^2 s, \tag{9}$$

where $D = d/dz$. The non-dimensional parameters in (6)–(9) are defined as

$B_T = g\alpha_0\Delta T_h/(Lf^2)$: thermal baroclinic number,
$B_S = g\beta\Delta S_h/(Lf^2)$: haline baroclinic number,
$G_T = g\alpha_0\,\Delta T_v/(Hf^2)$: thermal vertical stratification number,
$G_S = g\beta\Delta S_v/(Hf^2)$: haline vertical stratification number,
$E = A(1 + \kappa^{-2})/(H^2 f)$: Ekman number,
$\varepsilon = \alpha_1 H/\alpha_0$: thermobaric number.

We have assumed that the ocean is stably stratified in the vertical ($\partial\rho/\partial z < 0$). This means in the non-dimensional formulation that $G_T\,\{1 + \varepsilon\,(1 - 2z)\} + G_S > 0$ everywhere when we neglect the small horizontal temperature variation of the basic state.

An equation for the perturbation stream function is readily obtained from (6)–(9). We find that

$$\begin{aligned} &\{1 + (\omega + El^2)^2\}D^2\psi - 2il\{(1 + \varepsilon(1 - z))B_T + B_S\}D\psi \\ &-\left\{l^2(1 + \varepsilon(1 - z))G_T + l^2 G_S + l^2(\omega + El^2)^2 - il\varepsilon B_T\right\}\psi = 0. \end{aligned} \tag{10}$$

The boundary conditions for the stream function becomes

$$\psi = 0, \quad z = 0, 1. \tag{11}$$

When $E = 0$ and $\varepsilon = 0$, equation (10) governs the inviscid symmetric instability problem treated by Stone (1966). This is only a special case of the more general theory of the symmetric instability of the baroclinc vortex (Eliassen and Kleinschmidt, 1957). When $\varepsilon = 0$, the baroclinicity and the vertical stratification appears in (10) as $B \equiv B_T + B_S$ and $G \equiv G_T + G_S$, respectively. It is then sometimes convenient to replace B by the Richardson number Ri, that is, $Ri = G/B^2$. Applying the results from Stone (1966), we realize that the wall steepness κ in the Ekman number can be written approximately as $\kappa^{-2} = G$, that is, it is related to the vertical stability. When the TE is taken into account ($\varepsilon \neq 0$), the symmetry between temperature and salt in this problem disappears.

4. Analytical approach

With typical values of α_0 and α_1, we realize from the definition $\varepsilon = \alpha_1 H/\alpha_0$ that the thermobaric number will be of the order unity or less. This is because the layer depth, defined as the depth over which the main horizontal density variation occurs, usually is (much) smaller than the mean ocean depth of about 4000 m. This suggests that for most cases, the solutions could be written as series expansions after ε as a small parameter. Taking the wavenumber l, the vertical stratification parameters G_T, G_S and the Ekman number E to be given, we may write

$$
\begin{aligned}
\psi &= \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_{2+}\cdots \\
\omega &= \omega_0 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \cdots \\
B_T &= B_{T0} + \varepsilon B_{T1} + \varepsilon^2 B_{T2} + \cdots
\end{aligned}
\tag{12}
$$

We note from (10) that any contribution from the horizontal salinity gradient to symmetric instability is non-thermobaric, so we can take $B_S = B_{S0}$ in this analysis.

To O(ε^0), we have from (10) that

$$
\begin{aligned}
&\{1 + \left(\omega_0 + El^2\right)^2\}D^2\psi_0 - 2ilB_0D\psi_0 \\
&\quad - l^2\left\{G + \left(\omega_0 + El^2\right)^2\right\}\psi_0 = 0,
\end{aligned}
\tag{13}
$$

where $B_0 \equiv B_{T0} + B_{S0}$, and $G \equiv G_T + G_S$. Introducing $\sigma \equiv \omega_0 + El^2$ and applying that $\psi_0(z = 0) = 0$, the solution can be written as

$$
\psi_0 = C_0\left(\exp(ir_1z) - \exp(ir_2z)\right). \tag{14}
$$

Here,

$$
r_1, r_2 = \frac{l}{1+\sigma^2}\left(B_0 \pm \left\{B_0^2 - (1+\sigma^2)(G+\sigma^2)\right\}^{1/2}\right). \tag{15}
$$

The requirement $\psi_0(z = 1) = 0$ yields that $r_1 - r_2 = 2n\pi$, where n is a positive integer larger than zero. This condition determines the growth rate ω_0 of our solution. We find that the marginally stable solutions are given by $\omega_0 = 0$, that is, the instability is direct, and not oscillatory. The stability boundary for the most unstable vertical mode ($n = 1$), in the absence of the TE, is given by

$$
B_0 = \left(G + \frac{\pi^2}{l^2} + \left(G + 1 + \frac{2\pi^2}{l^2}\right)E^2l^4 + \left(1 + \frac{\pi^2}{l^2}\right)E^4l^8\right)^{1/2}. \tag{16}
$$

Values of $B_T + B_S$ above B_0 will cause the perturbations to grow exponentially in time to this order. To obtain the minimum value of B_0, we use the fact that the Ekman number E is a small parameter in this problem. Denoting the wavenumber that minimizes B_0 by l_c (the critical wavenumber), and assuming that also El_c^2 is a small quantity, we find from (16) that

$$
l_c = \left(\frac{\pi^2}{2(G+1)}\right)^{1/6} E^{-1/3}. \tag{17}
$$

This constitutes the most unstable wavenumber. We note that $El_c^2 \propto E^{1/3} \ll 1$, as anticipated above. The corresponding minimum, or critical, value of B_0 becomes

$$
B_{0c} = \left(G + \frac{3\pi^{4/3}}{2^{2/3}}(G+1)^{1/3}E^{2/3}\right)^{1/2}. \tag{18}
$$

We note from (17) that the most unstable perturbation cells are thin, but not infinitely thin as obtained from non-diffusive theory (Stone, 1966). The dependence on E of the critical baroclinicity parameter and the critical wavenumber are the same as those found by Weber (1980).

The normalized stream function for the marginally stable solution can be written

$$
\psi_0 = \sin(\pi z)\exp\left\{il\left(y + \frac{B_0}{1+E^2l^4}z\right)\right\}. \tag{19}
$$

We obtain from (2) that the isopycnals of the basic flow are lines parallel to $z = -\{(B_T + B_S)/(G_T + G_S)\}y$ at this order. For the most 'dangerous' marginally stable mode, we have from (18) that $B_{0c}^2 \approx G_T + G_S$. From (19), utilizing El_c^2 1, we find that the cell boundaries are approximately parallel to $z = -\{(B_T + B_S)/(G_T + G_S)\}y$, that is, they are aligned along the sloping isopycnals. This has motivated the term 'slantwise convection' for this problem (Emanuel, 1994).

To O(ε^1), we obtain from (10) that

$$
\begin{aligned}
L\{\psi_1\} &\equiv \left((1+E^2l^4)D^2 - 2ilB_0D - l^2(G+E^2l^4)\right)\psi_1 \\
&= -2El^2\omega_1D^2\psi_0 + 2il(B_{T1} + B_0(1-z))D\psi_0 \\
&\quad - \left(ilB_0 - l^2(1-z)G_T - 2El^4\omega_1\right)\psi_0.
\end{aligned}
\tag{20}
$$

The operator L is not self-adjoint. We denote the adjoint operator by $\tilde{L}$, and the solution of the adjoint problem by $\tilde{\psi}$. To obtain the adjoint problem, we follow Finlayson (1972). Letting angle brackets symbolize integration in the vertical from $z = 0$ to $z = 1$, we find from (20) by utilizing that $\psi_1(0) = \psi_1(1) = 0$,

$$
\left\langle\tilde{\psi}L\{\psi_1\}\right\rangle = \left\langle\psi_1\tilde{L}\{\tilde{\psi}\}\right\rangle + (1+E^2l^4)\left|\tilde{\psi}D\psi_1\right|_{z=0}^{z=1}, \tag{21}
$$

where $\tilde{L}\{\tilde{\psi}\} = ((1+E^2l^4)D^2 + 2ilB_0D - l^2(G+E^2l^4))\tilde{\psi}$. Requiring that $\tilde{\psi} = 0$ at $z = 0$, 1, we realize that the solution of the adjoint problem is just the complex conjugate of the present solution, that is, from (19),

$$\tilde{\psi} = \sin(\pi z) \exp\left\{-il\left(y + \frac{B_0}{1 + E^2 l^4} z\right)\right\}. \tag{22}$$

The solvability condition for the non-homogeneous equation (20) then becomes

$$\left\langle \tilde{\psi} L\{\psi_1\}\right\rangle = 0. \tag{23}$$

In (20), B_{T1} is the modification of the stability boundary introduced by the TE. From (23), we find that ω_1 is real. In the case of marginal stability ($\omega_1 = 0$), the solvability condition yields

$$B_{T1} = -\frac{1}{4B_0}\left[G_T + 2G_S + \frac{2\pi^2}{l^2} + \left(G_T + 2G_S + 2 + \frac{4\pi^2}{l^2}\right)E^2 l^4 + 2\left(1 + \frac{\pi^2}{l^2}\right)E^4 l^8\right], \tag{24}$$

where B_0 is given by (16). In this problem, ε is positive by definition. Then, if $B_{T1} < 0$, TE acts destabilizing, that is, it lowers the value of B_T necessary for instability. Conversly, if $B_{T1} > 0$, TE acts stabilizing. The sign of B_{T1} depends essentially on the sign of $G_T + 2G_S$. Since we consider a basic state that is stably stratified in the vertical, that is, $G_T + G_S > 0$, we have three possible combinations:

(i) $G_T > 0$, $G_S > 0$: TE always acts destabilizing,
(ii) $G_T < 0$, $G_S > 0$: TE acts destabilizing when $G_S > -G_T$ (always fulfilled),
(iii) $G_T > 0$, $G_S < 0$: TE acts destabilizing when $G_T > -2G_S$,

TE acts stabilizing when $-G_S < G_T < -2G_S$.

Concerning case (iii), there are occasions where freezing seawater and brine release may produce more salty water above less salty water. Under these circumstances the temperature will be close to, or at the freezing point, and the freezing process will trigger a convective plume as described by Torkildsen and Haugan (1999). However, except for freezing conditions, we typically find less salty water above saltier water in polar regions, that is, $G_S > 0$. Then TE acts destabilizing for all wavenumbers.

Close to the critical value, we have that $l \sim l_c \gg 1$, and $El_c^2 \ll 1$. Then (16) and (24) yield for the critical baroclinicity parameter $B_c \equiv (B_{T0} + B_{S0} + \varepsilon B_{T1})_c$ in a stably stratified ocean ($G = G_T + G_S > 0$):

$$B_c = G^{1/2}\left(1 - \frac{\varepsilon(G_T + 2G_S)}{4G}\right). \tag{25}$$

The critical wavenumber is given approximately by (17). In terms of a critical Richardson number, (25) can be written as

$$Ri_c = \frac{G}{B_c^2} = 1 + \frac{\varepsilon(G_T + 2G_S)}{2G}. \tag{26}$$

Accordingly, TE render the system unstable to symmetric perturbations even for Richardson numbers larger than 1 (when $G_S > 0$). Dynamically, TE acts to increase the basic geostrophic current shear with depth; see (4). Since growing symmetric disturbances extract their energy from the basic shear, it is obvious that TE must act destabilizing, as demonstrated by our analysis. The shape of the marginally stable cells will also change. But instead of calculating analytically the modified stream function to $O(\varepsilon^1)$, we solve (10) numerically by applying a traditional shooting method. In that case, comparison with our previously found stability boundary $B = B_{T0} + B_{S0} + \varepsilon B_{T1}$, given by (16) and (24), will serve as a test on the numerical computations. The application of a numerical procedure also renders the next step in the analytical solution to $O(\varepsilon^2)$ unnecessary.

5. Numerical solutions

The second-order differential equation (10) is converted to a set of first-order differential equations by setting $y_1 = \psi$ and $y_2 = D\psi$. We then have

$$\begin{aligned} Dy_1 &= y_2, \\ Dy_2 &= \frac{1}{1+\sigma^2}(2il\{(1 + \varepsilon(1-z))B_T + B_S\}y_2 \\ &\quad + \left\{l^2(1 + \varepsilon(1-z))G_T + l^2 G_S + l^2\sigma^2 - il\varepsilon B_T\right\}y_1), \end{aligned} \tag{27}$$

where y_1 and y_2 are complex quantities. As before, we define $\sigma \equiv \omega + El^2$. We look for marginally stable solutions, and put $\mathrm{Re}(\omega) = 0$. According to standard procedure, this problem is solved as an initial value problem, where the initial condition for y_1 is given by the boundary condition for ψ at $z = 0$, that is, $y_1 = 0$. We may choose the initial condition for y_2 arbitrarily. To ensure rapid convergence, we take $y_2 = 2\pi i$, which yields the exact stream function for $\varepsilon = 0$. For given values of $\mathrm{Im}(\omega)$, l, G_T, G_S, E, and B_S, we solve (27) by a fourth-order Runge–Kutta algorithm for various values of B_T until we obtain that $y_1(z = 1)$ is sufficiently close to zero.

We have formulated the symmetric instability problem in a general way, but when we present numerical solutions, we simplify and pick values of the non-dimensional parameters that best shed light on the basic physics of the problem. From field data at various locations in the polar regions, we find that the values of our parameters vary considerably from place to place. Since we are basically interested in exploring the TE, we let the density stratification be dominated by the temperature. Hence, we take $\Delta S_h \approx 0$, $\Delta S_v \approx 0$ in our example. A typical layer depth H will be 200–600 m. The thermobaric number ε will then be in the range 0.2–0.6. The eddy diffusivity is estimated in the range from 10 to 100 times larger than the common value of 1 cm^2 s^{-1} for the interior of the ocean. Accordingly, our Ekman number E will typically be of the order 10^{-3}, or less. From Aagard et al. (1985), we obtain that $\Delta T_v/H \sim 2.5 \times 10^{-4}$ K m^{-1} in several

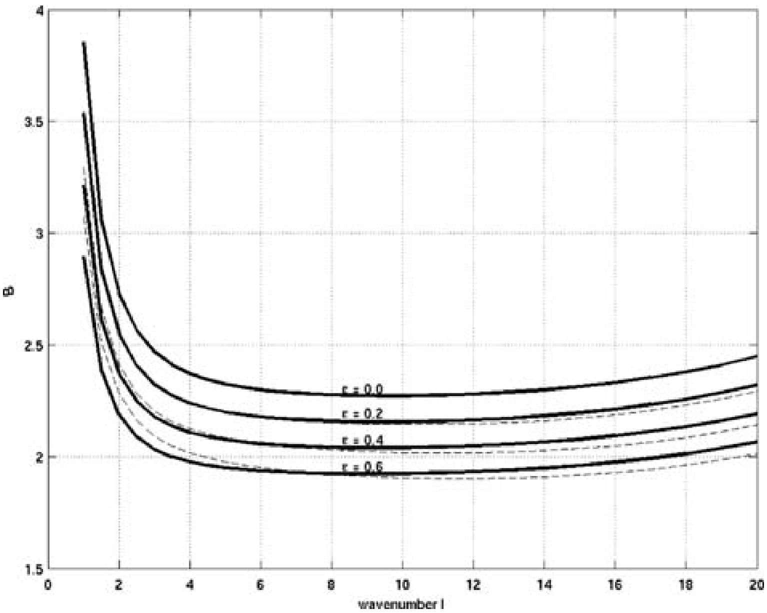

Fig. 1. Thermal baroclinic number $B = B_T$ for marginal stability vs. wavenumber l for various values of the thermobaric number ε, when $B_S = G_S = 0$, $G_T = 5$ and $E = 10^{-3}$. Solid lines: analytical results. Broken lines: numerical results.

areas of the Arctic Sea, while in the West Spitsbergen Current, it can be larger by two orders of magnitude. We here take $G_T = 5$ in our numerical examples, although this parameter can be considerably larger.

In Fig. 1, we have plotted B_T as function of the wavenumber l when $B_S = G_S = 0$, $G_T = 5$ and $E = 10^{-3}$ for various values of thermobaric number. We note that the analytical series expansion result $B_T = B_{T0} + \varepsilon B_{T1}$ converges very well towards the numerical result for wavenumbers around the critical value, which is close to 10. For smaller and larger wavenumbers, good convergence is only obtained for small ε. This is no surprise, since B_{T1} tends to minus infinity when l tends to zero, and when l tends to infinity. Actually, from the definitions in connection with (6)–(9), we have by increasing ε and keeping E constant, in fact increased the eddy viscosity from one numerical run to the next, which perhaps is not very realistic. To keep the eddy viscosity constant, we should have varied E according to $E \sim \varepsilon^{-2}$ in the various runs for $\varepsilon \neq 0$. However, to keep a simple picture that reveals the TE on the stability problem, we have chosen to do all our runs for $E = 10^{-3}$.

In Fig. 2, we have displayed numerical results for the stream function when $B_S = G_S = 0$, $G_T = 5$, $E = 10^{-3}$, $l = 10$ and $\varepsilon = 0$, while in Fig. 3, the values for B_S, G_S, G_T, E and l are the same, but now $\varepsilon = 0.4$. We note the asymmetry introduced by the TE, shifting the cell centres towards the lower part of the marginally unstable layer. The same effect is seen in buoyancy-driven convection (Løyning and Weber, 1997). Furthermore, we find that the tilt of the marginally stable cells increases with increasing values of ε. Finally, our calculations showed that, for given l, B_S, G_S, G_T and E, the smallest value of B_T occurred for $\mathrm{Im}(\omega) = 0$. Accordingly, the instability in this problem is direct.

6. Concluding remarks

By perturbing a stratified geostrophically balanced flow with respect to symmetric disturbances, or slantwise convection rolls, we have demonstrated that the TE destabilizes the problem when both temperature and salinity contribute positively to the vertical stability of the fluid. For vertical stability where the salinity contributes negatively, or more precisely when $-G_S < G_T < -2G_S$, TE acts to stabilize the flow. However, this situation is not likely to occur in polar waters where we usually have less salty water above saltier water (under freezing conditions, we may have the opposite situation, but then the temperature is close to, or at the freezing point, and the water column becomes vertically unstable). For not unrealistic temperature and salinity variations in polar waters, our results indicate that the TE may reduce the critical baroclinicity parameter by 10–20%.

We have also shown that TE changes the shape of the marginally stable slanting convection cells by shifting the cell centres towards the lower part of the fluid layer. This is analogous to Løyning and Weber (1997), where TE acts destabilizing in buoyancy-driven convection. In these two problems, the thermobaricity influences the dynamics in different ways. In buoyancy-driven convection, the growing perturbations extract their energy from the potential energy of the vertically unstable temperature part of the density distribution. In the slantwise convection case, the source of energy is related to the basic horizontal density gradient. Here, TE enhances the baroclinicity in the lower part of the fluid layer, promoting instability. The width of the most unstable slanting convection cells is determined by the effect of horizontal diffusion of momentum, heat and salt. In a turbulent ocean with equal eddy diffusion coefficients and a small Ekman number E, the critical cell width is proportional to $E^{1/3}$.

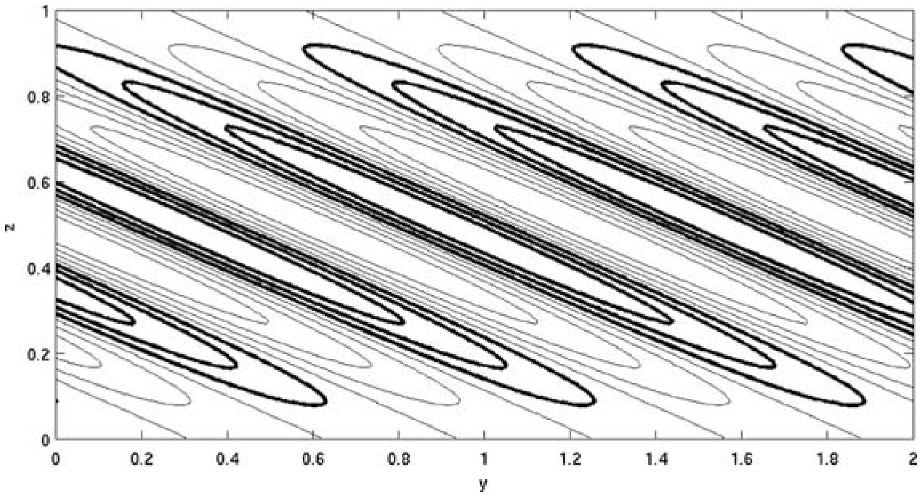

Fig. 2. The stream function for marginally stable solutions when $B_S = G_S = 0$, $G_T = 5$, $E = 10^{-3}$ and $l = 10$. The thermobaric effect is zero ($\varepsilon = 0$).

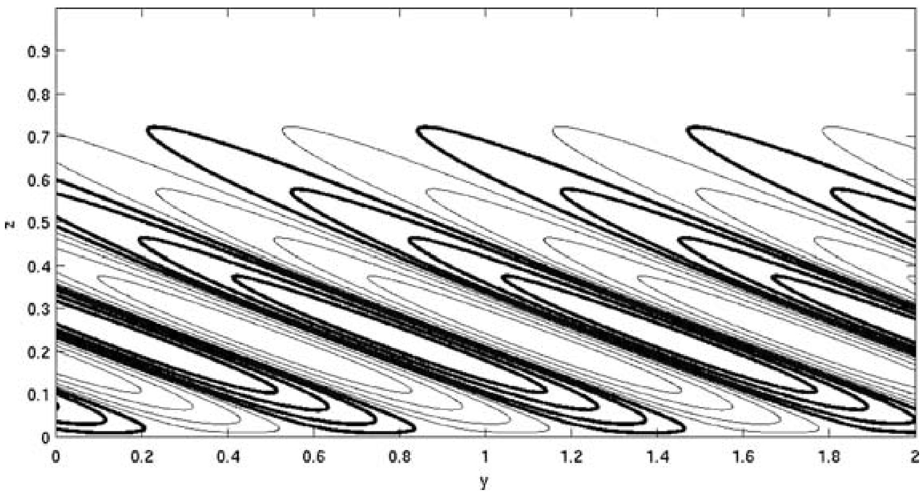

Fig. 3. The stream function for marginally stable solutions for the same parameters as in Fig. 2, but now with $\varepsilon = 0.4$.

The linear increase with depth of the thermal expansion coefficient in cold waters considered here also means that the mean expansion coefficient is higher than the surface value for this problem. Such an increase of the expansion coefficient acts destabilizing by itself. Re-doing our stability analysis with the (larger) mean coefficient $\alpha_m$obtained from (3), that is, $\alpha_m = \alpha_0 + \alpha_1 H/2$, the result for marginal stability follows directly from (16). It is then easy to show that the values for marginal stability in terms of the parameter $B_T = g\alpha_0 \Delta T_h/(Lf^2)$ in this case are found at higher values of B_T than the numerical results displayed in Fig. 1 (broken curves) for the same choice of parameters. This emphasizes the importance of the depth dependence of the expansion coefficient for the stability problem.

To the authors' knowledge, TEs on the stability of geostrophic ocean currents have not yet been observed in the field. Likely candidates for the manifestation of such phenomena would probably be the Antarctic Circumpolar Current, The West-Spitsbergen Current and the East-Greenland Current.

References

Aagard K., Swift, J. H. and Carmack E. 1985. Thermohaline circulation in the Arctic Mediterranean Seas. *J. Geophys. Res.* **90**, 4833–4846.

Akitomo K. 1999a. Open-ocean deep convection due to thermobaricity. 1. Scaling argument. *J. Geophys. Res.* **104**, 5225–5234.

Akitomo K. 1999b. Open-ocean deep convection due to thermobaricity. 2. Numerical experiments. *J. Geophys. Res.* **104**, 5225–5234.

Allen, J. S. and Newberger, P. A. 1996. Downwelling circulation on the Oregon Continental Shelf. Part I: response to idealizes forcing. *J. Phys. Oceanogr.* **26**, 2011–2035.

Allen, J. S. and Newberger, P. A. 1998. On symmetric instabilities in oceanic bottom boundary layers. *J. Phys. Oceanogr.* **28**, 1131–1151.

Eliassen A. and Kleinschmidt E. 1957. Dynamic Meteorology. In: *Handbuch der Physik* (ed S. Flügge). Springer-Verlag, Berlin, 1–154.

Emanuel, K. A. 1979. Inertial instability and mesoscale convective systems. part I: linear theory of inertial instability in rotating viscous fluids. *J. Atmos. Sci.* **36**, 2425–2449.

Emanuel, K. A. 1994. *Atmospheric Convection*. Oxford University Press, Oxford.

Finlayson, B. A. 1972. *The Method of Weighted Residuals and Variational Principles with Application in Fluid Mechanics, Heat and Mass Transfer*. Academic Press, New York.

Garwood, Jr., R., Isakari S, and Gallacher P. 1994. Thermobaric convection. In: *The Polar Oceans and Their Role in Shaping the Global Environment*, Geophys. Monogr. Ser. **85** (eds O. M. Johannesen, R. D. Muench and J. E. Overland), AGU, Washington, D.C., 199–209.

Gill, A. E. 1973. Circulation and bottom water production in the Weddel Sea. *Deep-Sea Res.* **20**, 111–140.

Gill, A. E. 1982. *Atmosphere-Ocean Dynamics*. Academic Press, New York.

Hoskins, B. J. 1974. The role of potential vorticity in symmetric stability and instability. *Quart. J. Roy. Meteor. Soc.* **100**, 480–482.

Killworth, P. D. 1977. Mixing on the Weddel Sea continental slope. *Deep-Sea Res.* **24**, 427–448.

Killworth, P. D. 1979. On “chimney” formations in the ocean. *J. Phys. Oceanogr.* **9**, 531–553.

Løyning, T. B. and Weber, J. E. 1977. Thermobaric effects on buoyancy-driven convection in cold seawater. *J. Geophys. Res.* **102**, 27.874–27.885.

McDougal, T. J. 1984. The relative roles of diapycnal and isopycnal mixing on subsurface water mass conversion, *J. Phys. Oceanogr.* **14**, 1577–1589.

McDougal, T. J. 1987a. Neutral surfaces. *J. Phys. Oceanogr.* **17**, 1950–1964.

McDougal, T. J. 1987b. Thermobaricity, cabbeling and water-mass conversion. *J. Geophys. Res.* **92**, 5448–5464.

Ooyama K. 1966. On the stability of the baroclinic circular vortex: a sufficient criterion for instability. *J. Atmos. Sci.* **23**, 43–53.

Stone, P. H. 1966. On non-geostrophic baroclinic instability. *J. Atmos. Sci.* **23**, 390–400.

Torkildsen, F. and Haugan, P. M. 1999. Modelling of deep water renewal through cold convective plumes. *Deep Sea Res. II* **46**, 1357–1383.

Walton, I. C. 1975. The viscous nonlinear symmetric baroclinic instability of a zonal shear flow. *J. Fluid Mech.* **68**, 757–768.

Weber, J. E. 1980. Symmetric instability of stratified geostrophic flow. *Tellus* **32**, 176–185.