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Zonally symmetric normal modes associated with the AO/NAM under a seasonally varying background climatology

By NAN ZHAO [1,2]* and MASAAKI TAKAHASHI [1], [1] *Center for Climate System Research, University of Tokyo, Kashiwa 277-8568, Japan* [2] *State Key Laboratory of Severe Weather, Chinese Academy of Meteorological Sciences, Beijing 100081, China*

ABSTRACT

This paper describes a numerical study of the role of zonally symmetric normal modes in the dynamics of the Arctic Oscillation (AO) or Northern Hemisphere Annular Mode (NAM) with seasonally varying climatological fields. A general theory of the mechanism for normal mode selection involving a fully developed non-linear interaction between different modes is proposed, and this explains not only the prominence of the AO/NAM in the Northern Hemisphere, but also the existence of the Antarctic Oscillation (AAO) in the Southern Hemisphere. Following this normal mode selection mechanism, zonally symmetric normal mode with spatial structures resembling those of the AO and AAO is extracted by use of ECMWF data and the linear operator of a two-dimensional model with a seasonally varying basic state. The interannual variability of this mode is also investigated in the light of this theory.

1. Introduction

The so-called Arctic Oscillation (AO) or Northern Annular Mode (NAM), a leading mode of Northern Hemisphere variability with zonal symmetry, a meridional dipole and a deep barotropic structure, has been the focus of much research in recent years. Systematic reviews of this topic can be found in, for example, Wallace (2000) and Thompson et al. (2002). The significance of this mode is attributed to its connection to many aspects of the world's weather and climate such as monthly mean variability, blocking patterns, severe weather events and the climate trends of recent decades. This mode is also found to have some influence on the atmospheric circulations in the stratosphere and the tropics. Many studies of the AO/NAM using data analysis and numerical models have revealed that the maintenance of this mode is closely associated with a feedback process between anomalies in the zonal-mean flow and eddy momentum fluxes (e.g. DeWeaver and Nigam, 2000; Lorenz and Hartmann, 2000, 2003; Kimoto et al., 2001; Eichelberger and Holton, 2002). Both baroclinic eddies and quasi-stationary waves in the Northern Hemisphere can interact with the zonal-mean flow and result in a fluctuation of the annular mode.

*Corresponding author.
e-mail: zhaon@cams.cma.gov.cn
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Although this theory depicts the primary aspect of the mechanism of AO excitation, fundamental issues of AO dynamics are still not well understood. The AO mode is usually represented by the leading EOF of Northern Hemisphere (20–90 N) variability (Thompson and Wallace 1998, 2000). However, this is not sufficient for an understanding of its dynamics, because EOF are regarded by many researchers to have only a statistical meaning and do not indicate a coordinated variability on a hemispheric scale due to some physical or dynamical process. Moreover, the theory of zonal mean flow-eddy interaction merely gives the zonal mean structure of the zonal wind, and does not guarantee a zonal coherence to the variation of the zonal wind along a certain latitude. Therefore, it does not mean a physical mode with a zonally symmetric structure can be organized throughout the hemisphere. We need a theory which is able to give a reasonable answer to the question of whether the zonally symmetric dipole structure on a hemispheric scale is a physical mode and, if so, how it is sustained.

On the other hand, zonally symmetric or annular *normal* modes, which have the most direct linkage to linear dynamics around the basic state of the atmosphere and interact with other modes due to non-linear dynamics, may play a key role in our understanding of the AO/NAM. Although it seems very natural to think so (because the normal mode, as a solution of the linear part of the dynamics equation, is definitely a physical and coherent mode), there is some reluctance to consider them because (i) there is a lack of knowledge about zonally symmetric normal modes on a planetary scale and (ii) traditional linear normal analysis on this spatial scale often gives damped and regularly oscillating normal modes, while the observed AO features involve persistent and irregular fluctuation on a broad range of time scales. This latter issue may be ignored if non-linear interactions between different physical modes and atmospheric waves are taken into account, and situation in this case will become more complex. In fact, mean flow-eddy interaction can be viewed as a non-linear interaction between annular normal modes with wavenumber = 0 and other modes with wavenumber ≥1. The resultant oscillation behaviour is far more varied than in the linear results. The former point is more serious issue to address, because theories of waves and instabilities of geophysical fluid so far give no reasonable explanation for this mode. It follows that the leading annular normal modes, their structures, stabilities, interactions with other modes and responses to external forcing could be significant subjects in the analysis of AO dynamics.

This paper is focused on the roles of annular or zonally symmetric normal modes in our understanding of the AO/NAM. This research on normal modes has two distinct features. First, in our analysis, we use a climatological background that varies with season rather than a steady-state climatological mean. This is because the normal mode of steady state conditions may possess a sufficiently long eigenperiod that it is comparable with (and thus influenced by) the annual period of the climatological state. The annual oscillation of the normal mode must therefore be considered. The observed AO shows remarkable seasonal changes, and appears to be more pronounced in winter and March than in other seasons, for example. Secondly, the roles of the leading annular normal modes are investigated under a non-linear framework rather than a simple linear analysis. In a linear framework, such modes are either stable and decay or are unstable but have regular or no oscillations, and are therefore be insufficient to explain the AO/NAM.

A hypothesis for the existence of the AO/NAM based on dynamical theory will be presented in Section 2. In Section 3, we will give a brief introduction to the numerical methods for extracting leading annular normal modes, which are believed to contribute to the AO/NAM, from a two-dimensional numerical model. Section 4 is devoted to a discussion of the calculation results and of the role of non-linear interactions between zonally symmetric normal modes and zonally asymmetric wave modes. A summary and further comments are provided in the last section.

2. A hypothesis for the AO/NAM

As proposed in the introduction, the role played by zonally symmetric or annular normal modes in the existence of the AO/NAM is investigated under a seasonally varying climatological background with a non-linear dynamical framework. Such a complex issue can be described mathematically as follows. We write the atmosphere model by the variables of disturbance (denoted by $\mathbf{u}$) around a seasonally varying climate state as

$$\frac{\partial \mathbf{u}}{\partial t} = L(t)\mathbf{u} + N(\mathbf{u}), \tag{2.1}$$

where $L(t)$ is a linear operator and $N(\mathbf{u})$ represents the non-linear part of the model. Since an annual cycle is incorporated in the basic state, $L(t)$ that explicitly depends on time t via basic state should satisfy a temporally periodic boundary condition

$$L(t+T) = L(t), \tag{2.2}$$

where $T = 1$ yr. Normal modes can be deduced from the linearized version of (2.1), namely

$$\frac{\partial \mathbf{u}}{\partial \mathrm{t}} = L(t)\mathbf{u} \tag{2.3}$$

For the periodically time-dependant eigenvalue problem in (2.3), the normal mode is usually assumed to have the following form

$$\mathbf{u} = \mathrm{e}^{\lambda t}\mathbf{v}, \tag{2.4}$$

where $\mathbf{v}(t) = \mathbf{v}(t+T)$ is annually periodic in time. This depicts the annual oscillation in the normal modes. Substituting (2.4) into (2.3), we have

$$-\frac{\partial \mathbf{v}}{\partial t} + L(t)\mathbf{v} = \lambda \mathbf{v}. \tag{2.5}$$

A new operator A is defined by

$$A = -\frac{\partial}{\partial t} + L(t). \tag{2.6}$$

So, a conventional eigenvalue problem can be proposed by

$$A\mathbf{v} = \lambda \mathbf{v} \tag{2.7}$$

with temporally periodic boundary condition $\mathbf{v}(t) = \mathbf{v}(t+T)$ and all the necessary spatial boundary conditions satisfied. Assuming it can be solved, we have

$$A\boldsymbol{\xi}_n = \lambda_n \boldsymbol{\xi}_n; \quad n = 1, 2, 3, \ldots. \tag{2.8}$$

We then obtain the different normal modes $\mathrm{e}^{\lambda_n t}\boldsymbol{\xi}_n$. Here, ξ_n describes the spatial structure of the nth mode and its change with season, and the factor $\mathrm{e}^{\lambda_n t}$ reflects its stability and eigenperiod of oscillation. The inner product between vector functions can be defined by

$$<,>= \int_0^T \int_\Omega (\cdot)\mathrm{d}\Omega \mathrm{d}t, \tag{2.9}$$

where Ω is the spatial domain in which our problem is considered. Hence, the adjoint operator of A is obtained as

$$A^* = \frac{\partial}{\partial t} + L^*(t). \tag{2.10}$$

And similarly, the eigenvalue problem of A^* is solved by

$$A^*\boldsymbol{\xi}_n^* = \bar{\lambda}_n\boldsymbol{\xi}_n^*; \quad n = 1, 2, 3, ..., \tag{2.11}$$

where $\bar{\lambda}_n$ is the conjugate complex number of λ_n.

The mathematical details so far can be studied from a standard text book (e.g. Iooss and Joseph, 1989). Hereafter, we will focus on how these can be linked to our understanding of the AO/NAM. Equation (2.1) can be rewritten as

$$-A\mathbf{u} = N(\mathbf{u}). \tag{2.12}$$

If the following properties

$$\langle A\mathbf{u}, \boldsymbol{\xi}_n^*\rangle = \langle \mathbf{u}, A^*\boldsymbol{\xi}_n^*\rangle = \lambda_n\langle \mathbf{u}, \boldsymbol{\xi}_n^*\rangle \tag{2.13}$$

of the nth adjoint eigenvector $\xi_n{}^*$ are exploited, the inner product of (2.12) with $\xi_n{}^*$ yields

$$\langle \mathbf{u}, \boldsymbol{\xi}_n^*\rangle = -\frac{\langle N(\mathbf{u}), \boldsymbol{\xi}_n^*\rangle}{|\lambda_n|\mathrm{e}^{i\varphi_n}}, \tag{2.14}$$

where the complex eigenvalue λ_n has been expressed by its norm $|\lambda_n|$ and phase angle φ_n. Eq. (2.14) is regarded by the authors as the key to understanding many aspects of AO/NAM dynamics.

Some conclusions may be drawn directly from (2.14), as the left-hand side is just the projection of the anomaly variation of the atmosphere onto each normal mode. This projection serves as a measure of the significance of this normal mode compared with others. From the right hand-side of the equation it is clear that the significance of a normal mode is jointly decided both by its eigenvalue λ_n and by the projections of the non-linear forcing, which comes from the non-linear interaction with other wave modes, onto this mode. It is well known that in a linear system the most unstable normal modes always have a tendency to grow first. However, in a fully developed unstable non-linear system like the atmosphere, the normal modes will interact and mix fully with each other, and the growth of any linearly unstable normal modes are subject to limitation. Hence, the rank of predominance of normal modes is no longer decided by the growth rate (or the real part of the eigenvalues), but by a new mechanism of mode selection detailed by eq. (2.14). For certain normal modes, the smaller the norms of its eigenvalue λ_n, or the larger the projections of the non-linear forcing onto it, the more pronounced it will be. This means that the slowest normal modes, which are often the ones with the largest spatial scales, may be a dominant component in the variability, if the projections of the non-linear forcing on these normal modes are large enough. Thus the spatial structures of these normal modes with largest projections on them must be captured by the leading EOFs which are usually explained as coherent structures with largest probability in occurrence and employed as the definition of the AO. Also, some previous studies (e.g. Simmons, Wallace and Branstator, 1983; Frederiksen and Bell, 1987; Frederiksen, and Branstator, 2001) demonstrate that atmospheric teleconnection patterns correspond to normal modes of linear models about climate states.

From this perspective, the dynamics of the AO/NAM can be understood as follows.

(i) The AO/NAM may be a combination of a few of the slowest zonally symmetric normal modes under this mechanism of mode selection. The leading annular normal modes, which have the largest spatial scale similar to the meridional dipole and zonally symmetric structure of the AO/NAM, usually also have the longest temporal scale or smallest values of $|\lambda_n|$. Such normal modes tend to be selected as the dominant modes constituting the AO/NAM, if the forcing resulting from non-linear interactions has projections onto these modes which are strong enough.

(ii) We can conclude that the main features of the spatial structure of the AO/NAM such as zonal symmetry, meridional dipole and the vertical barotropy may be attributed to the structures of the leading annular normal modes selected by the mechanism above.

(iii) The time scales of fluctuation of the AO/NAM are not decided by the eigenvalues of the corresponding normal modes, no matter whether these normal modes are linearly stable or unstable, oscillatory or non-oscillatory. But rather, they are decided by the time scales at which non-linear interaction between different normal modes or waves occurs.

3. Numerical implementation

In order to verify the above theory in a practical way, a numerical implementation is necessary. It is well known that eigenvalue problem of the linear operator from partial differential equations and that of the matrix from the corresponding finite difference equations may be quite different, and one must be careful in explaining the numerical results. In this section, a two-dimensional model is established based on a zonally symmetric climatological background from primitive equations, and is described in detail in Appendix. Here, we give a brief description of other important factors such as selection of the equilibrium, some numerical aspects and the forms of the linear operators or matrices.

The climatological state or the quasi-equilibrium in this study could be obtained from the zonal mean of observational data such as ECMWF data. However, fields of geopotential height, zonal wind and temperature obtained in this way usually do not exactly satisfy the geostrophic and hydrostatic relations and thus do not allow an equilibrium solution of the model equations. This may result in unreal normal modes if equilibrium is assumed and the model equations are linearized about it. Instead, we use only the zonal mean field of geopotential height from ECMWF data averaged over 9 yr from 1994 to 2002, and zonal wind and temperature are deduced from geopotential height by use of the geostrophic and hydrostatic relations, see (A.2) of Appendix. This ensure that the fields used are really in equilibrium and the model equations can then be written in the disturbance form about this equilibrium as in (2.1). Since we will deal with seasonally varying climatological fields in the following research, mean fields are prepared for each of the four seasons (DJF, MAM, JJA and SON, respectively). The structures of deduced fields of zonal wind and temperature are almost identical to those of the observations. The seasonal variation of the mean fields is clearly evident particularly in the fields of zonal wind.

We employ the same configuration for the model grid as that of the ECMWF data, with a meridional resolution of 2.5° and 15 vertical levels. In principal, the model domain for this research should be between 20 N and 90 N, because this is the region where the AO/NAM is situated and also there is a relatively small quantity of data to process. However, the artificial lateral boundary at 20 N is a major disadvantage, as it results in waves reflecting back to the domain and may produce unrealistic normal modes. In order to avoid this artificial lateral boundary, we choose the whole region between 90 S and 90 N as the model domain. However, the meridional resolution must be reduced to 5° so as to reduce the amount of data involved in solution of the eigenvalue problems. Even so, with carefully designed numerical experiments, normal modes at a comparable spatial scale with that of the AO/NAM are believed to be extracted correctly. Moreover, with the larger model domain investigation of the counterpart of the AO in the Southern Hemisphere, the Antarctic Oscillation (AAO), the connection between the tropics and the AO, and other zonally symmetric phenomena are also possible.

We use a centred-spatial difference scheme in the meridional direction. In order to damp the shortest resolvable wavelengths, numerical diffusion from a second-derivative smoother is introduced into the model. The filtering coefficient is set such that the e-fold damping time for the shortest resolvable wavelength (10°) is 5 d, so modes with a hemispheric spatial scale will have an e-fold damping time of more than 40 d. Vertical numerical diffusion from a second-derivative is also incorporated into the model as the main source of momentum dissipation. The vertical profile of the diffusion coefficient is selected in such way that the e-fold damping time for a wavelength of 100 hPa in the lower troposphere is 3 d, while that in the stratosphere exceeds 12 d. Momentum flux at the surface is also considered by using a bulk aerodynamic method with a global mean drag coefficient $C_{\mathrm{D}} = 2.0 \times 10^{-3}$. Newtonian cooling is incorporated in the model equations and a mean value of 1/100 $(\mathrm{d})^{-1}$ is selected for the cooling coefficient.

As can be inferred from (A.4) in Appendix, the linear operator $L(t)$ (t here indicates that L explicitly depends on time via the climatological background varying with season) can be expressed by the following matrix

$$L(t) = \begin{pmatrix} 0 & I_x - bA_2 & 0 \\ -I_y & 0 & A_4A_3 \\ 0 & eA_2 - d & 0 \end{pmatrix}. \tag{3.1}$$

The meanings of the symbols here can be found in Appendix. In the normal mode problem of (2.7), time t is discretized into four seasons, that is, winter (DJF), spring (MAM), summer (JJA) and autumn (SON). For convenience, hereafter they are numbered 1, 2, 3 and 4 in sequence. So if a centred-temporal difference scheme is used, eq. (2.7) can be temporally discretized as

$$\begin{aligned} -\frac{\mathbf{v}_2 - \mathbf{v}_4}{2\Delta t} + L_1\mathbf{v}_1 &= \lambda\mathbf{v}_1 \\ -\frac{\mathbf{v}_3 - \mathbf{v}_1}{2\Delta t} + L_2\mathbf{v}_2 &= \lambda\mathbf{v}_2 \\ -\frac{\mathbf{v}_4 - \mathbf{v}_2}{2\Delta t} + L_3\mathbf{v}_3 &= \lambda\mathbf{v}_3 \\ -\frac{\mathbf{v}_1 - \mathbf{v}_3}{2\Delta t} + L_4\mathbf{v}_4 &= \lambda\mathbf{v}_4, \end{aligned} \tag{3.2}$$

here L_i and $\mathbf{v}_i$ with $i = 1,2,3,4$ are $L(t)$ and the disturbance fields for the different seasons, respectively. In this way, the temporally periodic boundary condition for the annual cycle is automatically satisfied. Comparing (3.2) with (2.7), we have

$$A = \begin{pmatrix} L_1 & -J & 0 & J \\ J & L_2 & -J & 0 \\ 0 & J & L_3 & -J \\ -J & 0 & J & L_4 \end{pmatrix}, \tag{3.3}$$

where $J = (2\Delta t)^{-1}\, I$, I is the identity operator (or unit matrix) and $\Delta t = 1/4$ yr . Meanwhile,

$$\mathbf{v} = (\,\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ \mathbf{v}_4\,)^T. \tag{3.6}$$

It is necessary to discuss here the validity of this temporal difference scheme and the long time step of a quarter year to model the present problem. In the expression for normal mode $e^{\lambda t}\mathbf{v}$, the time dependence of $\mathbf{v}(t) = \mathbf{v}(t+T)$ depicts the slow seasonal modulation of the spatial patterns of the normal modes, while their temporal characteristics are mainly described by $e^{\lambda t}$. So, with an annually periodic boundary condition satisfied, t in eigenvalue problem (2.7) can be treated as an argument in space, and the centred-difference scheme is the best choice. Note that the computational stability of the centred-difference scheme in an eigenvalue problem with a periodic time boundary condition is essentially different from the leapfrog scheme of a time integration problem in which a forward iteration is needed. Since we are more interested in the season-to-season change of $\mathbf{v}(t)$ than in its higher superharmonics which are less prominent, the time step of a quarter year is sufficient capture this change approximately.

If the number of meridional grid points in the model domain is m and there are n levels, and noticing that the model has three variables, that is, zonal wind, meridional wind and temperature, then $\mathbf{v}_i$ is a $3mn$-dimensional vector and $\mathbf{v}$ is a $12mn$-dimensional vector. Correspondingly, the order of matrices Li and I are $3mn \times 3mn$, while that of A is $12mn \times 12mn$. So we reduce m to save time of computation. The eigenvalue problem in this study is solved by subroutines in the IMSL Math/Library, in which eigenvalues and corresponding eigenvectors are returned in the

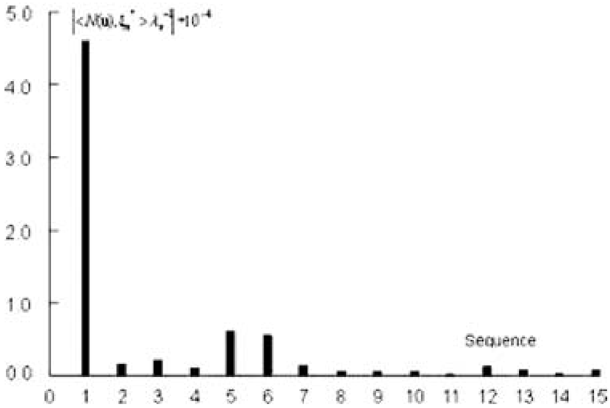

Fig. 1. The values of $| < N(\mathbf{u}), \boldsymbol{\xi}_n{}^* > \lambda_n{}^{-1} |$ (unitless) for the normal modes with sequence numbers from 1 to15 are shown, where time integration of the inner product is calculated over nine periods of the annual cycle from 1994 to 2002 and divided by 9. Three prominent modes, that is, numbers 1, 5 and 6, are extracted.

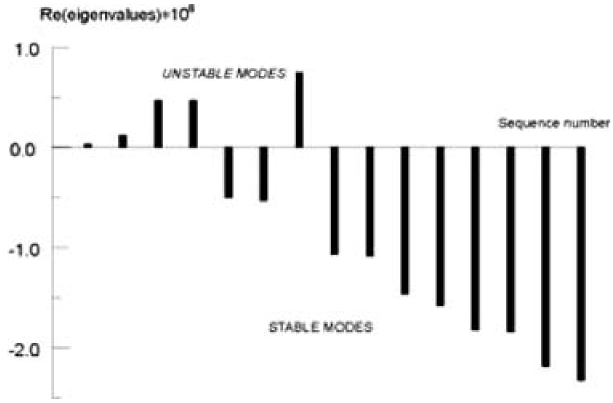

Fig. 2. The real parts (in units of s^{-1}) of the eigenvalues of the first 15 normal modes are shown to indicate the linear stabilities of these modes.

order of largest to smallest in complex magnitude. Since the leading mode is usually the mode with the largest temporal and spatial scales, the reverse order of smallest to largest in complex magnitude is adopted in this analysis. The sequence number of normal modes is given in this order hereafter.

4. Results and analysis

We discuss the *leading* normal modes that are associated with the AO/NAM. The accurate definition of 'leading' remains ambiguous. We have defined the sequence number of the normal modes in the previous section. However, this order need not be consistent with the rank of prominence of the normal mode because of the scenario for mode selection given in (2.14). Consequently, the major annular normal modes are ranked according to their prominence represented by the complex magnitude of the right-hand side of (2.14). However, if we want to calculate $<N(\mathbf{u}), \boldsymbol{\xi}_n{}^* >$, we must know the disturbance solution $\mathbf{u}(t)$in advance. In this study, we use the deviation of 6 hr ECMWF data over 9 yrs (1994–2002) from the model equilibrium given by the mean fields for each of the four seasons (DJF, MAM, JJA and SON). As a result, both stationary waves and normal modes which include different transient waves contribute to the non-linear interaction. The non-linear terms of $N(\mathbf{u})$ are given in eddy flux form in (A.7) of Appendix, so their physical meanings are clear and can easily be related to the concept of Eliassen–Palm flux (E–P flux) in the traditional theory of eddy-mean flow interaction. The normalized eigenvector $\boldsymbol{\xi}_n{}^*$ of adjoint operator A^* (the transpose of the matrix corresponding to operator *A)* can be obtained by solving eigenvalue problem for the same purpose.

Figure 1 shows the values of $| < N(\mathbf{u}), \boldsymbol{\xi}_n{}^* > \lambda_n{}^{-1} |$ for the normal modes with sequence numbers from 1 to 15, in which time integration of the inner product is calculated over 9 periods of the annual cycle from 1994 to 2002 and divided by 9. From these 15 modes, one can find that the most prominent mode is the mode with sequence numbers 1. In Fig. 2 the real parts of the corresponding eigenvalues are presented to indicate the linear stabilities of these modes. It can be seen that the most unstable mode is no longer the most prominent mode when non-linear interaction between different waves is considered. The most prominent normal mode, that is, mode number 1 is an unstable mode. In the remainder of this section, we will discuss this mode detailedly.

Linearly unstable mode 1 is the most important mode associated with the AO/NAM. Its structure is illustrated in Figs. 3 and 4 which show the latitude–altitude sections of zonal wind and temperature in this mode, respectively. In the extratropical regions of both hemispheres, the structure of this annular normal mode bears a resemblance to that of the observed AO and AAO. In spite of the computational error of the locations and the strength of the action centers (such as the greater distance of action centers from 45 N or 45 S in both hemispheres compared with the observations), deep dipole patterns of zonal wind on both sides of 45 N and 45 S are clearly seen in all seasons. We conclude that the AO and AAO, which are usually studied separately, are associated with two different segments of one normal mode that is approximately symmetric about the equator. Although the AO and AAO seem too distant to be connected to each other, this global structure of the annular normal mode can not be merely an artifact of the numerical analysis. The physical justification for such a mode in terms of the real atmosphere could be based on the following facts. (a) The two hemispheres by no means be isolated by the equator, so global mode of the atmospheric motion is permitted to exist in principal. (b) The properties of the symmetry about the equator due to the earth's rotation and the spherical geometry determine a zonal mean flow that is also approximately symmetric about the equator. This allows global normal modes with symmetric structure about the equator to

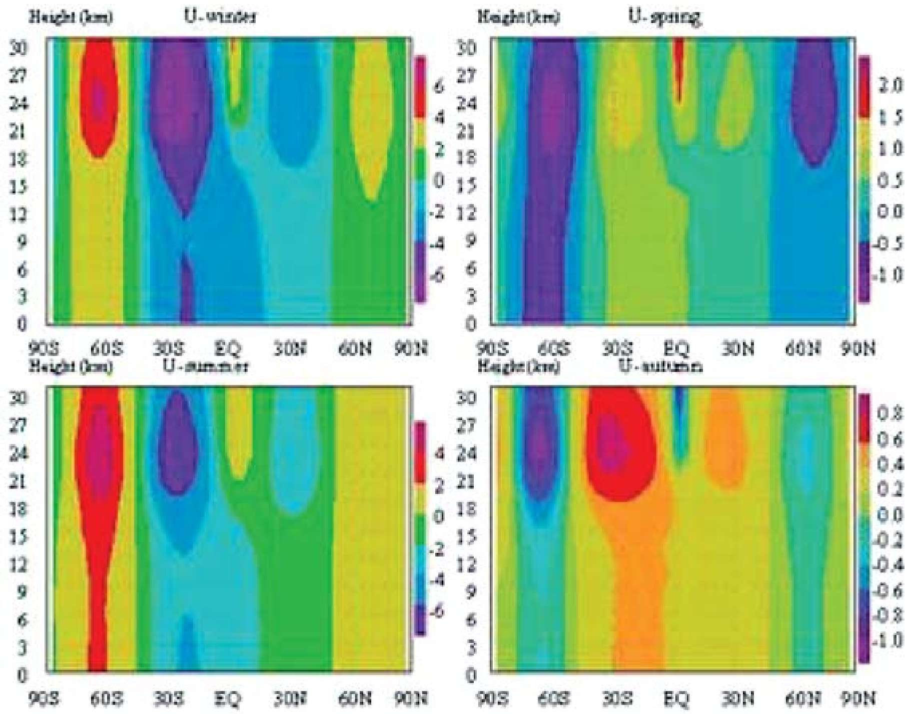

Fig. 3. The latitude–altitude sections of zonal wind in mode 1 in its winter, spring, summer and autumn phases (the normal mode is normalized, the scales for the colour bars are 10^{-2}).

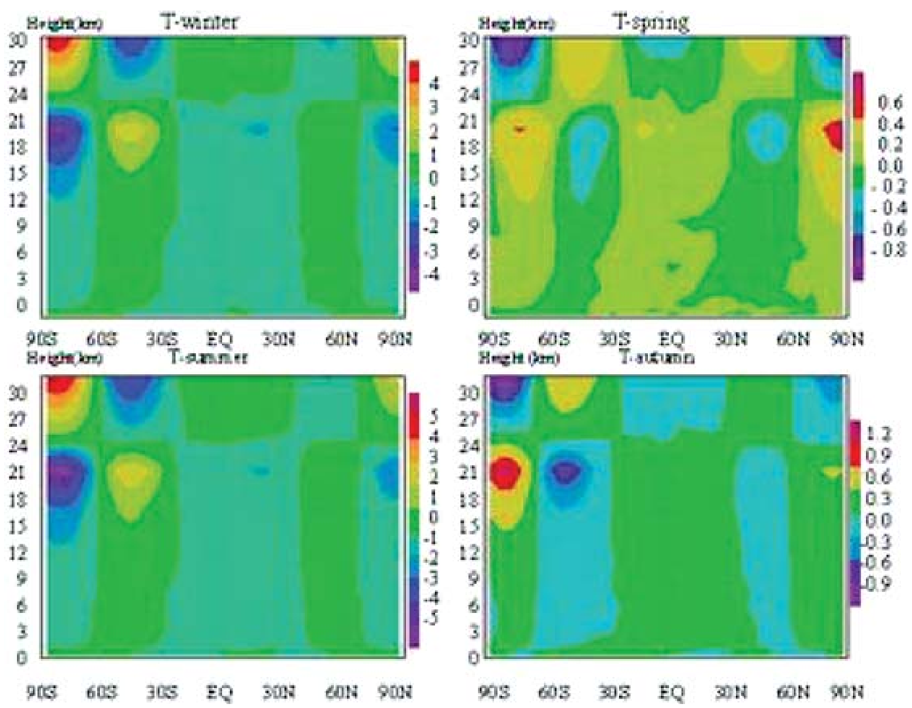

Fig. 4. The latitude–altitude sections of temperature in mode 1 in its winter, spring, summer and autumn phases (the normal mode is normalized, the scales for the colour bars are 10^{-2}).

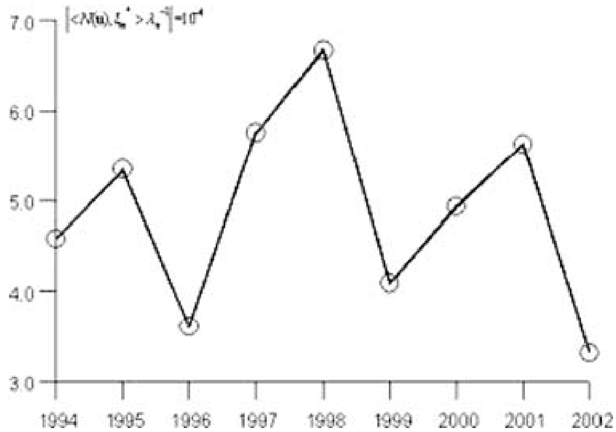

Fig. 5. The year-to-year variation of the values of $| < N(\mathbf{u}), \boldsymbol{\xi}_n{}^* > \lambda_n{}^{-1} |$ (unitless) for mode 1. The high values appearing in 1997 and 1998 may have something to do with the ENSO event of 1997/1998, while the low values in 1996 and 1999 correspond to La Niña years.

exist. So, if a normal mode can describe the spatial structure of the AO, it is quite possible that it describes the AAO as well because of this symmetry. Similar structures showing both the AO an AAO like Figs. 3 and 4 from EOF analysis can be found in Thompson, Lee and Baldwin (2002), which provides a support for our conclusion. Furthermore, the signature of the AO and AAO is also evident in the tropics: the westerly anomalies to the north of 45 N and south of 45 S are accompanied by a strengthening of the Trade Winds at low levels in the tropics, and easterly anomalies by a weakening. As two parts of one normal mode, the correlation between the tropospheric and stratospheric components of the AO is evident, and is the most basic aspect of the coupling between them. The seasonal change of this mode is also interesting: the phases of the AO and AAO in spring and autumn are opposite to those in winter and summer. This means that a semiannual oscillation of the mode can be expected to exist. An observational study by Baldwin and Dunkerton (2001) of the time-height development of the AO during the winter of 1998–1999 demonstrates a similar phase shift in spring and autumn. If time integration of the inner product of $< N(\mathbf{u}), \boldsymbol{\xi}_n{}^* >$ is computed for each year between 1994 and 2002 separately, the interannual variability of this normal mode can be investigated. Figure 5 gives the year-to-year variation of the values of $| < N(\mathbf{u}), \boldsymbol{\xi}_n{}^* > \lambda_n{}^{-1}|$ for mode 1. The high values appearing in 1997 and 1998 may have something to do with the ENSO event of 1997/1998, while the low values in 1996 and 1999 correspond to La Ni–a years. Unless the peak in 1995 may be associated with the weak ENSO event, the relationship with El Ni–o/La Ni–a is not evident in other years. It seems very speculative to infer from the shown 9 data points a connection to ENSO, but this at least implies that coupling between the atmosphere and the tropical Pacific Ocean can modulate the oscillation of mode 1 to some extent, although the existence of this mode or the AO/AAO depends merely on atmospheric processes. Some similar results suggesting this connection can be found in Dima et al (2002).

5. Summary and comments

We have performed a numerical study of the roles of zonally symmetric normal modes in the dynamics of the AO/NAM using a seasonally varying background climate. A theory of the mechanism for the ranking of normal modes by fully developed non-linear interaction between them is proposed, and this may account not only for the prominence of the AO/NAM in the Northern Hemisphere, but also for the existence of the AAO in the Southern Hemisphere. According to this normal mode selection mechanism, zonally symmetric normal mode with spatial structure resembling those of the AO and AAO is extracted by using ECMWF data and the linear operator of a two-dimensional model subjected to a seasonally varying basic state. Also, their interannual variations are investigated and linked to the ENSO.

In order to incorporate the seasonal change of the basic state, we have introduced a time-dependent eigenvalue problem in our analysis. A comparison with the results of a steady basic state would be helpful in understanding the present results. Figure 6 shows the structures of the zonal wind and temperature of the AO/AAO calculated as an eigenvector of L_1 (linear operator of winter). The pattern is similar to that in Fig. 3, which demonstrates that the normal modes and their season-to-season change obtained in the present study are not numerical artifacts.

It is no surprise to be able to describe more than one observational phenomena such as the AO and AAO in a single theoretical framework or that the AO and AAO are parts of a single normal mode, because these phenomena are usually identified by isolating the region where they are situated from the rest of the atmosphere. The present study treats all latitudes/altitudes and seasons as a whole, and thus provides an overview of the leading zonally symmetric mode observed globally, although there is still lots of room for doubts and interpretation.

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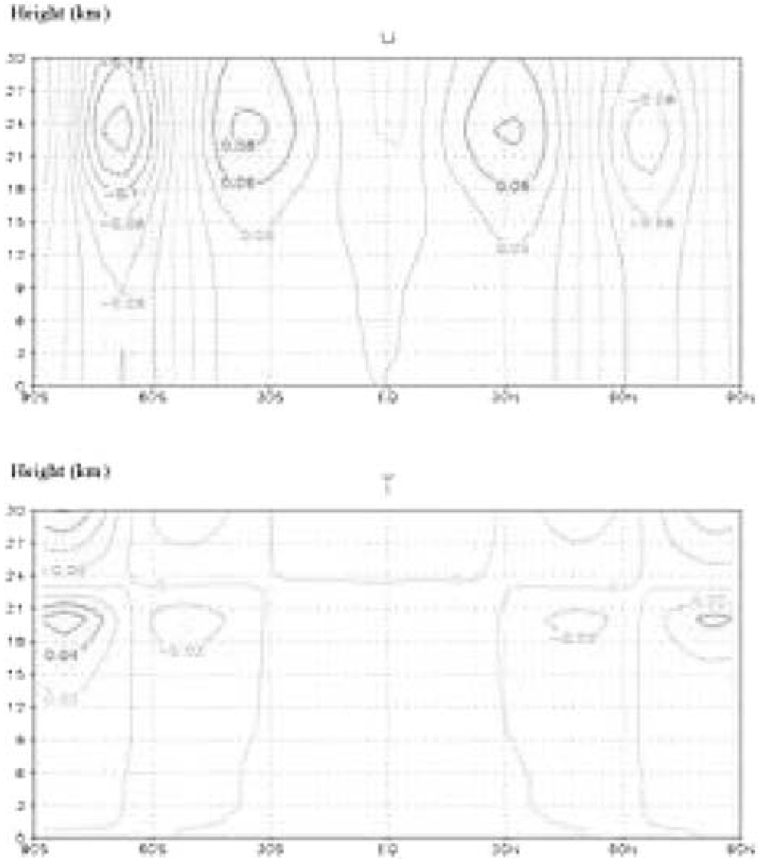

Fig. 6. The structures of zonal wind and temperature of the AO/AAO calculated as an eigenvector of L_1 (linear operator of winter). The normal mode is normalized.

7. Appendix

The set of model equations in the *p*-coordinate system is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{R\cos\varphi\partial\lambda} + v\frac{\partial u}{R\partial\varphi} + \omega\frac{\partial u}{\partial p} = \frac{\tan\varphi}{R}uv + fv - \frac{g\partial z}{R\cos\varphi\partial\lambda} + F_\lambda$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{R\cos\varphi\partial\lambda} + v\frac{\partial v}{R\partial\varphi} + \omega\frac{\partial v}{\partial p} = \frac{\tan\varphi}{R}u^2 - fu - \frac{g\partial z}{R\partial\varphi} + F_\varphi$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{R\cos\varphi\partial\lambda} + v\frac{\partial T}{R\partial\varphi} + \omega\frac{\partial T}{\partial p} = \kappa\frac{T}{p}\omega + \frac{Q(t)}{c_p}$$

$$\frac{\partial u}{R\cos\varphi\partial\lambda} + \frac{\partial v\cos\varphi}{R\cos\varphi\partial\varphi} + \frac{\partial\omega}{\partial p} = 0$$

$$T = -\frac{pg}{R_d}\frac{\partial z}{\partial p}$$

$$p = \rho R_d T. \quad \text{(A.1)}$$

All the symbols in the equations have their usual meanings. We suppose that the only effect of the zonally symmetric heating *Q(t),* which is an annually periodic function of time, is to maintain zonally symmetric fields of geopotential high gz_0 (also an annually periodic function of time) through the third and the fifth equations in (A.1). If the friction terms can be neglected, then system (A.1) has a zonally symmetric equilibrium which satisfies

$$\frac{\tan\varphi}{R}u_0^2 + fu_0 = -\frac{g\partial z_0}{R\partial\varphi}$$

$$T_0 = -\frac{pg}{R_d}\frac{\partial z_0}{\partial p} \quad \text{(A.2)}$$

$$v_0 = \omega_0 = 0,$$

where T_0 and u_0 are also annually periodic functions of time. The first equation is the spherical geostrophic relation and the second the hydrostatic balance. This equilibrium is employed as the climatological state. By linearizing (A.1) about this equilibrium and noting the zonal symmetry which allows solutions of the form

$$\begin{pmatrix} u \\ v \\ T \end{pmatrix} = \begin{pmatrix} u \\ v \\ T \end{pmatrix} e^{im\lambda}. \quad \text{(A.3)}$$

Equations in (A.1) become

$$\frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ T \end{pmatrix} = \begin{pmatrix} -ima - bA_1 & I_x - bA_2 & -imgcA_3 \\ -I_y & -ima & A_4A_3 \\ eA_1 & eA_2 - d & -ima \end{pmatrix}\begin{pmatrix} u \\ v \\ T \end{pmatrix}, \quad \text{(A.4)}$$

where we denote the following linear operators

$$A_1 = -\frac{im}{R\cos\varphi}\int_{p_s}^{p} (\,)\mathrm{d}p$$

$$A_2 = -\frac{1}{R\cos\varphi}\int_{p_s}^{p} \frac{\partial(\,)\cos\varphi}{\partial\varphi}\mathrm{d}p \quad \text{(A.5)}$$

$$A_3 = -\int_{0}^{p} \frac{R_d(\,)}{gp}\mathrm{d}p$$

$$A_4 = -\frac{g\partial}{R\partial\varphi}$$

and other quantities in (A.4) are denoted by

$$a = \frac{u_0}{R\cos\varphi}; b = \frac{\partial u_0}{\partial p}; c = \frac{1}{R\cos\varphi}; e = \kappa\frac{T_0}{p} - \frac{\partial T_0}{\partial p}$$

$$I_x \frac{\tan\varphi}{R}u_0 + f - \frac{\partial u_0}{R\partial\varphi}; I_y = 2\frac{\tan\varphi}{R}u_0 + f. \quad \text{(A.6)}$$

For zonally symmetric or annular modes, we let $m = 0$ in the equations above.

In order to assess the non-linear interaction among normal modes, we need to know the non-linear terms neglected in the linearization. These terms are given in eddy flux form as

$$N(u, v, T) = \begin{pmatrix} -\frac{\partial u^2}{R\cos\varphi\partial\lambda} - \frac{\partial uv}{R\partial\varphi} - \frac{\partial u\omega}{\partial p} + 2\frac{\tan\varphi}{R}uv \\ -\frac{\partial vu}{R\cos\varphi\partial\lambda} - \frac{\partial v^2}{R\partial\varphi} - \frac{\partial v\omega}{\partial p} + \frac{\tan\varphi}{R}(v^2 - u^2) \\ -\frac{\partial Tu}{R\cos\varphi\partial\lambda} - \frac{\partial Tv}{R\partial\varphi} - \frac{\partial T\omega}{\partial p} - \frac{\tan\varphi}{R}Tv + \frac{\kappa}{p}T\omega \end{pmatrix}. \quad \text{(A.7)}$$

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