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LETTER TO THE EDITOR

A note on the orientation and size of noctilucent cloud particles

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A recent paper by Bohren (1980) contains a discussion of the torque experienced by a non-spherical noctilucent cloud particle falling through the mesosphere. At the heights involved, the mean free path of the molecules in the atmosphere is much greater than any likely size of particle, and Bohren correctly states that a fluid dynamic analysis cannot be extrapolated to these conditions. He cites a recent analysis (Reid, 1975) as showing that the net couple acting on the particles must be zero. However, there are some limitations to the use of Reid's calculations (Reid, 1975) of the drag on particles falling through the atmosphere. His model considers specular reflection only of molecules. Epstein (1924) has shown that a model more in accord with observation is one in which the molecules striking a particle with a solid or liquid surface are diffusely reflected with a velocity distribution corresponding to the temperature of the surface. For specular reflection of an impinging molecule, the net momentum transfer is always along the normal to the surface. When a molecule is reflected diffusely, the total momentum transfer of the emerging molecules is also always along the normal, but the impinging molecules impart momentum to the particle with a component in the antidirection of the particle's velocity.

Consider the case of a cylinder falling with its axis at some angle θ to the vertical (see Fig. 1). Clearly, more molecules strike the forward face than strike the rear face. At first sight, the momenta transferred to these faces along the antidirection of V (which have a component in the direction of y) will give rise to a net torque, zero only when $\theta = 0$ or $\pi/2$. Further consideration shows the reduction in the number of molecules from the static case, $V = 0$, that hit the rear face gives a reduction in the momentum transfer in the direction of V, equivalent to an increase in the antidirection of V.

Epstein's analysis gives the momentum transfer by molecules impinging on a surface element of area dS, whose normal lies along the x axis, as having a component in any direction $(\alpha', \beta', \gamma')$

$$M^i \, \mathrm{d}S = -\tfrac{1}{2} N m \bar{c} V (\alpha\alpha' + \tfrac{1}{4}\beta\beta' + \tfrac{1}{4}\gamma\gamma') \, \mathrm{d}S.$$

In this, V is the speed of movement of dS in the direction (α, β, γ) through a stationary gas con-

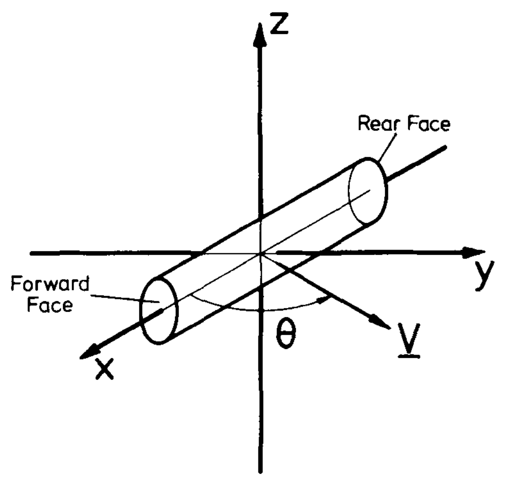

Fig. 1. Coordinates for a cylindrical particle moving through a stationary gas with velocity V (in the x,y plane). In the case of a particle falling through a static atmosphere, V is directed vertically downwards and the particle axis is inclined at the angle θ to the vertical.

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taining N molecules, each of mass m, per unit volume. The gas molecules have a Maxwellian velocity distribution with mean $\bar{c}$. It is assumed that $V \ll \bar{c}$. For specular reflection, one simply reverses the signs of α and V. The component of momentum imparted to the surface by the emitted molecules is

$$M_1^e \, \mathrm{d}S = -\tfrac{1}{2} Nm\bar{c}V(\alpha\alpha' - \tfrac{1}{2}\beta\beta' - \tfrac{1}{2}\gamma\gamma')\, \mathrm{d}S.$$

For diffuse reflection with accommodation (assuming the particle is a perfect thermal conductor), this magnitude of the transferred momentum is

$$M_2^e \, \mathrm{d}S = -\frac{\pi}{8} Nm\bar{c}V\alpha \, \mathrm{d}S$$

and it is directed along the normal to dS. The assumption of a conducting particle is shown by Epstein to be valid for spheres of radii much smaller than the mean free path in the gas.

That there is no net torque on a cylinder is shown by considering the momentum transferred to the two end faces in the y direction ($\alpha' = 0$, $\beta' = 1, \gamma' = 0$). For V as shown in Fig. 1, $\alpha = \cos\theta$, $\beta = \sin\theta$, $\gamma = 0$;

$$M^i \, \mathrm{d}S = -\tfrac{1}{2} Nm\bar{c}V \sin\theta \, \mathrm{d}S;$$

$$M_1^e \, \mathrm{d}S = +\tfrac{1}{2} Nm\bar{c}V \sin\theta \, \mathrm{d}S;$$

and

$$M_2^e \, \mathrm{d}S = 0.$$

For the rear face, $\alpha = \cos(\pi - \theta)$, $\beta = \sin(\pi - \theta)$, $\gamma = 0$; the three contributions to momentum transfer are exactly the same as for the front face.

Clearly, for specular reflection there is no y component of the net momentum transfer on either face; for diffuse reflection, the momentum transfer has a y component which is the same in both magnitude and direction on the two faces and there is no net torque.

The drag on any particle can be obtained by integration over the surface of the components in the direction of V ($\alpha' = \alpha, \beta' = \beta, \gamma' = \gamma$):

$$M^i \, \mathrm{d}S = -\tfrac{1}{2} Nm\bar{c}V(\cos^2\theta + \tfrac{1}{2}\sin^2\theta)\, \mathrm{d}S;$$

$$M_1^e \, \mathrm{d}S = -\tfrac{1}{2} Nm\bar{c}V(\cos^2\theta - \tfrac{1}{2}\sin^2\theta)\, \mathrm{d}S;$$

and

$$M_2^e \, \mathrm{d}S = -\frac{\pi}{8} Nm\bar{c}V \cos^2\theta \, \mathrm{d}S,$$

where θ is the angle between V and the normal to dS. For a sphere of radius r, the net momentum transfer is

$$M_1 = -\frac{4\pi}{3} Nm\bar{c}Vr^2;$$

$$M_2 = -\frac{\pi}{3}(4 + \pi/2)\, Nm\bar{c}Vr^2.$$

M_1 is the drag given by Reid. Observe that $M_2/M_1 = 1.39$; the extra drag when the molecules are diffusely, not specularly, reflected arises from the integral of $M_1^e \mathrm{d}S$ being zero, while that of $M_2^e \mathrm{d}S$ has the same sign as the integral of $M^i \mathrm{d}S$.

For a cylinder of radius r and length $2a$,

$$M_1 = -2\pi Nm\bar{c}Vr(r\cos^2\theta + a\sin^2\theta);$$

$$M_2 = -\frac{\pi}{2} Nm\bar{c}Vr\{(\pi/2 + 1)(r\cos^2\theta + a\sin^2\theta) + r + 2a\}.$$

Again, M_1 is the drag given by Reid. With accommodation, the drag M_2 is larger e.g. for a cylinder whose length is twice its diameter, and for $\theta = \pi/4$, $M_2/M_1 = 1.48$.

The particles will spin because of the Brownian rotational movement (Einstein, 1906). In the absence of drag, a particle having a moment of inertia I will spin with angular velocity

$$\dot{\theta} = \sqrt{kT/I}$$

The analysis developed by Langevin (1908) can be carried straight over to angular motion. It leads to the following expression for the rate of change of the mean squared rotation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\overline{\theta^2}) = 2kT/\delta + B\exp(-t\delta/I).$$

B is the integration constant. In this, $\dot{\theta}\delta$ is the angular deceleration by viscous drag or similar effects. In the Knudsen or free-molecular case, despinning of a particle occurs through the impinging molecules carrying off angular momentum from the surface. (We have seen above that there is no net angular momentum brought to the particle by any movement through the surrounding gas.) For a spherical particle of radius r,

$$\delta = \pi Nm\bar{c}r^4.$$

Using a model atmosphere appropriate to the summer mesosphere at 80 km altitude, i.e. with $T = 135$ K, $N = 1.4 \times 10^{21}$ m^{-3}, we have:

Radius of sphere	$\sqrt{kT/I}$	$2kT/\delta$	I/δ
0.01 μm	1.1×10^{8} s^{-1}	5.5×10^{12} s^{-1}	2.2×10^{-4} s
0.1 μm	3.5×10^{5} s^{-1}	5.5×10^{8} s^{-1}	2.2×10^{-3} s
1.0 μm	1.1×10^{3} s^{-1}	5.5×10^{4} s^{-1}	2.2×10^{-2} s

The numbers in this Table show that the noctilucent cloud particles will exhibit Brownian rotation with considerable net angular speed. To assume that the particles fall with random tumbling is a good assumption, whatever the size of the cloud particle.

There is some disagreement over the size of the scatterers in a noctilucent cloud. To some extent, this may be the result of different observational conditions. With observations at scattering angles in the neighbourhood of $\pi/2$, small particle (Rayleigh) scattering is likely to predominate. At small scattering angles (as when one looks from a spacecraft in the direction of the rising Sun), forward scattering from larger particles may be dominant.

In general, observations from sounding rockets are made with the former scattering angle, e.g. Tozer and Beeson (1974) and Witt *et al.* (1971). (Note that the data of Witt *et al.* (1976), which are cited by Bohren (1982) in this context, do not refer to a noctilucent cloud.) The data suggest an upper limit to particle size of around 0.1 μm. However, de Bary and Rössler (1974) deduce the presence of both small ("Aitken") and large ("Junge") particles from their rocket data. The spacecraft observations are exemplified by the observations of Avaste *et al.* (1981) and Belyayev *et al.* (1981) who estimate the particle size as being up to 0.7 μm. The *in situ* collections of particles of Hemenway and his colleagues (Hemenway *et al.*, 1964; Hallgren *et al.*, 1973) and Farlow *et al.* (1970) certainly do not rule out the presence on occasion of relatively quite large particles, perhaps of radii even greater than 1 μm.

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